To the theory of high-energy elastic nucleon collisions

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Abstract
The commonly used West and Yennie integral formula for the relative phase between the Coulomb and elastic hadronic amplitudes requires for the phase of the elastic hadronic amplitude to be constant at all kinematically allowed values of t. More general interference formula based on the eikonal model approach does not exhibit such limitation. The corresponding differences will be demonstrated and some predictions of different phenomenological models for elastic pp scattering at energy of 14 TeV at the LHC will be given. Special attention will be devoted to determination of luminosity from elastic scattering data; it will be shown that the systematic error might reach till 5 % if the luminosity is derived from the values in the center of the interference region with the help of West and Yennie formula.

1 Limited validity of West and Yennie integral formula

It has been shown in our earlier papers (see [1] and [2]) that the integral formula of West and Yennie [3] for the real relative phase between Coulomb and hadronic amplitudes

$$\alpha\Phi(s,t) = \mp\alpha\Big[\ln\left(\frac{-t}{s}\right) - \int_{-4p^2}^{0}\frac{d\tau}{|t-\tau|}\left(1-\frac{F^N(s,\tau)}{F^N(s,t)}\right)\Big] \tag{1}$$

requires for the hadronic amplitude $F^N(s,t)$ to have the constant phase at any kinematically allowed value of t; s being the value of the total CMS energy, t the four momentum transfer squared, p the value of the CMS momentum and $\alpha = 1/137.036$ the fine structure constant. The upper (lower) sign corresponds to the pp ($\bar{\rm p}$p) scattering. It follows in such a case

$$\int_{-4p^2}^{0}\frac{d\tau}{|t-\tau|}\Im\left(\frac{F^N(s,\tau)}{F^N(s,t)}\right) \;\equiv\; 0 \tag{2}$$

and further

$$\int\limits_{-4p^2}^{0}\frac{d\tau}{|t-\tau|}[\Re F^N(s,t)\Im F^N(s,\tau) - \Re F^N(s,\tau)\Im F^N(s,t)] \;\equiv\; 0. \tag{3}$$

Introducing then

$$F^N(s,t) \;=\; i|F^N(s,t)|e^{-i\zeta^N(s,t)}, \tag{4}$$

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it is possible to write further

$$\int_{-4p^2}^{t} d\tau \; \frac{\sin[\zeta^N(s,t)-\zeta^N(s,\tau)]}{t-\tau} \; |F^N(s,\tau)| - \int_{t}^{0} d\tau \; \frac{\sin[\zeta^N(s,t)-\zeta^N(s,\tau)]}{t-\tau} \; |F^N(s,\tau)| \;\equiv\; 0 \tag{5}$$

for any $t \in [-4p^2, 0]$. Both the integrals in Eq. (5) are proper integrals provided the first derivative of the hadronic phase $[\zeta^N(s,t)]'$ according to t variable is finite. It is evident that Eq. (5) is fulfilled if the phase is t independent, i.e., if

$$\zeta^N(s,t) \;=\; \zeta^N(s,\tau) \;\equiv\; \zeta^N(s). \tag{6}$$

It has been shown in Ref. [2] that Eq. (6) represents the unique solution of Eq. (5), if the relative phase between the Coul. and hadr. amplitudes is to be a real quantity as commonly assumed.

The problem of the t independence of hadronic phase was mentioned for the first time in Ref. [4]. However, this independence was used as an assumption only. Adding the other important assumption concerning purely exponential t dependence of the modulus of hadronic amplitude *in the whole kinematically allowed region of* t it was possible to perform analytically the integration in Eq. (1) (accepting also other approximations valid at asymptotic energies only - for detail see Ref. [1]). For the total elastic scattering amplitude the simplified West and Yennie formula

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) e^{i\alpha\Phi} + \frac{\sigma_{tot}}{4\pi} p\sqrt{s}(\rho + i) e^{Bt/2} \tag{7}$$

was then obtained. Here $f_1(t)$ and $f_2(t)$ are the dipole form factors (added by hand only), B is the constant diffractive slope, σ_{tot} the value of the total cross section and the constant $\rho = \Re F^N(s,0)/\Im F^N(s,0)$. These three quantities may depend on s only. The relative phase $\alpha\Phi(s,t)$ exhibits then logarithmic t dependence

$$\alpha\Phi(s,t) = \mp\alpha\left[\ln\left(\frac{-Bt}{2}\right) + \gamma\right], \tag{8}$$

where $\gamma = 0.577215$ is Euler's constant.

