

TOP THEORY - RESUMMED CALCULATIONS

Andrea Ferroglia

New York City College of Technology
NYCCT Center for Theoretical Physics

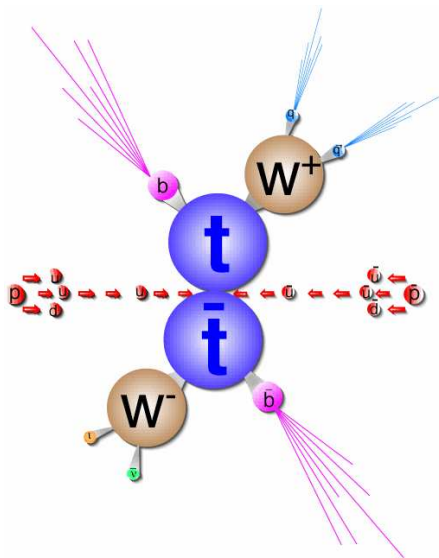
DESY Zeuthen, June 16, 2011



OUTLINE

- 1 TOP-QUARK PAIR PRODUCTION AT HADRON COLLIDERS
- 2 RESUMMATION AND APPROXIMATE NNLO RESULTS
- 3 OBSERVABLES

TOP QUARK PAIR PRODUCTION



TOP QUARK PAIR PRODUCTION

Key measurements at the Tevatron and the LHC include the top-quark pair production total cross section and differential distributions

$$p\bar{p}, pp \rightarrow t\bar{t}X$$

TOP QUARK PAIR PRODUCTION

Key measurements at the Tevatron and the LHC include the top-quark pair production total cross section and differential distributions

$$p\bar{p}, pp \rightarrow t\bar{t}X$$

The top quarks decay almost exclusively in a W boson and a b quark. The observed processes are

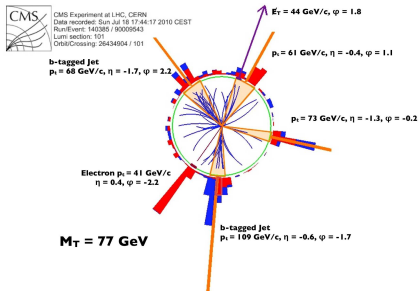
$$p\bar{p}, pp \rightarrow t\bar{t}X \rightarrow l_1^+ + l_2^- + j_b + j_{\bar{b}} + p_T^{\text{miss}} + (n \geq 0) \text{ jets}$$

$$p\bar{p}, pp \rightarrow t\bar{t}X \rightarrow l_1^\pm + j_b + j_{\bar{b}} + p_T^{\text{miss}} + (n \geq 2) \text{ jets}$$

$$p\bar{p}, pp \rightarrow t\bar{t}X \rightarrow j_b + j_{\bar{b}} + (n \geq 4) \text{ jets}$$

All these channels were observed and analyzed at the Tevatron

TOP-PAIR PRODUCTION AND THE LHC



- ▶ Next 2 years at the LHC
~ few 10^4 top pair events
- ▶ Fall 2010: First measurement of the total CS at ATLAS and CMS
- ▶ **June 2011: CMS 36 pb^{-1}**
(Di-lepton channel)

$$\sigma^{t\bar{t}} = 168 \pm 18 \pm 14 \pm 7 \text{ pb}$$

[arXiv:1105.5661](https://arxiv.org/abs/1105.5661)

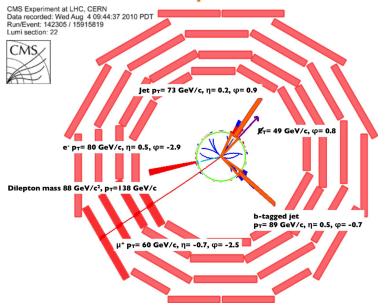
(Leptons + jets channel)

$$\sigma^{t\bar{t}} = 173_{-32}^{+39} \text{ pb}$$

[arXiv:1106.0902](https://arxiv.org/abs/1106.0902)

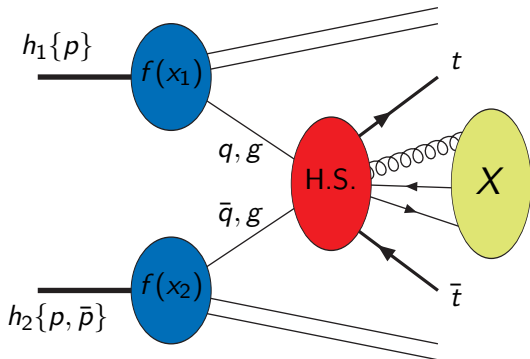
ATLAS 35 pb^{-1} (prelim.)

$$\sigma^{t\bar{t}} = 180 \pm 9 \pm 15 \pm 6 \text{ pb}$$



TOP QUARK PAIR HADROPRODUCTION

Top-quark pair production is a hard scattering process which can be computed in perturbative QCD

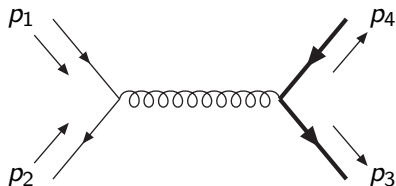


$$d\sigma_{h_1, h_2}^{t\bar{t}} = \sum_{i, j = q\bar{q}g} \int dx_1 dx_2 f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F) d\hat{\sigma}_{ij}(s, m_t, \alpha_s(\mu_R), \mu_F, \mu_R)$$

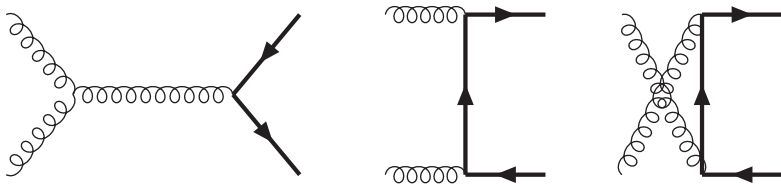
$$s_{\text{had}} = (p_{h_1} + p_{h_2})^2, \quad s = x_1 x_2 s_{\text{had}}$$

TREE LEVEL QCD PARTONIC PROCESSES

$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$

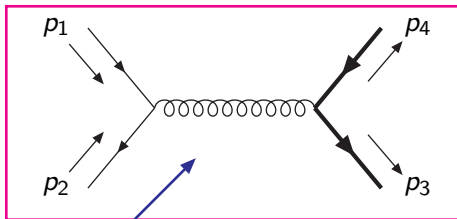


$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



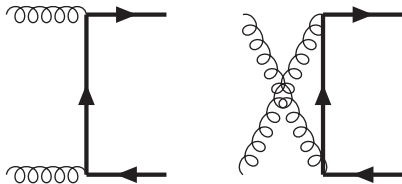
TREE LEVEL QCD PARTONIC PROCESSES

$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



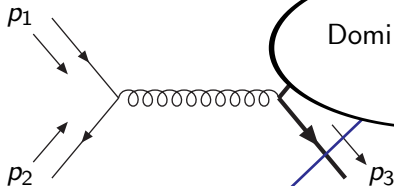
$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$

