Towards two loop Higgs masses in any model

[MDG and S. Paßehr, arXiv:1910.02094, accepted in EPJC]

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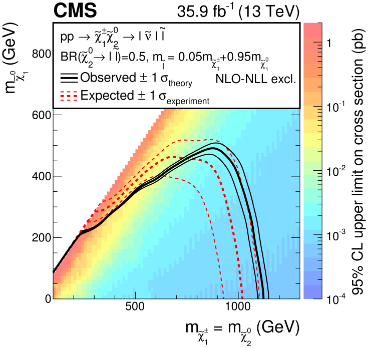
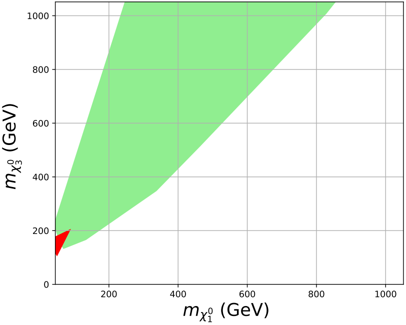
Constraining the electroweak sector of BSM theories

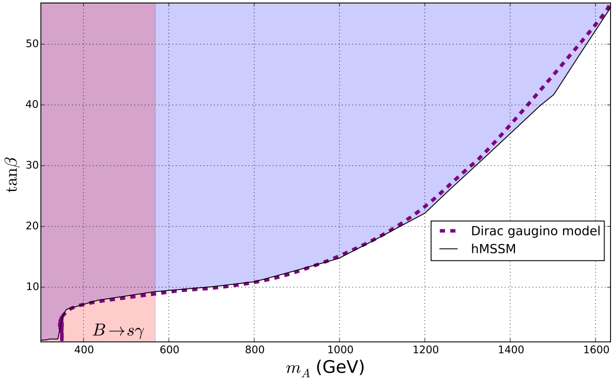
- Absence of evidence for new particles → diminishing chance of finding new <u>coloured resonances</u> at the LHC.
- But LHC is rather poor at probing electroweak-strength coupled particles, c.f.

Pessimistic simplified scenario vs "complete" scenario





Or more famously in the Higgs sector:



... hence the continued (or increasing) interest in THDM and other non-SUSY Higgs extensions

So now we are starting a new precision era.

Some parts of the toolbox are well developed:

- Collider constraints give us information up to a couple of TeV for coloured particles, but much less for electroweak (few hundred GeV). NLO corrections very important for SM, but for BSM

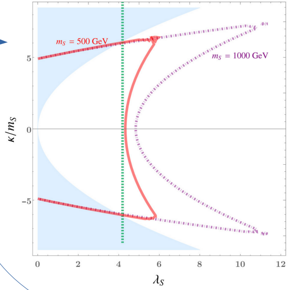
$$\sigma \sim m^4, \qquad \frac{\delta_{NLO}\sigma}{\sigma} \sim 1 \rightarrow \frac{\delta m}{m} \sim 20\%$$

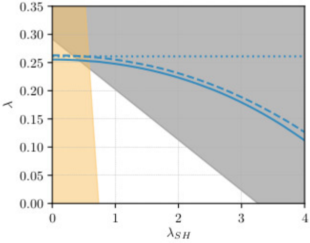
- Flavour constraints are very powerful, but again mainly for coloured states. E.g. 10s of TeV limits on four-fermion operators, vs 600 GeV for charged Higgs from $b \rightarrow s\gamma$ in THDM-II. Most can be calculated @ one loop [but many two-loop processes only known for MSSM (e.g. Barr-Zee diagrams]
- EWPT mostly calculable in e.g. SARAH at one-loop [but very little beyond one loop.]
- SMEFT program attempts to constrain models from precision (including EWPT). A lot of effort there in reinterpreting data, and going to one loop.

$$\mathcal{L} = \frac{1}{2}m_S^2 S^2 + \frac{1}{3}\kappa S^3 + \frac{1}{2}\lambda_S S^4 + \frac{1}{2}\lambda_{SH} S^2 |H|^2 + \frac{1}{2}\lambda |H|^4$$

But we can gain lots of information from the renormalisable terms that are not being (well) exploited:

- Unitarity
- Stability or instability scale of the electroweak vacuum
- The Higgs mass





This talk

Why calculate the Higgs mass: classic BSM perspective

For many years the standard example has been the MSSM for $\sim$ TeV-scale SUSY:

- Quartic predicted to be determined entirely by gauge couplings at tree level – in large M_H limit have

$$\lambda = \frac{1}{8}(g_Y^2 + g_2^2)\cos^2 2\beta = \frac{M_Z^2}{2v^2}\cos^2 2\beta$$

- Hence $\rightarrow m_h(\text{tree}) \leqslant M_Z$
- $\delta m_h^2(\text{loops}) \geqslant (125\text{GeV})^2 - (M_Z)^2 \geqslant (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$
- Can have $\delta m_h(\text{two loops}) \lesssim 10$ GeV $\rightarrow \delta m_h^2(\text{two loops}) \sim 15\% m_h^2$!

This has prompted much work on precision calculations of the Higgs mass in BSM theories.

