

PIER Graduate Week 2014  
6 – 9 Oct 2014

# Essentials of X-ray Physics

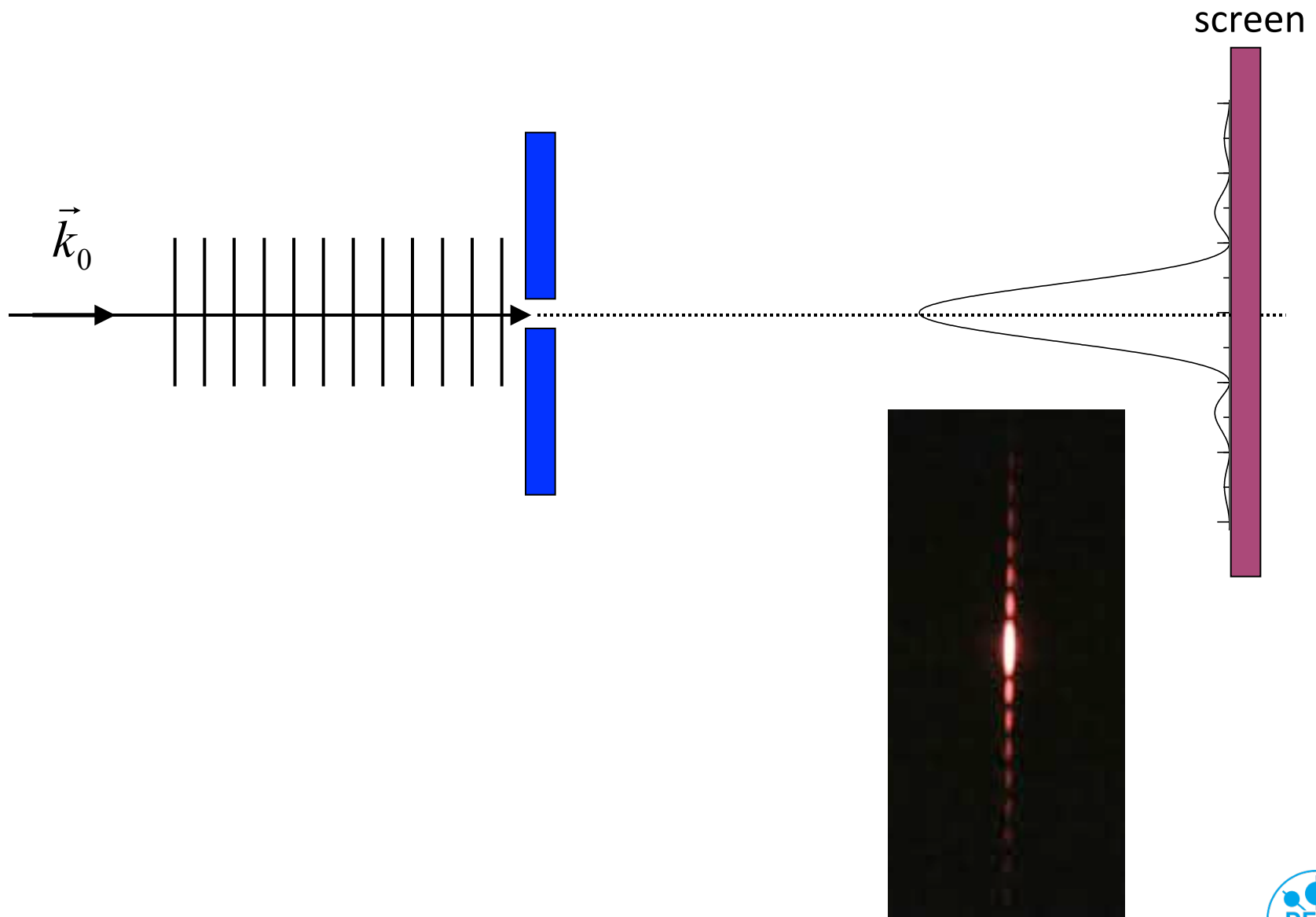
## Scattering, Imaging, Spectroscopy and beyond

### Part II

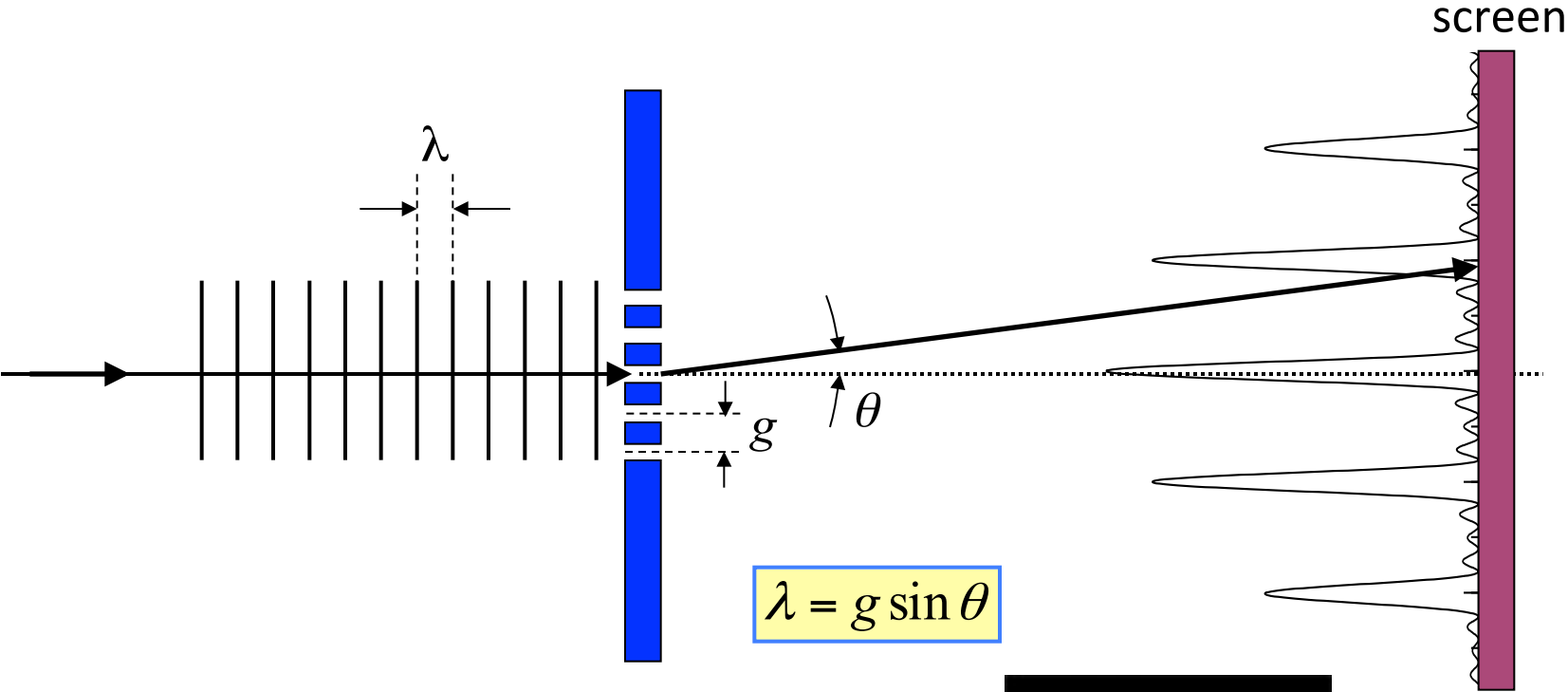
**Ralf Röhlsberger**

Deutsches Elektronen-Synchrotron DESY, Hamburg

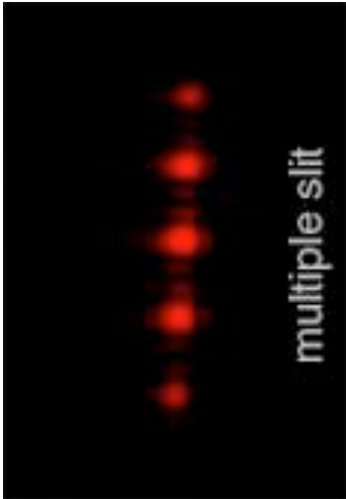
# Structure Determination using Light: Diffraction at a Single Slit