2 General eikonal model approach

The contemporary experimental data as well as the phenomenological models of high-energy elastic nucleon scattering show, however, convincingly that the quantity ρ cannot be t independent. Consequently, the West and Yennie approach [3] is not a convenient tool for description of interference between the Coulomb and elastic hadronic interactions of charged nucleons. However, the approach based on the eikonal model removes such troubles. The general formula for the total elastic scattering amplitude proposed in Ref. [5] may be valid at any s and t; it may be written as

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) + F^N(s,t)\left[1 \mp i\alpha G(s,t)\right], \tag{9}$$

where

$$G(s,t) = \int_{-4p^2}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}\left[f_1(t') f_2(t')\right] + \frac{1}{2\pi}\left[\frac{F^N(s,t')}{F^N(s,t)} - 1\right] I(t,t') \right\}, \tag{10}$$

and

$$I(t,t') = \int_0^{2\pi} d\Phi'' \frac{f_1(t'')f_2(t'')}{t''}, \quad t'' = t + t' + 2\sqrt{tt'}\cos\Phi''. \tag{11}$$

Instead of the t independent quantities B and ρ, it is now necessary to consider t dependent quantities being defined as

$$B(s,t) = \frac{d}{dt}\left[\ln\frac{d\sigma^N}{dt}\right] = \frac{2}{|F^N(s,t)|}\frac{d}{dt}|F^N(s,t)|, \qquad \rho(s,t) = \frac{\Re F^N(s,t)}{\Im F^N(s,t)}. \tag{12}$$

The total cross section is then given with the help of the optical theorem as

$$\sigma_{tot}(s) = \frac{4\pi}{p\sqrt{s}}\Im F^N(s,t=0). \tag{13}$$

3 Experimental data and West and Yennie formula

It is the differential cross section that is determined in corresponding experiments. In our normalization it equals

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2}|F^{C+N}(s,t)|^2. \tag{14}$$

In the past practically in all actual experiments the simplified West and Yennie elastic amplitude defined by Eqs. (7) - (8) has been used for the analysis of data at $|t| \leq 0.01$ GeV^2, in spite of the fact that the theoretical assumptions under which the amplitude was derived are not fulfilled at all kinematically allowed values of t but only in its narrow region in forward direction. Generally, some important discrepancies exist. The analysis of corresponding behavior has been performed in Ref. [6] where the general formula (Eqs. (9) - (11)) has been applied to data for pp scattering at energy of 53 GeV under different limitations of some free parameters specifying the modulus and the phase of the hadronic amplitude (as used in Ref. [5] - Eqs. (40) and (42)). The corresponding results were derived in Ref. [6]) and are represented in Fig. 1. First, only the phase has been fixed by putting $\tan\zeta^N(t) = \rho = 0.077$ according to the earlier fit [7] (based on West and Yennie formula), while the modulus parameters have been fitted. This fit has been compared to the case when only ρ has been assumed to be constant, but free; the optimum in such a case has been obtained with $\rho = 0.065$. In both the cases the experimental data have been represented by the square of the modulus fitted in the whole measured interval; see the solid line in Fig. 1. In addition to, in the other case the modulus formula has been limited to a simple exponential form (with two free parameters). Fundamental differences from experimental data exist now; only unsubstantial difference being obtained when ρ has been fixed ($\rho = 0.077$) and when it has been left free and fitted to $\rho = 0.021$; see dotted and dashed lines in Fig. 1. It is evident that the assumption of constant diffractive slope B is in strong contradiction to the experimental data. The curves corresponding to the fits with constant and t dependent quantities ρ are nearly the same, if the modulus is fitted and only weak dependence of ρ on t is allowed. Better χ^2 quantity is obtained if strong dependence on t is allowed.

model	σ_{tot} [mb]	σ_{el} [mb]	$B(0)$ [GeV^{-2}]	ρ	$\sqrt{<b^2_{tot}>}$ [fm]	$\sqrt{<b^2_{el}>}$ [fm]	$\sqrt{<b^2_{inel}>}$ [fm]
Islam	109.17	21.99	31.43	0.123	1.552	1.048	1.659
Petrov et al.2P	94.97	23.94	19.34	0.097	1.227	0.875	1.324
Petrov et al.3P	108.22	29.70	20.53	0.111	1.263	0.901	1.375
Bourrely et al.	103.64	28.51	20.19	0.121	1.249	0.876	1.399
Block et al.	106.74	30.66	19.35	0.114	1.223	0.883	1.336

