

Time-resolved Molecular Electron Dynamics

8-10-2014 & 9-10-2014



Max-Born-Institut

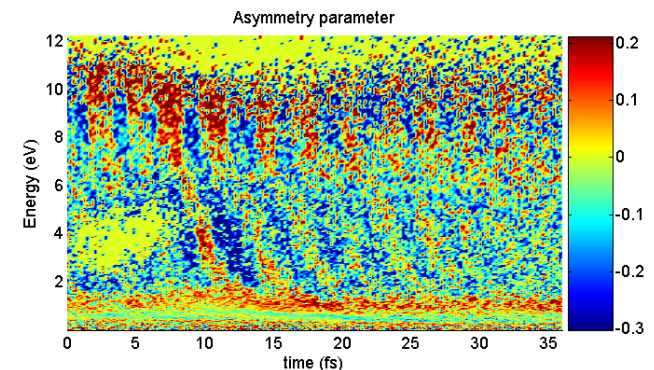
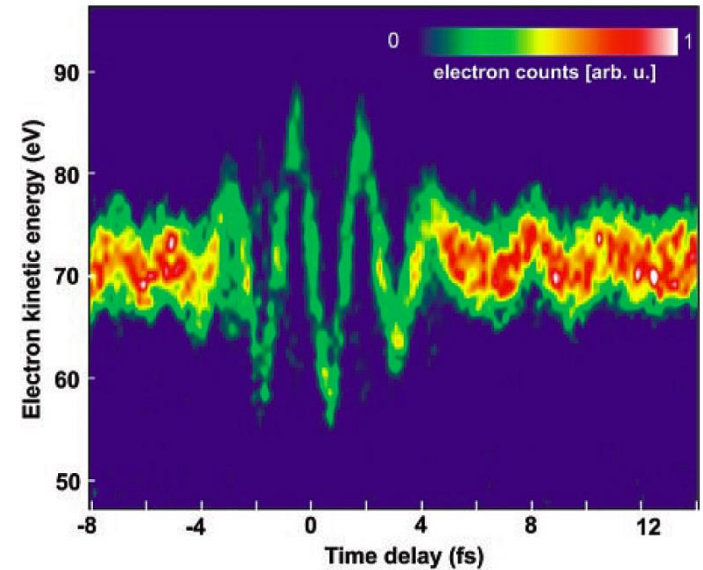
Marc Vrakking

marc.vrakking@mbi-berlin.de

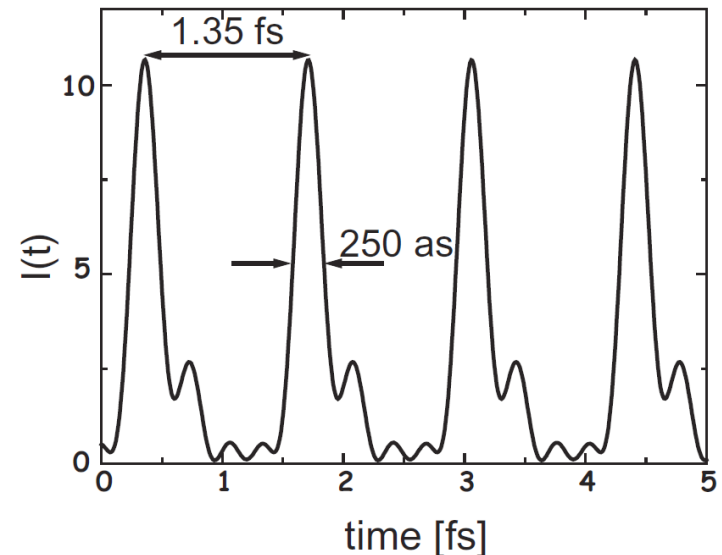
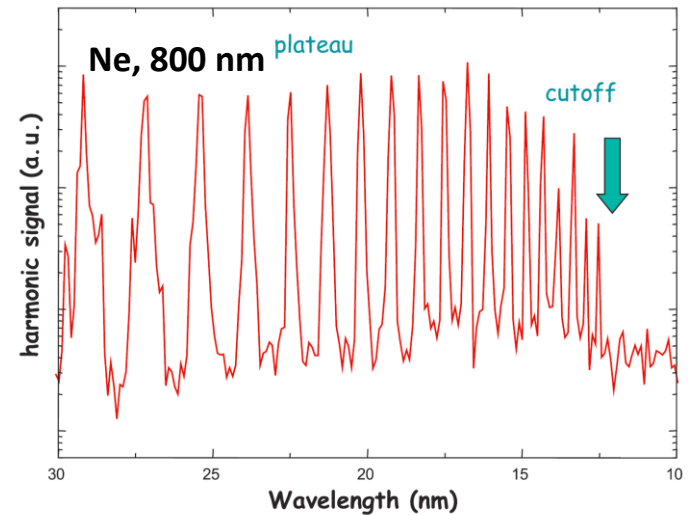
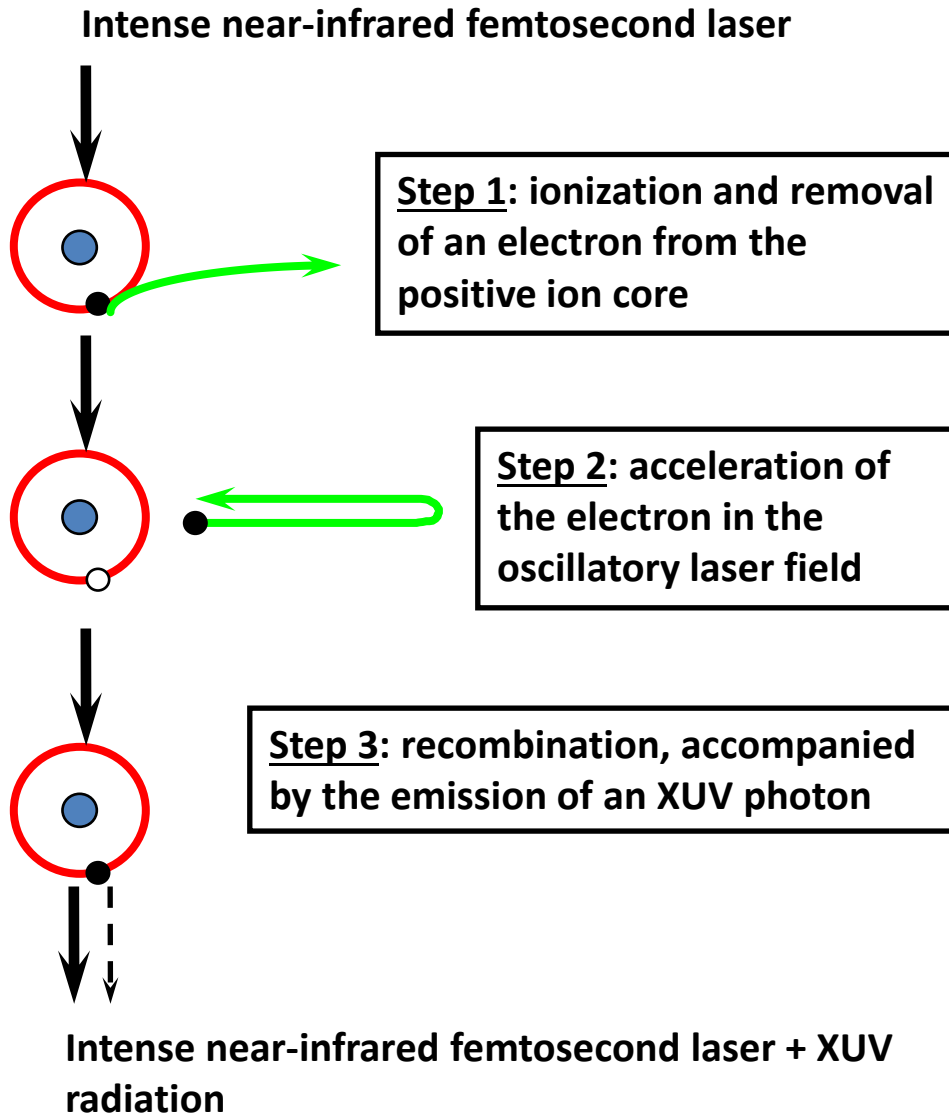
Today and tomorrow

- High harmonic generation and attosecond laser pulses
 - Atoms in intense laser fields
 - Generation of attosecond laser pulses
 - Characterization of attosecond laser pulses

- Attosecond pump-probe spectroscopy
 - Attosecond experiments – atoms
 - Attosecond experiments – molecules



Making attosecond laser pulses



1905: Simple photo-effect

132

6. *Über einen
die Erzeugung und Verwandlung des Lichtes
betreffenden heuristischen Gesichtspunkt;
von A. Einstein.*

Zwischen den theoretischen Vorstellungen, welche sich die Physiker über die Gase und andere ponderable Körper gebildet haben, und der Maxwell'schen Theorie der elektromagnetischen Prozesse im sogenannten leeren Raume besteht

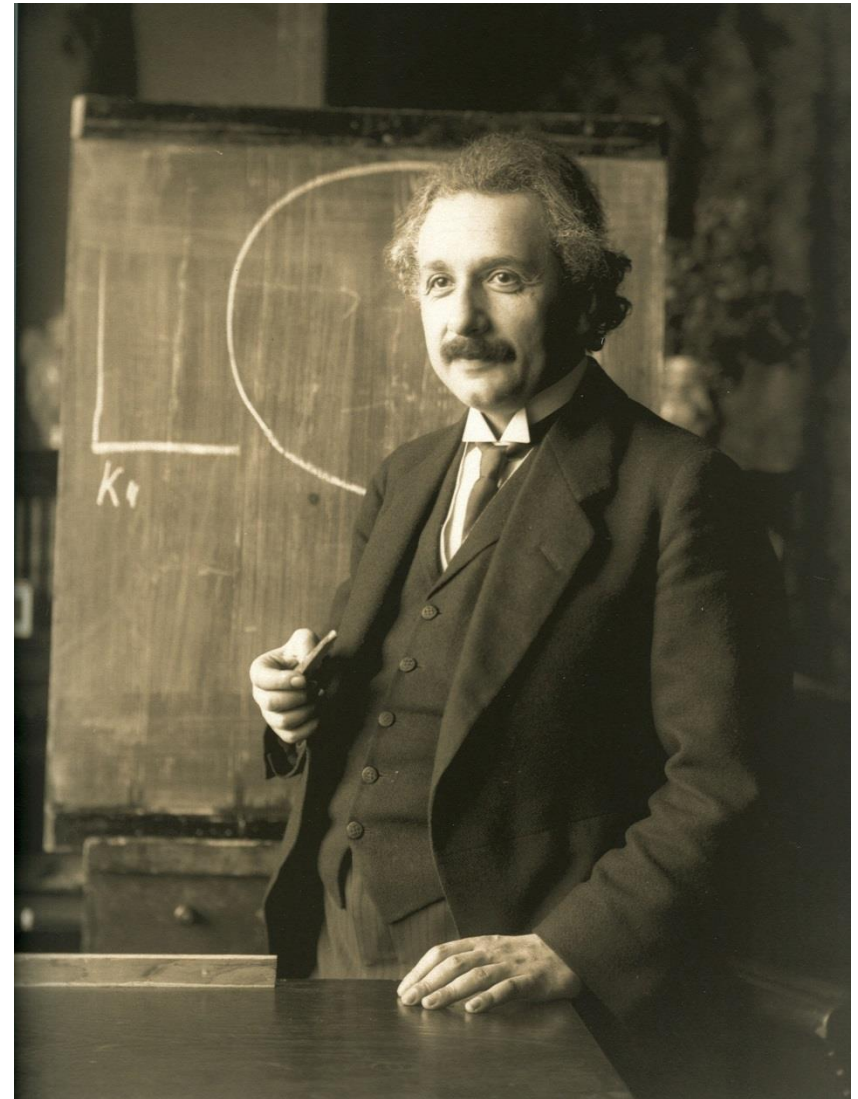
146

A. Einstein.

versehenes Elektron wird, wenn es die Oberfläche erreicht hat, einen Teil seiner kinetischen Energie eingebüßt haben. Außerdem wird anzunehmen sein, daß jedes Elektron beim Verlassen des Körpers eine (für den Körper charakteristische) Arbeit P zu leisten hat, wenn es den Körper verläßt. Mit der größten Normalgeschwindigkeit werden die unmittelbar an der Oberfläche normal zu dieser erregten Elektronen den Körper verlassen. Die kinetische Energie solcher Elektronen ist

$$\frac{R}{N} \beta v - P.$$

Ist der Körper zum positiven Potential Π geladen und von Leitern vom Potential Null umgeben und ist Π eben imetende einen Elektrizitätsverlust des Körpers zu verhindern



Multi-Photon Ionization

SOVIET PHYSICS JETP

VOLUME 23, NUMBER 1

JULY, 1966

MANY-PHOTON IONIZATION OF THE XENON ATOM BY RUBY LASER RADIATION

G. S. VORONOV and N. B. DELONE

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor August 27, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 50, 78-84 (January, 1966)

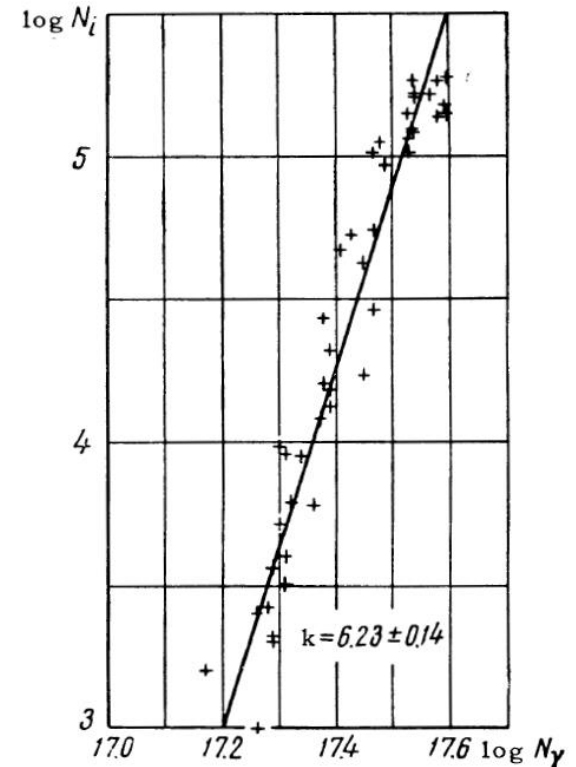
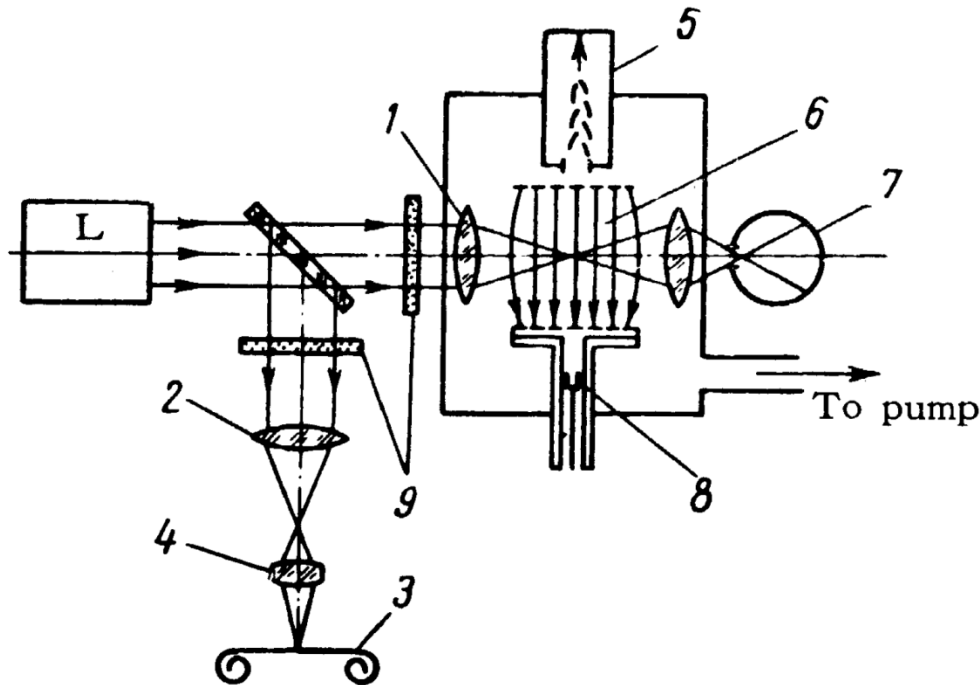


FIG. 2. Dependence of the number of ions formed N_i on the number of photons N_γ which have traversed the focusing region.