# Diffraction at a grating

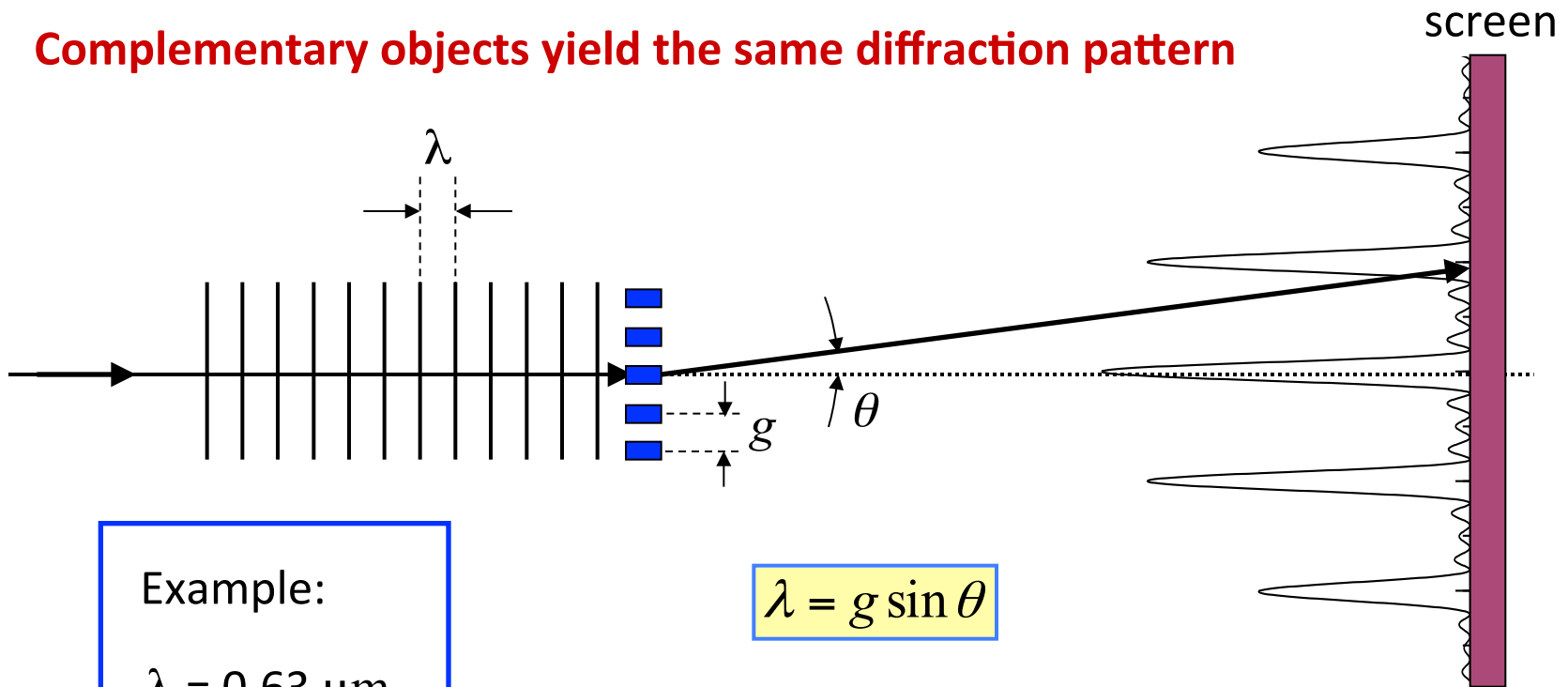


$$\lambda = g \sin \theta$$



# Theorem of Babinet

Complementary objects yield the same diffraction pattern



Example:

$$\lambda = 0.63 \mu\text{m}$$

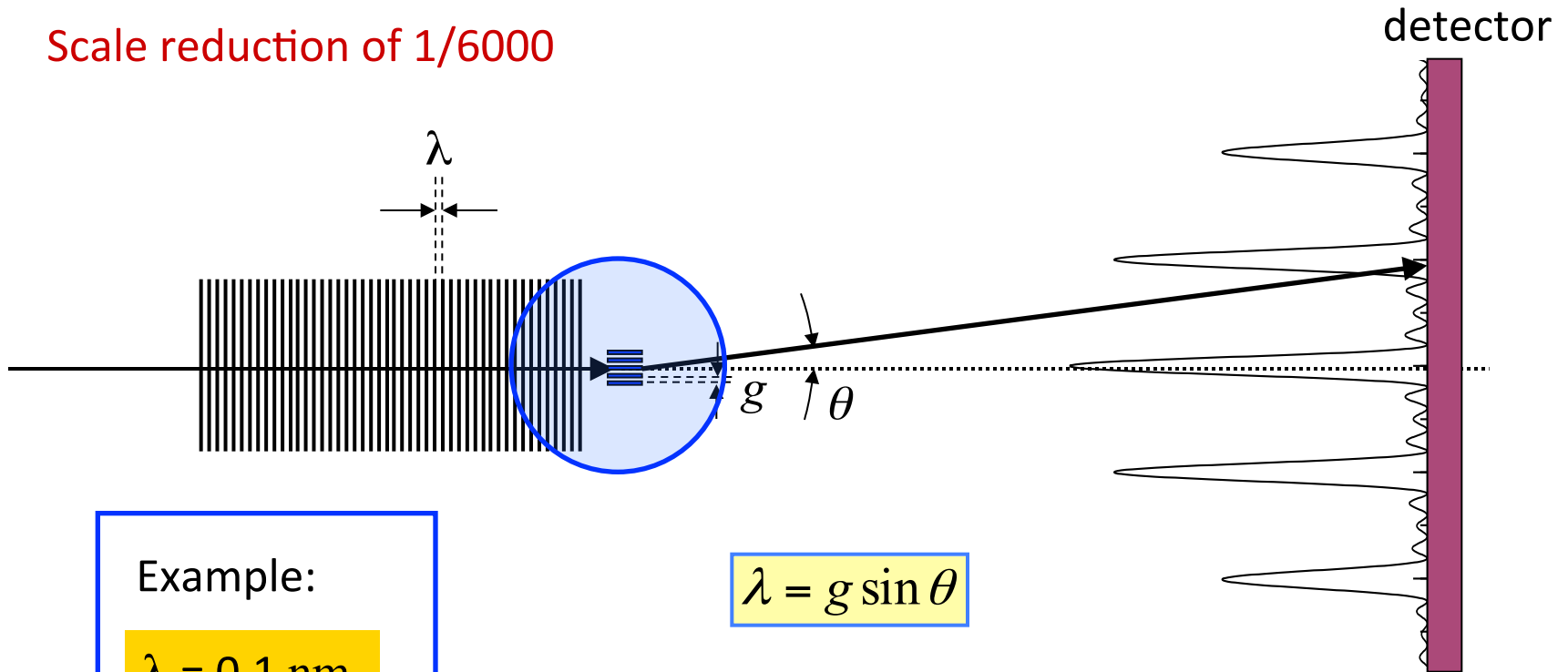
$$\theta = 10^\circ$$

$$g = 3.6 \mu\text{m}$$

$$\lambda = g \sin \theta$$

# Transition to x-rays

Scale reduction of 1/6000



Example:

$$\lambda = 0.1 \text{ nm}$$

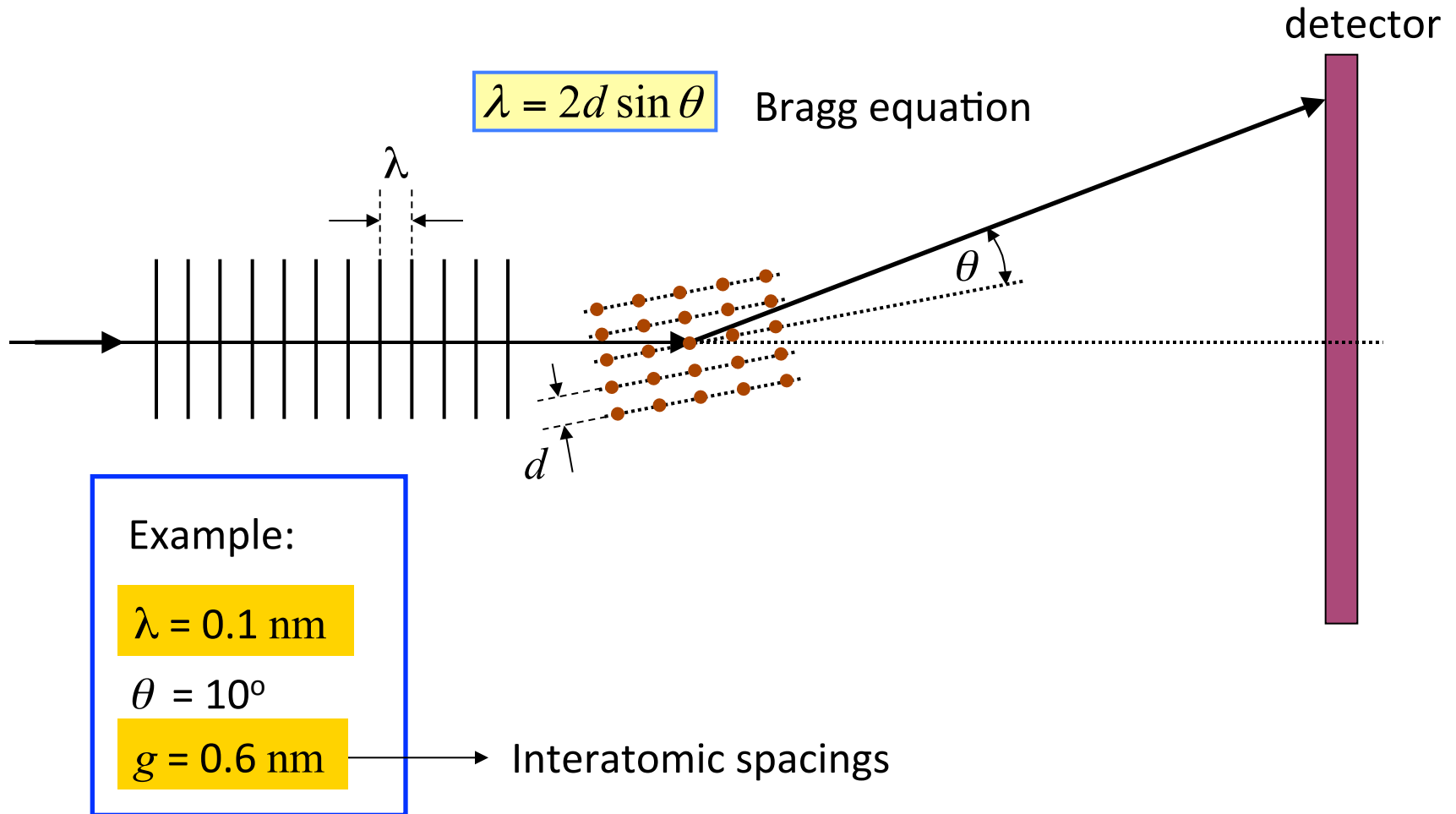
$$\theta = 10^\circ$$

$$g = 0.6 \text{ nm}$$

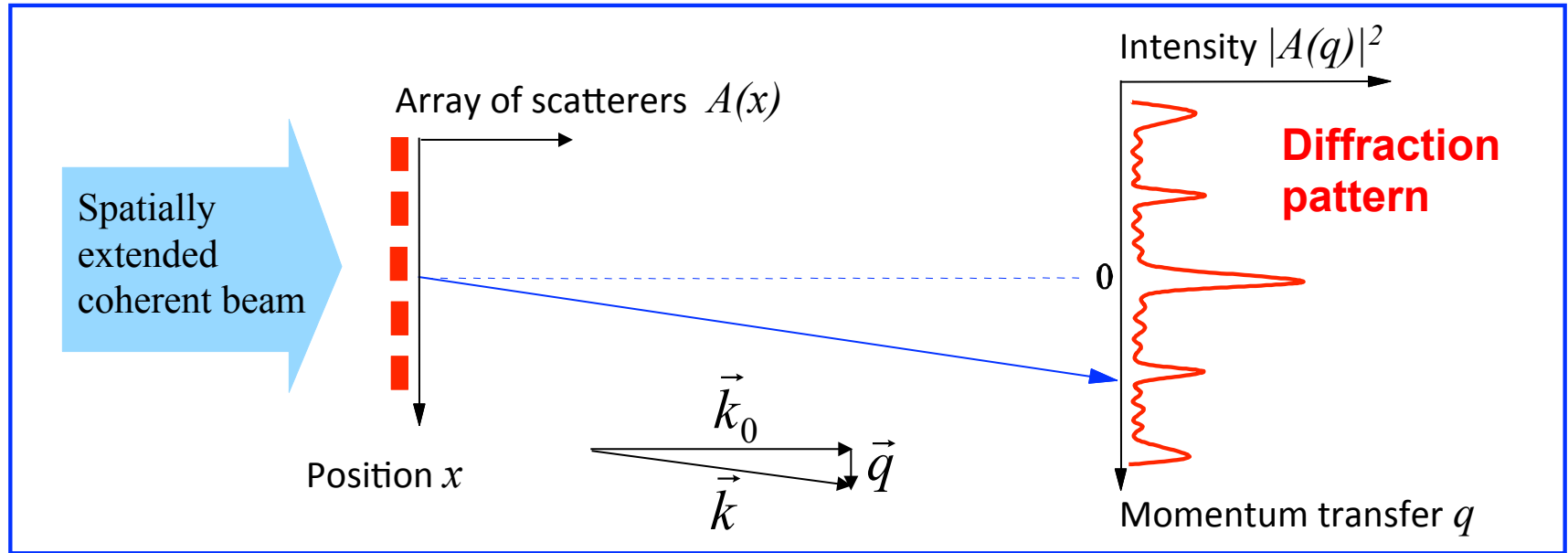
Interatomic spacings

$$\lambda = g \sin \theta$$

# X-ray diffraction



# Principles of structure determination



$$\vec{q} = \vec{k} - \vec{k}_0 \quad \text{momentum transfer}$$

Relation between  $A(x)$  and  $A(q)$ :

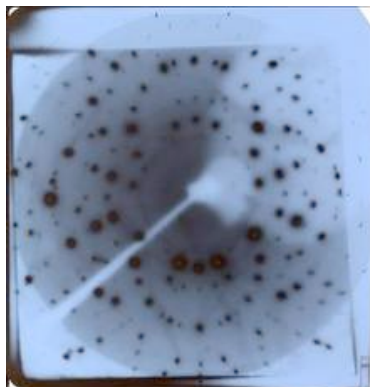
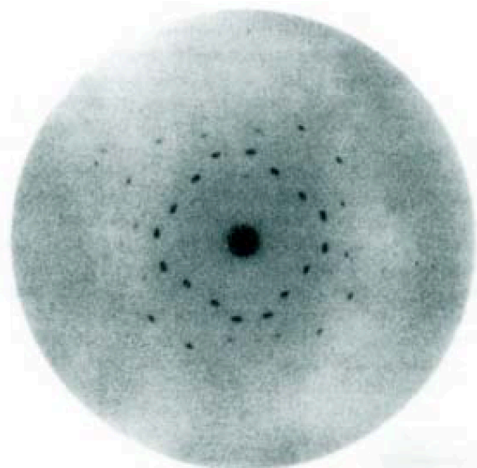
$$A(q) = \int A(x) e^{iqx} dx \quad (1)$$