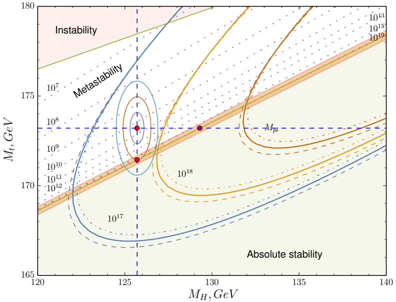
Non SUSY models

For Non-SUSY models (e.g. SM), cannot only use a tree-level spectrum:

- Maybe some scenarios predict the quartic (gauge-Higgs unification?)
- Maybe use the quartic couplings as inputs and scan $\rightarrow$ how large can they be? Quantum corrections can be (very) large!
- Need to extract the Higgs quartic/other couplings in the theory: again, quantum corrections can be large! (especially with hierarchies!)

$$\lambda = \frac{m_h^2}{2v^2} + \text{quantum corrections}$$

- SM quantum corrections are small, but need the Higgs quartic at two loops to check for vacuum stability (e.g. in `MR`)



Loop order	Butazzo et al (on-shell)	`SARAH`	`SMH` (Landau gauge)	λ with `mr`
Tree level	0.12917	0.12786	0.12786	0.12917
One loop	0.12774	0.12647	0.12580	0.12771
Two loops	0.12604	0.12619	0.12541	0.12601

- `SARAH` calculation is incomplete w.r.t other three.
- Since the RGEs are functions of $\log \mu$, changes in λ lead to exponential shifts of μ.

Example: THDM

E.g. in the THDM:

$$
\begin{aligned}
V_{\text{Tree}} =& \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_2^\dagger H_1|^2 \\
&+ m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \left(M_{12}^2 H_1^\dagger H_2 + \frac{1}{2} \lambda_5 (H_2^\dagger H_1)^2 + \text{h.c.} \right)
\end{aligned}
$$

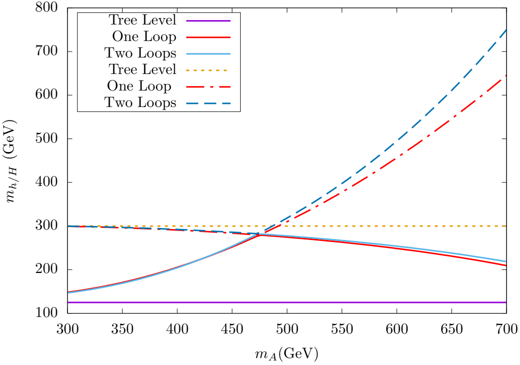
- With CP, have 8 parameters plus two expectation values minus 2 vacuum conditions and the weak vev $\rightarrow$ 7 free parameters.
- Can trade these for $m_h, m_H, m_A, m_{H^\pm}$, $\tan\beta$, $\tan\alpha$ and M_{12}^2:

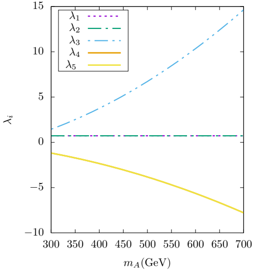
$$
\begin{aligned}
\lambda_1 =& \frac{1+t_\beta^2}{2(1+t_\alpha^2)v^2} \left(m_H^2 - M_{12}^2 t_\beta + t_\alpha^2 (m_h^2 - M_{12}^2 t_\beta) \right) \\
\lambda_2 =& \frac{1+t_\beta^2}{2(1+t_\alpha^2) t_\beta^3 v^2} \left(-M_{12}^2 - M_{12}^2 t_\alpha^2 + t_\beta (m_h^2 + m_H^2 t_\alpha^2) \right) \\
\lambda_3 =& \frac{1}{(1+t_\alpha^2) t_\beta v^2} \Big[m_h^2 t_\alpha + 2 m_{H^+}^2 (1+t_\alpha^2) t_\beta \\
& + m_h^2 t_\alpha t_\beta^2 - m_H^2 t_\alpha (1+t_\beta^2) - M_{12}^2 (1+t_\alpha^2)(1+t_\beta^2) \Big] \\
\lambda_4 =& \frac{1}{t_\beta v^2} \left(M_{12}^2 + m_A^2 t_\beta - 2 m_{H^+}^2 t_\beta + M_{12}^2 t_\beta^2 \right) \\
\lambda_5 =& \frac{1}{t_\beta v^2} \left(M_{12}^2 - m_A^2 t_\beta + M_{12}^2 t_\beta^2 \right)
\end{aligned}
$$

Unphysical couplings

- Main problem with this: it is very easy to have huge underlying unphysical couplings!

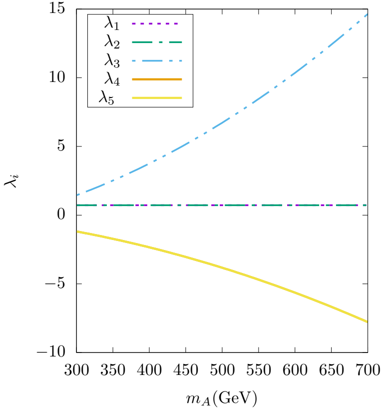
E.g. enforce the alignment limit of $\tan\alpha = -1/\tan\beta$, we can scan over the other parameters. If we take the Heavy Higgs mass to be 300 GeV and scan only over e.g. $m_A = m_{H^+}$ we find for loop corrections to masses:

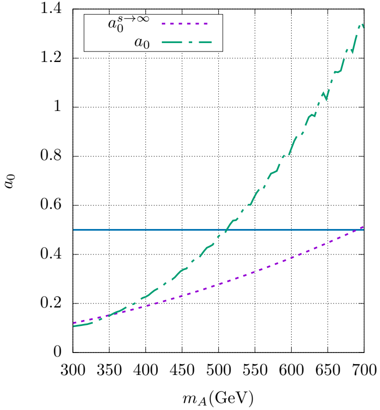




Unitarity

- Better check of perturbativity: use unitarity
- Naively $s \to \infty$ limit is enough because only quartic couplings





How to calculate the Higgs mass: EFT

When the new particles are heavy, we need to switch to EFT corrections to λ instead of computing

$$m^2_{\text{pole}} = m^2_{\text{tree}} + \Pi(m^2_{\text{pole}})$$

Can work in effective SM, THDM, SMEFT, ... but now need to compute matching conditions for

$$\delta\lambda_{EFT} = \lambda_{HET} + \delta\lambda$$

- Use the path integral to integrate out the heavy fields to derive a Wilsonian action:

$$\int [d\phi_H][d\phi_L] e^{iS[\phi_L,\phi_H]} \rightarrow \int [d\phi_L] e^{i\tilde{S}[\phi_L]}$$

  This generates a large set of diagrams to compute $\rightarrow$ generic expressions only known at one loop.

- Match the effective actions of both theories using the equations of motion on the full effective action to integrate out the heavy fields $\rightarrow$ still need to compute four-point diagrams, but takes care of combinatorics $\rightarrow$ again, only known at one loop.
- Match pole masses/physical quantities in both theories $\rightarrow$ can extract quartic couplings from only two-point amplitudes!

In the last approach, we have

$$2\lambda_{SM} v^2_{SM} + \Delta M^2_{SM} = m^2_{\text{tree}} + \Delta M^2_{HET}$$

so we can use a pole mass calculation to extract λ_{SM}. In principle have two loop corrections, modulo subtleties.

Pole mass calculations: standard approximations

At two loops, there are two standard approximations used in pole mass calculations:

- Gaugeless limit $\rightarrow$ set all couplings of broken gauge groups to zero. Dramatically reduces the number of diagrams to compute, and the complexity thereof.
- Effective potential limit $\rightarrow$ compute $\Pi_{hh}(0) \leftrightarrow \frac{\partial^2 V_{\text{eff}}}{\partial h^2}$. Allows us to use momentum free integrals, which are <u>much</u> faster to evaluate.

Beyond the SM, there are very few calculations beyond these limits at two loops (and none beyond them at three or four loops).

- In the MSSM, they strictly go hand in hand:

$$m^2_{\text{tree}} = M_Z^2 c_{2\beta}^2 \longrightarrow \Pi_{hh}(m^2_{\text{tree}}) = \Pi_{hh}(0) + \mathcal{O}(\alpha)$$

- In general theories, can make the argument that if the tree-level mass is small, this approximation is good; and if the tree-level mass is large, then the loop corrections are less important.

Partly as a result of the success of this approximation, and partly because of the difficulty, a complete calculation beyond the gaugeless/effective potential limit has never been done except in the SM.

State of the art for generic models

A summary of what can be done for the generic case:

	Conventional approach	SM	EFT matching
λ/Scalar masses	'Gaugeless EP' 2-loop	'Full' 2-loop/ partial 3 + 4 loop	'Gaugeless EP*' 2-loop
Gauge couplings	1-loop	2-loop	1-loop
Yukawas	1-loop	2-loop	1-loop
v	1-loop	2-loop	1-loop
RGEs	2-loop	3 or 4 loops	N/A

Despite the long history of precision calculations for the Higgs mass, two-loop electroweak corrections have been elusive:

- The SM EW results are quite recent: Butazzo et al in 2013; the packages `SMH` in 2014, `MR` in 2016 and `SMDR` in 2019 implement full two-loop results and strong three- and four-loop corrections.
- In the MSSM, strong three-loop results are available, and some $\mathcal{O}(\alpha_s \alpha)$ contributions [these all avoid the Goldstone Boson Catastrophe or GBC].
- Two-loop generic diagrams were known only to order $\mathcal{O}(g^2)$ in gauge couplings, due to [Martin, 03]; only in [MDG, Nickel, Staub, '15] we computed the tadpoles!

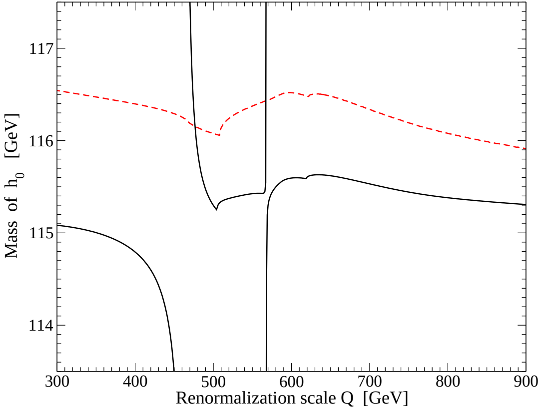
But naively

$$\left(\frac{\alpha(M_Z)}{s_W^2}\right)^2 \simeq 0.001, \qquad (\alpha_s)^3 \simeq 0.002$$

so two-loop electroweak effects could be as important as the three-loop strong ones!