Dominant at Tevatron
 $\sim 85\%$



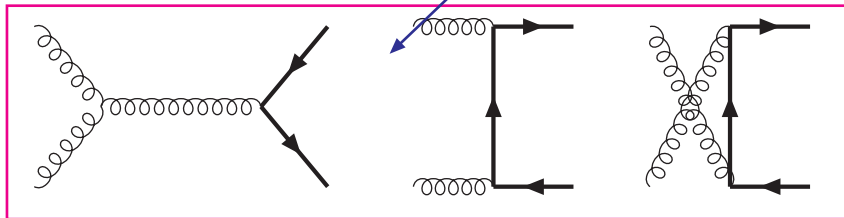
TREE LEVEL QCD PARTONIC PROCESSES

$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



Dominant at LHC
 $\sim 90\%$

$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



NLO CORRECTIONS

perturbative expansion:

$$d\hat{\sigma}_{ij} = \alpha_s^2 d\hat{\sigma}_{ij}^{(0)} + \alpha_s^3 d\hat{\sigma}_{ij}^{(1)} + \alpha_s^4 d\hat{\sigma}_{ij}^{(2)} + \dots$$

NLO CORRECTIONS

perturbative expansion:

$$d\hat{\sigma}_{ij} = \alpha_s^2 d\hat{\sigma}_{ij}^{(0)} + \alpha_s^3 d\hat{\sigma}_{ij}^{(1)} + \alpha_s^4 d\hat{\sigma}_{ij}^{(2)} + \dots$$

The NLO corrections to top-quark pair production have been a subject of active research for more than 20 years
(too many authors to list here!)

- NLO QCD corrections to the total cross section
- NLO QCD corrections to the distributions (p_T , rapidity, invariant mass, ...)
- Mixed QCD-EW corrections
- NLO with top decays in narrow width approximation
- NLO with top decays with off-shell top quarks (recent results by [Denner et al.](#) arXiv:1012.3975, and [Bevilacqua et al.](#) arXiv:1012.4230)
- Beyond fixed order calculations: **NLL resummation of threshold effects**

Resummation = organization of large logarithms in perturbative expansion

$$\begin{aligned}
 \hat{O} &= \overbrace{1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3)}^{\text{NNLO}} \\
 &= \underbrace{\exp\left(\underbrace{Lg_1(\alpha_s L)}_{LL} + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right)}_{\text{NLL}} \underbrace{C(\alpha_s)}_{\text{constants}} \\
 &\quad + \text{suppressed terms}
 \end{aligned}$$

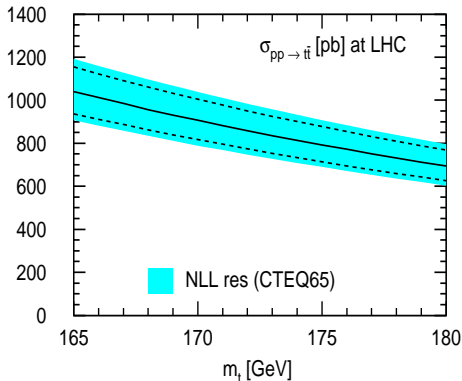
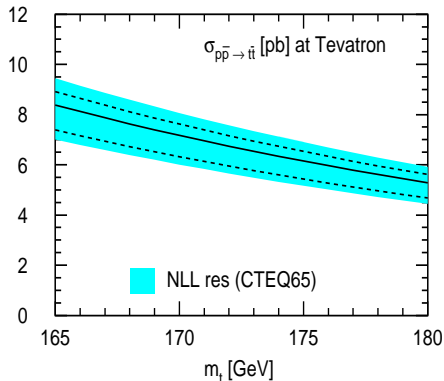
$L = \ln\left(\frac{\text{"high" scale}}{\text{"low" scale}}\right)$ The arguments depends on the observable

Resummation reduces the theoretical uncertainties on a given observable

UNCERTAINTY ON THE NLO+NLL CROSS SECTION

The partonic cross sections involve terms like $\ln(1 - 4m^2/s)$ which become large near threshold and must be resummed

Berger, Contopanagos ('95-'98), Kidonakis, Sterman ('97), Bonciani *et al.* ('98), Kidonakis *et al.* ('01), Kidonakis, Vogt ('03), Banfi, Laenen ('05), Cacciari *et al.* ('08) Czakon *et al.* ('09)

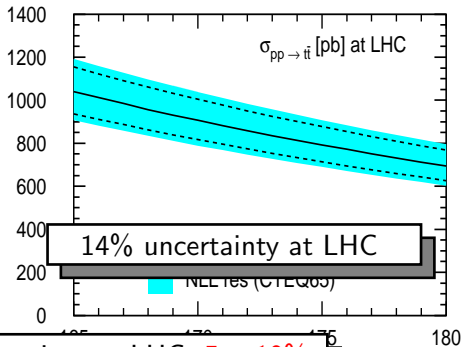
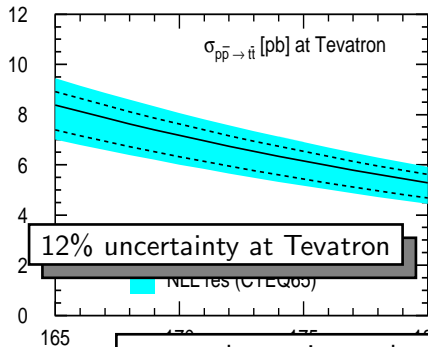


S. Moch and P. Uwer ('08)

UNCERTAINTY ON THE NLO+NLL CROSS SECTION

The partonic cross sections involve terms like $\ln(1 - 4m^2/s)$ which become large near threshold and must be resummed

Berger, Contopanagos ('95-'98), Kidonakis, Sterman ('97), Bonciani *et al.* ('98), Kidonakis *et al.* ('01), Kidonakis, Vogt ('03), Banfi, Laenen ('05), Cacciari *et al.* ('08) Czakon *et al.* ('09)



expected experimental uncertainty at LHC 5 – 10%

S. Moch and P. Uwer ('08)

BEYOND NLO+NLL

In order to take full advantage of the LHC potential, one needs to obtain theoretical predictions that go
beyond NLO+NLL accuracy

In order to take full advantage of the LHC potential, one needs to obtain theoretical predictions that go beyond NLO+NLL accuracy

Full NNLO Calculations

Calculations are carried out in a perturbative expansions in powers of α_s

Goal: calculate all of the corrections of $\mathcal{O}(\alpha_s^2)$ (NNLO) to the tree-level process

In order to take full advantage of the LHC potential, one needs to obtain theoretical predictions that go beyond NLO+NLL accuracy

Full NNLO Calculations

Calculations are carried out in a perturbative expansions in **powers** of α_s

Goal: calculate all of the corrections of $\mathcal{O}(\alpha_s^2)$ (NNLO) to the tree-level process

NNLL Resummation Approximate NNLO

Not all corrections of order α_s^n have the same relevance

Large logarithms $\ln(r) \sim 1/\alpha_s$ are present

Goal: to resum corrections $\propto \alpha_s^n \ln^m(r)$ to all orders in α_s

In order to take full advantage of the LHC potential, one needs to obtain theoretical predictions that go beyond NLO+NLL accuracy

this talk

Full NNLO Calculations

Calculations are carried out in a perturbative expansions in **powers of α_s**

Goal: calculate all of the corrections of $\mathcal{O}(\alpha_s^2)$ (NNLO) to the tree-level process