Task: Determine  $A(x)$  from measured  $|A(q)|^2$

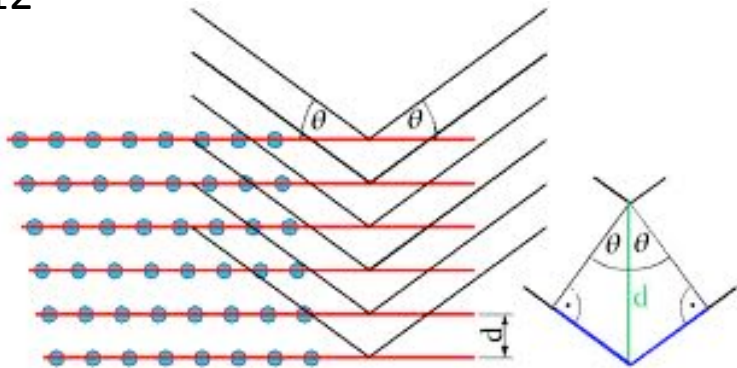
Problem: Eq. (1) cannot be simply inverted, because the phase is lost  
(Phase problem of crystallography)

# X-ray Diffraction from Crystalline Structures

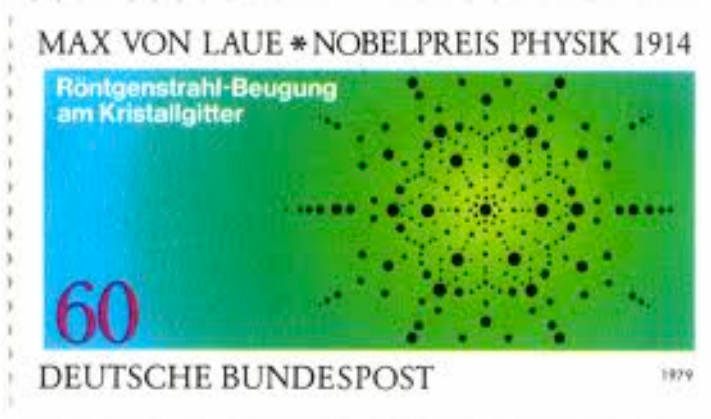
Apparently, x-rays are preferentially scattered in particular directions



First diffraction patterns obtained by Max v. Laue in 1912



Max von Laue  
(1879 – 1960)



**Today: X-ray diffraction of synchrotron radiation for structure determination**



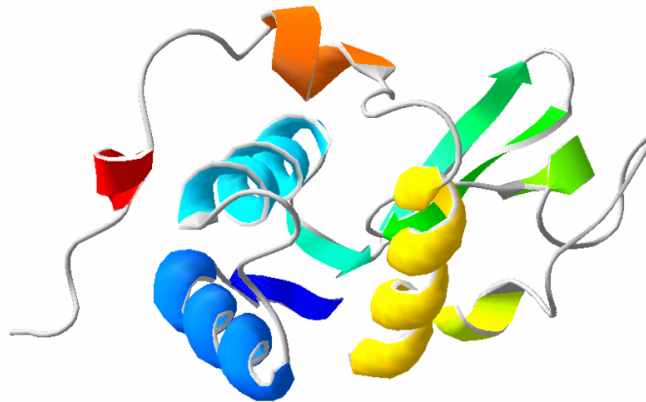
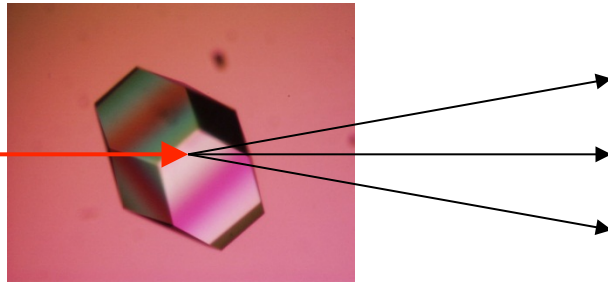


# Structure Determination of Proteins

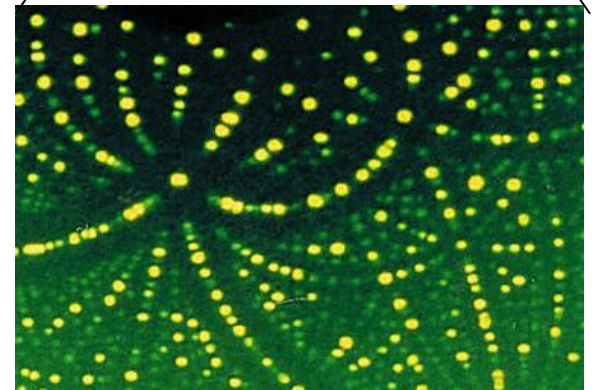
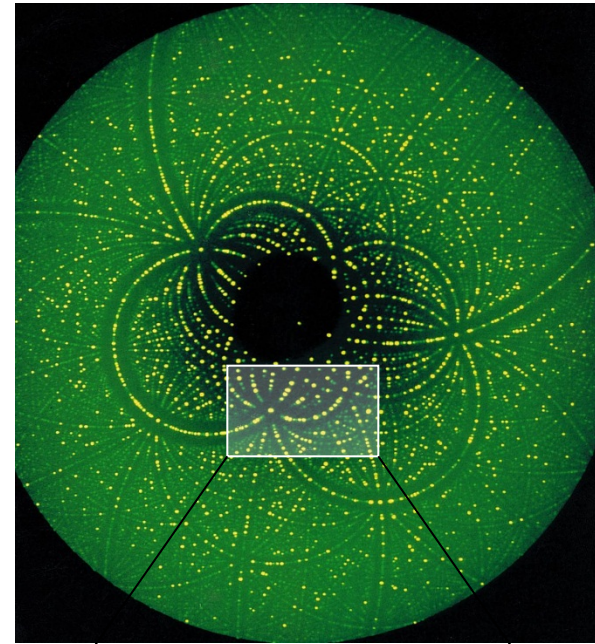
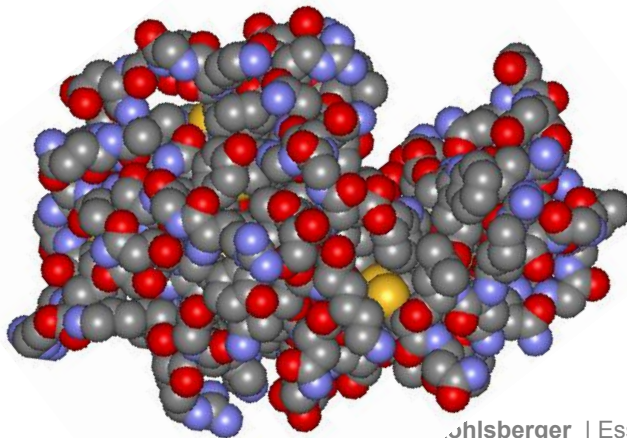
## Diffraction pattern

X-rays

Protein crystal:  
**Lysozyme**  
(enzyme from egg white)