Notable exception in the MSSM: the GBC

A notable attempt was made in the effective potential approach on the full MSSM potential – From S. Martin [hep-ph/0211366]:



Solid line: including EW effects, dashed line without

This shows both the GB catastrophe near $Q = 568$ GeV and the 'Higgs boson catastrophe' near 463 GeV.

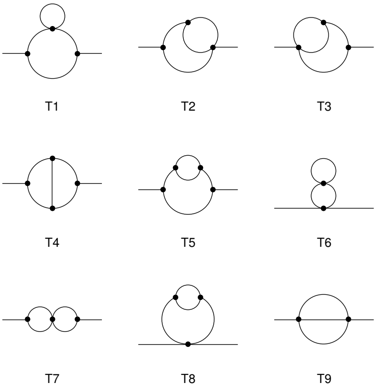
After this EW corrections were abandoned until recently.

Calculating the full generic two-loop result

To go away from the gaugeless limit we need the full self-energies/tadpoles.

There are 9 irreducible self-energy topologies:

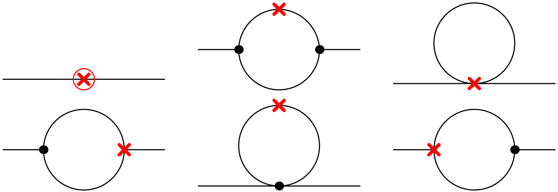
$1 \rightarrow 1$



Topologies 2 and 3 are equal for real scalar self-energies.

Renormalising the result

- The typical approach to renormalisation in explicit models is to compute unrenormalised diagrams, counterterms, and insertions separately: we add topologies



- This would mean giving the result only in terms of unrenormalised loop integrals, or the ϵ^0 pieces, and also just giving the result of these diagrams: the user would have to compute the counterterms for each model.
- This is simple, but inefficient: there are also many cancellations between these diagrams and the genuine two-loop integrals, in particular terms of the form

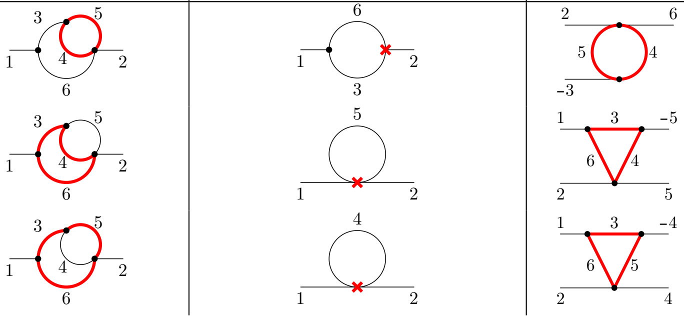
$$\frac{1}{\epsilon}\int d^d q \frac{1}{q^2 - m_1^2}\,\frac{1}{(q-p)^2 - m_2^2}$$

- Indeed it is known that the $\mathcal{O}(\epsilon)$ pieces of the subdivergences cancel out in the loop integrals when we use the basis of functions available in `TSIL`.

BPHZ method

Instead we use the BPHZ method of renormalising, where we subtract off the subdivergences:

Selected sub-loop	Diagram with counterterm	Counterterm



The forest formula ensures that this is equivalent.

Classes level

Next we need to populate the topologies with fields and evaluate them. For this we need generic vertices; a generic QFT looks like

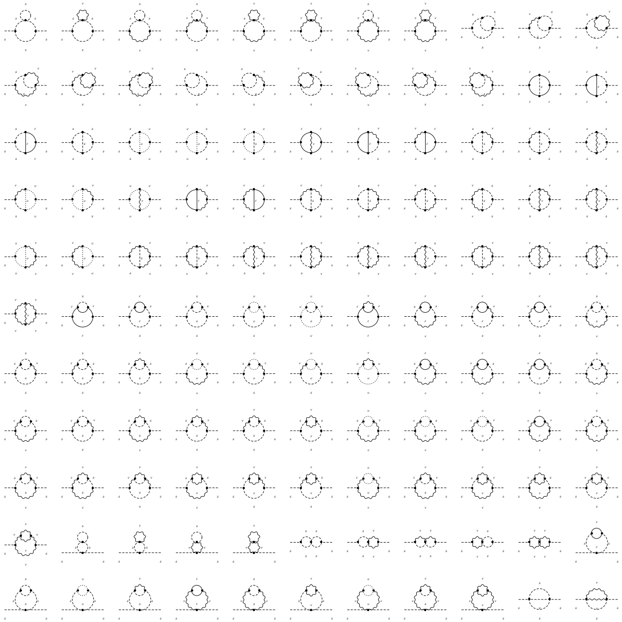
$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{SF} + \mathcal{L}_{SV} + \mathcal{L}_{FV} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{S\text{ghost}} \, .$$