NNLL Resummation Approximate NNLO

Not all corrections of order α_s^n have the same relevance
Large logarithms $\ln(r) \sim 1/\alpha_s$ are present

Goal: to resum corrections $\propto \alpha_s^n \ln^m(r)$ to all orders in α_s

NLL Resummation

in collaboration with

V. Ahrens, M. Neubert, B. Pecjak, and L. L. Yang



arXiv:1105. 5824[hep-ph]

arXiv:1103.0550 [hep-ph]

JHEP 1009 (2010) 097 (arXiv:1003.5827 [hep-ph])

Phys.Lett. B687 (2010) 331-337 (arXiv:0912.3375 [hep-ph])

SOFT LIMITS

partonic process:

$$p_i(p_1) + p_j(p_2) \longrightarrow t(p_3) + \bar{t}(p_4) + X(k) \quad (i, j \in \{q, \bar{q}, g\})$$

Name	Observable	Soft Variable
Production Threshold	σ	$\beta = \sqrt{1 - \frac{4m_t^2}{s}}$
Single Particle Inclusive (1PI)	$\frac{d\sigma}{dp_T dy}$	$s_4 = (p_4 + k)^2 - m_t^2$
Pair Invariant Mass (PIM)	$\frac{d\sigma}{dM_{t\bar{t}} d\cos\theta}$	$(1 - z) = 1 - \frac{M_{t\bar{t}}^2}{s}$

parto

Large logarithms appear in the partonic cross section if
the “soft variable” is small

Name	Observable	Soft Variable
Production Threshold	σ	$\beta = \sqrt{1 - \frac{4m_t^2}{s}}$
Single Particle Inclusive (1PI)	$\frac{d\sigma}{dp_T dy}$	$s_4 = (p_4 + k)^2 - m_t^2$
Pair Invariant Mass (PIM)	$\frac{d\sigma}{dM_{t\bar{t}} d\cos\theta}$	$(1 - z) = 1 - \frac{M_{t\bar{t}}^2}{s}$

SOFT LIMITS

parto

Large logarithms appear in the partonic cross section if the “soft variable” is small

Name	Observable	Soft Variable
Physically, the soft limit coincides with events in which the final state is $t\bar{t}$ + soft emission		
Pair Invariant Mass (PIM)	$\frac{d\sigma}{dM_{t\bar{t}}d\cos\theta}$	$(1 - z) = 1 - \frac{M_{t\bar{t}}^2}{s}$

SOFT LIMITS

parto

Large logarithms appear in the partonic cross section if the “soft variable” is small

Name	Observable	Soft Variable
Physically, the soft limit coincides with events in which the final state is $t\bar{t}$ + soft emission		
$\beta \rightarrow 0$ is a special case of 1PI and PIM and $\langle \beta \rangle \sim 0.5$ \implies we focused on PIM and 1PI kinematics		

Str

In PIM, in the limit $z \rightarrow 1$ (threshold region) there are three different scales

$$s, M^2, m_t^2 \gg s(1-z)^2 \gg \Lambda_{\text{QCD}}^2$$

FACTORIZATION

In PIM, in the limit $z \rightarrow 1$ (threshold region) there are three different scales

$$s, M^2, m_t^2 \gg s(1-z)^2 \gg \Lambda_{\text{QCD}}^2$$

In the threshold region the partonic cross section factors into hard functions and soft functions (matrices in color space)

Kidonakis, Sterman ('97)

$$d\hat{\sigma} \sim \text{Tr} \left[\mathbf{H}(M, m_t, \cos \theta, \mu) \mathbf{S}(\sqrt{s}(1-z), m_t, \cos \theta, \mu) \right]$$

FACTORIZATION

In PIM, in the limit $z \rightarrow 1$ (threshold region) there are three different scales

$$s, M^2, m_t^2 \gg s(1-z)^2 \gg \Lambda_{\text{QCD}}^2$$

In the threshold region the partonic cross section factors into hard functions and soft functions (matrices in color space)

Kidonakis, Sterman ('97)

$$d\hat{\sigma} \sim \text{Tr} \left[\mathbf{H}(M, m_t, \cos \theta, \mu) \mathbf{S}(\sqrt{s}(1-z), m_t, \cos \theta, \mu) \right]$$

The same factorization formula can be obtained using the language of Soft-Collinear Effective Theory

FACTORIZATION

In PIM, in the limit $z \rightarrow 1$ (threshold region) there are three different

$$\text{An equivalent formula is valid in 1PI kinematics}$$
$$d\hat{\sigma} \sim \text{Tr}[\mathbf{H}(s', t'_1, u'_1, m_t, \mu) \mathbf{S}(s_4, s', t'_1, u'_1, m_t, \mu)]$$

In the threshold region the partonic cross section factors into hard functions and soft functions (matrices in color space)

Kidonakis, Sterman ('97)

$$d\hat{\sigma} \sim \text{Tr}[\mathbf{H}(M, m_t, \cos \theta, \mu) \mathbf{S}(\sqrt{s}(1-z), m_t, \cos \theta, \mu)]$$

The same factorization formula can be obtained using the language of Soft-Collinear Effective Theory

FACTORIZATION

In PIM, in the limit $z \rightarrow 1$ (threshold region) there are three different

$$\text{An equivalent formula is valid in 1PI kinematics}$$
$$d\hat{\sigma} \sim \text{Tr}[\mathbf{H}(s', t'_1, u'_1, m_t, \mu)\mathbf{S}(s_4, s', t'_1, u'_1, m_t, \mu)]$$

In the threshold region the partonic cross section factors into hard functions and soft functions (matrices in color space)

Kidonakis, Sterman ('97)

H \rightarrow virtual corrections, same for PIM and 1PI

S \rightarrow real corrections, different in PIM and 1PI

H and **S** satisfy RG equations

which one can solve to resum large corrections

The of

RESUMMED FORMULAS

$$d\hat{\sigma} \propto \exp [4a_{\gamma\phi}(\mu_s, \mu_f)] \text{Tr} \left[\mathbf{U}(\mu_h, \mu_s) \mathbf{H}(\mu_h) \mathbf{U}^\dagger(\mu_h, \mu_s) \tilde{\mathfrak{s}}(\partial_\eta, \mu_s) \right] \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \lambda^{-1+2\eta}$$

E_s = energy of the soft radiation / depends on the distribution

$$2E_s \equiv m_t \lambda = m_t \underbrace{\frac{s_4}{m_t \sqrt{s_4 + m_t^2}}}_{\lambda_{\text{1PI}_{\text{SCET}}}}$$

$$2E_s \equiv M \lambda = M_{t\bar{t}} \underbrace{\frac{1-z}{\sqrt{z}}}_{\lambda_{\text{PIM}_{\text{SCET}}}}$$

RESUMMED FORMULAS

$$d\hat{\sigma} \propto \exp [4a_{\gamma\phi}(\mu_s, \mu_f)] \text{Tr} \left[\mathbf{U}(\mu_h, \mu_s) \mathbf{H}(\mu_h) \mathbf{U}^\dagger(\mu_h, \mu_s) \tilde{\mathbf{s}}(\partial_\eta, \mu_s) \right] \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \lambda^{-1+2\eta}$$