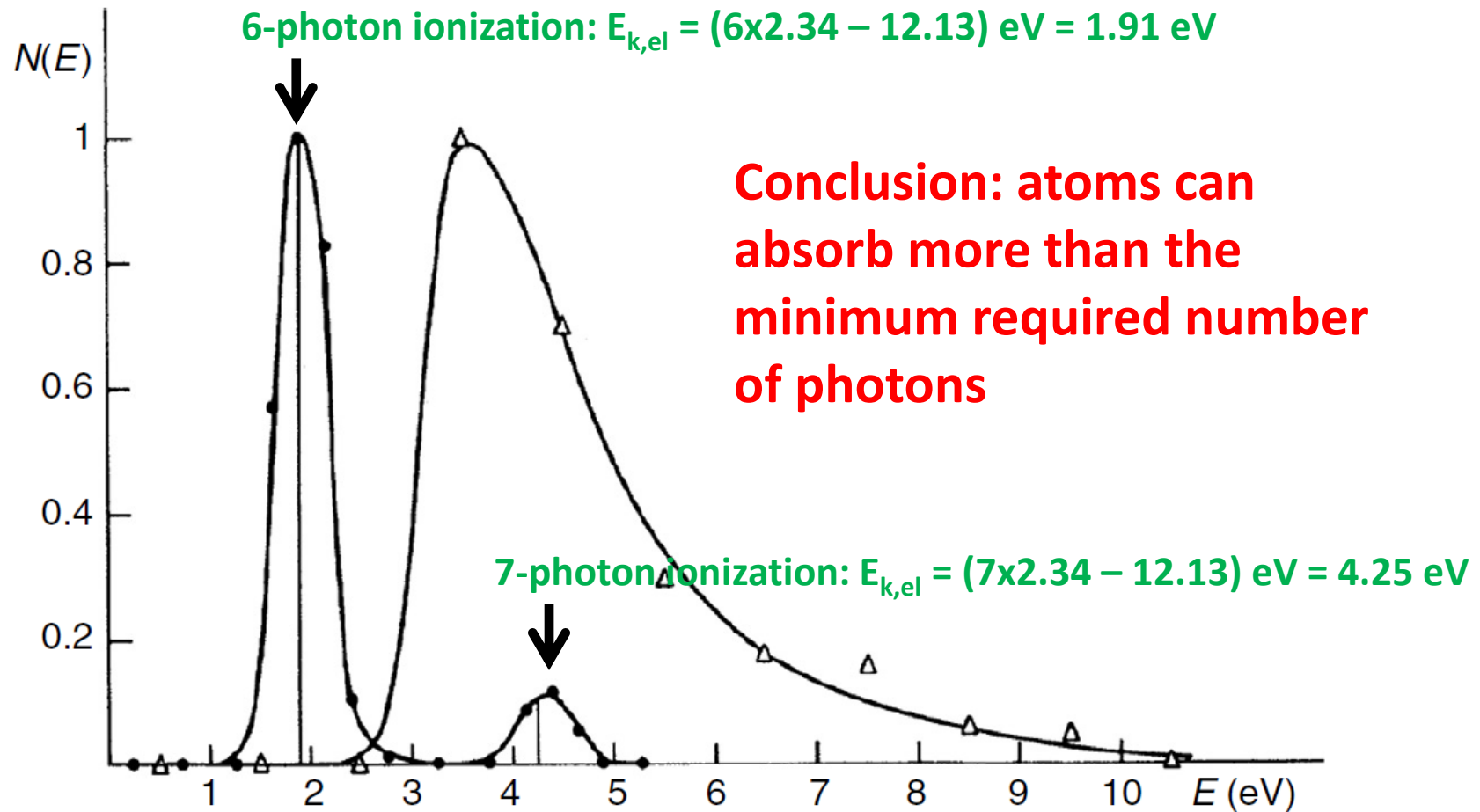
Multi-Photon Ionization

According to perturbation theory, the rate of n-photon absorption is given as

$$W_{fi}^{(n)} = \frac{2\pi}{\hbar} (2\pi\alpha\hbar)^n \overset{\text{laser}}{\text{intensity}} I_{laser}^n \overset{\text{final state}}{\text{density}} \left| \overset{\text{N-photon}}{\text{matrix}} \tilde{T}_{fi}^{(n)} \right|^2 \rho_f(E_f) \overset{\text{element}}{\quad}$$

Therefore, the lowest-order perturbation theory (LOPT) ionization rate is proportional to $I_{laser}^{n_0}$, where n_0 is the minimum number of photons needed

Above-Threshold Ionization

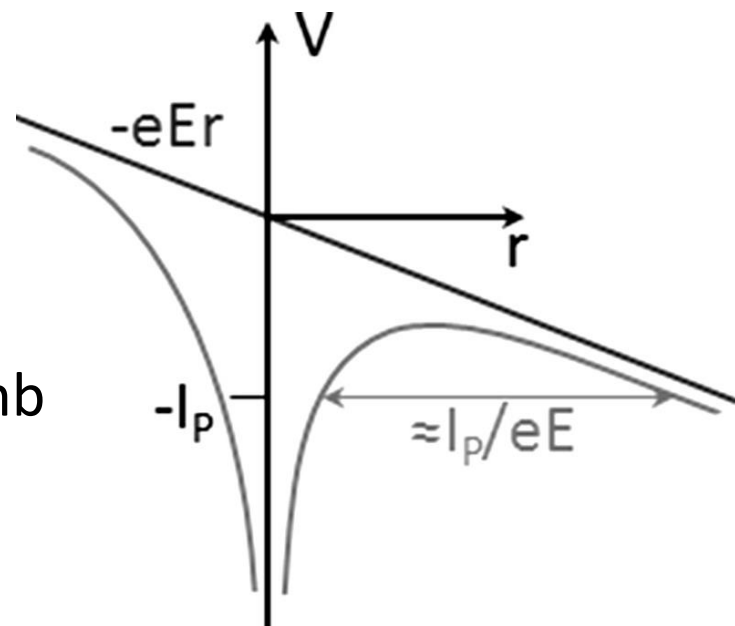


Energy spectra of electrons produced by multiphoton ionization of xenon atoms, for two photon energies. Triangles: $\omega = 1.17 \text{ eV}$; Circles: $\omega = 2.34 \text{ eV}$

Distortion of the Coulomb potential

$$V(z) = -\frac{1}{z} - E_{laser}z$$

For sufficiently strong laser fields the electron can tunnel through the Coulomb + laser field potential, or pass over the saddlepoint in this potential



Saddlepoint: $\frac{\partial V(z)}{\partial z} = \frac{1}{z^2} - E_{laser} = 0$

$$z_{saddle} = \frac{1}{\sqrt{E_{laser}}}; V(z_{saddle}) = -2\sqrt{E_{laser}}$$

Over-the-barrier ionization: $IP = -2\sqrt{E_{laser}}; E_{laser} = \frac{IP^2}{4}; I_{laser,OTB} = \frac{IP^4}{16}$

Distortion of the Coulomb potential

Over-the-barrier
ionization:

$$I_{laser,OTB} = \frac{IP^4}{16}$$

$$1 \text{ a.u.} = 3.51 \times 10^{16} \text{ W/cm}^2$$

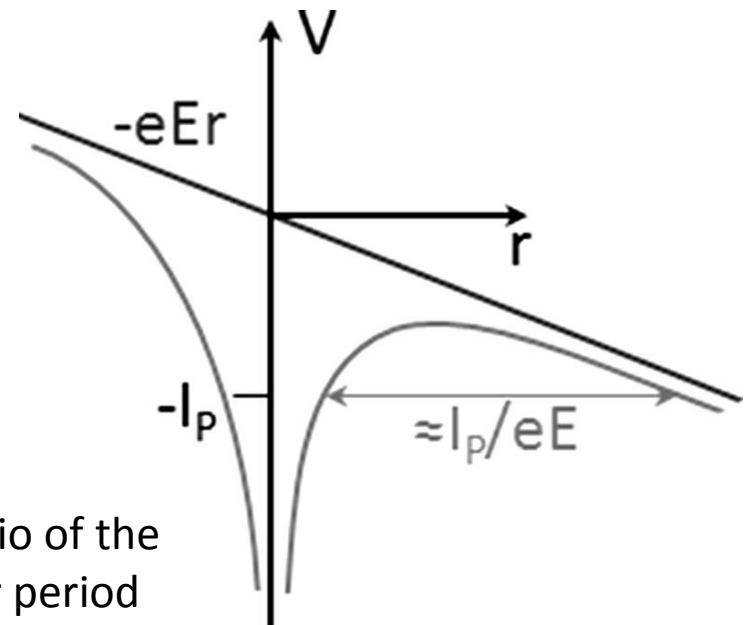
Hydrogen atom (IP=0.5 a.u.) : $I_{laser} = 0.0039 \text{ a.u.} = 1.4 \times 10^{14} \text{ W/cm}^2$

Below $I_{laser,OTB}$ the electron can escape the atom by tunneling through the Coulomb + laser electric field potential, provided that the potential is sufficiently quasi-static

This condition is expressed by the Keldysh parameter γ

$$\gamma = \sqrt{\frac{IP}{2U_p}} = \frac{\tau_{tunneling}}{\tau_{laser}} \ll 1$$

The Keldysh parameter can be interpreted as the ratio of the time it takes the electron to tunnel out and the laser period



Tunneling formulas

Provided suitable approximations are made, the rate of tunnel ionization can be described by simple formulas

Strong field approximation:

Assume that after the ionization process the interaction of the electron with the core is negligible, and that the photoelectron only interacts with the laser electric field

Adiabatic approximation:

Assume that in the presence of the laser field the atom remains in the lowest available state, and that no population is transferred to excited states

Single active electron approximation:

Assume only the most weakly bound electron is ionized

Tunneling formulas

In the adiabatic approximation, the ionization rate at time t , when the electric field equals $E(t)$, is given by $\Gamma_{DC}(E(t))$

Ground state hydrogenic atom:

$$\Gamma_{DC}(E) = \frac{4Z^5}{E} \exp\left(-\frac{2Z^3}{3E}\right)$$

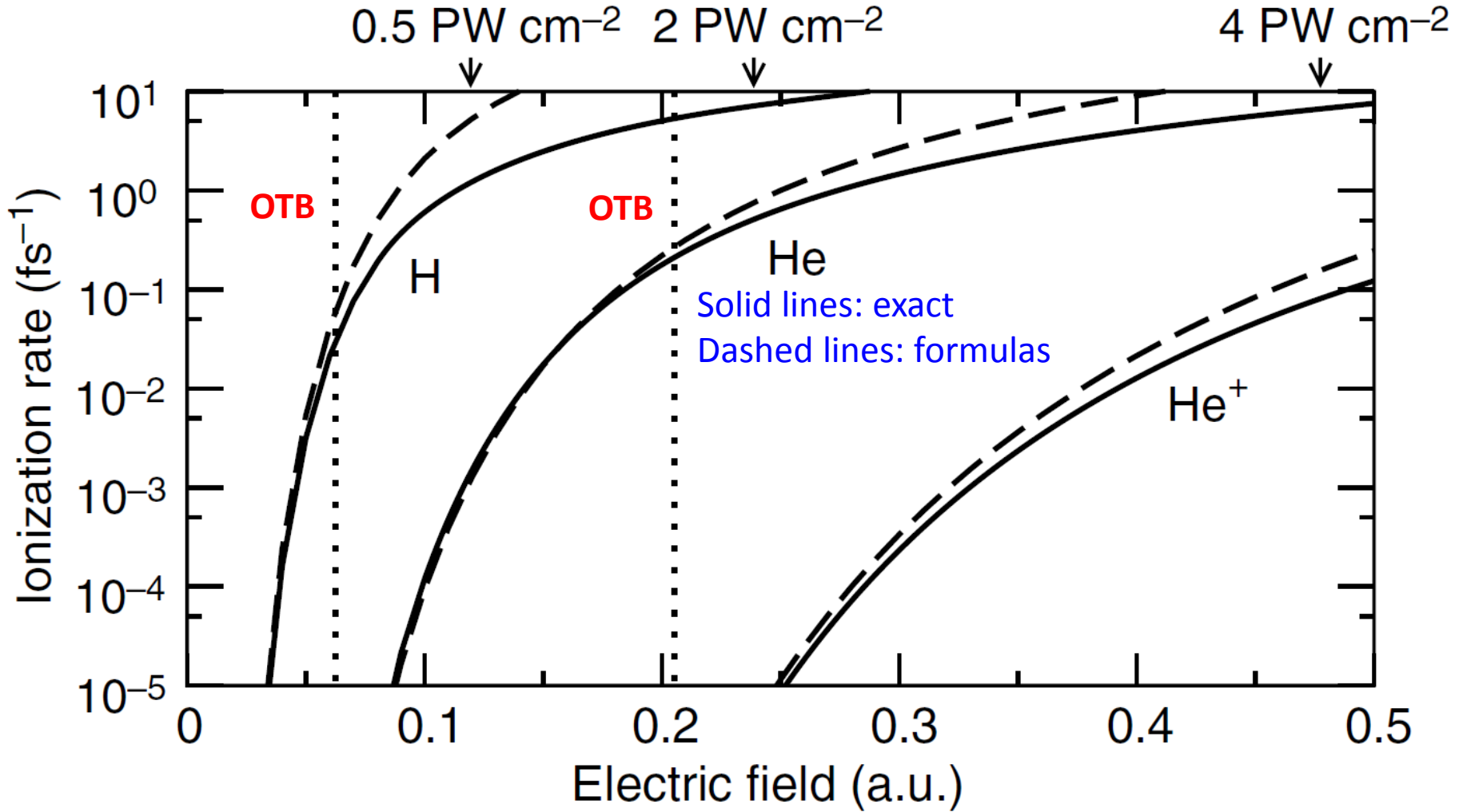
Non-hydrogenic atoms in arbitrary states:

$$\Gamma_{DC}(E) = C_{as}^2 A(l, m) \frac{\kappa^2}{2} \left(\frac{2\kappa^3}{E}\right)^{\frac{2Z_c - |m| - 1}{\kappa}} \exp\left(-\frac{2\kappa^3}{3E}\right)$$

$\kappa = (2IP)^{1/2}$
↓

$$A(l, m) = \frac{2l + 1}{2^{|m|}} \frac{(l + |m|)!}{|m|! (l - |m|)!} \quad C_{as} = \left[\frac{2^{2n^*}}{n^* \Gamma(n^* + l + 1) \Gamma(n^* - l)} \right]^{1/2}$$

Tunneling formulas



C.J. Joachain, N.J. Kylstra and R.M. Potvliege, *Atoms in Intense Laser Fields*, (Cambridge University Press, 2012)

After ionization: Propagation assuming the strong-field approximation (SFA)

Assume that the electron does not feel the ion anymore as soon as it has tunneled out (strong field approximation)

Assume, moreover, that the Coulomb-free motion starts with $v=0$ at $r=0$, and that the laser amplitude is constant

$$a(t) = (qE_0/m_e)\cos\omega t$$

$$v(t) = v_0\sin\omega t + v_{0z}$$

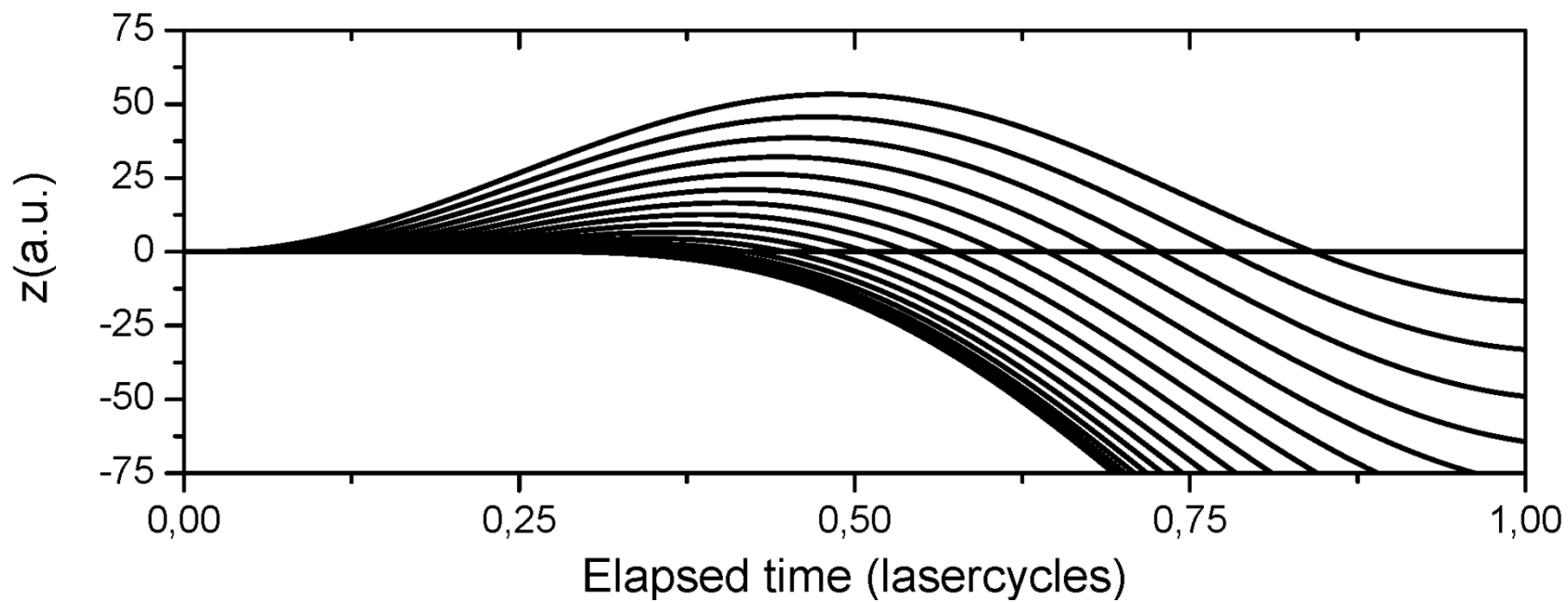
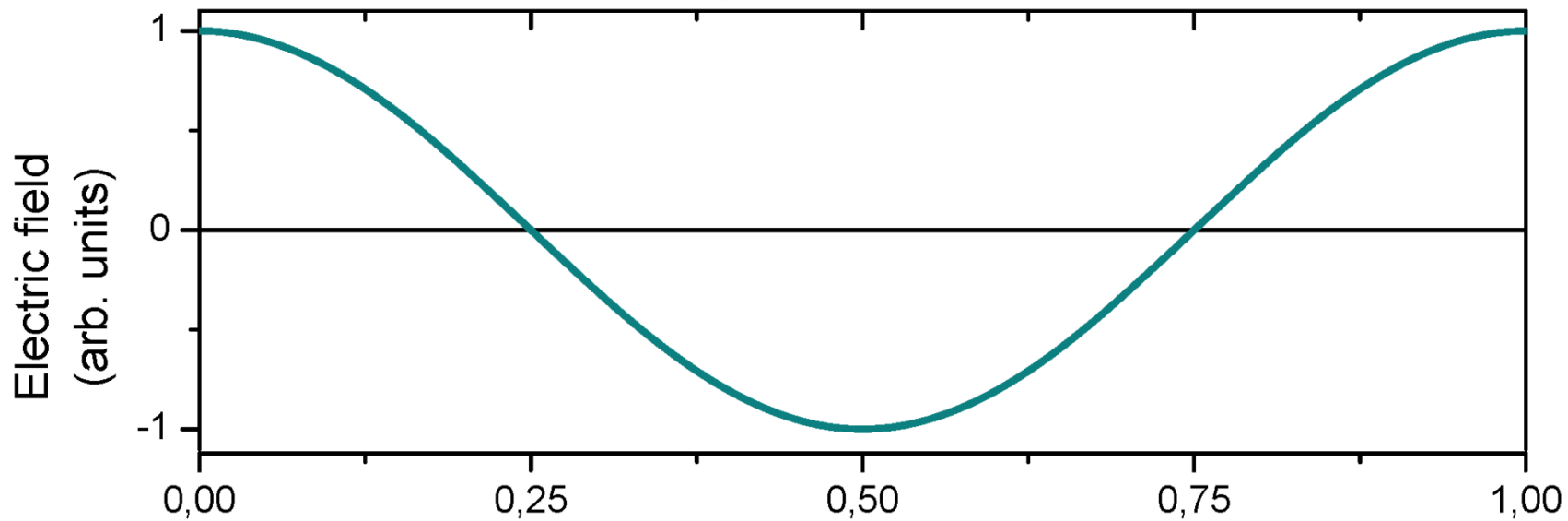
$$v_0 = qE_0/m_e\omega$$