where

$$\begin{aligned}
\mathcal{L}_S &\equiv -\frac{1}{6}\, a_{ijk}\, \Phi_i\, \Phi_j\, \Phi_k - \frac{1}{24}\, \lambda_{ijkl}\, \Phi_i\, \Phi_j\, \Phi_k\, \Phi_l \, , \\
\mathcal{L}_{SF} &\equiv -\frac{1}{2}\, y^{IJk}\, \psi_I\, \psi_J\, \Phi_k - \frac{1}{2}\, y_{IJk}\, \overline{\psi}^I\, \overline{\psi}^J\, \Phi_k \, , \\
\mathcal{L}_{FV} &\equiv g_I^{aJ}\, A_\mu^a\, \overline{\psi}^I\, \overline{\sigma}^\mu\, \psi_J \, , \\
\mathcal{L}_{SV} &\equiv \frac{1}{2}\, g^{abi}\, A_\mu^a\, A^{\mu b}\, \Phi_i + \frac{1}{4}\, g^{abij}\, A_\mu^a\, A^{\mu b}\, \Phi_i\, \Phi_j + g^{aij}\, A_\mu^a\, \Phi_i\, \partial^\mu \Phi_j \, , \\
\mathcal{L}_{\text{gauge}} &\equiv g^{abc}\, A_\mu^a\, A_\nu^b\, \partial^\mu A^{\nu c} - \frac{1}{4}\, g^{abe}\, g^{cde}\, A^{\mu a}\, A^{\nu b}\, A_\mu^c\, A_\nu^d + g^{abc}\, A_\mu^a\, \omega^b\, \partial^\mu \overline{\omega}^c \, , \\
\mathcal{L}_{S\,\text{ghost}} &\equiv \xi\, \hat{g}^{abi}\, \Phi_i\, \overline{\omega}^a\, \omega^b \, .
\end{aligned}$$

This is in terms of real scalars and vectors, Weyl fermions, and ghosts.
Actually for the computer algebra we shall use four-component Majorana fermions.

Brute force



Evaluating

- Generate the amplitudes with `FeynArts`
- Evaluate the two-loop amplitudes in general gauge using `TwoCalc`. Gives results in terms of scalar integrals and tensors

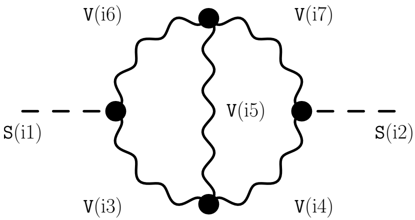
$$Y^{j_1\cdots j_o}_{i_1\cdots i_n} = \int \frac{\mathrm{d}^d q_1\, \mathrm{d}^d q_2}{\left[\imath\, \pi^2\, (2\,\pi\,\mu)^{d-4}\right]^2} \frac{k^2_{j_1}\cdots k^2_{j_o}}{\left(k^2_{i_1} - m^2_{i_1}\right)\cdots\left(k^2_{i_n} - m^2_{i_n}\right)}$$

- We perform the BPHZ renormalisation in MS' and DR' schemes using our own code to calculate counterterms and match them into the insertions, and `FormCalc, OneCalc` to evaluate the insertion diagrams.
- We keep the result in an unexpanded form because the integral reduction differs depending on whether there are IR singularities/special values of the momenta/degenerate masses
- We have derived all the necessary integral reductions, either from in `TwoCalc`, by `TARCER`, or in many cases by hand, to reduce to a basis of integrals that can be evaluated in `TSIL`.

Gauge choice

We give the results explicitly in Feynman gauge:

- Not because of the GBC (this still exists for theories with genuine Goldstone bosons!)
- Nor reducing number of diagrams (Landau gauge has fewer classes)
- Because the expressions are much shorter: in particular

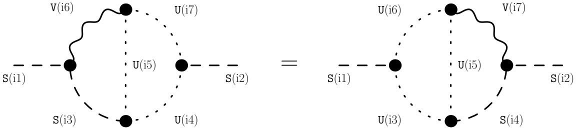


Gauge	Integrals
Feynman	6
Landau	896
R_ξ	924

However, we have the expressions for Landau/general gauge, which we will be able to use later to demonstrate gauge independence.

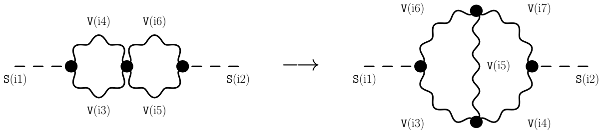
Simplifying

- Initially we have 121 self-energy (and 25 tadpole) diagrams.
- We can trivially reduce the number of self-energies to 92 from relating topologies 2 and 3 and also exchanges not identified by `FeynArts`:



- We can further reduce this to 58 classes! First we do this by exchanging quartic vector couplings for products of triple vector couplings:

$$i\frac{\partial^4\mathcal{L}}{\partial A^{\mu_a}\,\partial A^{\mu_b}\,\partial A^{\mu_c}\,\partial A^{\mu_d}} = -2ig^{abe}g^{cde}\eta^{\mu_a\mu_b}\eta^{\mu_c\mu_d}$$
$$-2ig^{ace}g^{bde}\eta^{\mu_a\mu_c}\eta^{\mu_b\mu_d} - 2ig^{ade}g^{cbe}\eta^{\mu_a\mu_d}\eta^{\mu_c\mu_b}$$



Ghost busting

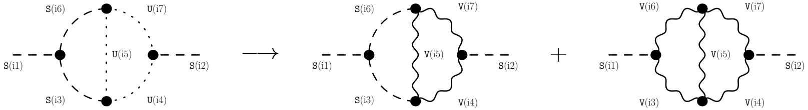
The ghost-ghost-vector couplings are trivially given by g^{abc}, but the scalar-ghost-ghost couplings are more subtle:

$$\hat{g}^{abi} = \frac{1}{2}\, g^{abi} - \frac{1}{2}\, g^{abc}\, (F_D)^c_i$$

where

$$(F_D)^a_j = \begin{cases} 0\,, & a \quad > N_G \\ 0\,, & j \quad > N_G \\ m_a\, \delta_{aj}\,, & a, j \quad \leqslant N_G \end{cases}$$

i.e. it depends on whether the scalar is a Goldstone or not!
But with some care we can bust all of the ghosts, e.g.