E_s = energy of the soft radiation / depends on the distribution

$$2E_s \equiv m_t \lambda = m_t \underbrace{\frac{s_4}{m_t^2}}_{\lambda_{\text{PI}}} \quad 2E_s \equiv M\lambda = M_{t\bar{t}} \underbrace{(1-z)}_{\lambda_{\text{PI}}}$$

RESUMMED FORMULAS

$$d\hat{\sigma} \propto \exp [4a_{\gamma\phi}(\mu_s, \mu_f)] \text{Tr} \left[\mathbf{U}(\mu_h, \mu_s) \mathbf{H}(\mu_h) \mathbf{U}^\dagger(\mu_h, \mu_s) \tilde{\mathbf{s}}(\partial_\eta, \mu_s) \right] \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \lambda^{-1+2\eta}$$

E_s = energy of the soft radiation / depends on the distribution

$$2E_s \equiv m_t \lambda = m_t \underbrace{\frac{s_4}{m_t \sqrt{s_4 + m_t^2}}}_{\lambda_{\text{1PI}_{\text{SCET}}}} \quad 2E_s \equiv M \lambda = M_{t\bar{t}} \underbrace{\frac{1-z}{\sqrt{z}}}_{\lambda_{\text{PI}_{\text{SCET}}}}$$

RG-impr. PT	log accuracy	Γ_{cusp}	γ^h, γ^ϕ	$\mathbf{H}, \tilde{\mathbf{s}}$
LO	NLL	2-loop	1-loop	tree-level
NLO	NNLL	3-loop	2-loop	1-loop

All the pieces for the NNLL calculation are now available

Ahrens, AF, Neubert, Pecjak, and Yang ('10-'11)

APPROXIMATE NNLO

The resummed formulas can be re-expanded

$$d\hat{\sigma}_{\text{NNLO}} = D_3 P_3(\lambda) + D_2 P_2(\lambda) + D_1 P_1(\lambda) + D_0 P_0(\lambda) + C_0 \delta(\lambda) + R(\lambda)$$

with

$$P_n(z) = \left[\frac{\ln^n(1-z)}{(1-z)} \right]_+ \quad P_n(s_4) = \left[\frac{\ln^n(s_4/m_t^2)}{s_4} \right]_+$$

- It was possible to calculate D_3, D_2, D_1, D_0 and the scale dependence of C_0 (D_3, D_2, D_1 first obtained by Kidonakis, Leanen, Moch, and Vogt ('01))
- Keeping the exact form of the energy re-organizes the threshold expansion, so that some formally subleading terms are kept compared to other calculations (Kidonakis et.al.), which use PIM and 1PI

$$\frac{\ln(z)}{1-z} \quad \text{in PIM}_{\text{SCET}} \quad \frac{\ln\left(1 + \frac{s_4}{m_t^2}\right)}{s_4} \quad \text{in 1PI}_{\text{SCET}}$$

DYNAMICAL THRESHOLD ENHANCEMENT

Does the soft limit provide a good approximation of the **exact** result?

ex. invariant mass distribution

$$z = \frac{M^2}{s} \quad \tau = \frac{M^2}{s_{\text{had}}}$$

$$\frac{d\sigma}{dM} = \frac{8\pi\beta}{3M} \int_{\tau}^1 \frac{dz}{z} \sum_{ij=(q\bar{q}, gg, \bar{q}q)} \overbrace{f_{ij}\left(\frac{\tau}{z}, \mu\right)}^{\text{parton luminosity}} \underbrace{C_{ij}(z, \dots, \mu)}_{\text{partonic distribution}}$$

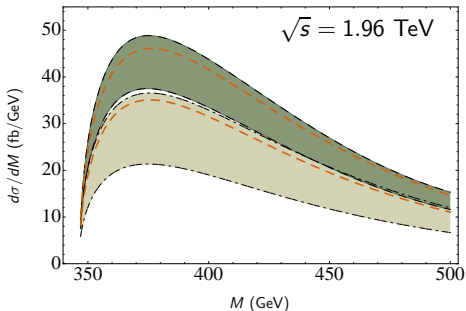
The limit $z \rightarrow 1$ provides a good approximation to the complete result if

a) $\tau \sim 1$; ... but the interesting region is $\tau < 0.3$

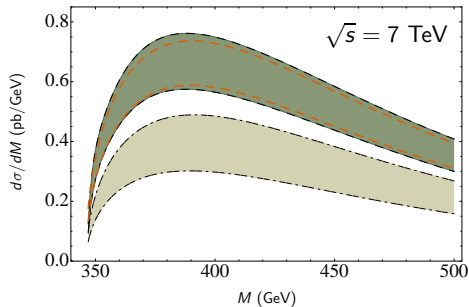
b) $f_{ij} \rightarrow 0$ for $z \rightarrow \tau$; **Dynamical Threshold Enhancement**

DYNAMICAL THRESHOLD ENHANCEMENT

Tevatron



LHC

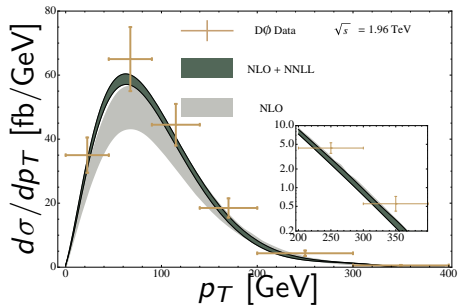
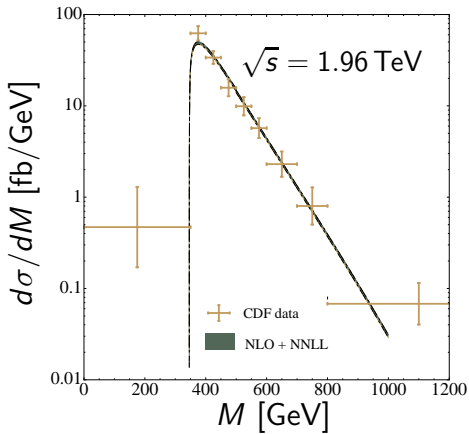


- ▶ Exact NLO result (dark grey band) obtained with MCFM (Campbell, Ellis)
- ▶ The NLO threshold expansion \rightarrow band between the dashed lines ($200 \text{ GeV} \leq \mu \leq 800 \text{ GeV}$; close to $M/2 \leq \mu \leq 2M$)
- ▶ The threshold expansion agrees quite well with the exact result, even in the low invariant mass region

Observables

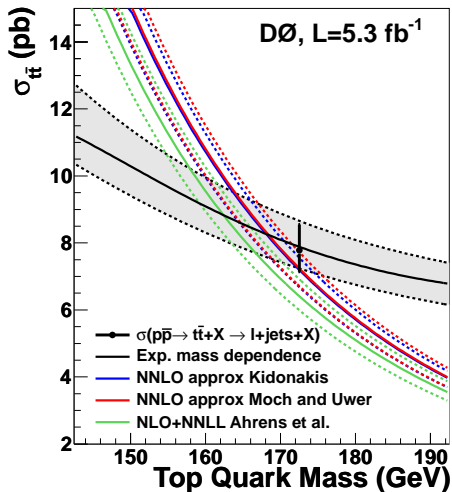
Differential Cross Sections,
Total Cross Section,
Forward-Backward Asymmetry