$$z(t) = z_0(-\cos\omega t) + v_{0z}t + z_{0z}$$

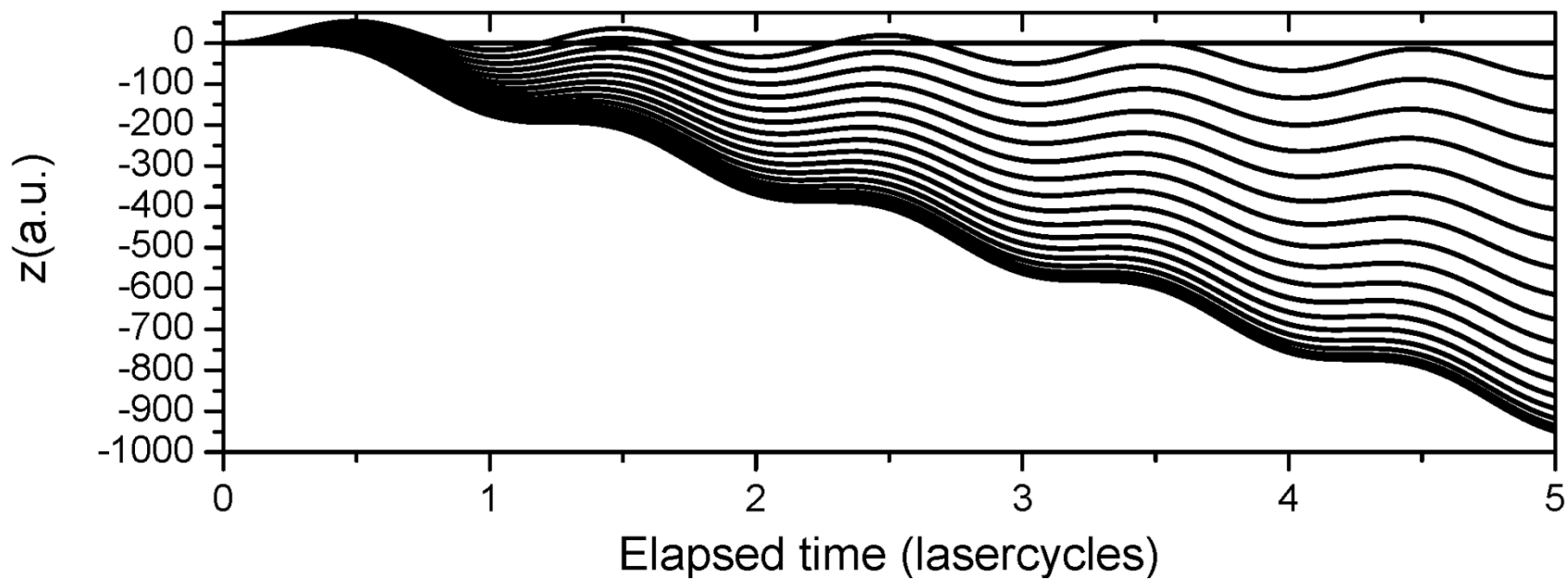
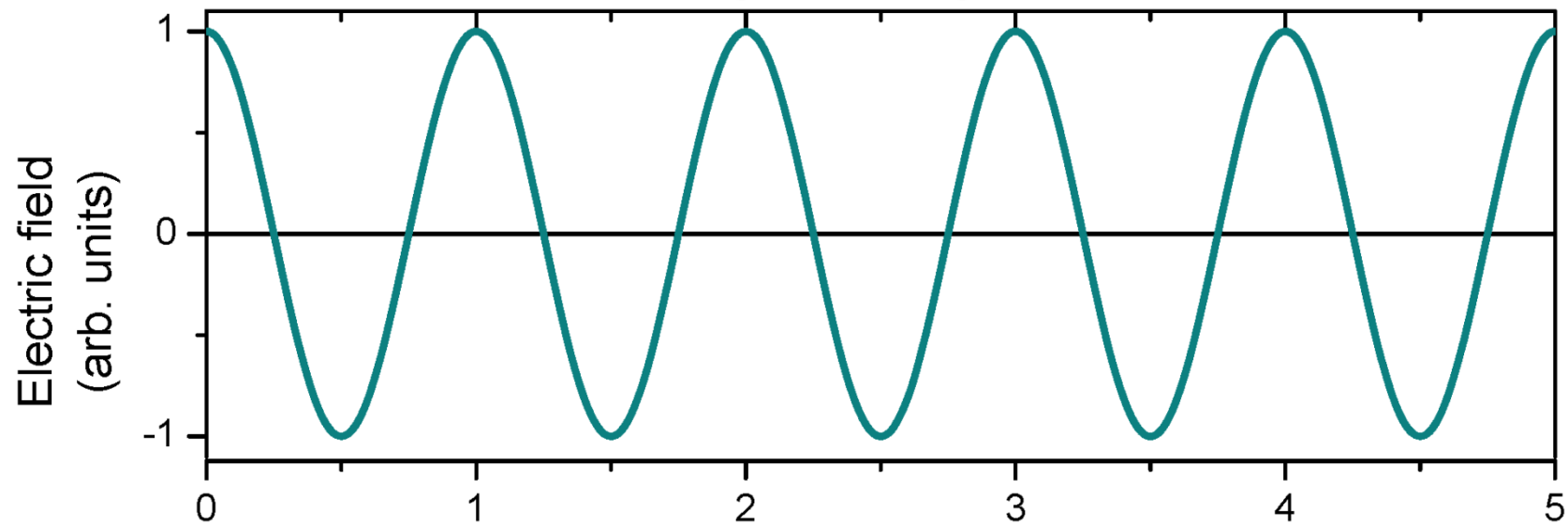
$$z_0 = qE_0/m_e\omega^2$$

N.B. $E(t) = -dA(t)/dt$, i. e. $v_0 = qA_0/m_e$

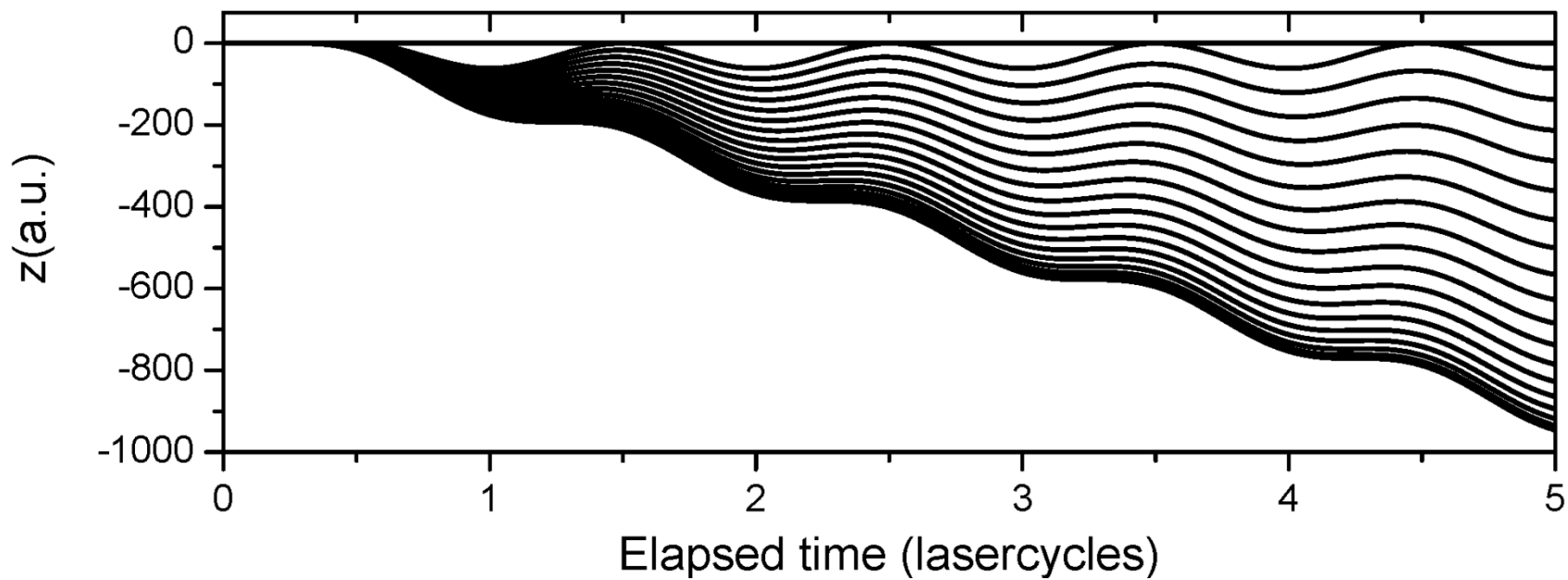
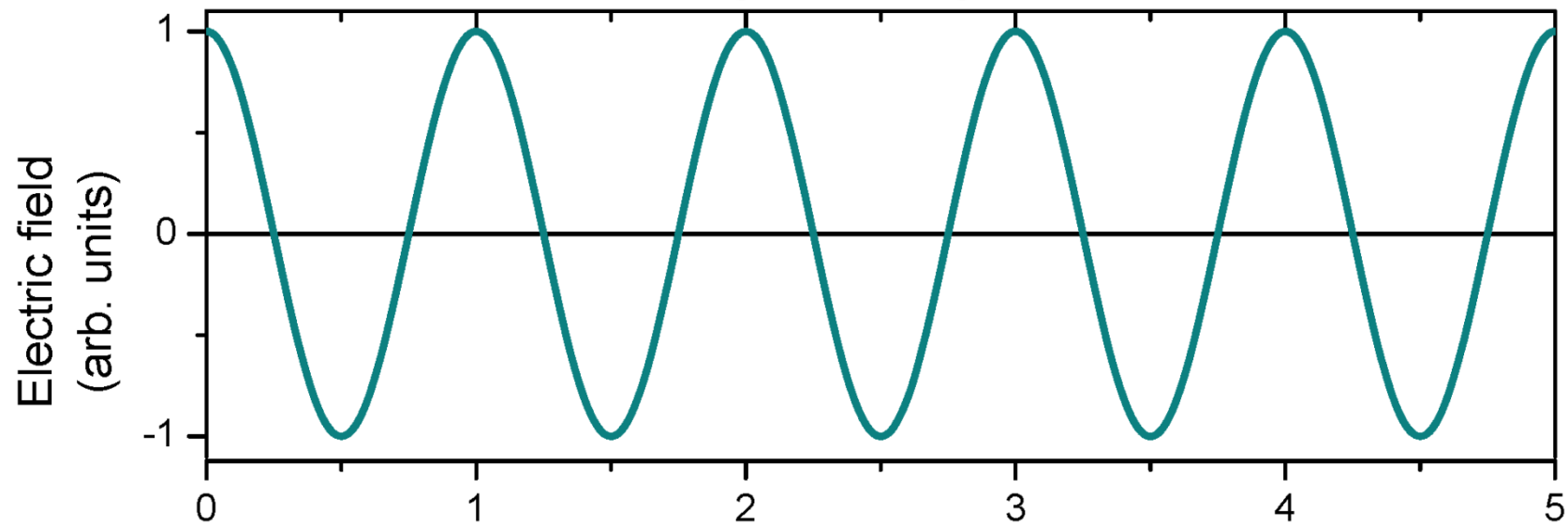
$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$



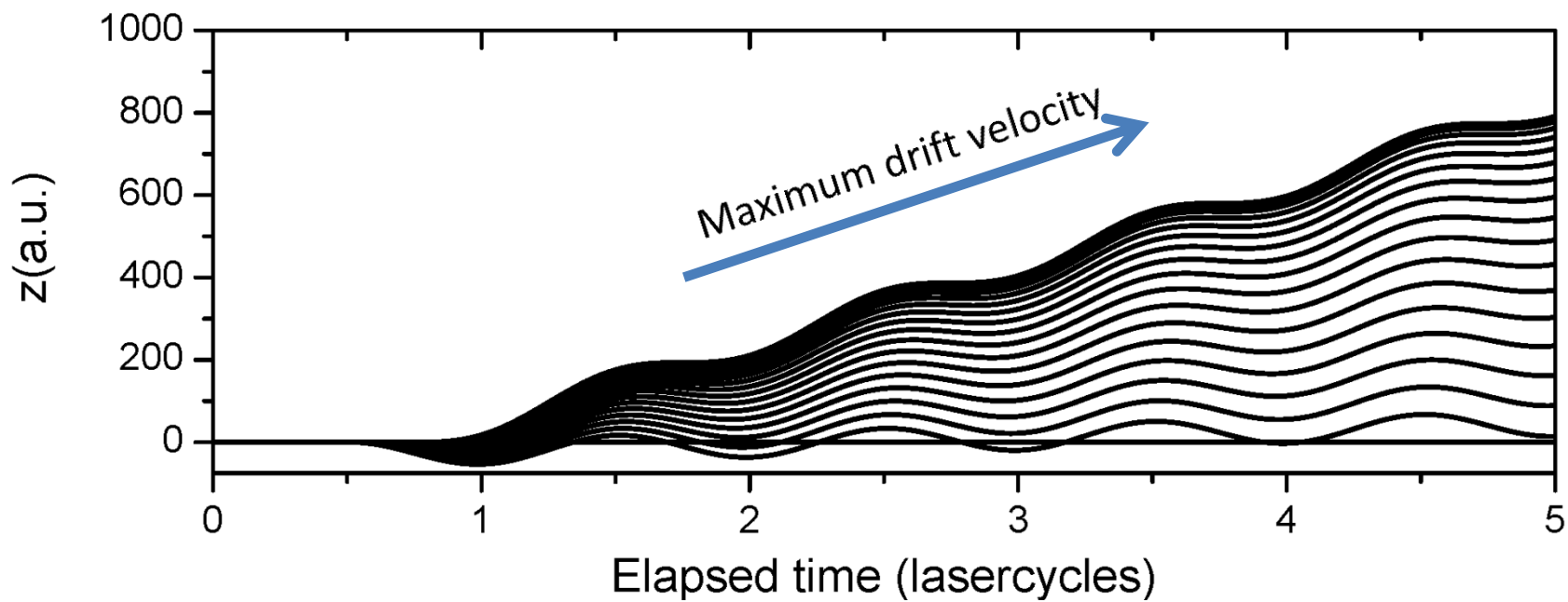
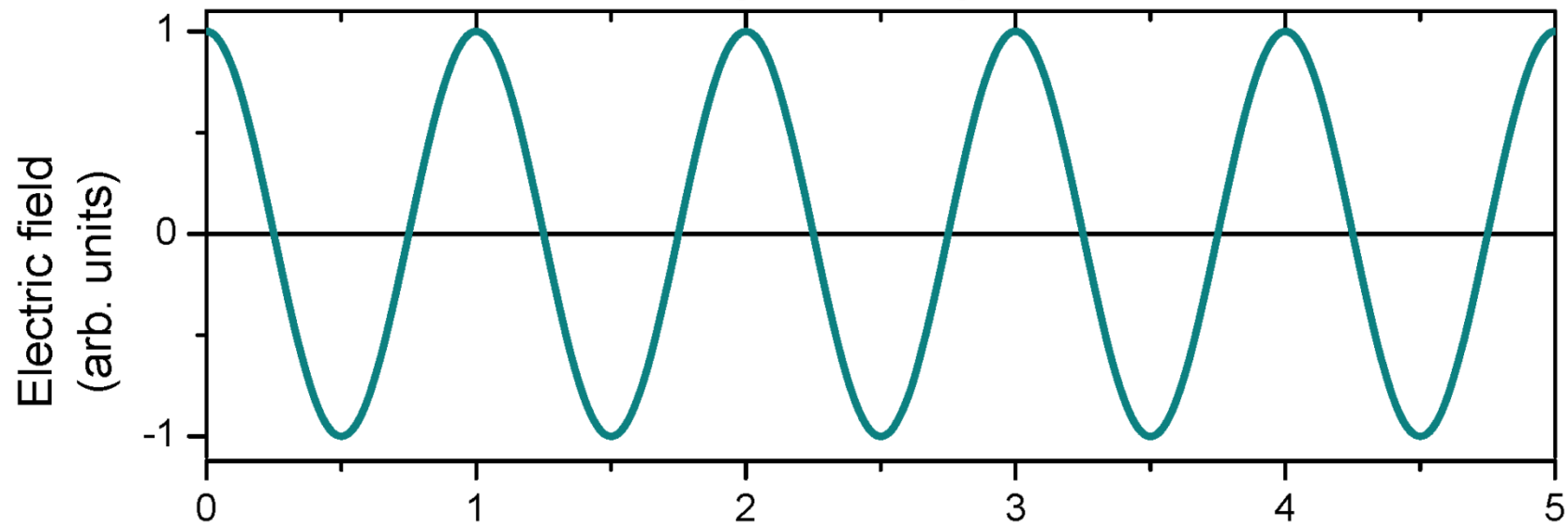
$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$



