

NLO jet production in k_T -factorisation

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Motivation

- We want to understand the strong force (QCD)
 - per se as one of the fundamental forces in nature
 - as background at collider experiments
- soft energy scale \rightsquigarrow confinement \rightsquigarrow no free quarks/ gluons observable, but **jets** of hadronized particles
- hard energy scale \rightsquigarrow asymptotic freedom \rightsquigarrow access via perturbative QCD
- factorization to disentangle soft from hard physics

Motivation - need for BFKL resummation

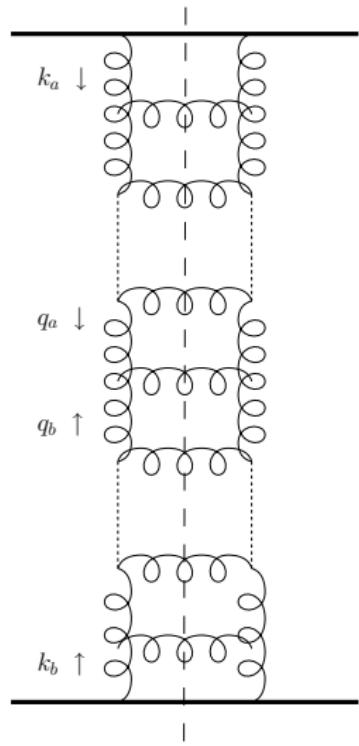
perturbative QCD = expansion in coupling α_s

- large but ordered scales (e.g. $s \gg |t| \gg \Lambda_{\text{QCD}}^2$) \rightsquigarrow large logs ($\log s/t$) for each additional emission in multi Regge kinematics \rightsquigarrow compensating smallness of α_s
- need to resum terms $\sim (\alpha_s \log s/t)^n$
 \rightsquigarrow LO **Balitsky-Fadin-Kuraev-Lipatov** equation ['75-'78]
- resummation of terms $\sim \alpha_s(\alpha_s \log s/t)^n$
 \rightsquigarrow NLO BFKL equation ['98]

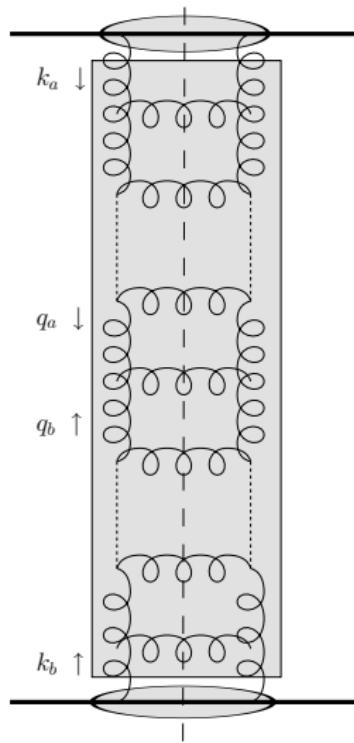
Outline

- ① Motivation and Introduction ✓
- ② Jet production vertex at central rapidity
 - Jet production at LO
 $\gamma^*\gamma^*$, pp , unintegrated gluon density
 - Jet production at NLO
 $\gamma^*\gamma^*$, pp , unintegrated gluon density
- ③ Summary

Total cross section at LO BFKL



Total cross section at LO BFKL

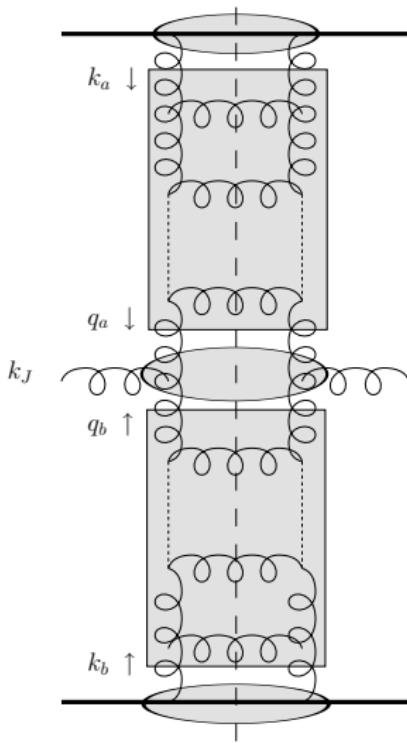


$$\sigma(s) = \int \frac{d^2 \mathbf{k}_a}{2\pi \mathbf{k}_a^2} \int \frac{d^2 \mathbf{k}_b}{2\pi \mathbf{k}_b^2} \Phi_A(\mathbf{k}_a) \Phi_B(\mathbf{k}_b) \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b).$$