INVARIANT MASS DISTRIBUTION AND p_T DISTRIBUTION VERSUS TEVATRON DATA



Normalization and shape of the distributions are consistent with data

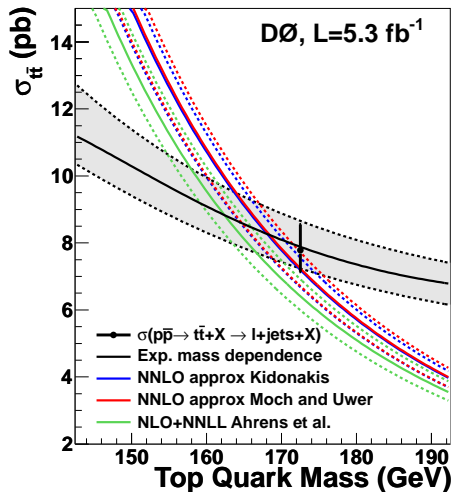
THE TOTAL INCLUSIVE CROSS SECTION



- Measured at the Tevatron and at the LHC
- Comparison theory/experiments can be used to extract m_t
[Langenfeld *et al.*, ('09), D0 ('11)]

Figure from D0 in lepton +jets channel,
arXiv:1101.0124

THE TOTAL INCLUSIVE CROSS SECTION



- Measured at the Tevatron and at the LHC
- Comparison theory/experiments can be used to extract m_t
[Langenfeld *et al.*, ('09), D0 ('11)]
- Can be calculated by integrating the differential distributions
WARNING: different soft limit neglect different types of power corrections

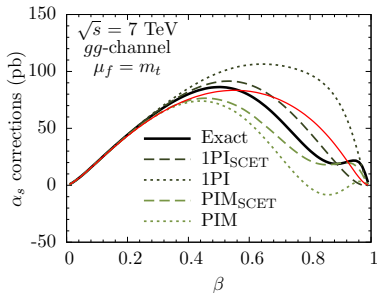
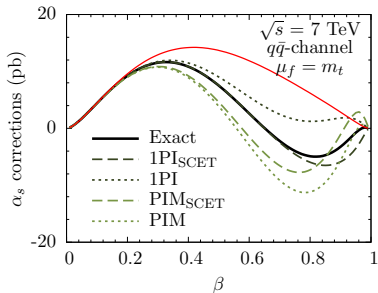
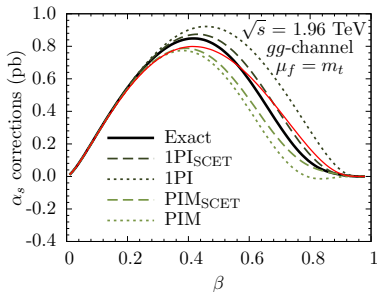
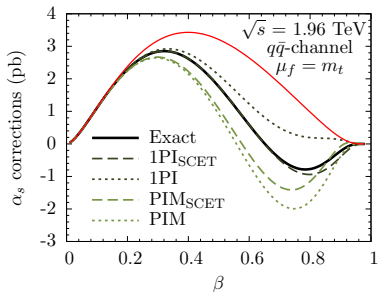
Figure from D0 in lepton +jets channel,
arXiv:1101.0124

THE β DISTRIBUTION AND THE TOTAL CS

$$\frac{d\sigma}{d\beta} = \frac{1}{s} \frac{8\beta}{(1-\beta^2)^2} \sum_{ij} \underbrace{f_{ij}\left(\frac{s}{s_{\text{had}}}, \mu_f\right)}_{\text{parton luminosities}} \alpha_s^2 \underbrace{f_{ij}\left(\frac{4m_t^2}{s}, \mu_f\right)}_{\text{partonic c. s.}} \quad s = \frac{4m_t^2}{(1-\beta^2)}$$

- ▶ Can be calculated both from PIM and 1PI kinematics, or directly for $\beta \rightarrow 0$ [Langenfeld *et al*, Czakon *et al*, Beneke *et al* ('09)]
- ▶ For $\beta \rightarrow 0$ gluon emission is soft, 1PI, PIM, and exact QCD must agree
- ▶ For larger β , different calculational schemes neglect different kinds of power corrections: it is interesting to compare the various approximations with the exact result at NLO

PIM vs. 1PI AT NLO



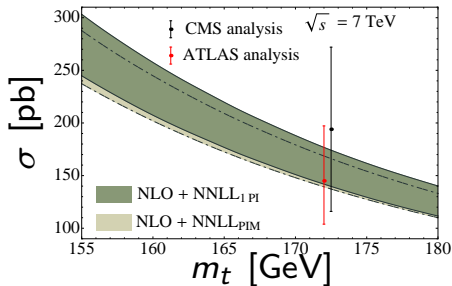
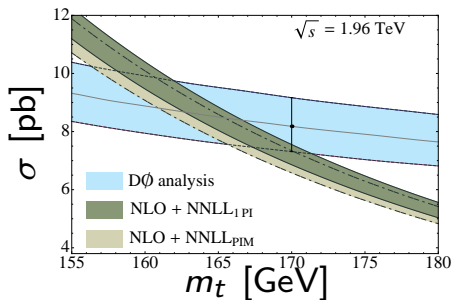
THE TOTAL CROSS SECTION IN DIFFERENT SCHEMES

$m_t/2 < \mu_f < 2m_t$, $m_t = 173.1$ GeV, MSTW2008 pdfs, only scale uncertainties

(c. s. in pb)	Tevatron	LHC (7 TeV)
$\sigma_{\text{NLO}, q\bar{q}+gg}$	$6.80^{+0.27}_{-0.73}$	160^{+5}_{-15}
$\sigma_{\text{NLO leading 1PI}_{\text{SCET}}$, (1PI)	$6.79^{+0.20}_{-0.70}$ ($7.23^{+0.45}_{-0.86}$)	163^{+0}_{-11} (183^{+6}_{-18})
$\sigma_{\text{NLO leading PIM}_{\text{SCET}}$, (PIM)	$6.42^{+0.42}_{-0.76}$ ($6.20^{+0.28}_{-0.69}$)	152^{+7}_{-15} (143^{+1}_{-12})
$\sigma_{\text{NNLO approx 1PI}_{\text{SCET}}$, (1PI)	$6.63^{+0.00}_{-0.27}$ ($7.06^{+0.00}_{-0.29}$)	155^{+3}_{-2} (180^{+3}_{-8})
$\sigma_{\text{NNLO approx PIM}_{\text{SCET}}$, (PIM)	$6.62^{+0.05}_{-0.40}$ ($6.46^{+0.18}_{-0.45}$)	155^{+8}_{-8} (148^{+14}_{-11})
$\sigma_{\text{NLO+NNLL, 1PI}_{\text{SCET}}$	$6.55^{+0.16}_{-0.14}$	150^{+7}_{-7}
$\sigma_{\text{NLO+NNLL, PIM}_{\text{SCET}}$	$6.46^{+0.18}_{-0.19}$	147^{+7}_{-6}