These results stem from:

- Gauge transformations of scalars

$$\delta R_i = \alpha^a \theta^a_{ij} R_j \longrightarrow \text{define } F^a_i \equiv \theta^a_{ji} v_j = -\theta^a_{ij} v_j$$

- Ghost couplings to scalars

$$\mathcal{L}_{\text{ghost}} = -\bar{c}^a \frac{\delta G^a}{\delta \bar{\alpha}^b} c^b \supset \bar{c}^a \xi F^a_i \theta^b_{ij} \hat{R}_j c^b$$

- Goldstone's theorem, $V_0(R_i + \alpha^a \delta^a_i) = V_0(R_i)$:

$$\alpha^a \theta^a_{ij} R_j \frac{\partial V}{\partial R_i} = 0\,, \qquad \frac{\partial(\alpha^a \delta^a_i)}{\partial R_j} \frac{\partial V_0}{\partial R_i} + \alpha^a \delta^a_i \frac{\partial^2 V_0}{\partial R_i \partial R_j}$$

$$0 = -\bar{\alpha}^a F^a_i \frac{\partial^2 V_0}{\partial \hat{R}_i \partial \hat{R}_j}\,.$$

- Mass matrix of goldstone bosons is given by:

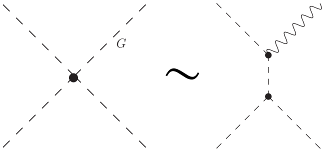
$$\mathcal{L}_\xi \supset -\frac{\xi}{2} F^a_i F^a_j \hat{R}_i \hat{R}_j\,.$$

- Masses of gauge bosons are $m^2_{ab} \equiv F^a_i F^b_i$

More about Goldstones

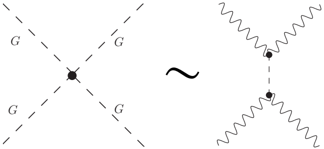
Can actually derive relations for all Goldstone boson couplings, e.g.

$$\lambda_{G_a jkl} = \frac{1}{m_a}\left[g^{aij}\, a_{ikl} + g^{aik}\, a_{ijl} + g^{ail}\, a_{ijk}\right] \longrightarrow$$



and $$\lambda_{G_a G_b G_c G_d} = \frac{m^2_{0,i}}{4\, m_a\, m_b\, m_c\, m_d}\left[g^{abi}\, g^{cdi} + g^{aci}\, g^{bdi} + g^{adi}\, g^{bci}\right]$$

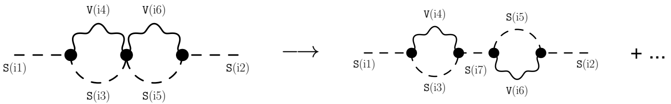
$$\longrightarrow$$



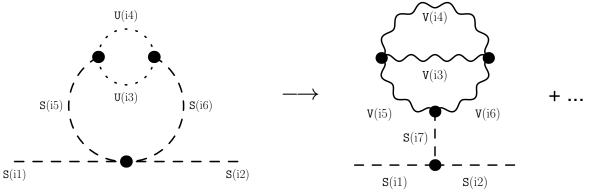
- In principle we could sum over all Goldstone bosons $\rightarrow$ get explicitly gauge invariant result (up to tadpoles ...) [work in progress: have done this at one loop]
- For now we only remove them where necessary in the ghost diagrams – results for now should treat Goldstone propagators as if they were any other scalar.

Special cases

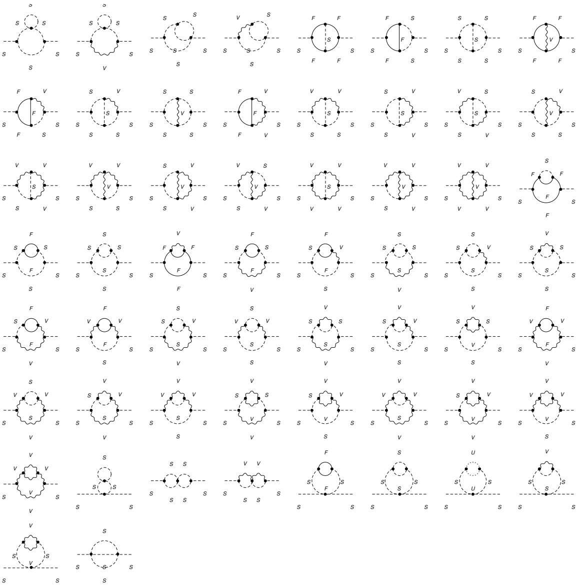
We find two amusing special cases which can either be left unreduced or lead to a non-1PI-irreducible topology:



or one with an “internal” propagator:



58 remaining classes



TLDR

While we gave closed-form expressions in the paper, most efficient way to deliver is in a code: `TLDR` at

