Note on 1609.06320 (Arvanitaki-Dimopoulos-Gorbenko-Huang-Van Tiburg model)

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- The Higgs VEV v affects the vacuum structure via the stabilization of 5th dimension.
- When we choose 4D cosmological constant $\Lambda_4 \sim (\text{meV})^4$, $v \simeq v^*$ is chosen naturally.
- The size of v^* is determined by mass parameters of new fermions.

1 Model

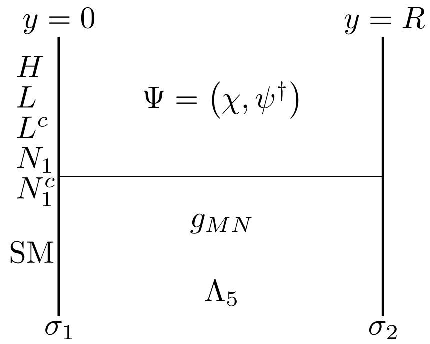

Figure 1: (Figure 2 of 1609.06320)

See Fig. 1

- The Standard model particles live in the brane at $y = 0$.
- New fermions $N_1(1,1)_0$, $N_1^c(1,1)_0$, $L(1,2)_{1/2}$, $L^c(1,2)_{-1/2}$ in the brane at $y = 0$.

- A bulk fermion $\Psi = (\chi, \psi^\dagger)(1,1)_0$.
- mixing term between N_1^c and χ.

$$\mathcal{L}_1 = M_0 L L^c + M_1 N_1 N_1^c + Y H L N_1^c + Y^c H^\dagger L^c N_1 + c.c. \tag{2.7}$$
$$+ \mu^{1/2} N_1^c \chi + c.c. \tag{2.8}$$

The mixing term becomes important when $v \simeq v_*$, where

$$v_* \equiv \sqrt{\frac{M_0 |M_1|}{Y Y^c}}. \tag{2.9}$$

By integrating out heavy fermions, we obtain

$$\mathcal{L}_1 = m_1(v) \tilde{N}_1 \tilde{N}_1^c + \mu^{1/2} \tilde{N}_1^c \chi + c.c. \tag{2.10}$$

where

$$m_1(v) \simeq M_1 - \frac{Y Y^c v^2}{M_0}. \tag{2.11}$$

1.1 Boundary condition of bulk fermion

$$\Psi = \begin{pmatrix} \chi \\ \psi^\dagger \end{pmatrix}. \tag{1}$$

at $y = R$, they impose

$$\partial_5 \psi^\dagger|_R = 0, \qquad \chi|_R = 0. \tag{2.5}$$

at $y = 0$ (and in the limit of $\mu = 0$), they impose

$$\psi^\dagger|_0 = 0, \qquad \partial_5 \chi|_0 = 0. \tag{2.5}$$

the brane interaction affects the boundary condition at $y = 0$ as

$$[-\partial^2 + |m_1(v)|^2]\psi^\dagger + i\mu\bar{\sigma}^\mu \partial_\mu \chi = 0. \tag{2.12}$$

Rough behavior of the above boundary condition is

$$\begin{cases} \psi^\dagger|_{0_+} \simeq 0 & (|m_1(v)| \lesssim \mu \ \ \& \qquad\qquad R \ll 1/\mu \qquad) \\ \chi|_{0_+} \simeq 0 & (|m_1(v)| \lesssim \mu \ \ \& \quad 1/\mu \ll R \ll \mu/|m_1(v)|^2) \\ \psi^\dagger|_{0_+} \simeq 0 & (|m_1(v)| \lesssim \mu \ \ \& \ \ \mu/|m_1(v)|^2 \ll R \qquad) \\ \psi^\dagger|_{0_+} \simeq 0 & (|m_1(v)| \gtrsim \mu) \end{cases} \tag{2.13 \& 2.14}$$

Something special happens if $|m_1(v)| \lesssim \mu$ ***and*** $1/\mu \ll R \ll \mu/|m_1(v)|^2$ ***are satisfied.***