$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$



$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$



Maximum drift velocity

$$v_{\infty}(t_{\text{ionization}}) = \int_{t_{\text{ionization}}}^{\infty} a(t) dt =$$

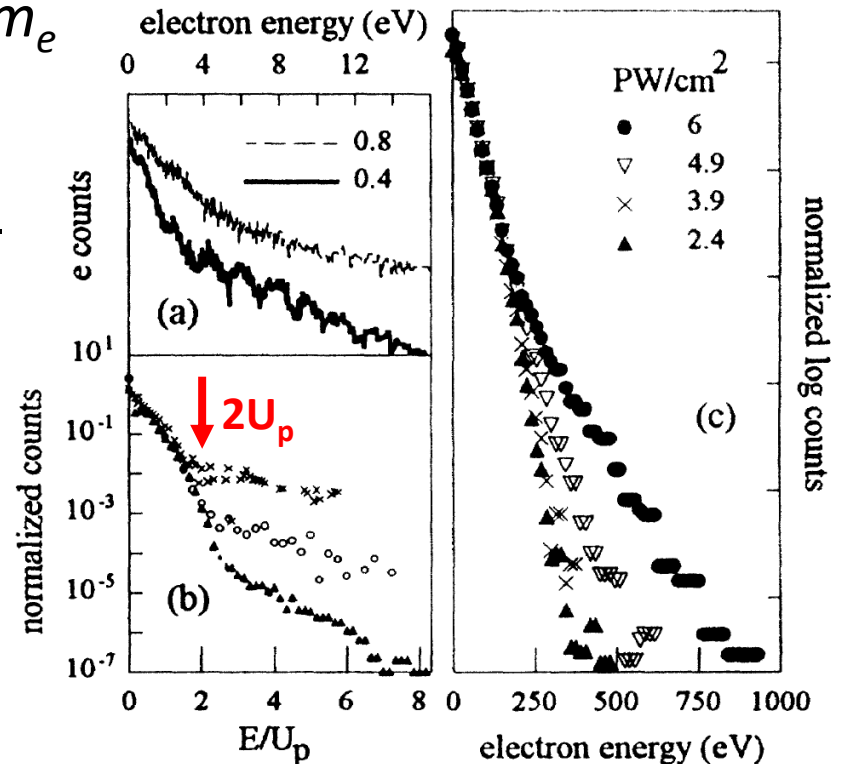
$$\int_{t_{\text{ionization}}}^{\infty} dt \left(\frac{qE(t)}{m_e} \right) \cos \omega t \sim \frac{qE(t)}{m_e \omega} \sin \omega t \Big|_{t_{\text{ionization}}}^{\infty} =$$

$$= \frac{-qE_0}{m_e \omega} \sin(\omega t_{\text{ionization}}) = qA(t_{\text{ionization}})/m_e$$

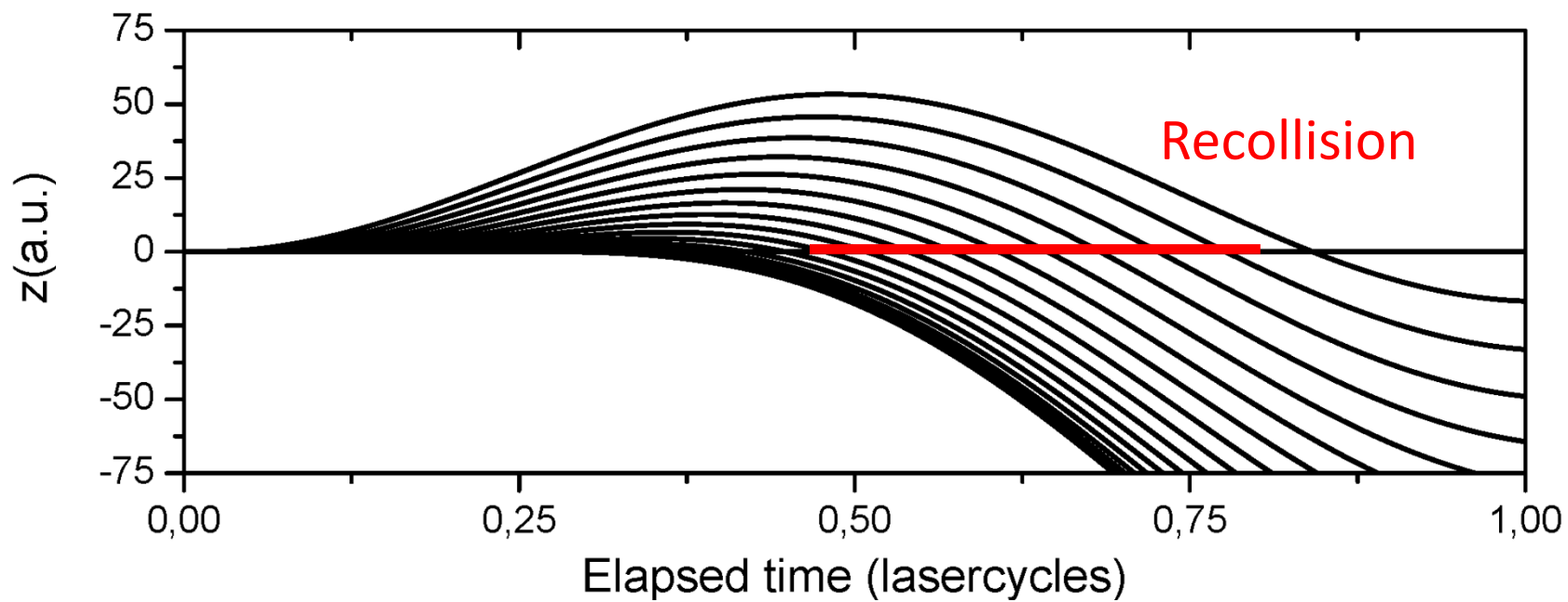
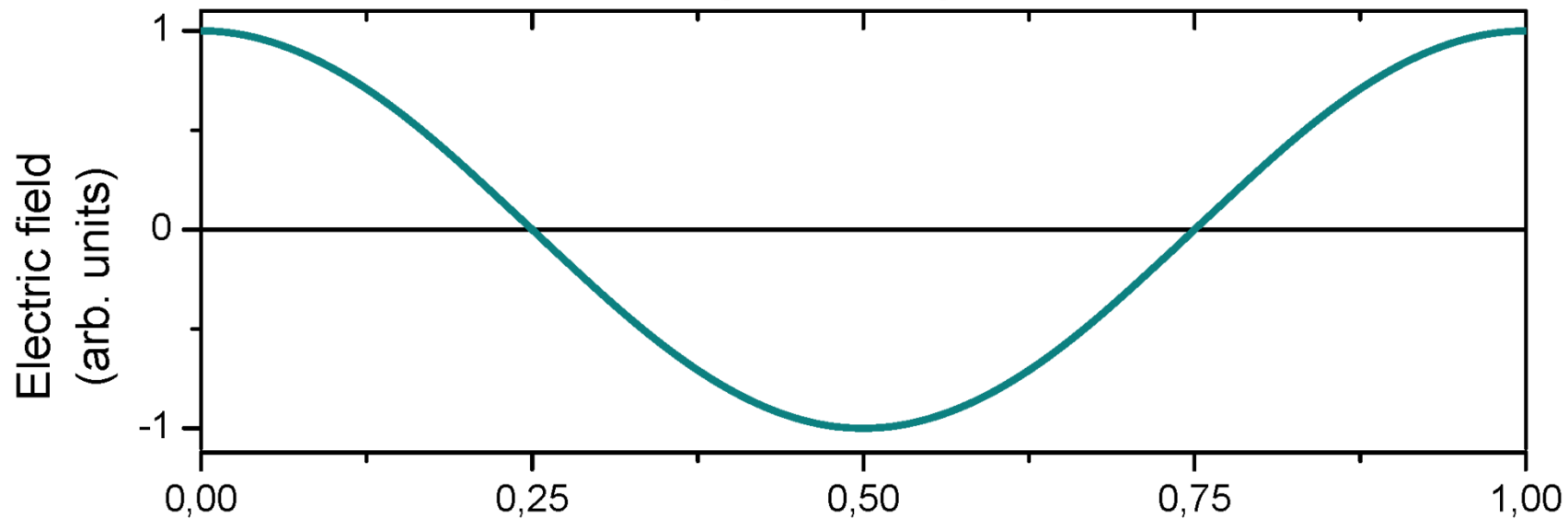
$$E_{k,max} = \frac{1}{2} m_e v_{max}^2 = \frac{q^2 E_0^2}{2m_e \omega^2} = \frac{q^2 A_0^2}{2m_e}$$

$2U_p$

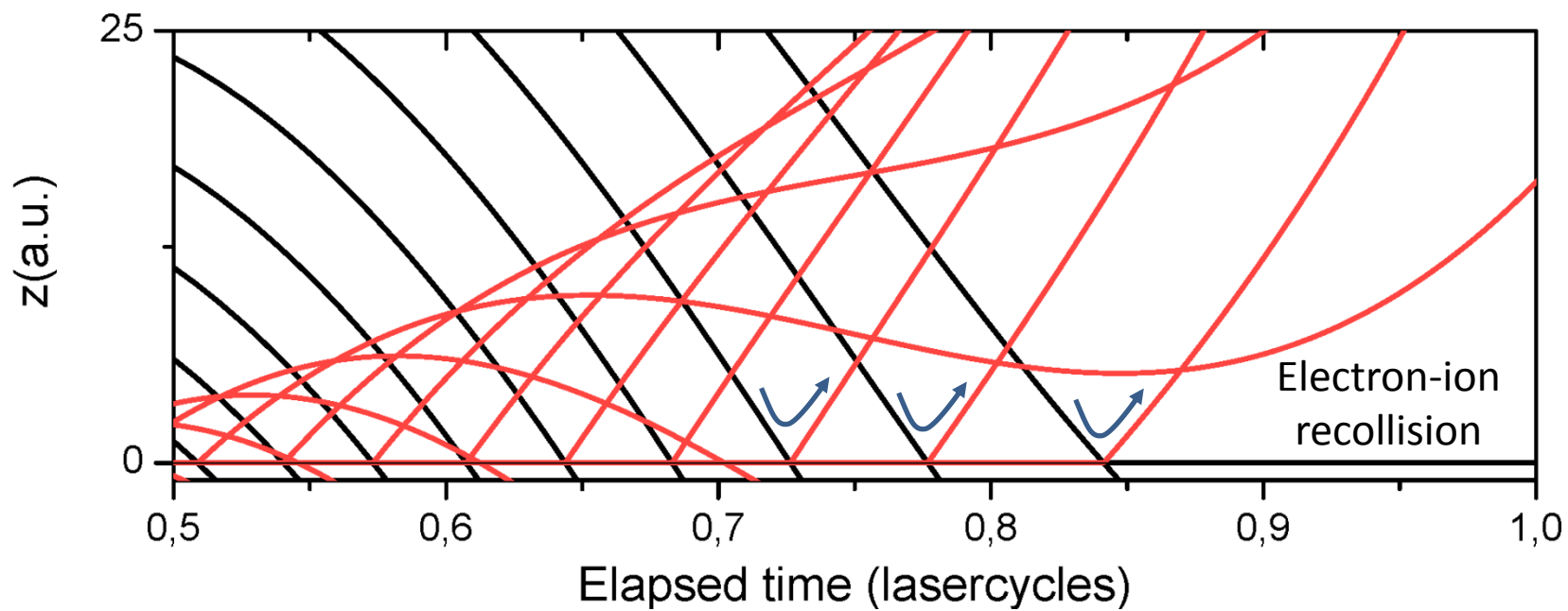
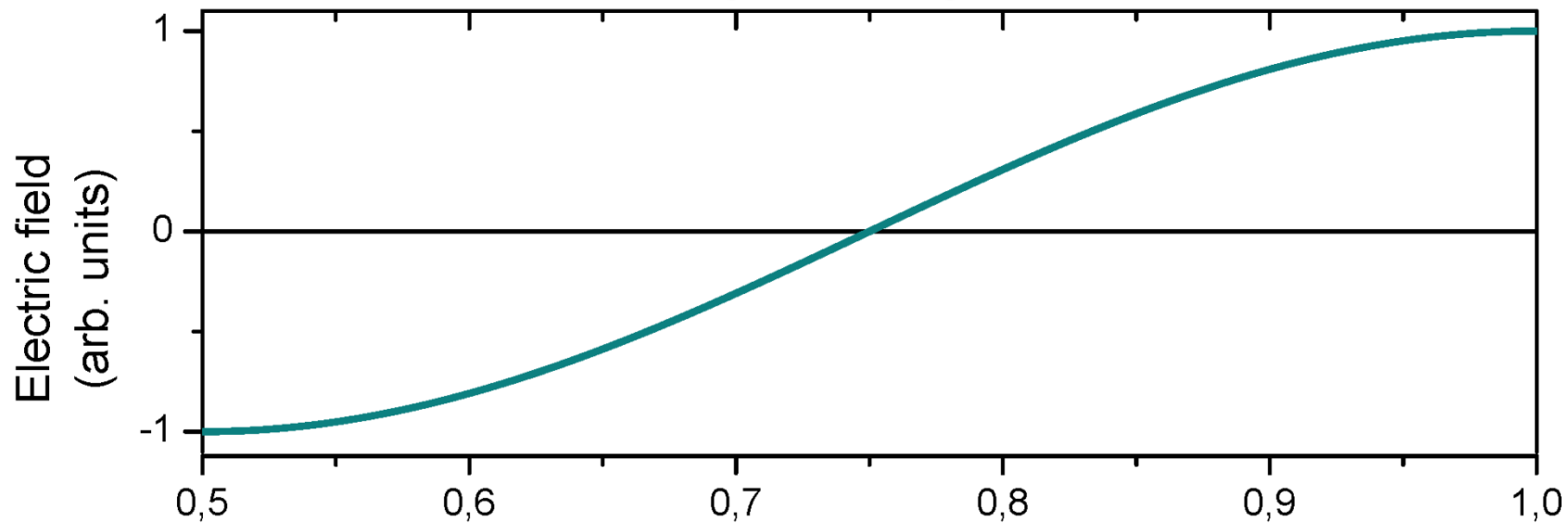
The photoelectron spectrum also contains energies $> 2U_p$



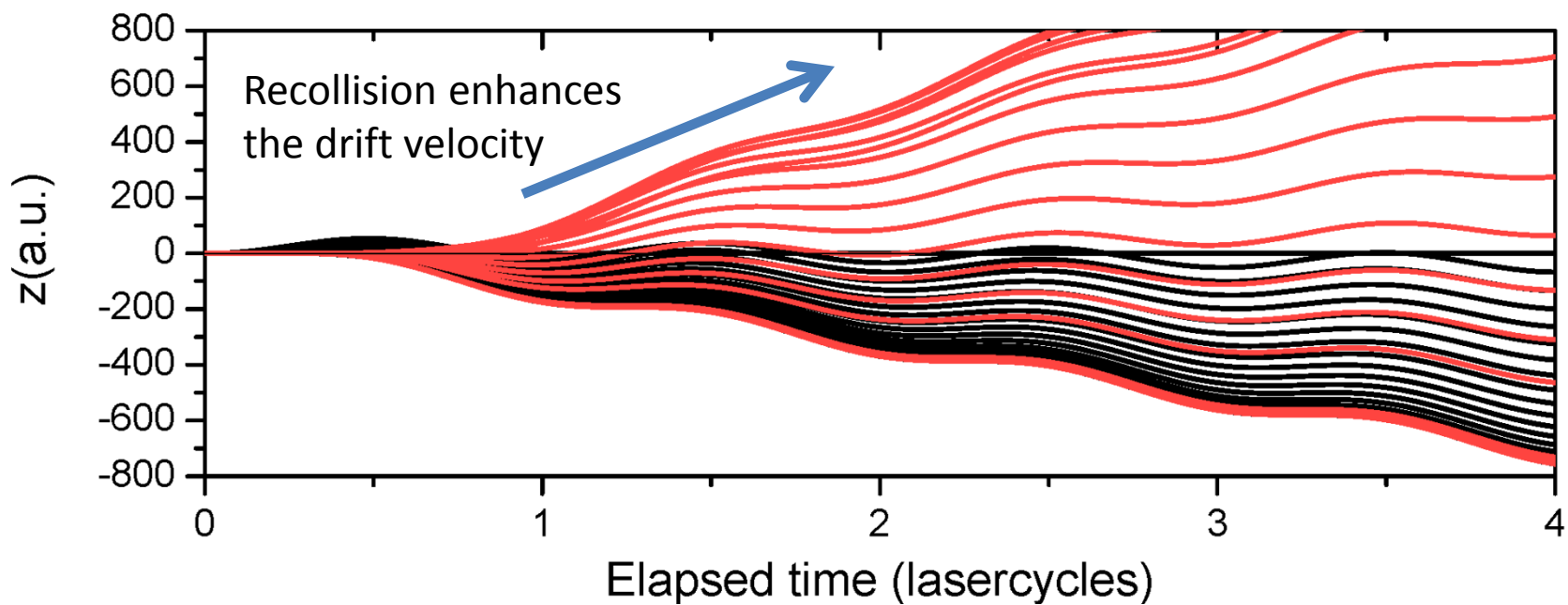
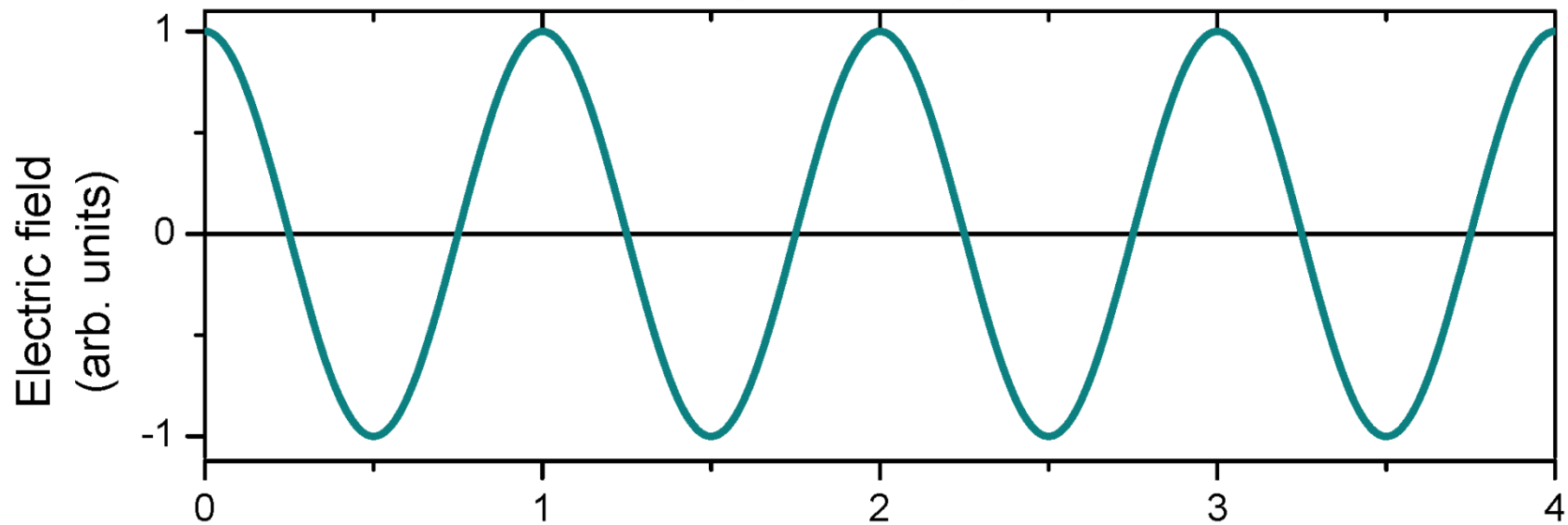
$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$