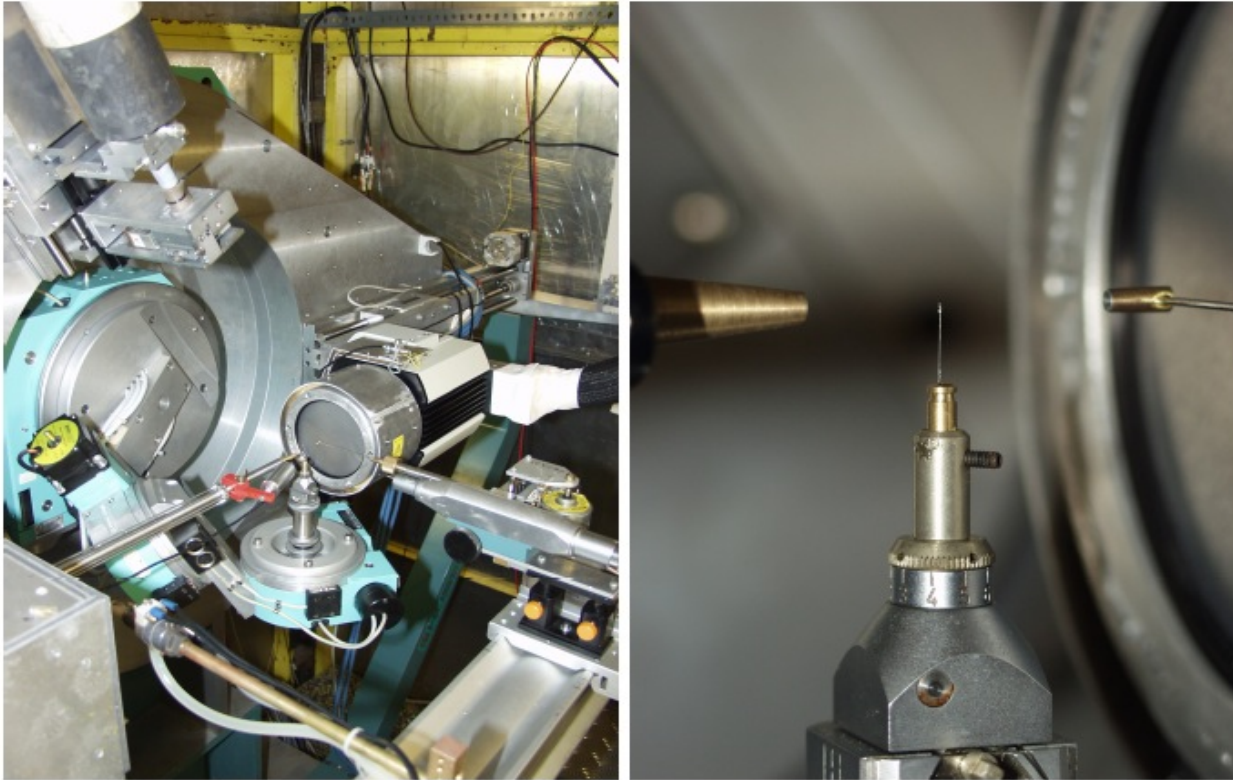


Carbon: Grey  
Nitrogen: Blue  
Oxygen: Red  
Sulphur: Yellow



# Structure determination of Proteins: Instrumentation

## Single crystal diffractometers

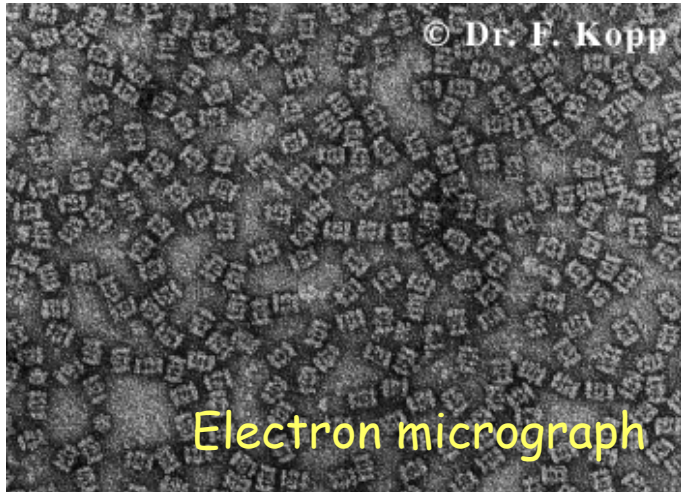


Single crystal diffractometer in  $\kappa$ -geometry with CCD and scintillation counter. Crystal mounted on a glass fiber. The  $\kappa$ -diffractometer has 3 rotations for the crystal and one for the detector.



# The Proteasome

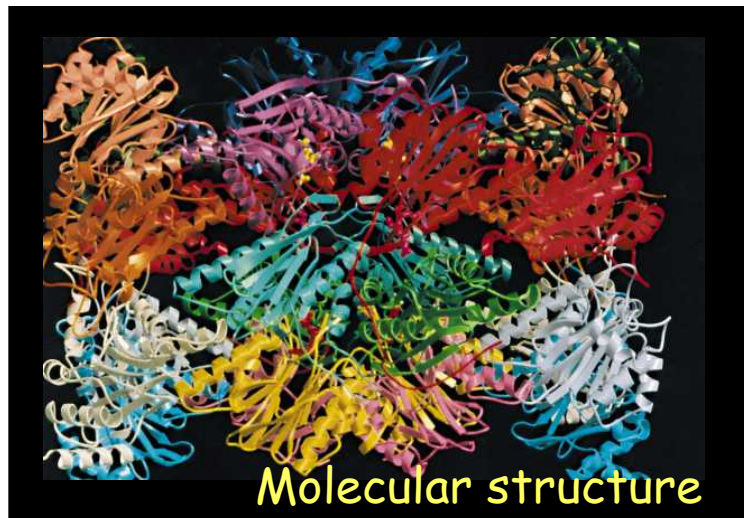
(cuts proteins into peptides and amino acids)



# The Ribosome

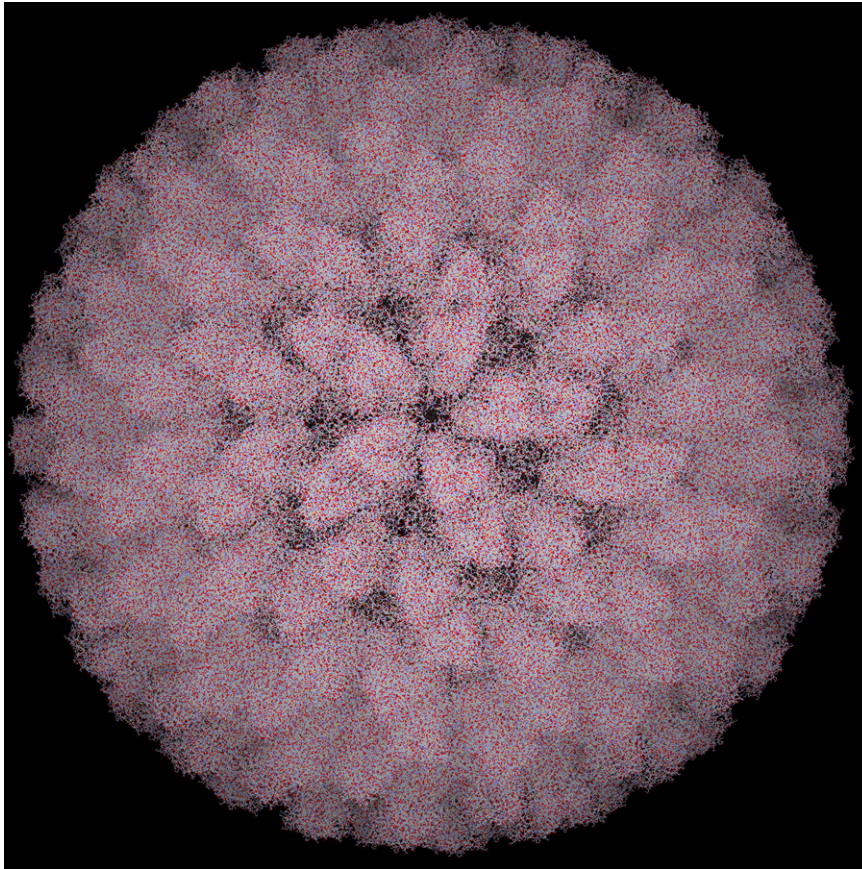
(synthesis of proteins)