1.2 Casimir energy from fermion

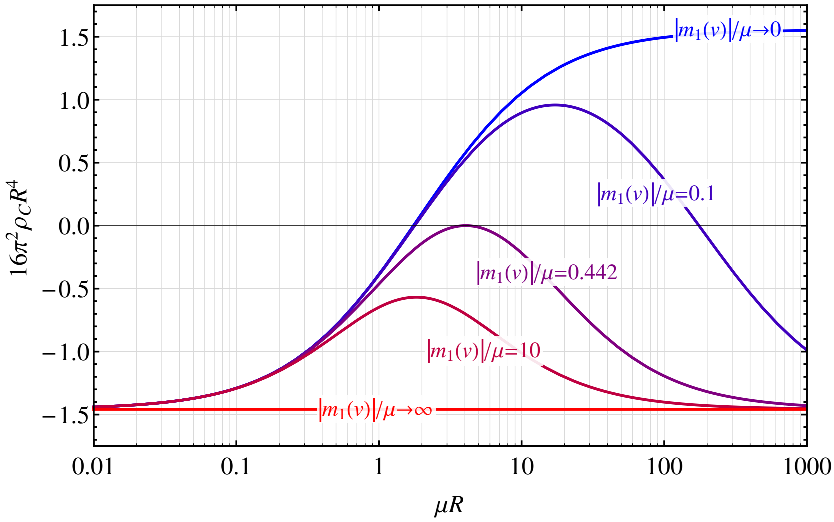

Figure 2: $16\pi^2\rho_C R^4$ (Figure 5 of 1609.06320)

$$T_N^M = \frac{\beta}{R^5}\mathrm{diag}(1,1,1,1,-4). \tag{2.18}$$

The dimensionless parameter β is a function of $m_1(v)$, μ, and R.

$$\beta \simeq \frac{1}{32\pi^2}\begin{cases} -\frac{45\zeta(5)}{32} & (\qquad R \ll 1/\mu \qquad) \\ +\frac{3\zeta(5)}{2} & (\qquad 1/\mu \ll R \ll \mu/|m_1(v)|^2) \\ -\frac{45\zeta(5)}{32} & (\mu/|m_1(v)|^2 \ll R \qquad) \end{cases} \tag{2.22}$$

See also Fig. 2. ***Casimir energy flips the sign when*** $|m_1(v)| \lesssim \mu$ ***and*** $1/\mu \ll R \ll \mu/|m_1(v)|^2$ ***are satisfied.***

2 Landscape

New dimenfion full parameters

- the brane fermion masses : M_1, M_2
- the mixing term between the bulk and brane fermions : μ
- the brane tensions : σ_1, σ_2

- 5D cosmological constant : Λ_5

They scans m_H^2, σ_1, and σ_2 in the landscape, because they are not protected by symmetries. The number of possible values are denoted as $\mathcal{N}_{m_H^2}$, $\mathcal{N}_{\sigma_1}$, and $\mathcal{N}_{\sigma_2}$. They assume the possible value of m_H^2, σ_1, and σ_2 are uniformly distributed.

They fix M_1, M_2, and μ. (those parameter is protected by chiral symmetry)

Also, they do not scan Λ_5 just for simplicity.

4D cosmological constant Λ_4 is a function of σ_1, σ_2, and the Higgs vev v.

3 Cosmological constant & stabilization

We take the following metric.

$$ds^2 = a(y)^2[-dt^2 + e^{2\mathcal{H}t}d\vec{x}^2] + dy^2.$$

(see eq 3.1) 4D cosmological constant is given as

$$\Lambda_4 = 3\mathcal{H}^2 M_{\rm Pl}^2.$$

We are interested in the case with $\mathcal{H} = 0$. In this case,

$$ds^2 = a(z)^2(dx^2 + dz^2), \tag{2.23}$$

$$L = z_2 - z_1 = \int_0^R \frac{dy}{a(y)}. \tag{2.24}$$

3.1 $v \gg v_*$

When we can neglect Casimir energy contribution, we obtain

$$3\frac{a''}{a} + 3\frac{a'^2}{a^2} - \frac{3\mathcal{H}^2}{a^2} + \frac{\Lambda_5}{M_5^3} = -\frac{\sigma_1}{M_5^3}\delta(y) - \frac{\sigma_2}{M_5^3}\delta(y-R), \tag{3.3}$$

$$6\frac{a'^2}{a^2} - \frac{6\mathcal{H}^2}{a^2} + \frac{\Lambda_5}{M_5^3} = 0. \tag{3.4}$$

By integrating Eq. 3.3,

$$\left.\frac{a'}{a}\right|_0 = -\frac{\sigma_1}{6M_5^3}, \tag{3.5}$$

$$\left.\frac{a'}{a}\right|_R = +\frac{\sigma_2}{6M_5^3}. \tag{3.6}$$

By using 3.4 & 3.5, we obtain

$$\mathcal{H}^2 = \frac{1}{6M_5^3}\left(\Lambda_5 + \frac{\sigma_1^2}{6M_5^3}\right) \equiv \frac{\lambda_1}{6M_5^3}. \tag{3.7}$$

$\mathcal{H} = 0$ when $\sigma_1 = \sigma_*$,

$$\sigma_* = \sqrt{-6M_5^3\Lambda_5}. \tag{3.9}$$

3.1.1 Tuning for C.C.

For small $\mathcal{H}$,

$$\Lambda_4 = 3M_{\rm Pl}^2\mathcal{H}^2 \simeq \frac{M_{\rm Pl}^2\sigma_*}{6M_5^6}(\sigma_1 - \sigma_*)$$