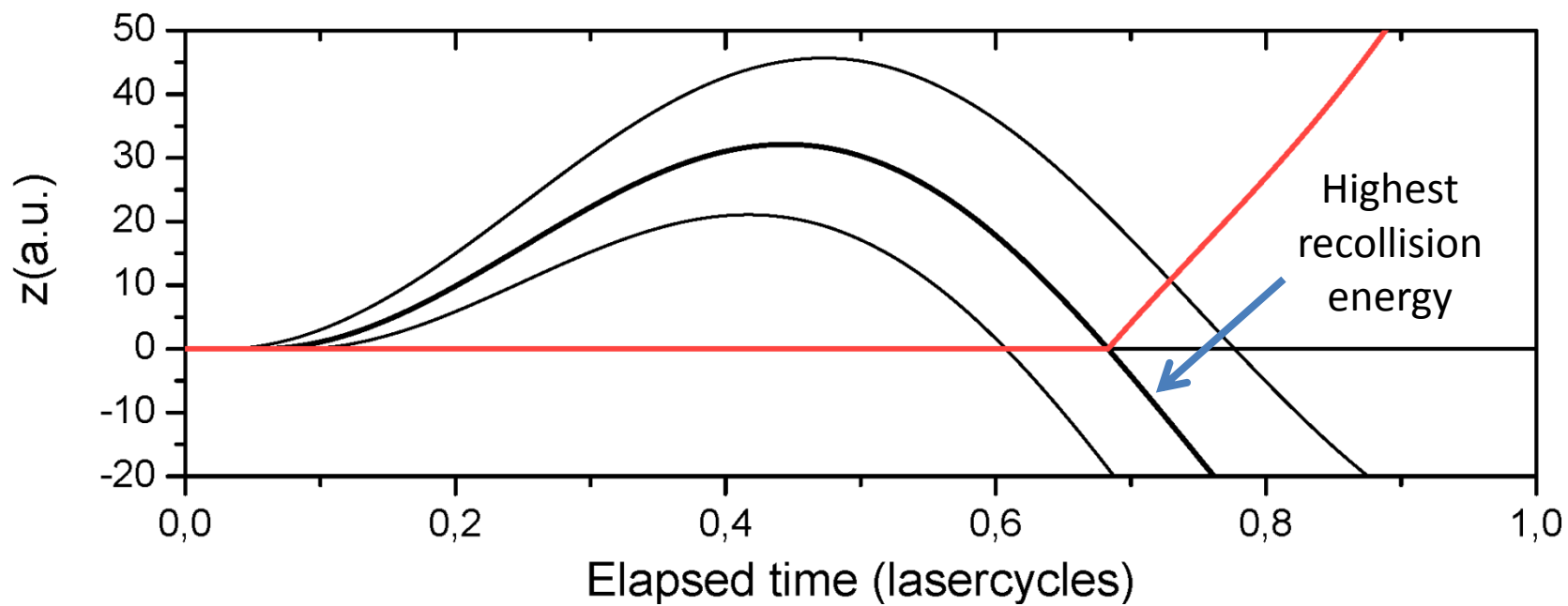
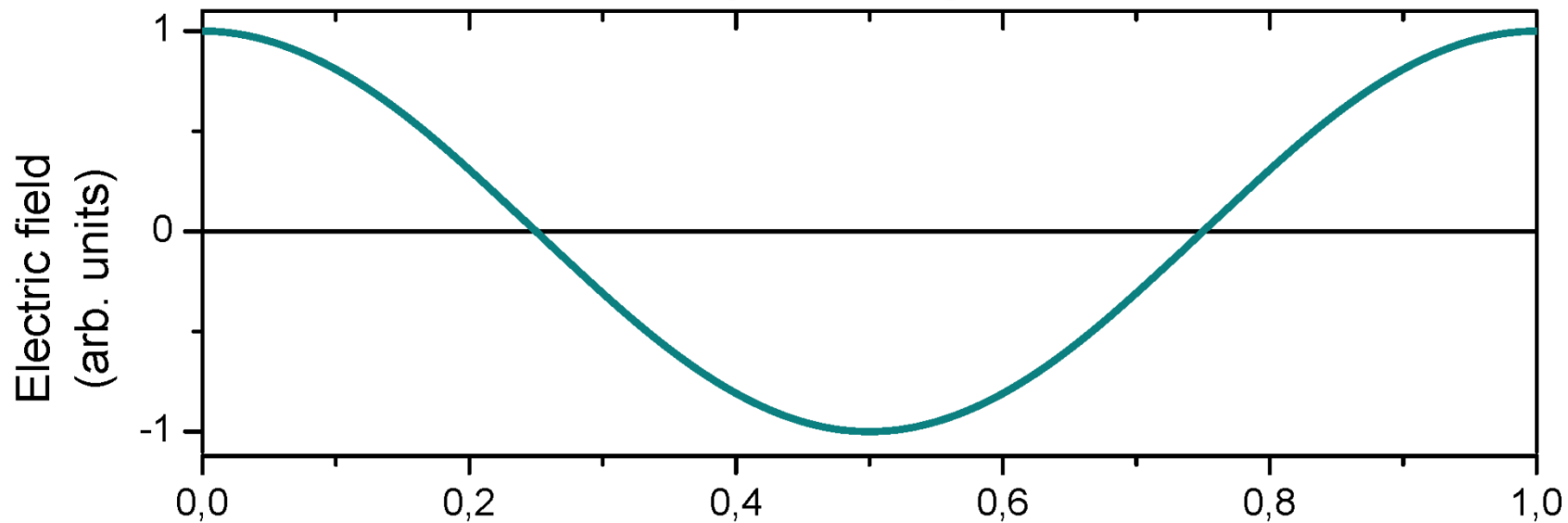
$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$

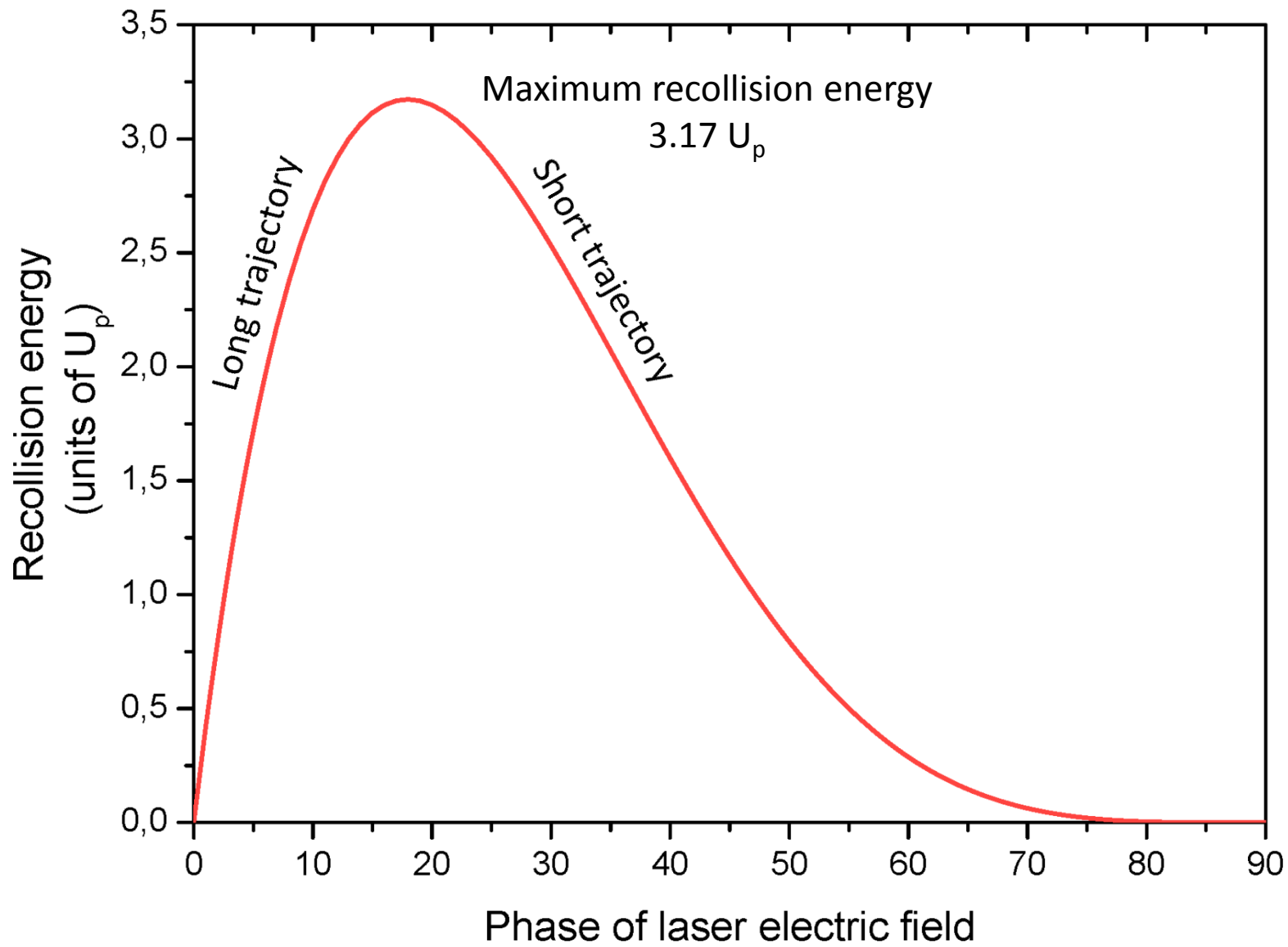


$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$



$$F = F_0 \cos(\omega t); F_0 = 0.1 \text{ a.u.}$$



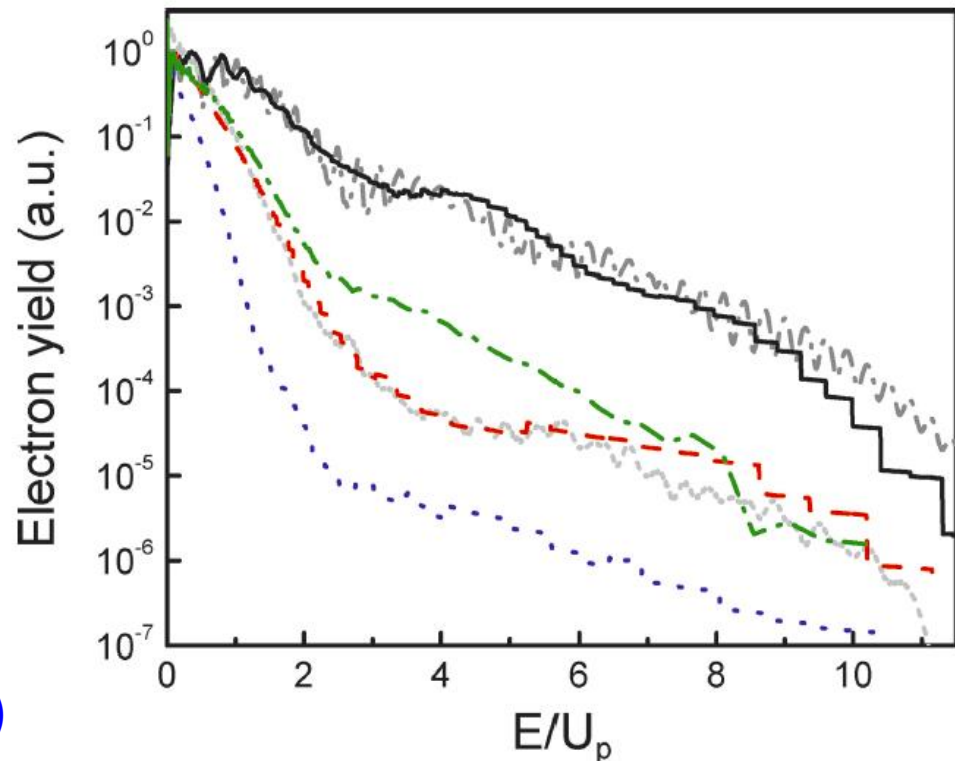


Recollision trajectories

The highest possible return energy is $3.17 U_p = 0.7925 q^2 A_0^2 / m_e$, corresponding to a velocity of $1.259 q A_0 / m_e$. This recollision occurs near a zero crossing of the field.

When the recollision flips the sign of the velocity, the field increases the velocity to apprx. $-2.259 q A_0 / m_e$, corresponding to an energy of $10.2 U_p$

ATI of Argon at 0.8 μm (black line), 1.3 μm (green line), 2 μm (red line), and 3.6 μm (blue line) at an intensity of 0.08 PW/cm².



Canonical momentum

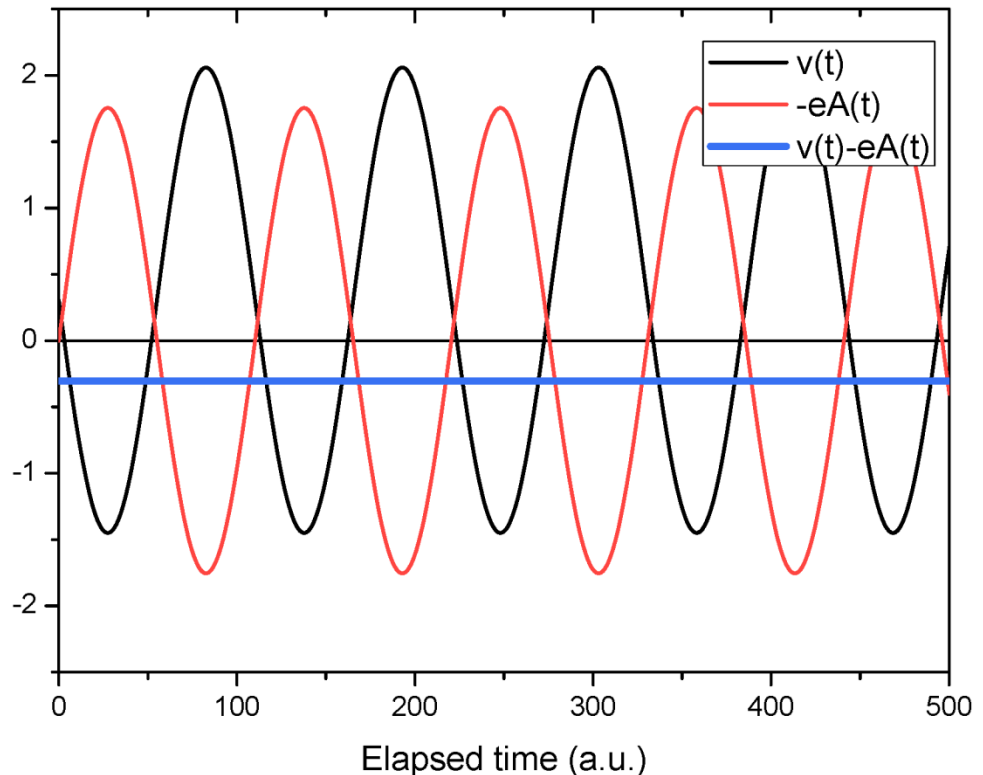
$$E(t) = E_0 \cos(\omega t) = -\frac{dA(t)}{dt} \quad A(t) = -A_0 \sin(\omega t) + \text{constant} \quad A_0 = \frac{E_0}{\omega}$$

$$v(t) = v_0 + \int_{t_0}^t a(t) dt = v_0 + \int_{t_0}^t \frac{-eE(t)}{m} dt = v_0 + \frac{e}{m} (A(t) - A(t_0))$$