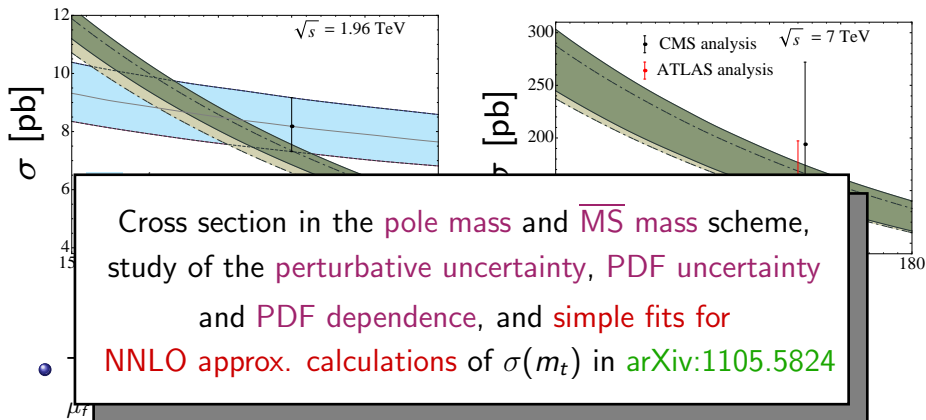
- PIM and 1PI do not agree well with each other Kidonakis, Laenen, Moch, Vogt ('01); Kidonakis ('10)
- PIM_{SCET} and 1PI_{SCET} are consistent with each other and exact result at NLO Ahrens, AF, Neubert, Pecjak, Yang ('10, '11)
- NLO+NNLL and approximate NNLO consistent with each other

THE TOTAL CROSS SECTION



- The theoretical predictions include scale and PDF uncertainties, $\mu_f = 400$ GeV in the figure
- 1PI_{SCET} and PIM_{SCET} give consistent results
 averages $\sigma_{\text{TEV}} = 6.63^{+0.07+0.63}_{-0.41-0.48}$ pb , $\sigma_{\text{LHC}}(7 \text{ TeV}) = 155^{+8+14}_{-9-14}$ pb
 (approx. NNLO, MSTW2008 PDFs, $\mu_f = m_t$, scale uncertainty, PDFs + α_s)

THE TOTAL CROSS SECTION

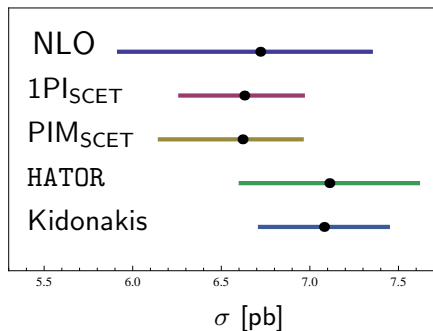


- 1P_{SCET} and PIM_{SCET} give consistent results
 averages $\sigma_{\text{TEV}} = 6.63^{+0.07+0.63}_{-0.41-0.48} \text{ pb}$, $\sigma_{\text{LHC}}(7 \text{ TeV}) = 155^{+8+14}_{-9-14} \text{ pb}$
 (approx. NNLO, MSTW2008 PDFs, $\mu_f = m_t$, scale uncertainty, PDFs + α_s)

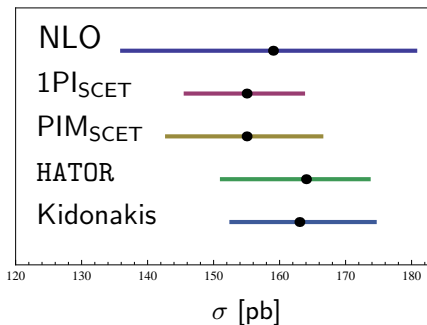
COMPARISONS

$m_t = 173.1$ GeV, $mt/2 < \mu_f = \mu_r < 2mt$, MSTW2008 90% CL
PDF and scale uncertainties added in quadrature

Tevatron



LHC 7 TeV



- $1PI_{SCET}$ and PIM_{SCET} NNLO approx. calculations [Ahrens et al '10, '11](#)
- Kidonakis: NNLO approx 1PI formulas ($m_t = 173$ GeV) [Kidonakis '10](#)
- HATOR: production threshold formulas [Aliev et al '10](#)

THE FORWARD-BACKWARD ASYMMETRY AT THE TEVATRON

The **forward-backward asymmetry** originates from the difference in the top-quark production rates in the forward and backward hemisphere in $p\bar{p}$ collisions

$$A_{\text{FB}}^i \equiv \frac{N(y_t^i \geq 0) - N(y_t^i \leq 0)}{N(y_t^i \geq 0) + N(y_t^i \leq 0)}$$

- The measured asymmetry in the lab frame (CDF 5.3 fb⁻¹)

$$A_{\text{FB}}^{p\bar{p}} = 0.150 \pm 0.050 \text{ stat} \pm 0.024 \text{ syst}$$

- The measurements in the $t\bar{t}$ frame are

$$A_{\text{FB}}^{t\bar{t}} = 0.158 \pm 0.072 \text{ stat} \pm 0.017 \text{ syst} \quad (\text{CDF } 5.3 \text{ fb}^{-1})$$

$$A_{\text{FB}}^{t\bar{t}} = 0.08 \pm 0.04 \text{ stat} \pm 0.01 \text{ syst} \quad (\text{DO } 4.3 \text{ fb}^{-1})$$

THE FORWARD-BACKWARD ASYMMETRY AT THE TEVATRON

The **forward-backward asymmetry** originates from the difference in the top- q and top- \bar{q} cross sections in $p\bar{p}$ collisions

AFNPY ('10) partonic frame	A_{FB} ($\mu_f = m_t$)	A_{FB} ($\mu_f = 400 \text{ GeV}$)
"LO" QCD	$0.074^{+0.007}_{-0.006}$	$0.067^{+0.005}_{-0.005}$
"LO" + NNLL	$0.073^{+0.011}_{-0.007}$	$0.066^{+0.006}_{-0.005}$

- The $A_{FB}^{t\bar{t}} = 0.076^{+0.008}_{-0.005}$ LO QCD ; $A_{FB}^{t\bar{t}} = 0.080^{+0.007}_{-0.005}$ LO QCD + EW
Bernreuther and Si ('10)

- The measurements in the $t\bar{t}$ frame are

$$A_{FB}^{t\bar{t}} = 0.158 \pm 0.072 \text{ stat} \pm 0.017 \text{ syst} \quad (\text{CDF } 5.3 \text{ fb}^{-1})$$

$$A_{FB}^{t\bar{t}} = 0.08 \pm 0.04 \text{ stat} \pm 0.01 \text{ syst} \quad (\text{DO } 4.3 \text{ fb}^{-1})$$

THE FORWARD-BACKWARD ASYMMETRY AT THE TEVA

The forward top-quad collision

in $p\bar{p}$

AFNPY ('11) Laboratory frame	A_{FB} ($\mu_f = m_t$)	A_{FB} ($\mu_f = 400 \text{ GeV}$)
"LO" QCD	$0.048^{+0.005}_{-0.004}$	$0.044^{+0.004}_{-0.003}$
"LO" + NNLL	$0.049^{+0.002}_{-0.002}$	$0.046^{+0.003}_{-0.003}$

The predicted LO asymmetry is $A_{FB}^{p\bar{p}} = 0.051^{+0.007}_{-0.003} \%$

Kühn and Rodrigo ('08), Bernreuther and Si ('10)