`http://tldr.hepforge.org`

- Renormalised generic expressions, in terms of “bare” loop integrals rather than just the ϵ^0 part
- $\to$ functions and rules for performing taking the finite part for any mass/momentum configuration
- e.g. the first diagram:

```
{1, SSS[i1 , i3 , i4 , 1] SSS[i2 , i3 , i5 , 1] SSSS[i4 , i5 , i6 , i6 , 1] (−(1/2)
   T[Df[k1, MS[i4 ]] Df[k1, MS[i5 ]] Df[k2, MS[i3 ]] Df[k3, MS[i6 ]]] + (
    MS[i6]^2 TOneLoop[Df[k1, MS[i4 ]] Df[k1, MS[i5 ]] Df[k2, MS[i3 ]]])/(2 del))}
```

then typing `expandanint[unexpSS[[1]]]`:

```
B0fin[0, 0, MS[i6]^2] ((B0fin[p2, MS[i3]^2, MS[i5]^2] MS[i6]^2 SSS[i1 , i3 , i4 ,
   1] SSS[i2 , i3 , i5 , 1] SSSS[i4 , i5 , i6 , i6 , 1])/(2 MS[i4]^2 − 2 MS[i5]^2) −
   (B0fin[p2, MS[i3]^2, MS[i4]^2] MS[i6]^2 SSS[i1 , i3 , i4 , 1] SSS[i2 , i3 , i5 ,
   1] SSSS[i4 , i5 , i6 , i6 , 1])/(2 (MS[i4]^2 − MS[i5]^2)))
```

There should be no `B0del` functions unless there are IR divergences ...

Check in the Standard Model

Have performed analytic checks:

- Separate explicit diagrammatic calculation, using (modified) `SM` model file in `FeynArts` with explicit counterterms.
- ... recall all one-loop counterterms in SM can be obtained from self-energies, e.g.

$$\mathtt{C[S[1],S[1],S[1],S[1]]} == -3\,\mathtt{I}\,\mathtt{EL}^2\mathtt{MH}^2/(4\,\mathtt{SW}^2\mathtt{MW}^2)*\{\{1, 2\,\mathtt{dZe1} - 2\,\mathtt{dSW1}/\mathtt{SW}$$
$$+\mathtt{dMHsq1}/\mathtt{MH}^2 + \mathtt{EL}/(2\,\mathtt{SW}\,\mathtt{MW}\,\mathtt{MH}^2)\mathtt{dTH1} - \mathtt{dMWsq1}/\mathtt{MW}^2 + 2\,\mathtt{dZH1}\}\}$$

- In Landau gauge $\rightarrow$ exactly reproduce the expressions in `SMH` [interesting conclusion about treatment of tadpoles].
- Redo diagrammatic calculation in Feynman gauge using same technique $\rightarrow$ compare with results using `TLDR`.
- Exact agreement using “brute force” 121 classes, and set of 58.

Prospects for the (N)MSSM

To use the results for the (N)MSSM:

- Need to map generic diagrams to model $\rightarrow$ adapt diagrammatic routines from SARAH.
- $\rightarrow$ needs efficient way to manage `TSIL` evaluation (in progress)

For a fixed order calculation:

- Need Z self energy ... and muon decay!

Maybe it is better to use the EFT pole-mass matching:

- Just need Z self energy
- Or in fact, maybe not even that: could just use $\Pi_{hh}(0)$ and $\Pi'_{hh}(0)$.

Both approches also need an overhaul of the mass calculation in `SARAH/SPheno`: switch to perturbative evaluation rather than iterative, [maybe also tadpole method?]

Treatment of tadpoles

The “Martin” approach:

- Fix vacuum expectation values and adjust masses order by order, cf in SM.

$$V_{\text{tree}} = \mu^2|H|^2 + \lambda|H|^4, \qquad \mu^2 = -\lambda v^2 - \frac{1}{v}\frac{\partial \Delta V}{\partial h}$$

- Working at minimum of quantum potential $\rightarrow$ tadpole diagrams cancel.
- Gauge invariance of result is not manifest
- In SM, the Goldstone boson mass is $m_G^2 = \mu^2 + \lambda v^2 + \text{gauge} - \text{fixing}$

$$m_G^2 \underset{\text{Landau gauge}}{=} \mathcal{O}(1-\text{loop})$$

- ... may be negative! And (particularly in Landau gauge) have IR divergences

$$0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}} \qquad \left\{A(x) \equiv x(\log x/Q^2 - 1)\right\}$$

$$+ \underbrace{\frac{\log\frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v A(m_G^2) + \frac{4\lambda^3 v^3}{M_h^2}A(M_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}}$$