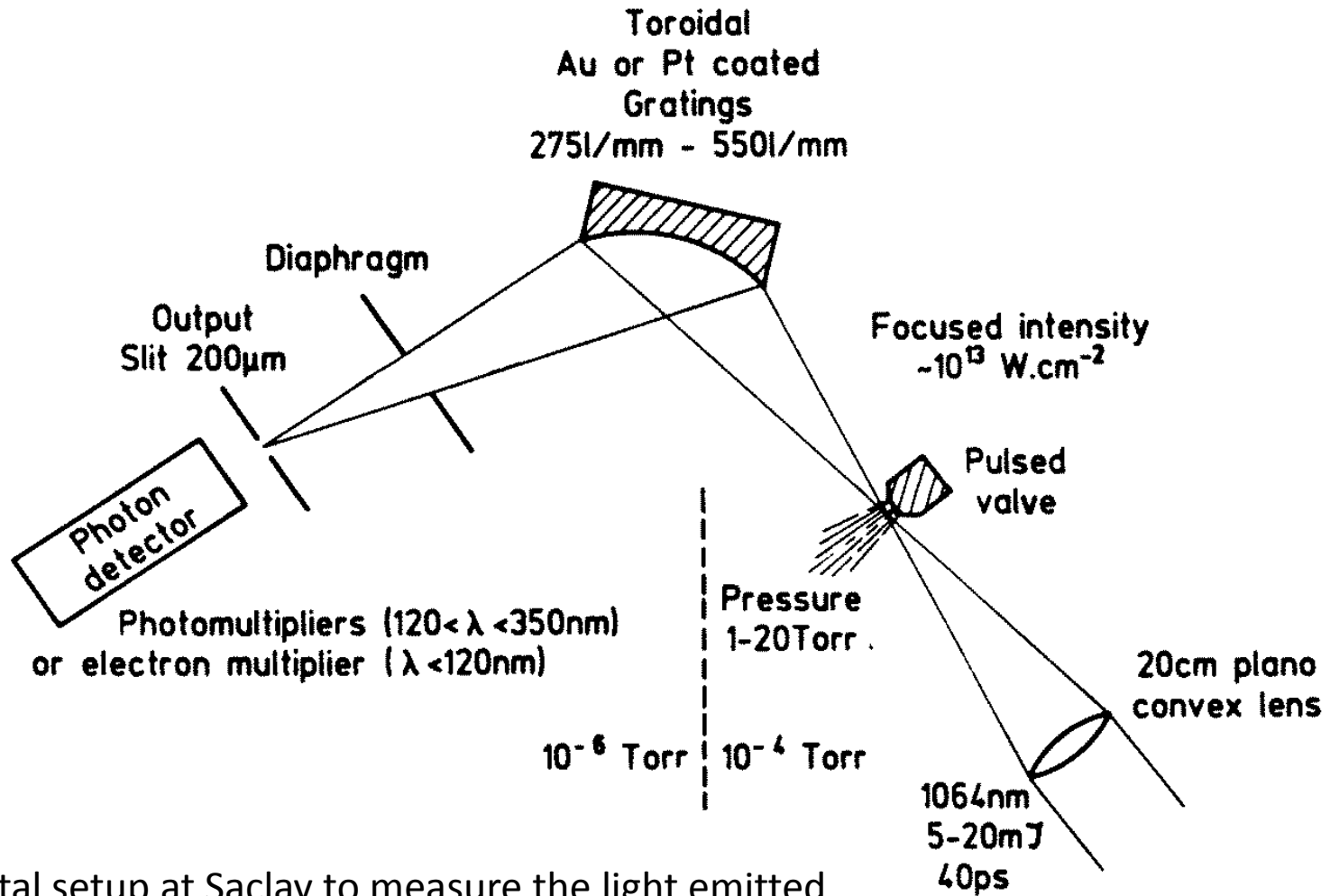
$$v(t) - \frac{e}{m} A(t) = v_0 - \frac{e}{m} A(t_0)$$

In a strong laser field
the conserved
quantity is the
canonical
momentum:

$$\mathbf{p} = m\mathbf{v}(t) - e\mathbf{A}(t)$$



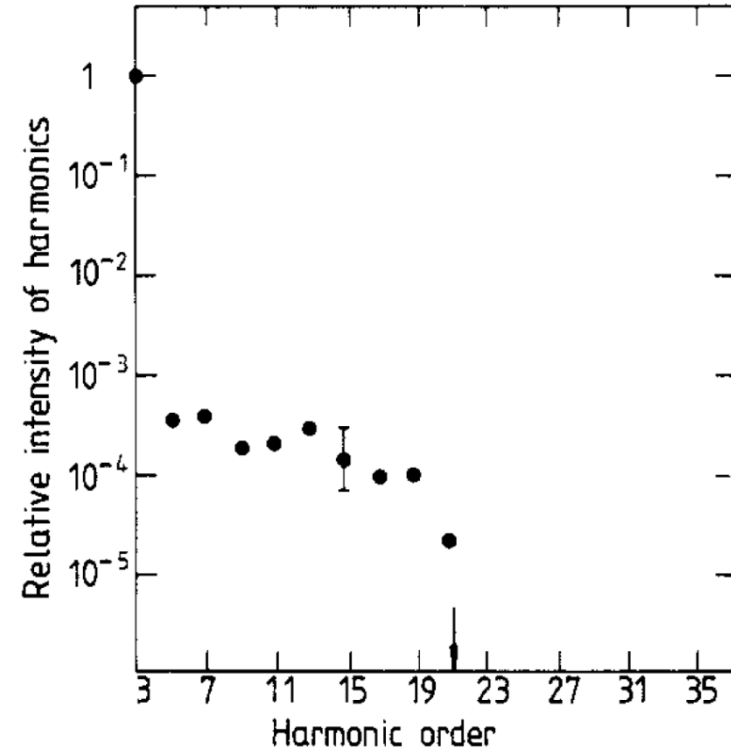
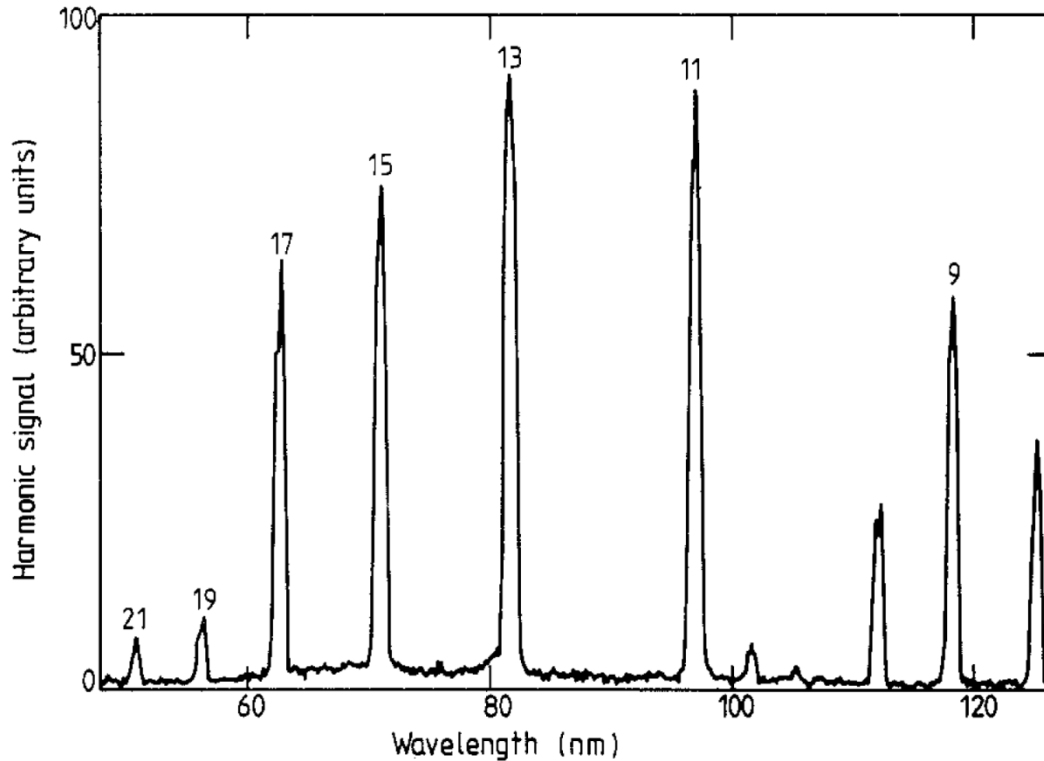
Mid-1980's: studies of ATI



Experimental setup at Saclay to measure the light emitted during above-threshold ionization experiments

A. L'Huillier et al., in 'Atoms in Intense Laser Fields', edited by Gavrila and Muller, (Academic Press, 1992)

Discovery of High-Harmonic Generation (HHG)

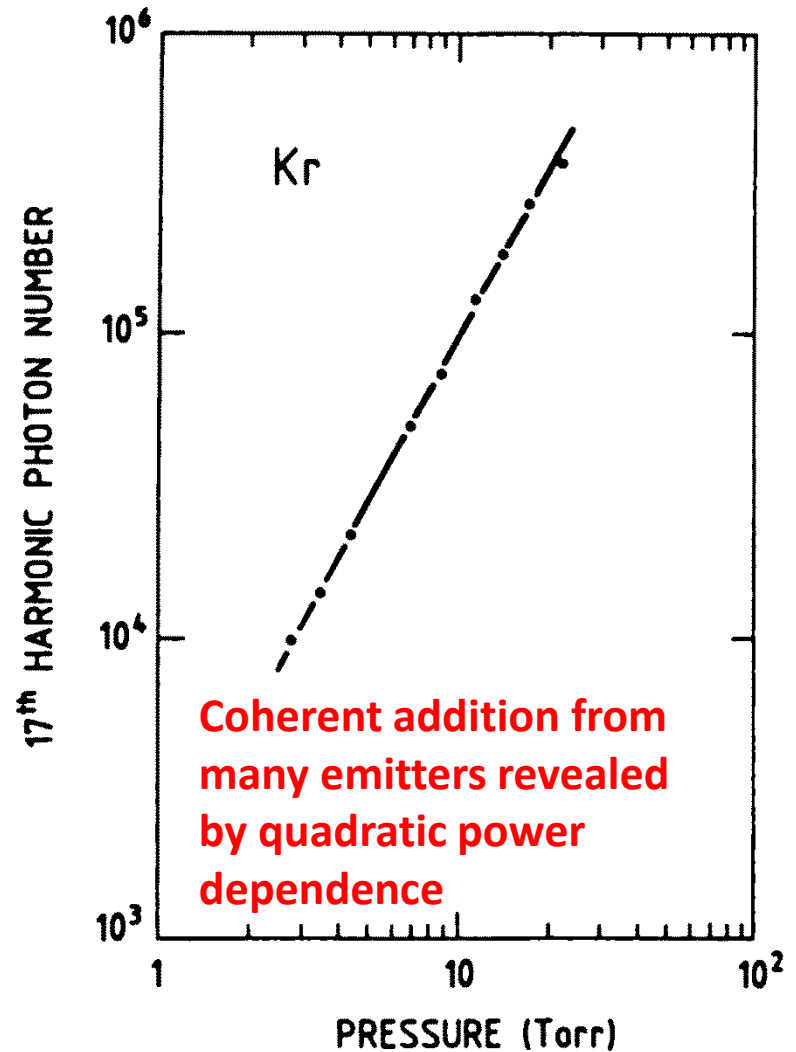
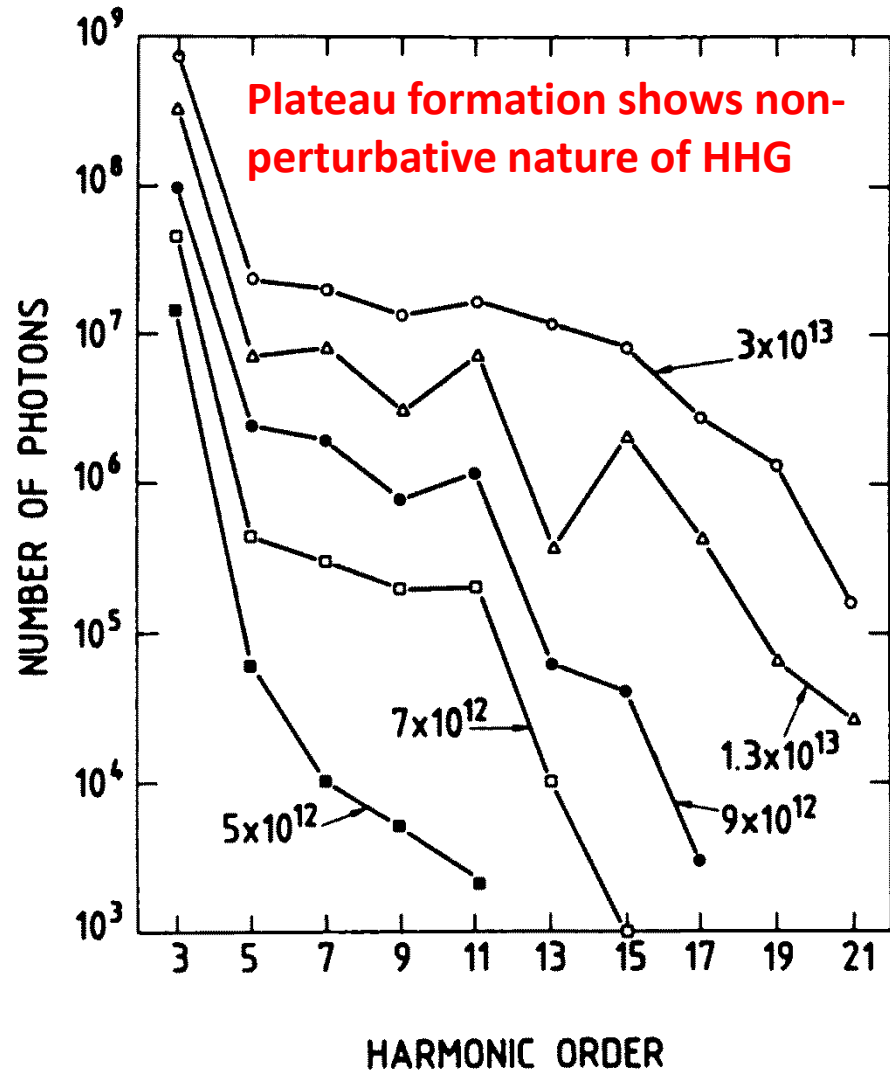


High-harmonic generation in Xe using a 30 ps, 1064 nm laser focused to ca. 10^{13} W/cm²

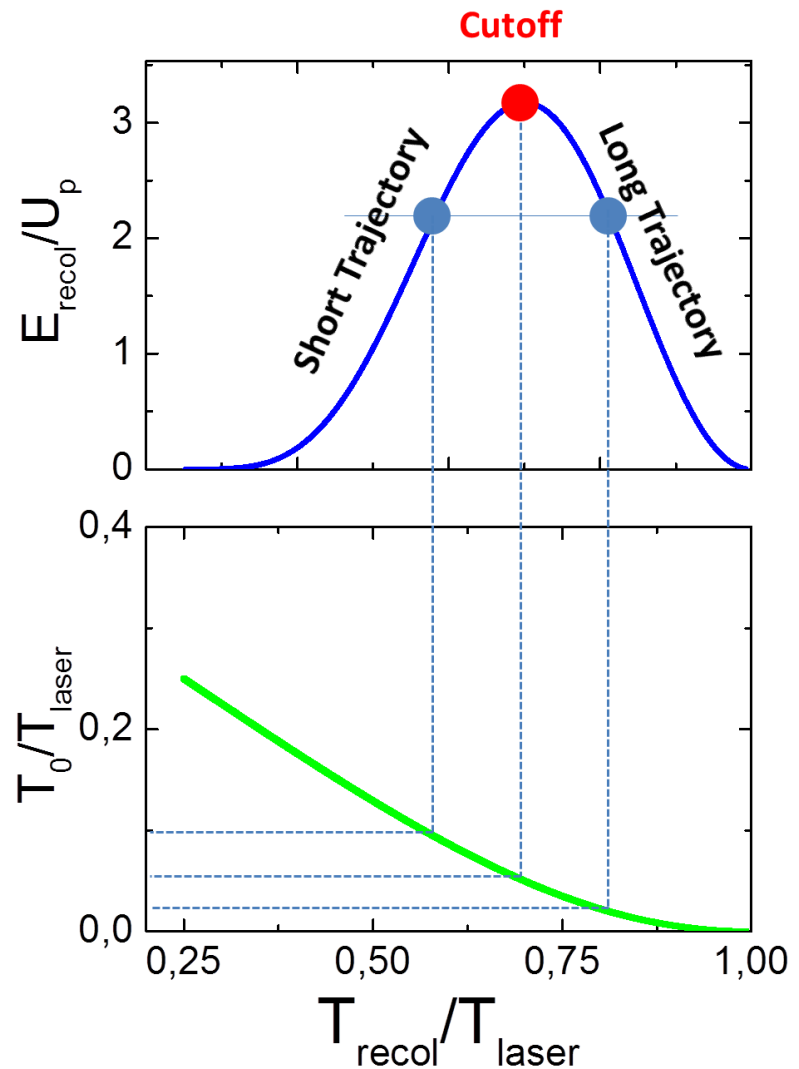
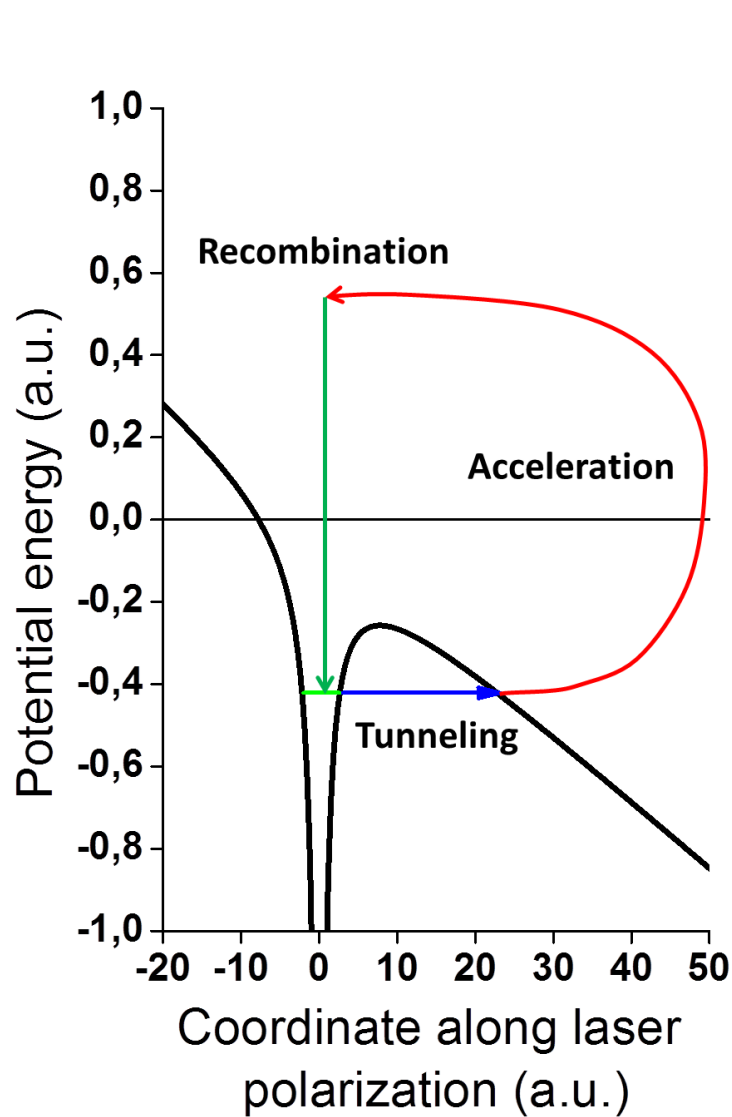
[M. Ferray et al., J. Phys. B 21 L31 \(1988\)](#)