C.C. does not depend on σ_2***.*** By using $M_{\rm Pl}^2 \simeq M_5^3/k$ and $k = \sqrt{-\Lambda_5/6M_5^3}$.

$$\Lambda_4 \simeq \sigma_1 - \sigma_* \tag{3.8}$$

In this case, we need to tune σ_1 ***in a precision of*** $(\text{meV})^4$ ***to be consistent with observation.***

3.2 $v \simeq v_*$

When $v \simeq v_*$ and $L \gg 1/\mu$, the Casimir energy becomes important:

$$3\frac{a''}{a} + 3\frac{a'^2}{a^2} - \frac{3\mathcal{H}^2}{a^2} + \frac{\Lambda_5}{M_5^3} + \frac{\beta}{M_5^3L^5a^5} = -\frac{\sigma_1}{M_5^3}\delta(y) - \frac{\sigma_2}{M_5^3}\delta(y-R), \tag{3.11}$$

$$6\frac{a'^2}{a^2} - \frac{6\mathcal{H}^2}{a^2} + \frac{\Lambda_5}{M_5^3} - \frac{\beta}{4M_5^3L^5a^5} = 0. \tag{3.12}$$

See also [2–4].

For $\mathcal{H} = 0$, we obtain

$$L^5 = \frac{4\beta}{\lambda_1}, \tag{3.16}$$

where

$$\lambda_1 = \Lambda_5 + \frac{\sigma_1^2}{6M_5^3}.$$

3.2.1 Tuning for $L \lesssim 1/\mu$

In order to get $L > 1/\mu$,

$$\Lambda_5 + \frac{\sigma_1^2}{6M_5^3} \lesssim \mu^5.$$

Expanding σ_1 around σ_*,

$$\frac{\Lambda_5}{\sigma_*}(\sigma_1 - \sigma_*) \lesssim \mu^5.$$

By using $\sigma_* = \sqrt{-6M_5^3\Lambda_5}$ and $k = \sqrt{-\Lambda_5/6M_5^3}$,

$$\sigma_1 - \sigma_* \lesssim \frac{\mu^5}{k} \sim \frac{\mu^4}{\gamma}, \tag{2}$$

where $\gamma = e^{kR} \simeq a(0)/a(R)$.[1]

3.2.2 Tuning for C.C.

For $\mathcal{H} = 0$,

$$\sigma_2^{\mathcal{H}=0}(\sigma_1) = -\sigma_1 - \frac{5}{2^{3/5}}\beta^{1/5}\lambda_1^{4/5}. \tag{3.21}$$

For non-zero $\mathcal{H}$ and fixed σ_1,

$$\sigma_2 = \sigma_2^{\mathcal{H}=0}(\sigma_1) + \Lambda_4 \tag{3.25}$$

we need to tune $\sigma_1+\sigma_2$ ***in a precision of*** $(\mathrm{meV})^4$ ***to be consistent with observation.***

4 Anthropic principle & Higgs VEV

$v \sim v_*$ need tuning of m_H^2 at the level of v_*^2/M_{UV}^2. In order to get $\Lambda_4 \sim (\mathrm{meV})^4$ with $v \sim v_*$ vacua, we need $L > 1/\mu$ and this requires tuning of σ_1 at the level of μ^4/γ. then,

$$\mathcal{N}_{\sigma_1} \gtrsim \underbrace{\frac{M_{\mathrm{UV}}^2}{v_*^2}}_{v\sim v_*} \times \underbrace{\frac{M_{\mathrm{UV}}^4}{\mu^4/\gamma}}_{L>1/\mu}. \tag{4.3}$$

[1] $a(y) = e^{-ky}$ and $R = (1/k)\log\gamma$. Then,

$$L = \int_0^R \frac{dy}{a(y)} = \frac{\gamma - 1}{k} \sim \frac{\gamma}{k}. \tag{3}$$

In this case, by using eq. 4.3,

$$\Lambda_4^{\rm II,min} \sim \frac{M_{\rm UV}^4}{\mathcal{N}_{\sigma_1}} \lesssim \frac{\mu^4}{\gamma}\frac{v_*^2}{M_{\rm UV}} \sim {\rm keV}^4 \left(\frac{\mu}{\rm GeV}\right)^4 \left(\frac{10^4}{\gamma}\right) \left(\frac{10^{12}\ {\rm GeV}}{M_{\rm UV}}\right)^2 . \quad (4.6)$$

Thus, when we assume $\Lambda_4 \sim ({\rm meV})^4$ is chosen by anthoric principle, this explains $v \sim v^*$.

References

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