- with impact factors Φ
- Green's function f_ω obeys BFKL equation

$$\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b) = \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) + \int d^2 \mathbf{k} \mathcal{K}(\mathbf{k}_a, \mathbf{k}) f_\omega(\mathbf{k}, \mathbf{k}_b)$$

Jet production at LO BFKL



$$\frac{d\sigma}{d^2\mathbf{k}_{Jet} dy_{Jet}} = \int \frac{d^2\mathbf{k}_a}{2\pi\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{2\pi\mathbf{k}_b^2} \Phi_A(\mathbf{k}_a) \Phi_B(\mathbf{k}_b)$$

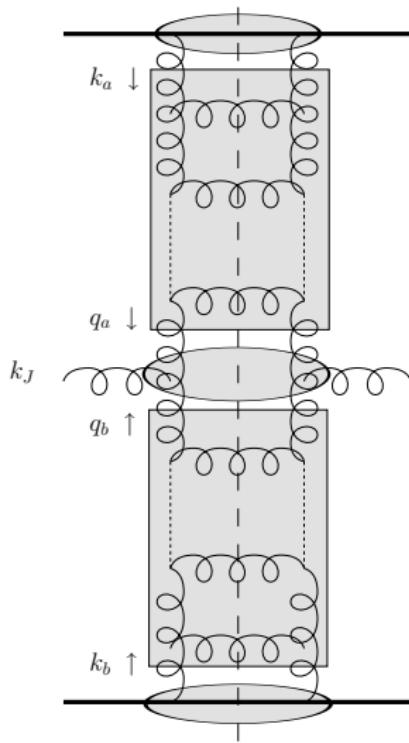
$$\times \int d^2\mathbf{q}_a \int d^2\mathbf{q}_b \int \frac{d\omega}{2\pi i} \left(\frac{s_{AJ}}{s_0} \right)^\omega f_\omega(\mathbf{k}_a, \mathbf{q}_a)$$

$$\times \mathcal{V}(\mathbf{q}_a, \mathbf{q}_b; \mathbf{k}_{Jet}, y_{Jet})$$

$$\times \int \frac{d\omega'}{2\pi i} \left(\frac{s_{BJ}}{s'_0} \right)^{\omega'} f_{\omega'}(-\mathbf{q}_b, -\mathbf{k}_b)$$

with the LO emission vertex

$$\mathcal{V} = \mathcal{K}_{\text{real}}^{(LO)}(\mathbf{q}_a, -\mathbf{q}_b) \delta^{(2)}(\mathbf{q}_a + \mathbf{q}_b - \mathbf{k}_{Jet}).$$

$\gamma^*\gamma^*$ scattering (LO)

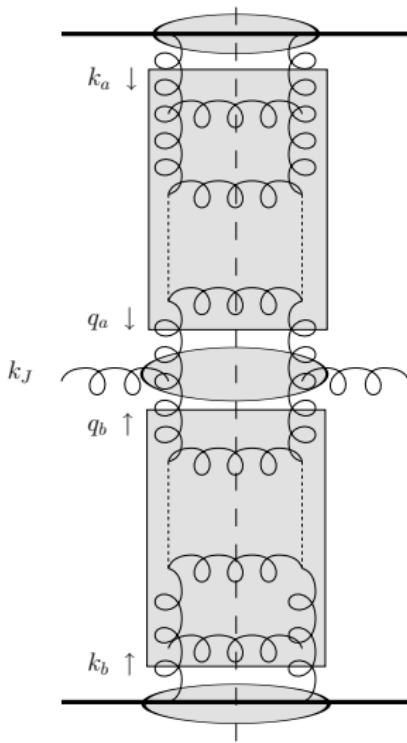
- impact factors and jet provide hard scale as well
- symmetric situation, choose s_0 as

$$s_0 = |\mathbf{k}_a| |\mathbf{k}_{\text{Jet}}|, \quad s'_0 = |\mathbf{k}_{\text{Jet}}| |\mathbf{k}_b|$$

- natural language of rapidities:

$$\left(\frac{s_{AJ}}{s_0} \right)^\omega = e^{\omega(y_A - y_{\text{Jet}})}$$

pp scattering (LO)