+ similar observations around the time in the Rhodes-group using 248 nm driver lasers

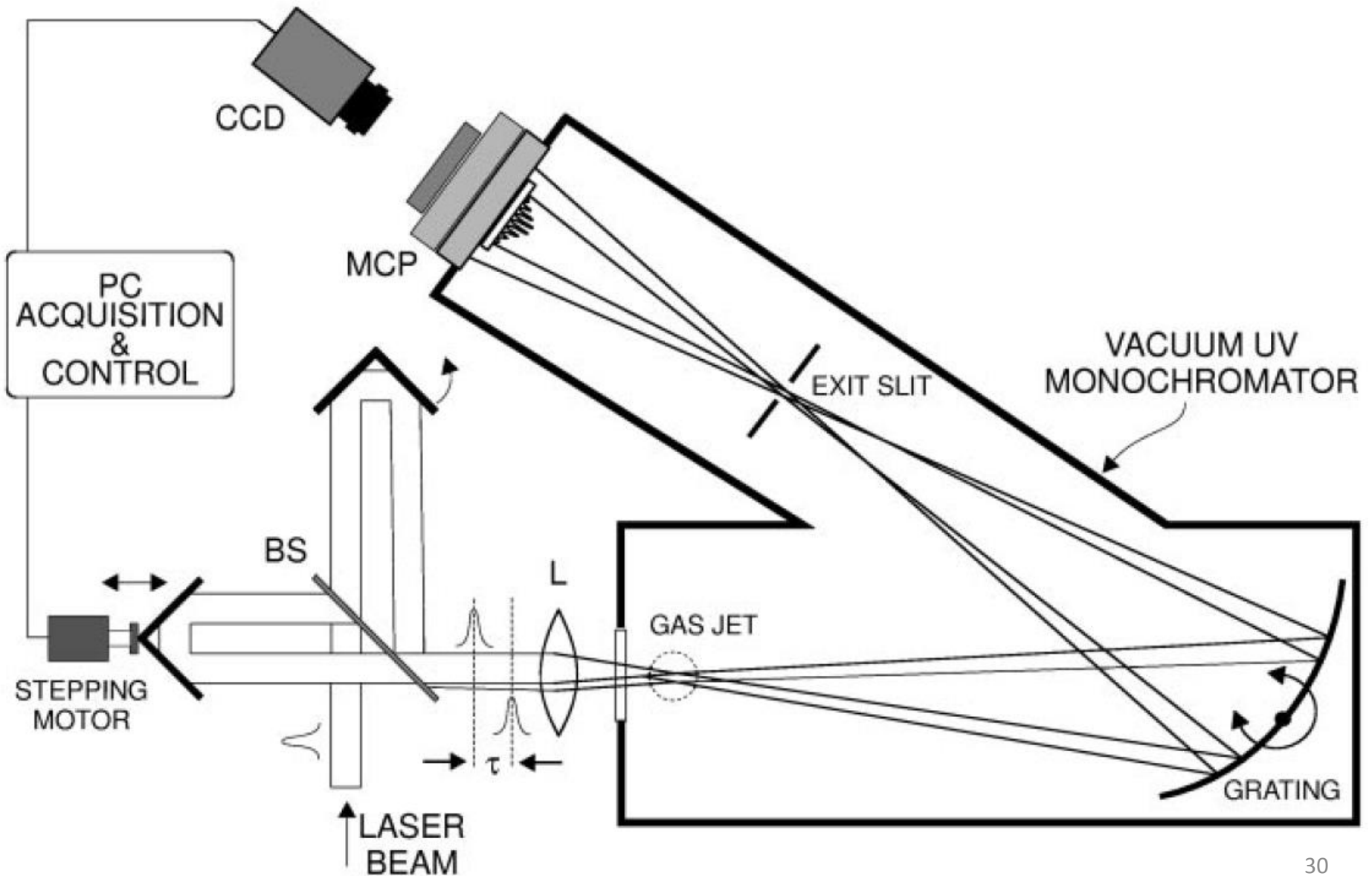
Intensity and pressure dependence



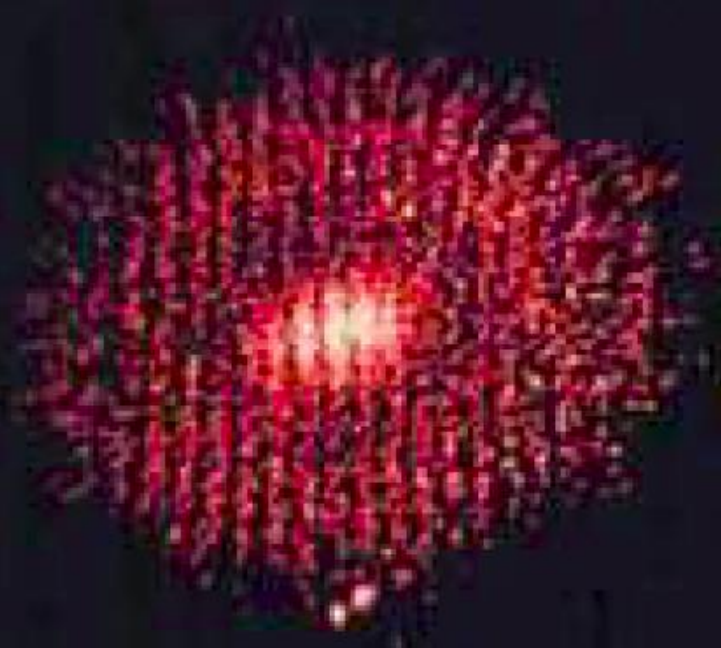
Cut-off law: $E_{cutoff} = IP + 3.17 U_p$



HHG via short and long trajectories

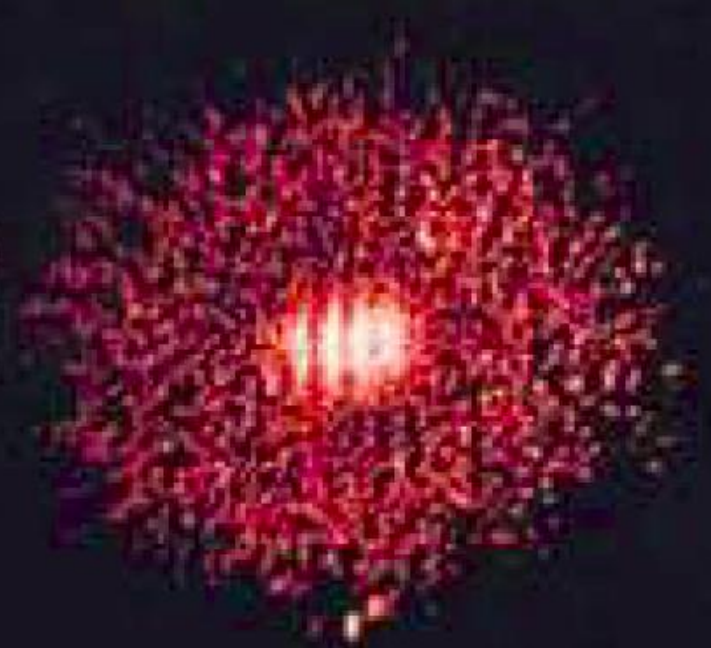


15 th harmonic (53 nm)



0 fs

(delay)



15 fs

Phase-matching

In the HHG medium, the driver laser and the generated harmonics move with a different phase velocity

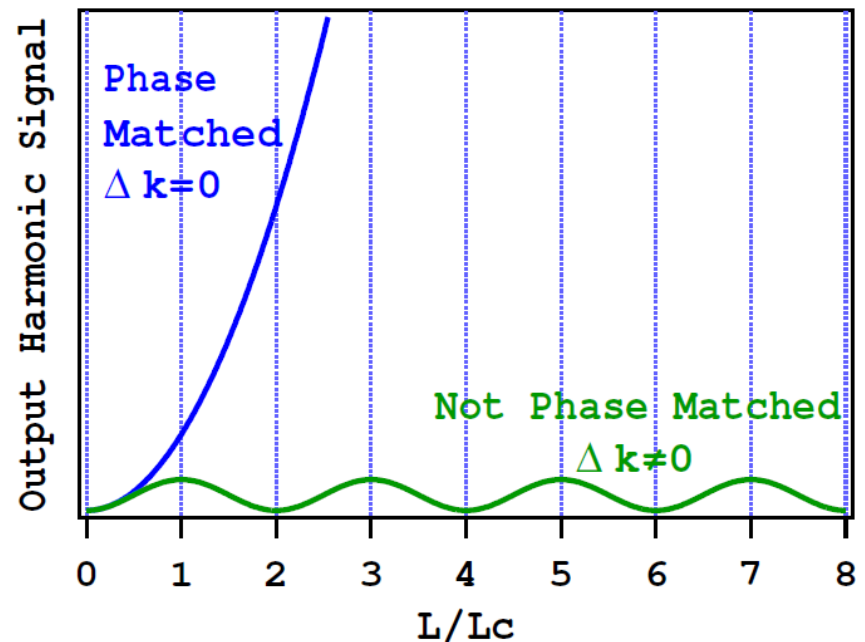
$$v_{phase} = \lambda v = \frac{\lambda \omega}{2\pi} = \frac{\omega}{k}$$

laser frequency
laser wavevector

$$v_1 = \frac{\omega}{k_1} \quad v_q = \frac{q\omega}{k_q}$$

Phase matching: $\Delta k_q = k_q - qk_1 = 0$

Coherence length: $L_c = \frac{\pi}{\Delta k}$



Phase-matching

\sim pressure

$$\Delta k = \Delta k_a + \Delta k_{fe} + \Delta k_{foc} + \Delta k_{traj}$$

\sim intensity

In the HHG medium, phase matching is affected by

(i) The density of neutral gas $\Delta k_a = \frac{q\omega}{2\varepsilon_0 c} N [\alpha_{pol}(q\omega) - \alpha_{pol}(\omega)]$

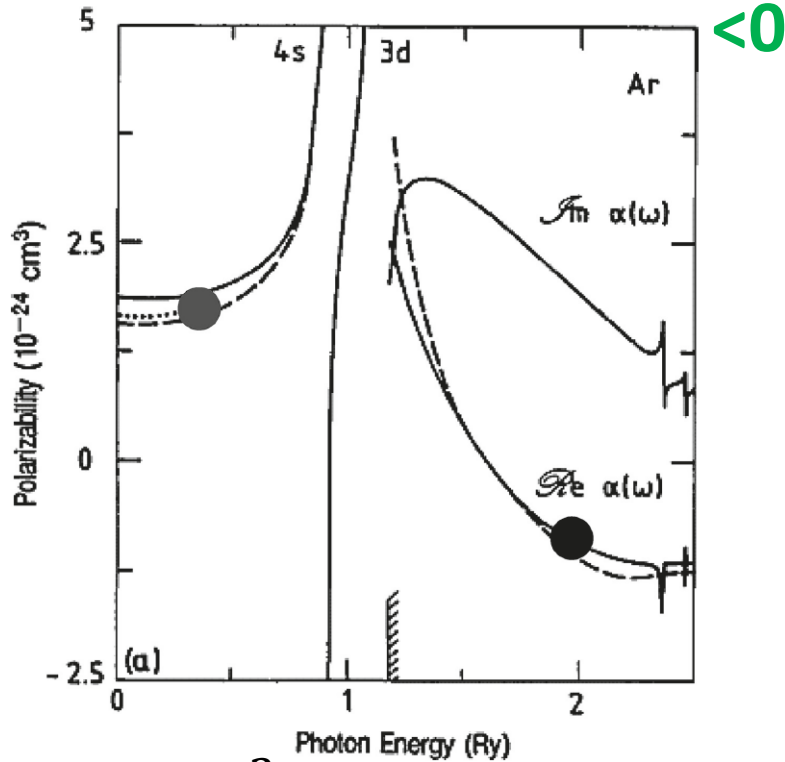
(ii) The density of free electrons $\Delta k_{fe} = \frac{qe^2}{2\varepsilon_0 cm\omega} N_e$

(iii) The laser focusing (no waveguiding assumed) $\Delta k_{foc} \sim \frac{q}{z_0}$

(iv) The electron trajectories $\Delta k_{traj} = \alpha_{traj} \frac{\partial I}{\partial z}$

Serendipity in Phase-matching

$$\Delta k_a = \frac{q\omega}{2\epsilon_0 c} N [\alpha_{pol}(q\omega) - \alpha_{pol}(\omega)]$$

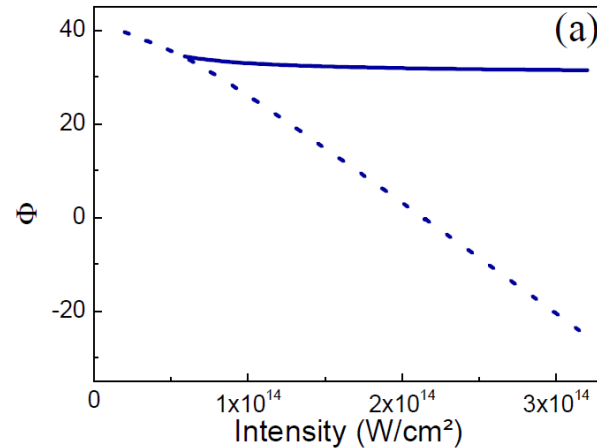


$$\Delta k_{fe} = \frac{qe^2}{2\epsilon_0 cm\omega} N_e > 0$$

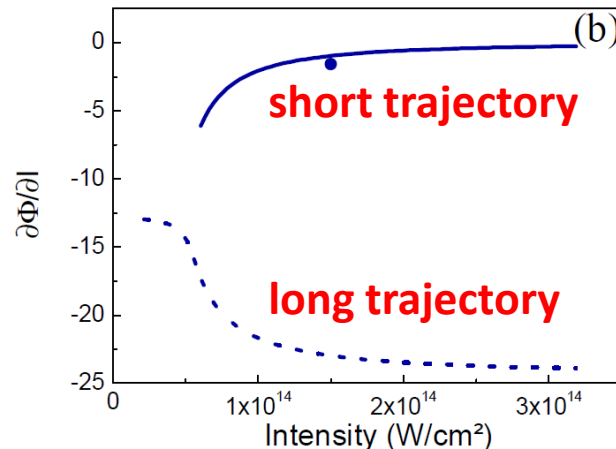
L'Huillier et al., JOSA B 7, 529 (1990)

$$\Delta k_{traj} = \alpha_{traj} \frac{\partial I}{\partial z}$$

$$\alpha_{traj} = -\frac{\partial \Phi}{\partial I} > 0$$

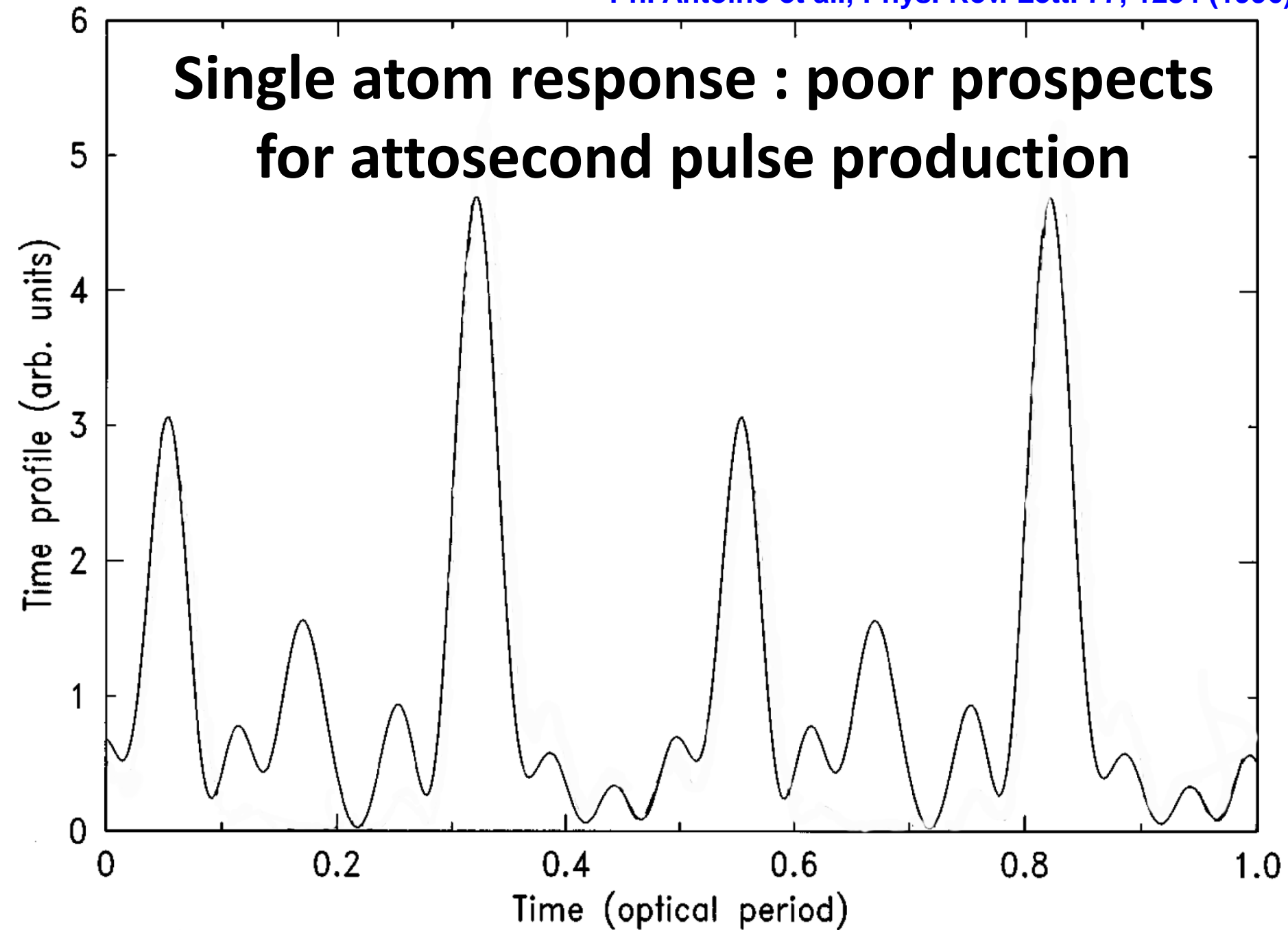


Short and long trajectory have different phase-matching conditions!