Table 1: The values of basic parameters predicted by different models

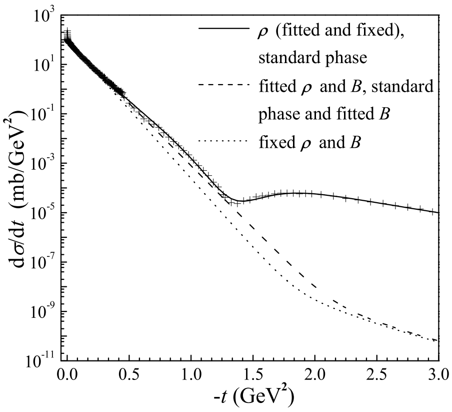

Fig. 1: $\frac{d\sigma}{dt}$ for pp scattering at 53 GeV; all graphs correspond to constant ρ.

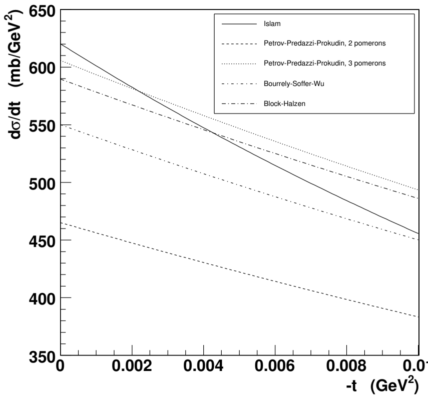

Fig. 2: $\frac{d\sigma}{dt}$ predictions at low $|t|$ for pp scattering at 14 TeV corresponding to different models

4 Model predictions for pp elastic scattering at the LHC

In connection with the TOTEM experiment that will investigate the elastic pp scattering at energy of 14 TeV [8] we have studied the predictions of four models proposed by the following authors: Islam, Luddy and Prokhudin [9], Petrov, Predazzi and Prokhudin (with hadronic amplitude corresponding to the exchange of two, resp. three pomerons - labelled as 2P, resp. 3P) [10], Bourrely, Soffer and Wu [11] and Block, Gregores, Halzen and Pancheri [12]. All the given models have contained some free s dependent parameters that have been established in these papers by fitting the experimental data on pp differential cross sections at several lower energies. Using these fitted values we have established the dynamical quantities: the total cross section, momentum transfer distribution $\frac{d\sigma}{dt}$, the t dependent diffractive slope $B(t)$ and the t dependent quantity $\rho(t)$ at higher energy values. The values of σ_{tot}, σ_{el}, $B(0)$ and $\rho(0)$ for 14 TeV can be found in Table 1. The corresponding model predictions are shown in Figs. 2 - 5. It is evident that the predictions of all the models differ rather significantly. Fig. 2 shows different predictions for values of the differential cross sections at small $|t|$ values. Great differences concern the values for total cross sections that are in direct relation to the values of differential cross section at $t = 0$; they run from 95 mb to 110 mb and differ rather significantly from the value predicted by COMPETE collaboration [13] $\sigma_{tot} = 111.5 \pm 1.2 {}^{+4.1}_{-2.1}$ mb which has been determined by extrapolation of the fitted lower energy data with the help of dispersion relations technique. The predictions of $\frac{d\sigma}{dt}$ values for higher values of $|t|$ are shown in Fig. 3. Let us point out especially the second diffractive dip demonstrated by Bourrely, Soffer and Wu momentum transfer distribution. The different predictions for t dependence of the diffractive slopes $B(t)$ are shown in Fig. 4. Fig. 5 exhibits the t dependence of the quantity $\rho(t)$. Table 1 contains also the values of the root-mean-squares calculated for each of the analyzed models with the formulas published in Ref. [14].

5 Luminosity estimation on the basis of pp elastic scattering at the LHC

The accurate determination of the elastic amplitude is very important in the case when the luminosity $\mathcal{L}$ is to be calibrated on the basis of elastic process; it holds that [15]

$$\frac{1}{\mathcal{L}}\frac{dN_{el}}{dt} = \frac{\pi}{sp^2}|F^{C+N}(s,t)|^2, \tag{15}$$

where $\frac{dN_{el}}{dt}$ is the counting rate established experimentally at the given t. The so called Coulomb calibration in the region of smallest $|t|$ where the Coulomb amplitude is dominant (reaching nearly 100 %) can be hardly realized due to technical limitations. The approach allowing to avoid corresponding difficulties may be based on Eq. (15), when the elastic counting rate can be, in principle, measured at any t which can be reached, and the total elastic scattering amplitude $F^{C+N}(s,t)$ may be determined with required accuracy at any $|t|$, too. Then the luminosity $\mathcal{L}$ could be determined using Eq. (15). However, in this case it is very important which formula