- The measured asymmetry in the lab frame (CDF 5.3 fb^{-1})

$$A_{FB}^{p\bar{p}} = 0.150 \pm 0.050 \text{ stat} \pm 0.024 \text{ syst}$$

- The measurements in the $t\bar{t}$ frame are

$$A_{FB}^{t\bar{t}} = 0.158 \pm 0.072 \text{ stat} \pm 0.017 \text{ syst} \quad (\text{CDF } 5.3 \text{ fb}^{-1})$$

$$A_{FB}^{t\bar{t}} = 0.08 \pm 0.04 \text{ stat} \pm 0.01 \text{ syst} \quad (\text{DO } 4.3 \text{ fb}^{-1})$$

THE FORWARD-BACKWARD ASYMMETRY AT THE TEVATRON

The **forward-backward asymmetry** originates from the difference in the top-quark production rates in the forward and backward hemisphere in $p\bar{p}$ collisions

$$A_{\text{FB}}^i \equiv \frac{N(y_t^i \geq 0) - N(y_t^i \leq 0)}{N(y_t^i \geq 0) + N(y_t^i \leq 0)}$$

- The measured asymmetry in the lab frame (CDF 5.3 fb⁻¹)

$$A_{\text{FB}}^{p\bar{p}} = 0.150 \pm 0.050 \text{ stat} \pm 0.024 \text{ syst}$$

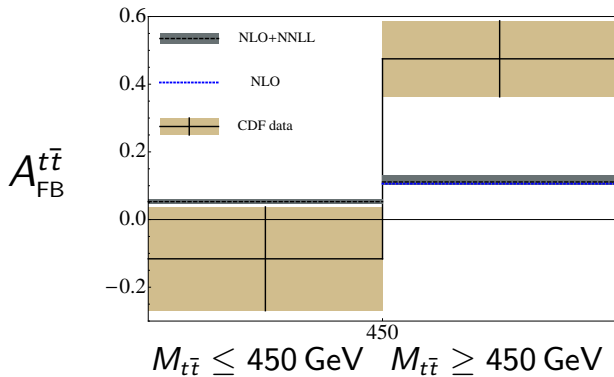
- The measurements in the $t\bar{t}$ frame are

$$A_{\text{FB}}^{t\bar{t}} = 0.158 \pm 0.072 \text{ stat} \pm 0.017 \text{ syst} \quad (\text{CDF } 5.3 \text{ fb}^{-1})$$

$$A_{\text{FB}}^{t\bar{t}} = 0.08 \pm 0.04 \text{ stat} \pm 0.01 \text{ syst} \quad (\text{DO } 4.3 \text{ fb}^{-1})$$

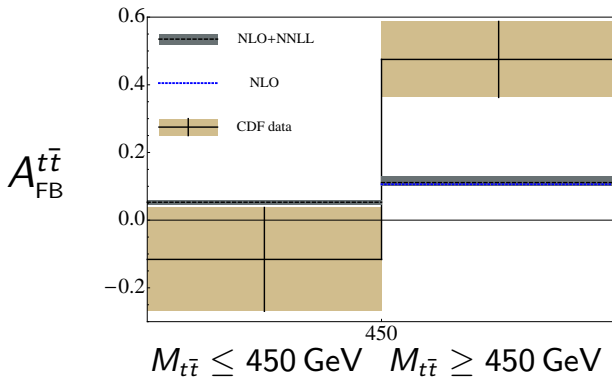
Theory and experiment agree at about 2σ

INVARIANT MASS DEPENDENT ASYMMETRY



$$A_{\text{FB}}^{t\bar{t}}(m_1, m_2) = \frac{\int_{m_1}^{m_2} dM_{t\bar{t}} \left[\left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_F - \left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_B \right]}{\int_{m_1}^{m_2} dM_{t\bar{t}} \left[\left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_F + \left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_B \right]}$$

INVARIANT MASS DEPENDENT ASYMMETRY



- Measured in the $t\bar{t}$ frame [CDF arXiv:1101.0034](#)
- $M_{t\bar{t}} \leq 450$ GeV: compatible with LO within 1σ
- $M_{t\bar{t}} \geq 450$ GeV: larger than LO by 3.4σ
- Higher order corrections are not the answer ([V. Ahrens et al](#) preliminary)

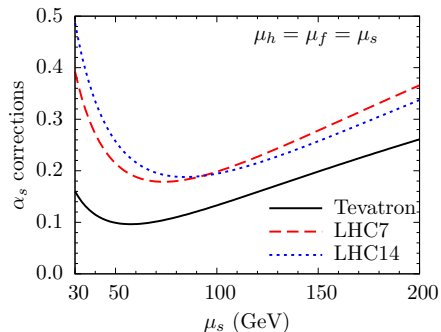
SUMMARY & CONCLUSIONS

- Top pairs at the LHC \implies need to go **beyond NLO+NLL** accuracy
- **NNLL Resummed / Approximate NNLO results** for double differential distributions are available in two different threshold limits: 1PI_{SCET} and PIM_{SCET} . The two approaches kinematic schemes provide consistent results
- **Phenomenology**: the NNLL resummed / approximate NNLO results used to obtain predictions for
 - ▶ **Total Cross Section**
 - ▶ **Forward-Backward Asymmetry**
 - ▶ **FB Asymmetry for $M_{t\bar{t}} \geq 450$ GeV**

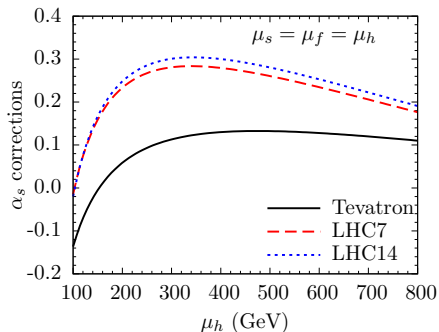
Backup Slides

SCALES IN THE RESUMMED CROSS SECTION

1PI kinematics, total cross section

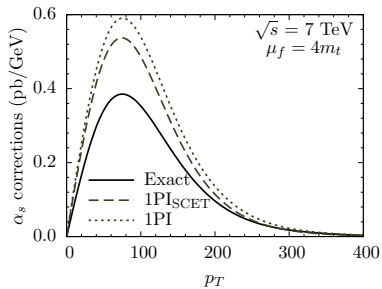
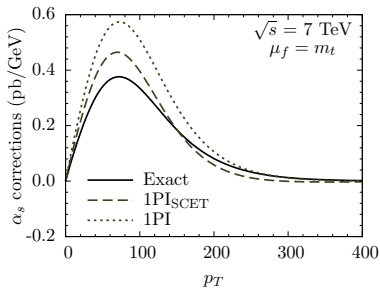
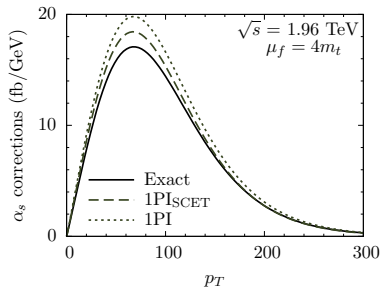
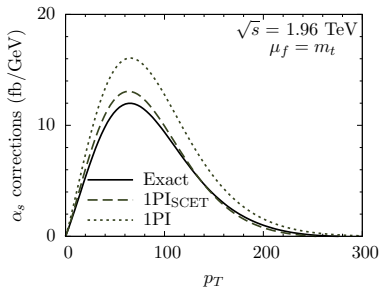