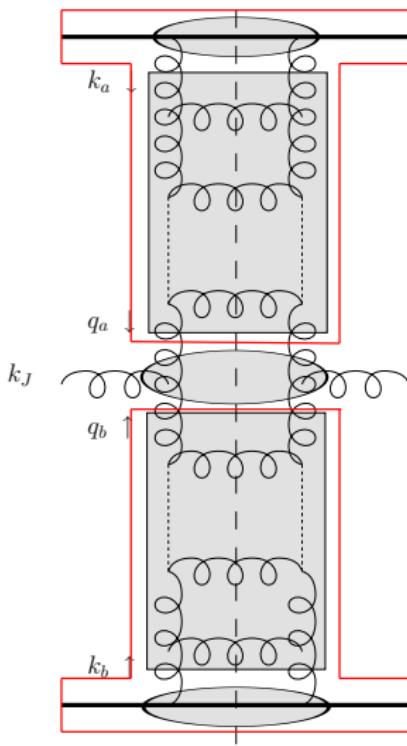
- only jet provides hard scale
- asymmetric situation, choose s_0 as

$$s_0 = \mathbf{k}_{Jet}^2, \quad s'_0 = \mathbf{k}_{Jet}^2$$

- natural language of longitudinal momentum fractions

$$\left(\frac{s_{AJ}}{s_0}\right)^\omega = \left(\frac{1}{x_1}\right)^\omega$$

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$$\left(\frac{s_{AJ}}{s_0}\right)^\omega = \left(\frac{1}{x_1}\right)^\omega$$

Unintegrated gluon density

define the unintegrated gluon density

$$g(x, \mathbf{k}) = \int \frac{d^2 \mathbf{q}}{2\pi \mathbf{q}^2} \Phi_P(\mathbf{q}) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} x^{-\omega} f_\omega(\mathbf{q}, \mathbf{k})$$

which obeys the evolution equation

$$\frac{\partial g(x, \mathbf{q}_a)}{\partial \ln 1/x} = \int d^2 \mathbf{q} \mathcal{K}(\mathbf{q}_a, \mathbf{q}) g(x, \mathbf{q})$$

Then cross section can be written in k_T factorization

$$\begin{aligned} \frac{d\sigma}{d^2 \mathbf{k}_{\text{Jet}} dy_{\text{Jet}}} &= \\ &\int d^2 \mathbf{q}_a \int d^2 \mathbf{q}_b \textcolor{red}{g(x_1, \mathbf{q}_a) g(x_2, \mathbf{q}_b)} \mathcal{V}(\mathbf{q}_a, \mathbf{q}_b; \mathbf{k}_{\text{Jet}}, y_{\text{Jet}}) \end{aligned}$$

Changes at NLO BFKL

Q: Can we just replace the LO expressions for impact factors, kernel and Green's function by their NLO counterparts?

Changes at NLO BFKL

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A: No!

- real Kernel $\mathcal{K}_{\text{real}}$ contains at NLO two particle production
 - jet algorithm
 - separation MRK \leftrightarrow QMRK \rightsquigarrow scale s_Λ
- energy scale s_0 is now a relevant parameter

Jet definition

- remember: at LO $\mathcal{K}_{\text{real}} \sim \begin{array}{c} | \\ - \end{array} \sim \mathcal{V}$
- at NLO $\mathcal{K}_{\text{real}} \sim \begin{array}{c} \curvearrowleft \\ + \end{array} \int \begin{array}{c} \curvearrowleft \\ \times \end{array}$
- for $\begin{array}{c} \curvearrowleft \\ \times \end{array}$ two possibilities:
 - both together form a jet
 - one forms the jet, other one unresolved
- define distance in rapidity-azimuthal angle space
 $R_{12} = \sqrt{(y_1 - y_2)^2 + (\phi_1 - \phi_2)^2}$
 - $\theta(R_0 - R_{12}) : \begin{array}{c} \curvearrowleft \\ \square \end{array}$
 - $\theta(R_{12} - R_0) : \begin{array}{c} \curvearrowleft \\ \times \end{array}$
- open integration to extract jet

$$\mathcal{V} \sim \begin{array}{c} \curvearrowleft \\ + \end{array} \int \begin{array}{c} \curvearrowleft \\ \square \end{array} + \int \begin{array}{c} \curvearrowleft \\ \times \end{array}$$

Subtraction term

- real and virtual parts with different $x_{1,2}$ configurations \rightsquigarrow different $g(x_1, q_a)g(x_2, q_b)$ \rightsquigarrow cancellation of divergences?

$$\mathcal{V} = \text{ } \triangleright \text{ } + \int \text{ } \triangleleft \text{ } + \int \text{ } \triangleleft^x$$

Subtraction term

- real and virtual parts with different $x_{1,2}$ configurations \rightsquigarrow
different $g(x_1, q_a)g(x_2, q_b)$ \rightsquigarrow cancellation of divergences?