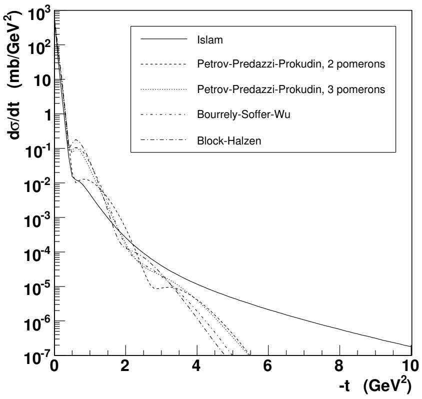

Fig. 3: $\frac{d\sigma}{dt}$ predictions for pp scattering at 14 TeV corresponding to different models

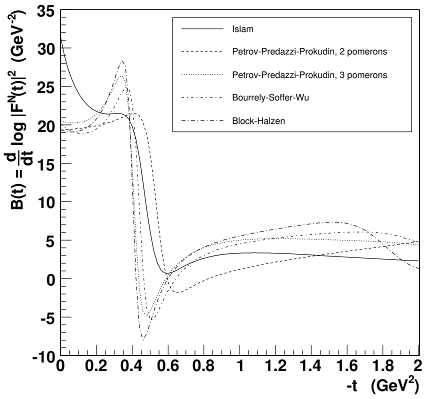

Fig. 4: t dependence of the slope predictions for pp scattering at 14 TeV for different models

for the total elastic amplitude $F^{C+N}(s,t)$ is made use of. In the following we will demonstrate possible differences which can be obtained at different t values in comparison with the commonly used West and Yennie approach. For this reason let us calculate the quantity

$$R(t) = \frac{|F^{C+N}_{eik}(s,t)|^2 - |F^{C+N}_{WY}(s,t)|^2}{|F^{C+N}_{eik}(s,t)|^2}, \tag{16}$$

where $F^{C+N}_{eik}(s,t)$ is the total elastic scattering eikonal model amplitude calculated for an investigated hadronic amplitude $F^N(s,t)$ while $F^{C+N}_{WY}(s,t)$ is the West and Yennie total elastic scattering amplitude calculated for the same hadronic amplitude. The calculation was performed

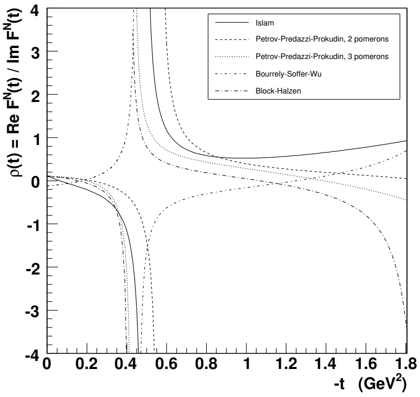

Fig. 5: $\rho(t)$ predictions for pp scattering at 14 TeV corresponding to different models

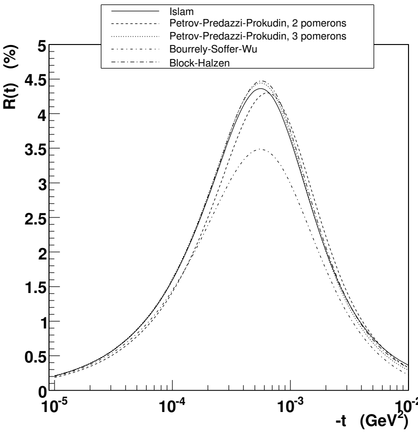

Fig. 6: $R(t)$ quantity predictions for pp scattering at 14 TeV corresponding to different models

for pp elastic scattering at the LHC energy of 14 TeV; in the former case Eqs. (9) - (11) were used, in the latter one Eqs. (7) - (8). The t dependence of $R(t)$ for different models is shown in Fig. 6. The maximum values lie approximately at $t = -0.006$ GeV^2, showing that the differences of physically consistent eikonal models from West and Yennie formula may be almost 5 $\%$. In the preceding case only the models with a weak dependence of $\rho(t)$ on t have been considered. Yet larger difference may be obtained when the cases with weak and strong dependences will be compared; i.e., the cases for central and peripheral distribution of elastic hadron scattering - see the analysis of pp scattering at ISR energies [14]. It means that the luminosity determined for the central and peripheral distributions of elastic pp scattering at LHC energy of 14 TeV may be burdened by a non-negligible mutual systematic error.

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