The problem extends to two-loop self energies etc

To solve the GBC/respect perturbation series, expand perturbatively:

$$\mu^2 = -\lambda v^2 - \frac{1}{v}\frac{\partial \Delta V(\mu)}{\partial v} = (\mu^2)^{\text{tree}} - \frac{1}{v}\frac{\Delta V((\mu^2)^{\text{tree}})}{\partial v} + \frac{1}{v^2}\left[\left(\frac{\partial^2 \Delta V}{\partial v \partial \mu^2}\right)\left(\frac{\partial \Delta V}{\partial v}\right)\right]_{\mu^2=(\mu^2)^{\text{tree}}}$$

$$\to (\mu^2)^{(1)} = -\frac{1}{v}T^{(1)}\Big|_{\mu^2=-\lambda v^2}$$

$$(\mu^2)^{(2)} = \underbrace{-\frac{1}{v}T^{(2)}\Big|_{\mu^2=-\lambda v^2} + \frac{1}{v}T^{(1)}\left[\frac{\partial T^{(1)}}{\partial m_G^2}\right.}_{\text{IR divergences cancel}} + \left.\underbrace{\frac{\partial T^{(1)}}{\partial m_h^2}}_{\text{IR safe}}\right]_{\mu^2=-\lambda v^2}$$

Find that this equivalent to shifts in the tadpoles

$$\Delta T^{(2)} = -\frac{1}{v}T^{(1)}\left[\frac{\partial T^{(1)}}{\partial m_G^2} + \frac{\partial T^{(1)}}{\partial m_h^2}\right]$$

and also for self-energies:

$$\Delta \Pi^{(2)} \supset -\frac{1}{v}T^{(1)}\left[\frac{\partial \Pi^{(1)}}{\partial m_G^2} + \frac{\partial \Pi^{(1)}}{\partial m_h^2}\right]$$

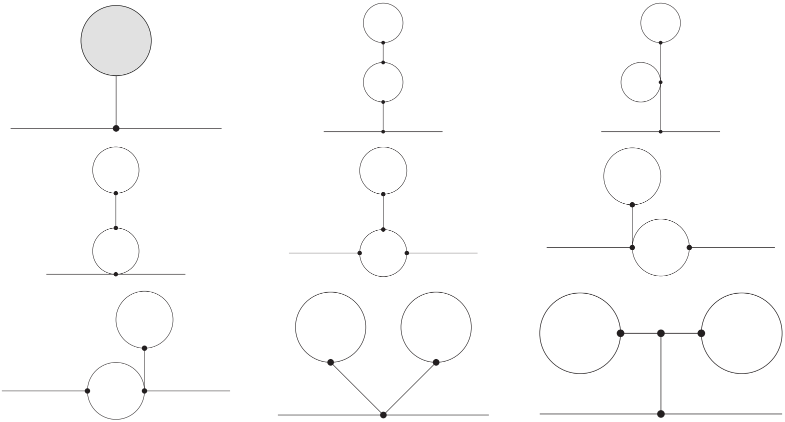
New: this is equivalent to including a counterterm for the μ^2 parameter modulo an important subtlety in Landau gauge/gaugeless limit.

Alternative treatment of tadpoles

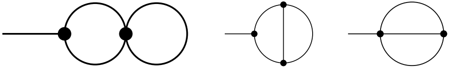
Alternative approaches that are not often used in BSM:
Jägerlehner-Fleischer $\leftrightarrow$ vev renormalisation; and $\texttt{MR}$ Kniehl et al:

- Work with tree-level expectation values and masses: $m_G^2 = 0$ in Landau gauge.
- Include all internal (but still 1PI) diagrams).
- Result for self-energy is gauge independent.
- But now we need to include reducible diagrams in all processes, e.g. Z self-energy ...

Tadpole diagrams



The blob represents the three genuine two-loop tadpole topologies:



Gauge invariance vs hierarchy problem

The interesting feature of the Kniehl et al approach to tadpoles is the explicit gauge independence of the result.

However, it emphasises the hierarchy problem:

- Consider solving the tadpole equation instead for the vev:

$$v^2 = -\frac{\mu^2}{\lambda} - \frac{T(v^2)}{\lambda} = -\frac{\mu^2 + \delta\mu^2}{\lambda} + \mathcal{O}(v^2)$$

- In the Martin approach this is fixed, but Kniehl et al approach is equivalent to solving perturbatively for the vev ...
- $\delta\mu^2$ is not protected from quantum corrections, so even in $\sim$ TeV scale SUSY, get

$$\delta v^2 \sim \frac{(\mathrm{TeV})^2}{16\pi^2\lambda} \sim (200\ \mathrm{GeV})^2$$

- So we then get

$$m^2_{\mathrm{pole}} = m^2_{\mathrm{tree}} + 2\lambda\delta v^2 + \Pi_{\mathrm{no\ internal\ tadpoles}}(m^2_{\mathrm{pole}}) \longrightarrow \frac{\delta m^2}{m^2_{\mathrm{tree}}} \sim 0.8$$

- Can view this as a measure of fine-tuning: if we calculate mass with this method and find large shifts, the theory is fine-tuned.
- ... in such cases, need to switch to EFT matching (work in progress).

Conclusions

Now have a complete generic two-loop calculation for scalar self energies and tadpoles, available as a package.

But this is just the start. Future steps include:

- Implementation in SARAH plus code for linking to `TSIL`. Or maybe standalone?
- Vector [and fermion?] self-energies.
- ... Will be immediately useful for charged/coloured scalars, not just the Higgs/heavy neutral scalars.
- Eliminate all <u>Goldstone bosons</u> from the calculation, sum with reducible diagrams to get an explicitly gauge-invariant result
- Apply to EFT calculation (through pole-matching).
- Matching with SMEFT to include $\delta\rho$
- Essential part of calculation of Higgs decays, ...
- Longer term: also four-fermi interaction at zero momentum for muon decays ...