$$\mathcal{V} = \left(\textcolor{brown}{\triangleright} + \int \textcolor{red}{\llcorner} \right) + \int \left(\textcolor{brown}{\llbracket} - \textcolor{red}{\llbracket} \right) + \int \left(\textcolor{brown}{\llcorner^*} - \textcolor{red}{\llbracket^*} \right)$$

- add singular part of 2 particle production (in x configuration of virtual part) times $0 = 1 - \theta(R_0 - R_{12}) - \theta(R_{12} - R_0)$
- first bracket: analytical cancellation of divergences
- second and third bracket: numerical cancellation of divergences

$\gamma^*\gamma^*$ scattering (NLO)

- NLO calculation of the kernel was performed in framework with hard scale impact factors
- \rightsquigarrow can keep (in principle) LO formula with NLO impact factors, Green's functions, jet vertex

pp scattering (NLO)

proton: soft scale jet: hard scale

- in asymmetric situation: necessity of scale change
 $s_0 = |\mathbf{k}_a| |\mathbf{k}_{Jet}| \rightarrow s_0 = \mathbf{k}_{Jet}^2$
- symmetric change $s_0 = |\mathbf{k}_a| |\mathbf{k}_{Jet}| \rightarrow s_0 = f_1(|\mathbf{k}_a|)f_2(|\mathbf{k}_{Jet}|)$
could be compensated by change in only the impact factors
and the vertex
- asymmetric change effects complete evolution; now from a
soft scale to a hard scale

pp scattering - consequences of scale change

- modified Kernel for evolution of Green's function

$$\tilde{\mathcal{K}}(\mathbf{q}_1, \mathbf{q}_2) = \mathcal{K}(\mathbf{q}_1, \mathbf{q}_2) - \frac{1}{2} \int d^2\mathbf{q} \mathcal{K}^{(LO)}(\mathbf{q}_1, \mathbf{q}) \mathcal{K}^{(LO)}(\mathbf{q}, \mathbf{q}_2) \ln \frac{\mathbf{q}^2}{\mathbf{q}_2^2}$$

$$\omega \tilde{f}_\omega(\mathbf{k}_a, \mathbf{q}_a) = \delta^{(2)}(\mathbf{k}_a - \mathbf{q}_a) + \int d^2\mathbf{q} \tilde{\mathcal{K}}(\mathbf{k}_a, \mathbf{q}) \tilde{f}_\omega(\mathbf{q}, \mathbf{q}_a)$$

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- modified proton impact factor

$$\tilde{\Phi}(\mathbf{k}_a) = \Phi(\mathbf{k}_a) - \frac{1}{2} \mathbf{k}_a^2 \int d^2\mathbf{q} \frac{\Phi^{(LO)}(\mathbf{q})}{\mathbf{q}^2} \mathcal{K}^{(LO)}(\mathbf{q}, \mathbf{k}_a) \ln \frac{\mathbf{q}^2}{\mathbf{k}_a^2}.$$

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- \Rightarrow new NLO unintegrated gluon distribution

$$g(x, \mathbf{k}) = \int d^2\mathbf{q} \frac{\tilde{\Phi}_P(\mathbf{q})}{2\pi\mathbf{q}^2} \int \frac{d\omega}{2\pi i} \tilde{f}_\omega(\mathbf{k}, \mathbf{q}) x^{-\omega}$$

$$\frac{\partial g(x, \mathbf{q}_a)}{\partial \ln 1/x} = \int d^2\mathbf{q} \tilde{\mathcal{K}}(\mathbf{q}_a, \mathbf{q}) g(x, \mathbf{q}).$$

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- modified vertex

$$\tilde{\mathcal{V}}(\mathbf{q}_a, \mathbf{q}_b) = \mathcal{V}(\mathbf{q}_a, \mathbf{q}_b)$$

$$\begin{aligned} & - \frac{1}{2} \int d^2\mathbf{q} \mathcal{K}^{(LO)}(\mathbf{q}_a, \mathbf{q}) \mathcal{V}^{(LO)}(\mathbf{q}, \mathbf{q}_b) \ln \frac{\mathbf{q}^2}{(\mathbf{q} - \mathbf{q}_b)^2} \\ & - \frac{1}{2} \int d^2\mathbf{q} \mathcal{V}^{(LO)}(\mathbf{q}_a, \mathbf{q}) \mathcal{K}^{(LO)}(\mathbf{q}, \mathbf{q}_b) \ln \frac{\mathbf{q}^2}{(\mathbf{q}_a - \mathbf{q})^2}. \end{aligned}$$

Summary

We constructed a jet vertex

- in NLO k_T factorization
- explicitly free of divergences
- implications for definition of uPDFs at NLO
- kept track of dependence on all scales involved