The 30S subunit of the ecoli ribosome

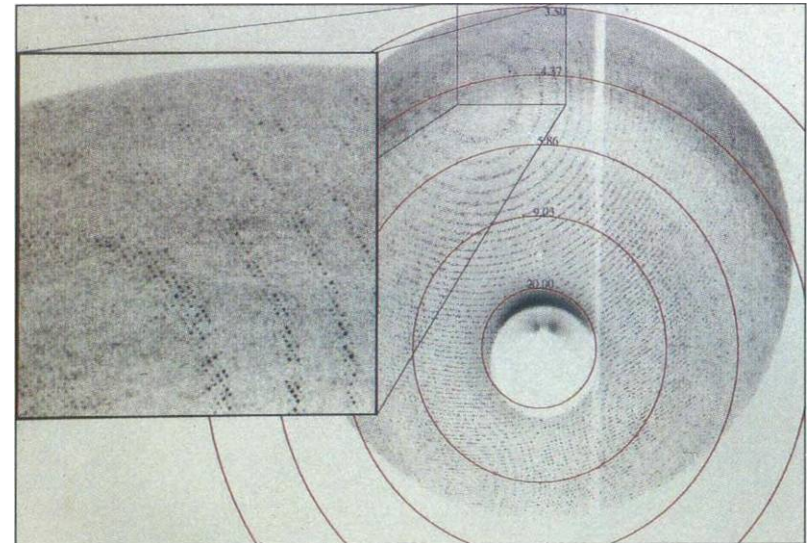




# Very Large Biomolecules (e.g. Viruses)



## Example: Blue Tongue Virus

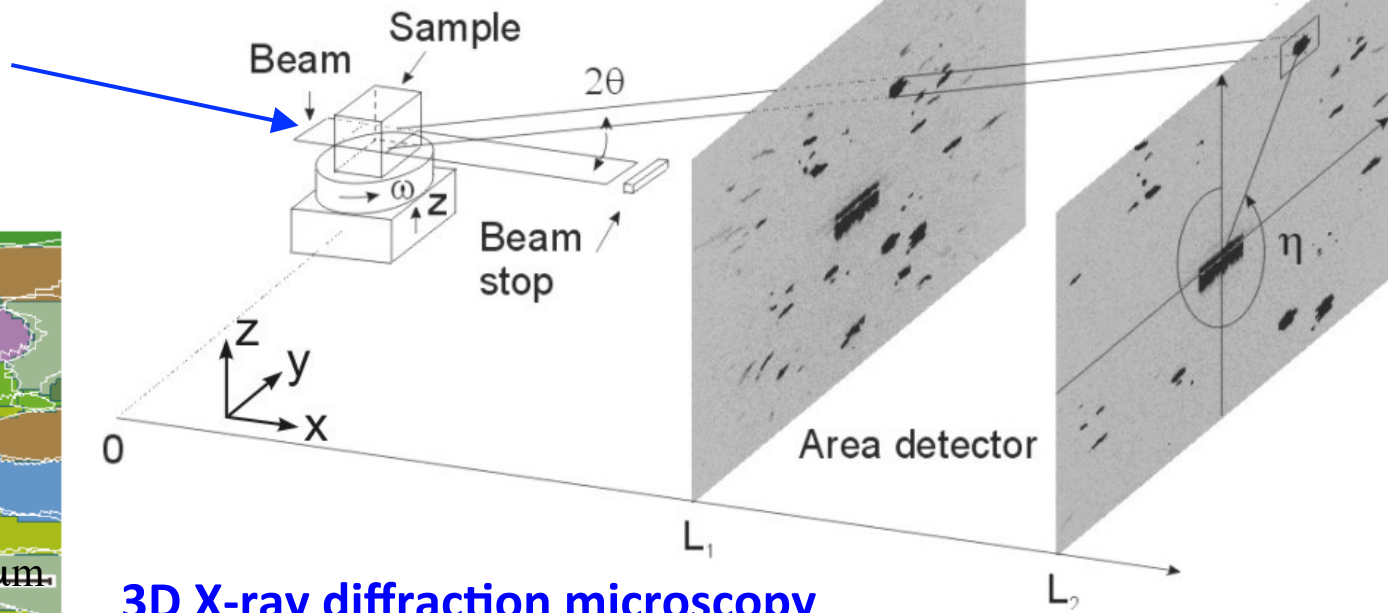
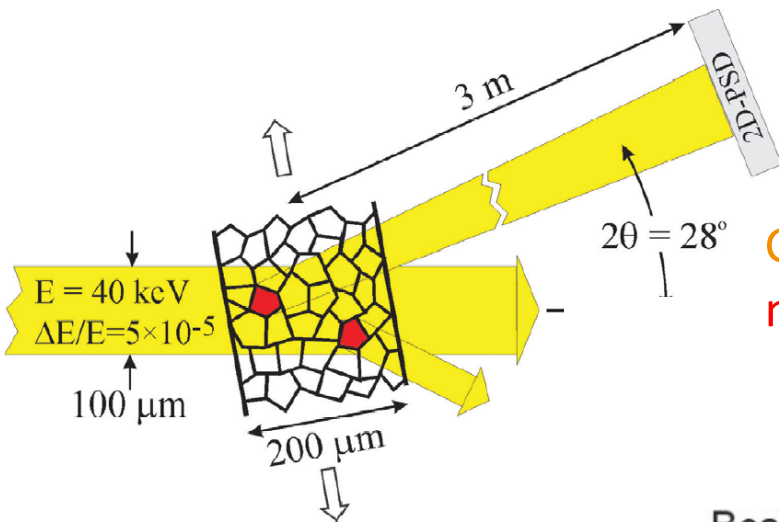


J.M. Grimes et al., Nature 395, 470-478 (1998)

# Materials Science: Grain Structure of Metals and Alloys

How does the microstructure of a material change upon mechanical load and deformations ?

Grain properties to be determined: position, morphology, orientation, deformation, composition