NLO corrections (%) from soft
function: $\mu_s \sim 50 - 90$ GeV

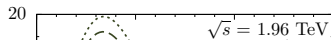
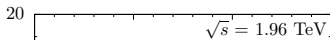


NLO corrections (%) from hard
function: $\mu_h \sim 400$ GeV

P_T DISTRIBUTION AT THE LHC

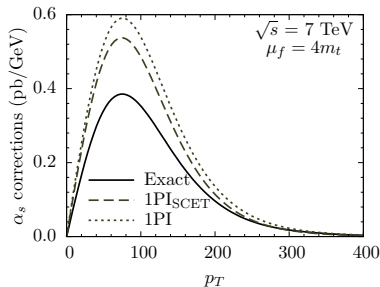
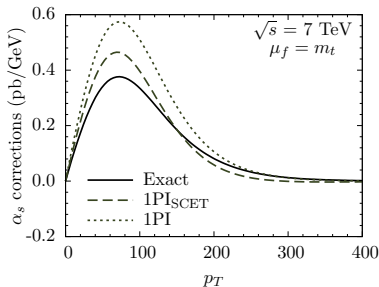


P_T DISTRIBUTION AT THE LHC



1PI_{SCET} does not work well for intermediate values of p_T at the LHC:

$m_t^2 \gg s_4$ is not valid in that region



TOTAL CROSS SECTION TABLES

POLE MASS	Tevatron		LHC7	
	MSTW	CTEQ	MSTW	CTEQ
LO	$6.66^{+2.95+(0.34)}_{-1.87-(0.27)}$	$5.45^{+2.16+0.33(0.29)}_{-1.42-0.27(0.24)}$	$122^{+49+(6)}_{-32-(7)}$	$100^{+35+9(7)}_{-24-8(7)}$
NLO	$6.72^{+0.41+0.47(0.37)}_{-0.76-0.45(0.24)}$	$6.77^{+0.40+0.50(0.43)}_{-0.74-0.40(0.34)}$	$159^{+20+14(8)}_{-21-13(9)}$	$148^{+18+13(11)}_{-19-12(10)}$
NNLO app.	$6.63^{+0.07+0.63(0.33)}_{-0.41-0.48(0.25)}$	$6.91^{+0.09+0.53(0.46)}_{-0.44-0.43(0.36)}$	$155^{+8+14(8)}_{-9-14(9)}$	$153^{+8+13(11)}_{-8-12(10)}$

Total cross sections in pb for $m_t = 173.1$ GeV with MSTW2008 and CTEQ6.6 PDFs. The first error results from the perturbative uncertainty from both scale variations and the difference between PIM and IPI kinematics, the second one accounts for the combined PDFs+ α_s uncertainty. The numbers in parenthesis show the PDF uncertainty only.

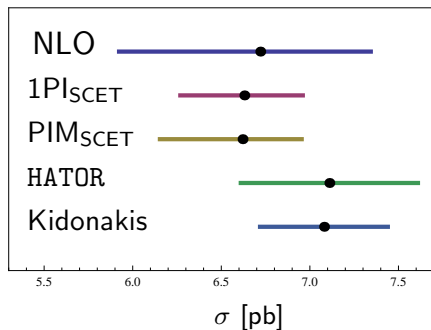
MS MASS	Tevatron		LHC7	
	MSTW	CTEQ	MSTW	CTEQ
LO	$8.82^{+3.91+(0.44)}_{-2.48-(0.35)}$	$7.24^{+2.86+0.46(0.40)}_{-1.89-0.38(0.32)}$	$160^{+64+(8)}_{-42-(9)}$	$131^{+45+11(9)}_{-31-10(8)}$
NLO	$7.33^{+0.11+0.50(0.40)}_{-0.49-0.47(0.25)}$	$7.39^{+0.10+0.57(0.50)}_{-0.48-0.45(0.39)}$	$179^{+11+15(10)}_{-19-14(10)}$	$167^{+10+15(12)}_{-17-13(11)}$
NNLO app.	$6.64^{+0.11+0.58(0.33)}_{-0.40-0.43(0.23)}$	$6.92^{+0.12+0.52(0.46)}_{-0.43-0.42(0.37)}$	$157^{+9+13(8)}_{-9-13(9)}$	$154^{+9+13(11)}_{-9-12(10)}$

Total cross sections in pb in the $\overline{\text{MS}}$ scheme, for $\overline{m}(\overline{m}) = 164.1$ GeV. The first error results from the perturbative uncertainty from both scale variations and the difference between PIM and IPI kinematics, the second one accounts for the combined PDFs+ α_s uncertainty. The numbers in parenthesis show the PDF uncertainty only.

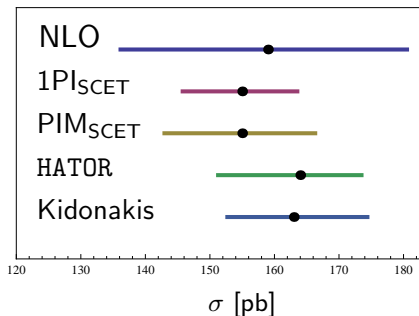
COMPARISONS

$m_t = 173.1$ GeV, $mt/2 < \mu_f = \mu_r < 2mt$, MSTW2008 90% CL
PDF and scale uncertainties added in quadrature

Tevatron



LHC 7 TeV



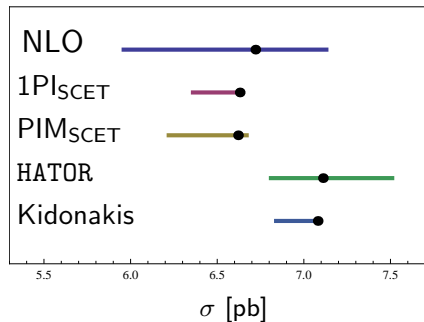
- 1PIS_{SCET} and PIM_{SCET} NNLO approx. calculations [Ahrens et al '10, '11](#)
- Kidonakis: NNLO approx 1PI formulas ($m_t = 173$ GeV) [Kidonakis '10](#)
- HATOR: production threshold formulas [Aliev et al '10](#)

COMPARISONS

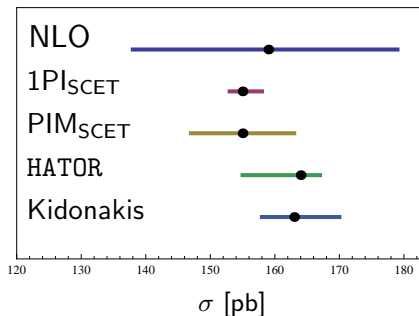
$m_t = 173.1$ GeV, $mt/2 < \mu_f = \mu_r < 2mt$, MSTW2008 90% CL

Only scale uncertainty

Tevatron



LHC 7 TeV



- 1PI_{SCET} and PIM_{SCET} NNLO approx. calculations [Ahrens et al '10, '11](#)
- Kidonakis: NNLO approx 1PI formulas ($m_t = 173$ GeV) [Kidonakis '10](#)
- HATOR: production threshold formulas [Aliev et al '10](#)