## 3D X-ray diffraction microscopy

H. F. Poulsen et al., J. Appl. Cryst. 34, 751 (2001)



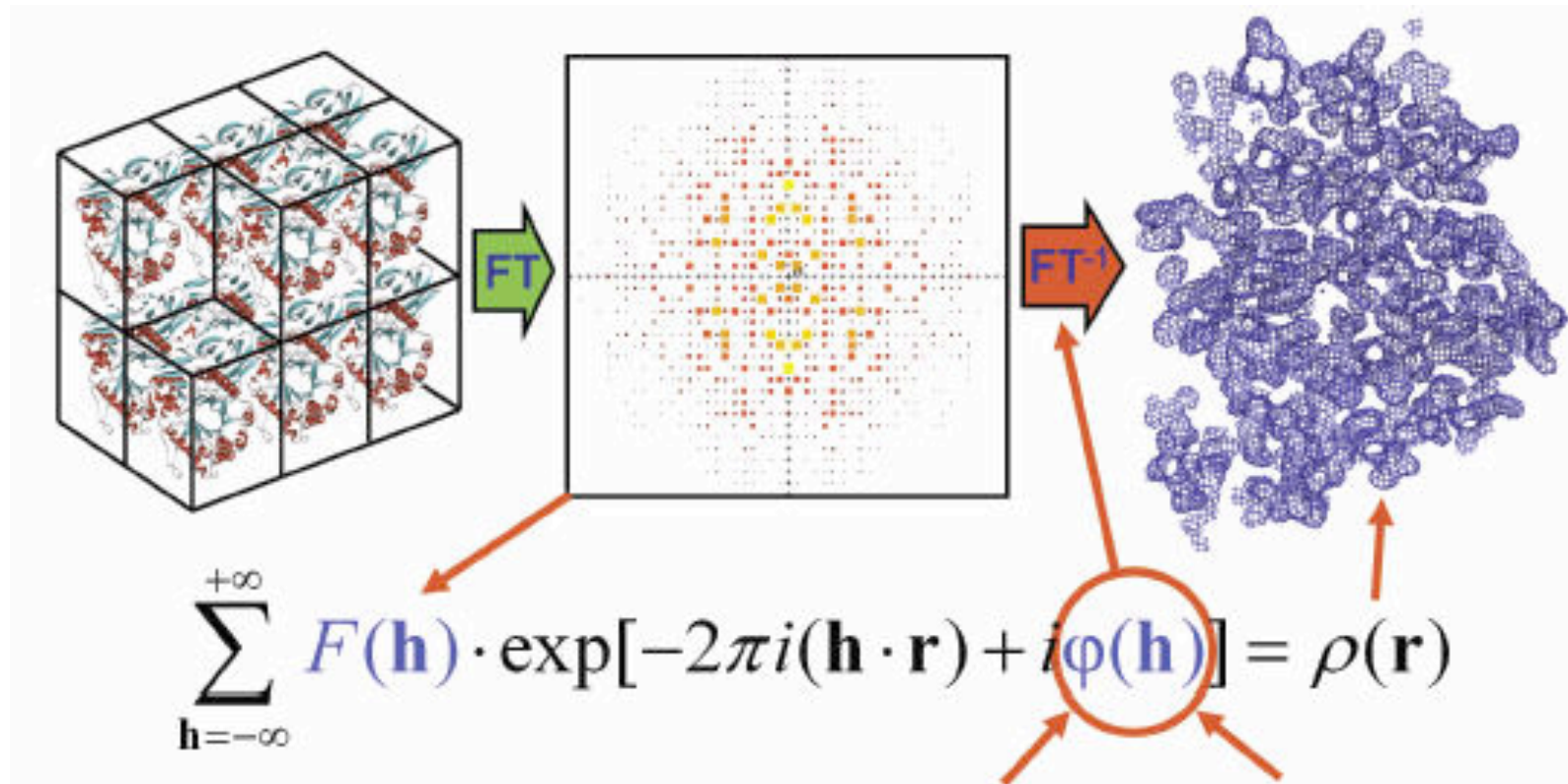


# The phase problem of crystallography

Crystallized molecules

Diffraction pattern

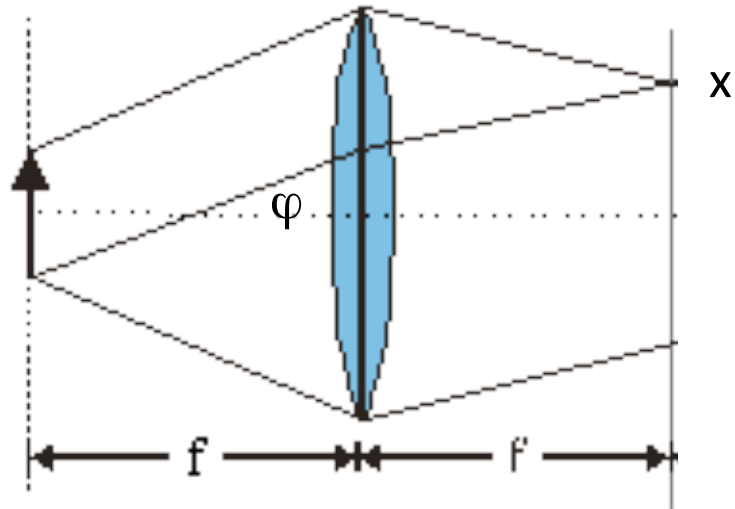
Electron density of the molecule



**Why don't we take a lens to obtain an image ?**

# Fouriertransformation (FT) with a lens

Object                      Fourierplane

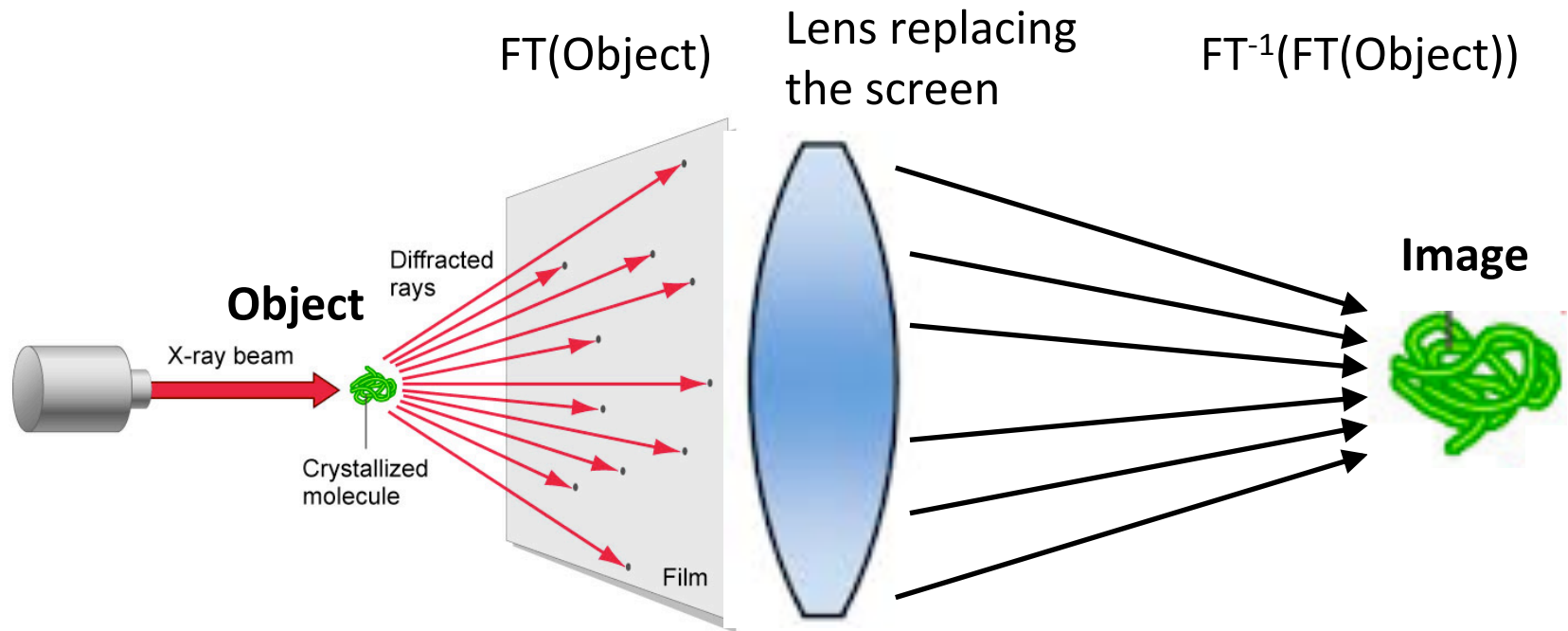


**Object in the focal plane  
→ FT(Object) in the Fourierplane**

A lens focusses parallel rays from an object in the focal plane on a point in the Fourier plane

→ Mapping of angle  $\varphi$  to coordinate  $x$   
= Fouriertransformation of the object

# Fourier transformation with a lens



**Can we apply this for x-rays ?**



## Index of refraction

$$n = 1 - \delta$$

$$\delta = \frac{\rho_e r_0 \lambda^2}{4\pi}, \quad \rho_e \equiv \text{electron density}$$
$$\approx 10^{-5} - 10^{-6} \quad \text{for } \lambda = 0.1 \text{ nm}$$

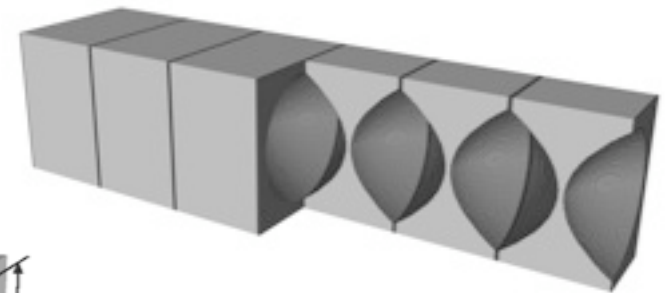
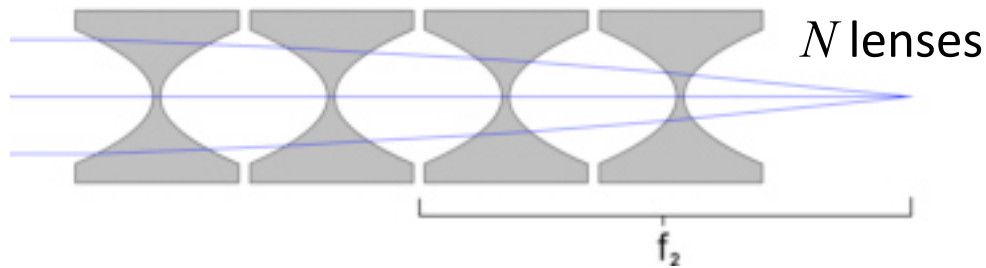
**For x-rays, every medium is optically thinner than vacuum !**

# X-ray Lenses

$$n = 1 - \delta$$

$n < 1 \rightarrow$  Concave lenses are focussing:

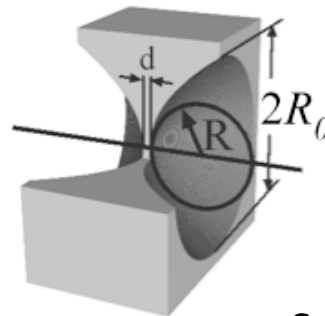
$\Theta \approx$  a few mrad



Focal length : 
$$f = \frac{R}{2\delta N}$$

**Numerical aperture**  $NA = n \sin \Theta$

**Resolution** 
$$d = \frac{\lambda}{2 NA}$$



Source: [www.xray-lens.de](http://www.xray-lens.de)



# X-ray reflection from surfaces

For x-rays, every medium is optically thinner than vacuum !

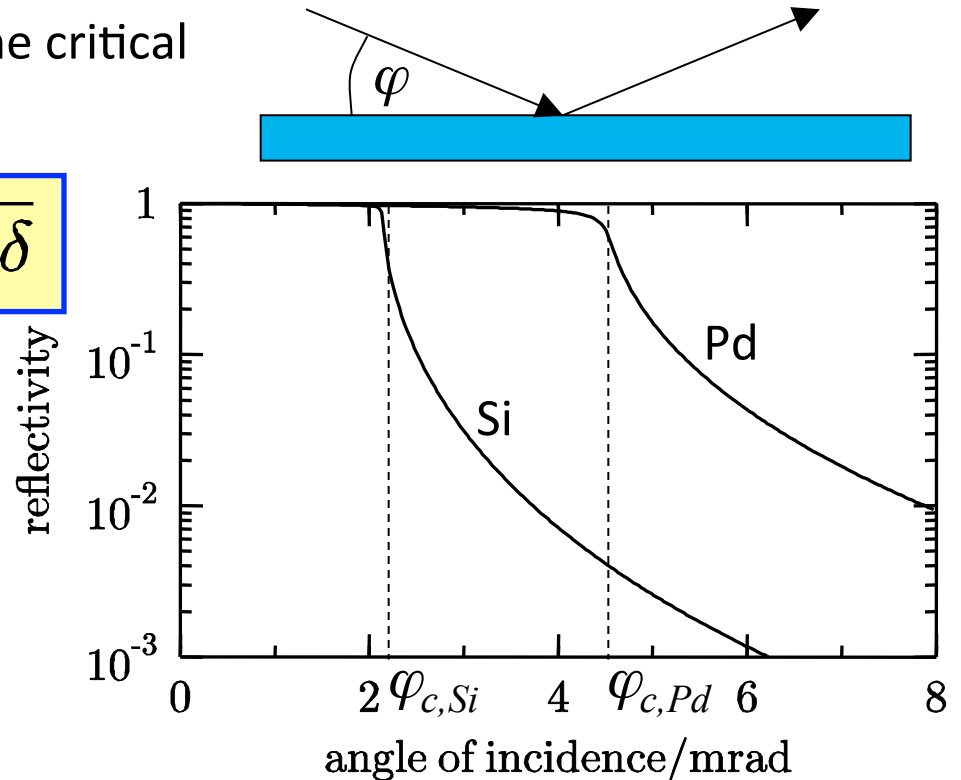
X-rays incident on a surface below the critical angle are totally reflected

Critical angle of total reflection

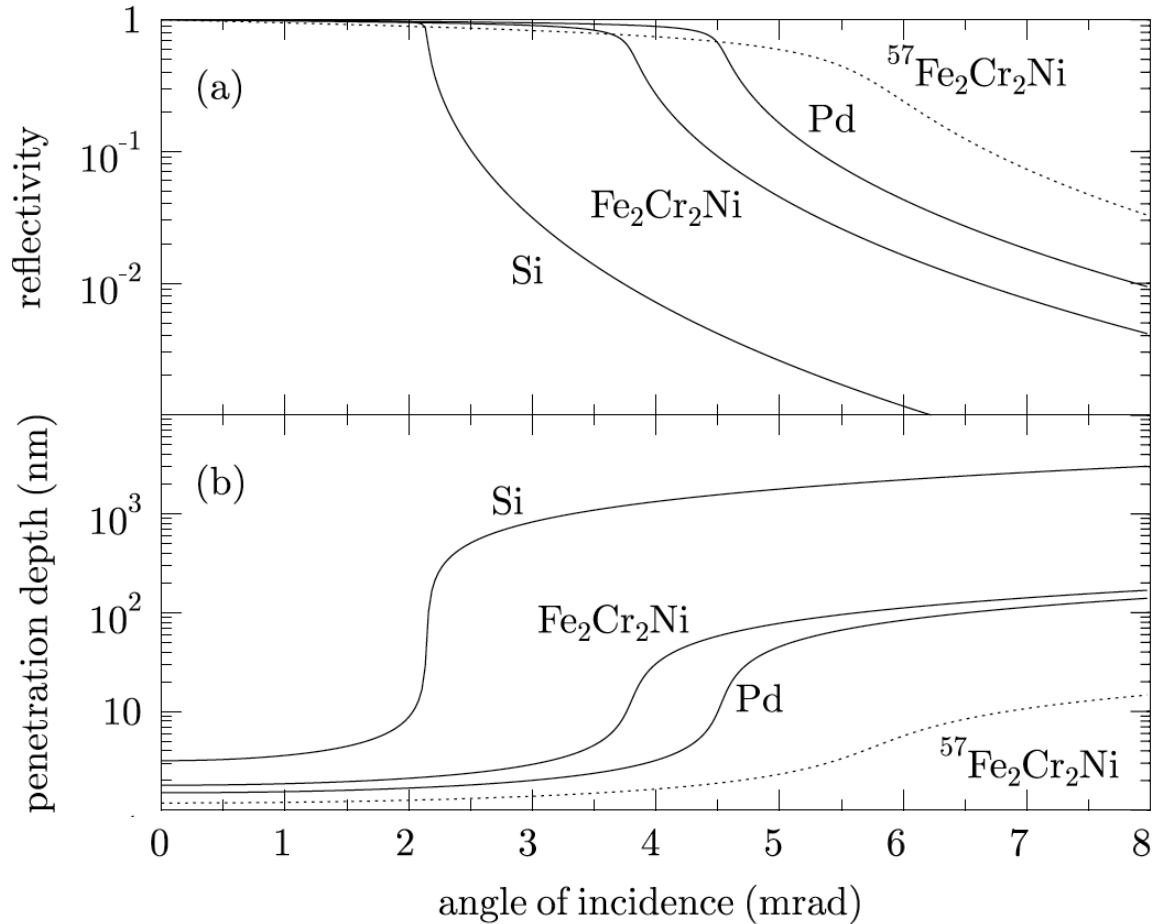
$$\varphi_c = \sqrt{2\delta}$$

For angles  $\varphi < \varphi_c$  the penetration depth of hard x-rays is only a few nm.

X-rays can be used for the study of structures at surfaces



# X-ray Grazing Incidence and Penetration Depth



$E = 14.4 \text{ keV}$