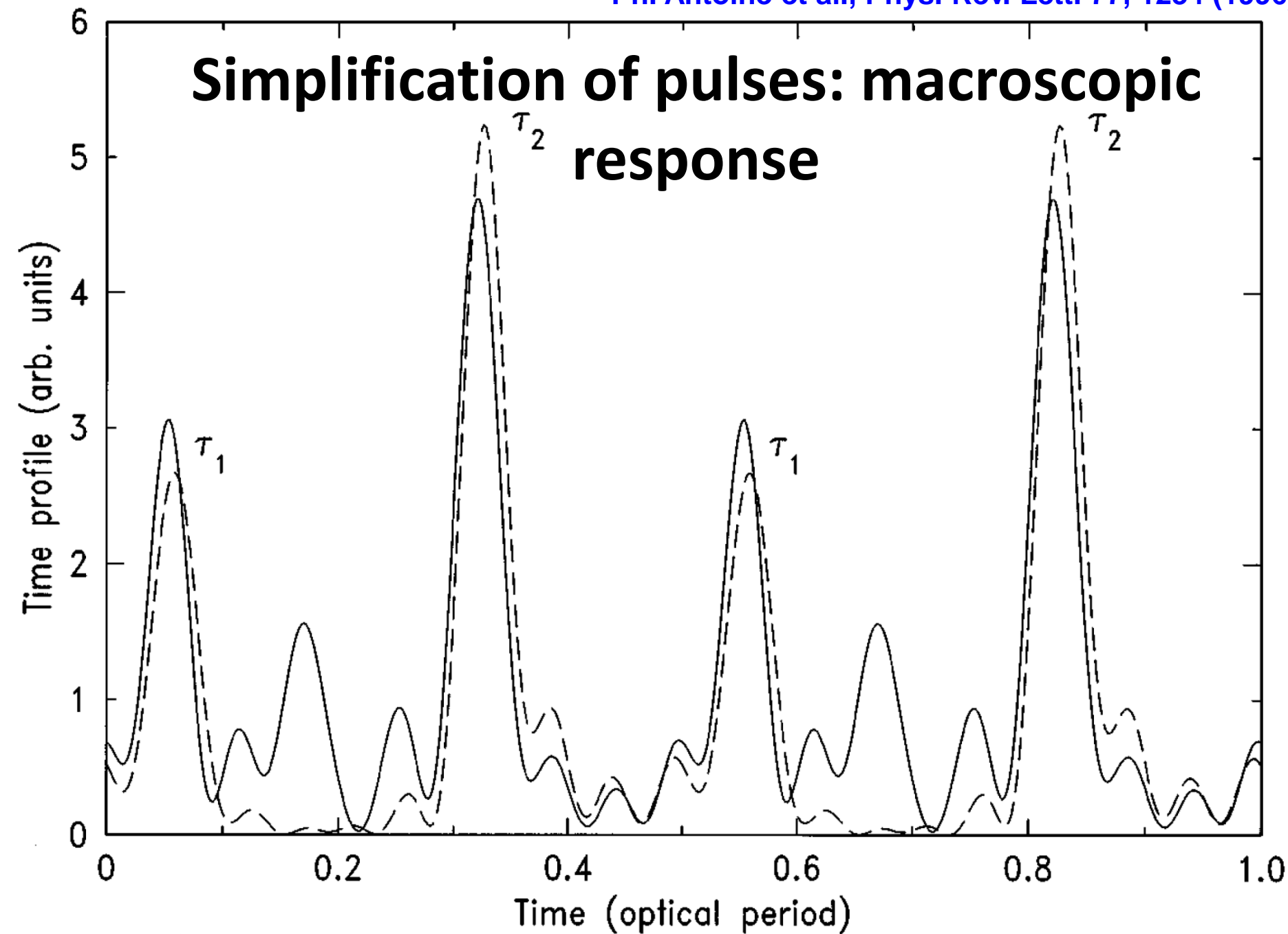


Varju et al.,
J. Mod. Opt.
2, 379 (2004)

Single atom response : poor prospects for attosecond pulse production



Simplification of pulses: macroscopic response



Generation of attosecond pulses by HHG

Science **292**, 1689 (2001);
DOI: 10.1126/science.1059413



P. M. Paul,¹ E. S. Toma,² P. Breger,¹ G. Mullot,³ F. Augé,³
Ph. Balcou,³ H. G. Muller,^{2*} P. Agostini¹

In principle, the temporal beating of superposed high harmonics obtained by focusing a femtosecond laser pulse in a gas jet can produce a train of very short intensity spikes, depending on the relative phases of the harmonics. We present a method to measure such phases through two-photon, two-color photoionization. We found that the harmonics are locked in phase and form a train of 250-attosecond pulses in the time domain. Harmonic generation may be a promising source for attosecond time-resolved measurements.

Attosecond metrology

M. Hentschel^{*†}, R. Kienberger^{*†}, Ch. Spielmann^{*}, G. A. Reider^{*}, N. Milosevic^{*}, T. Brabec^{*}, P. Corkum[‡], U. Heinzmann[§], M. Drescher[§] & F. Krausz^{*}

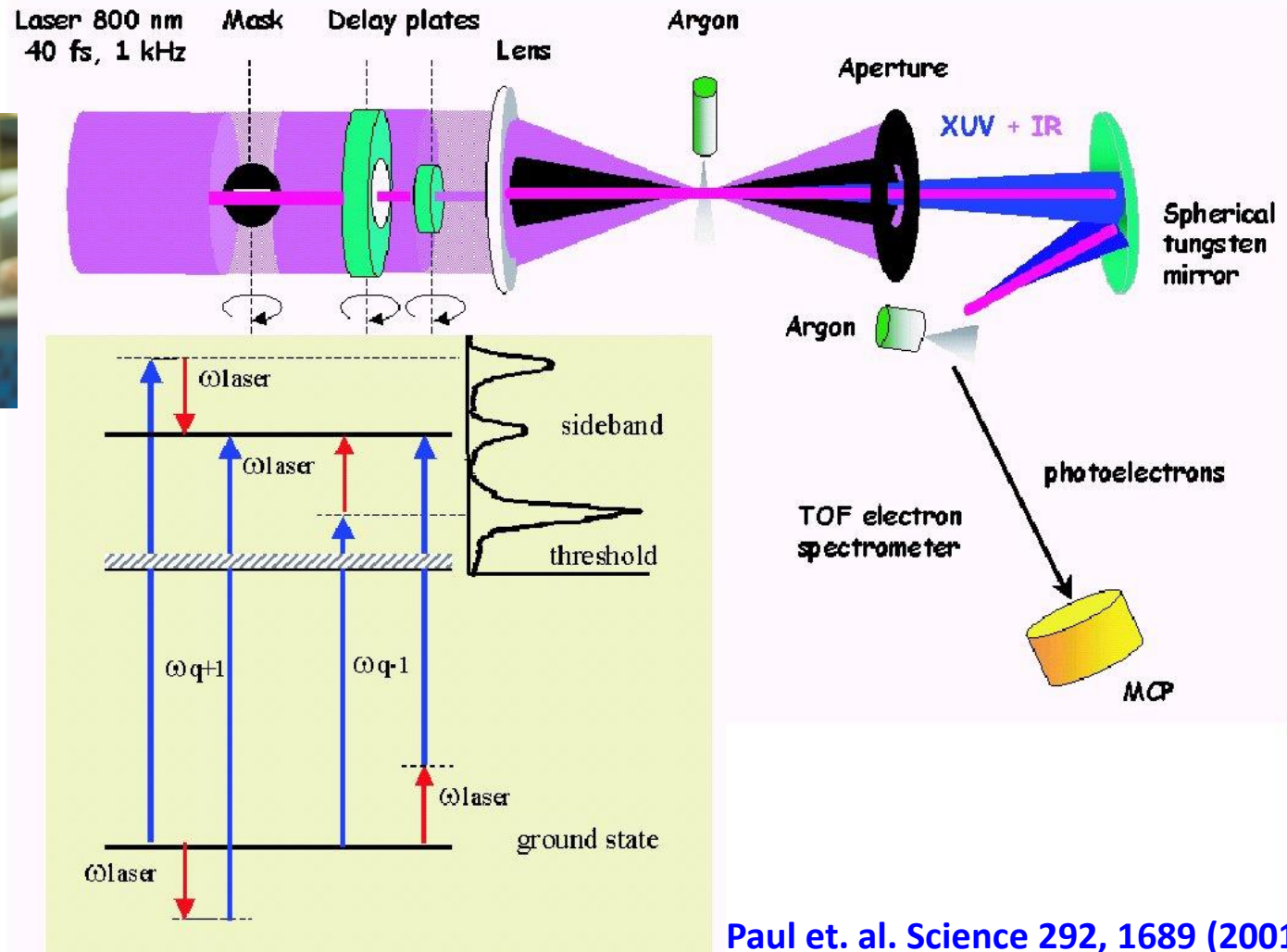
The generation of ultrashort pulses is a key to exploring the dynamic behaviour of matter on ever-shorter timescales. Recent developments have pushed the duration of laser pulses close to its natural limit—the wave cycle, which lasts somewhat longer than one femtosecond ($1 \text{ fs} = 10^{-15} \text{ s}$) in the visible spectral range. Time-resolved measurements with these pulses are able to trace dynamics of molecular structure, but fail to capture electronic processes occurring on an attosecond ($1 \text{ as} = 10^{-18} \text{ s}$) timescale. Here we trace electronic dynamics with a time resolution of $\leq 150 \text{ as}$ by using a subfemtosecond soft-X-ray pulse and a few-cycle visible light pulse. Our measurement indicates an attosecond response of the atomic system, a soft-X-ray pulse duration of $650 \pm 150 \text{ as}$ and an attosecond synchronism of the soft-X-ray pulse with the light field. The demonstrated experimental tools and techniques open the door to attosecond spectroscopy of bound electrons.

RABBITT

Reconstruction of Attosecond harmonic Beating By Interference of Two-photon Transitions

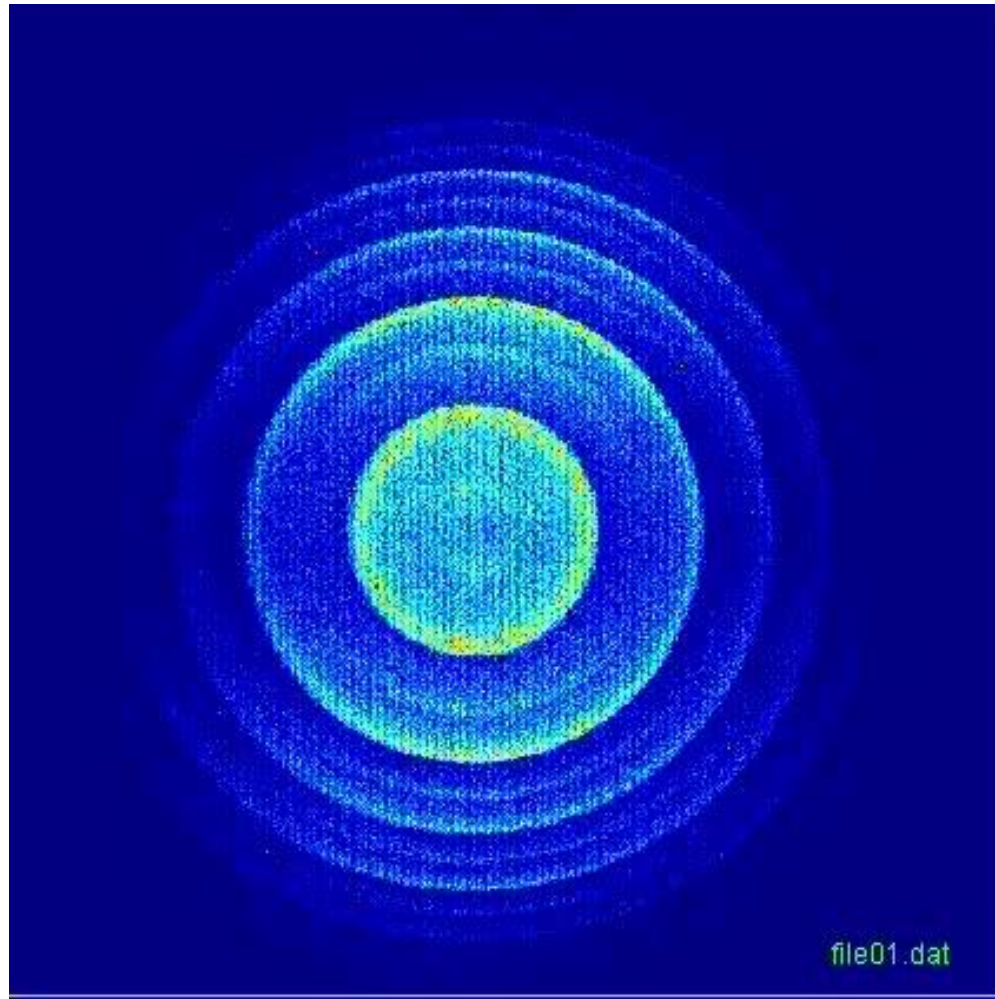


Harm-Geert Muller



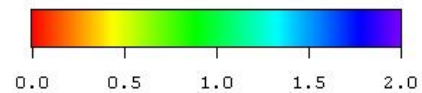
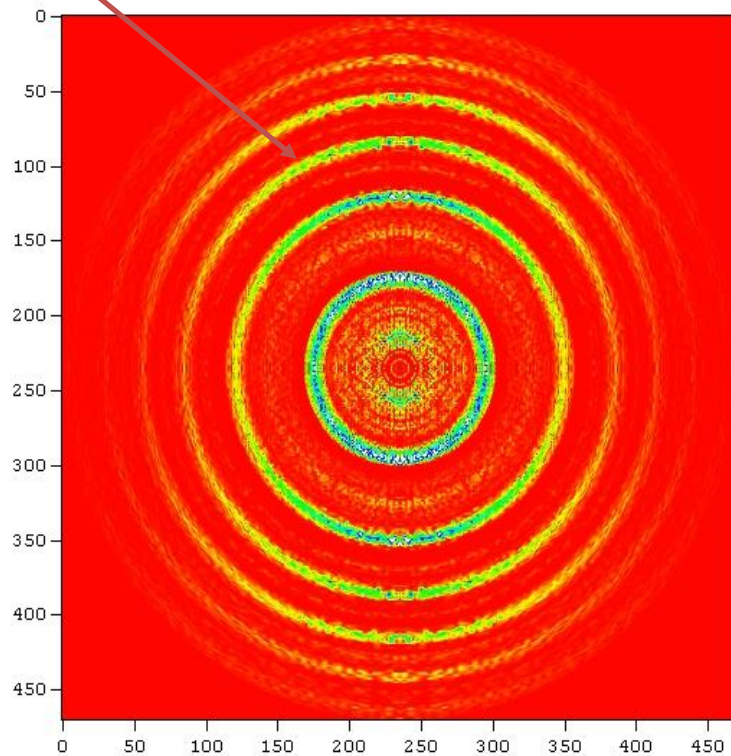
Paul et. al. Science 292, 1689 (2001)

Raw Photoelectron Images vs Time-delay

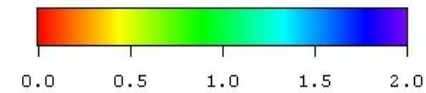
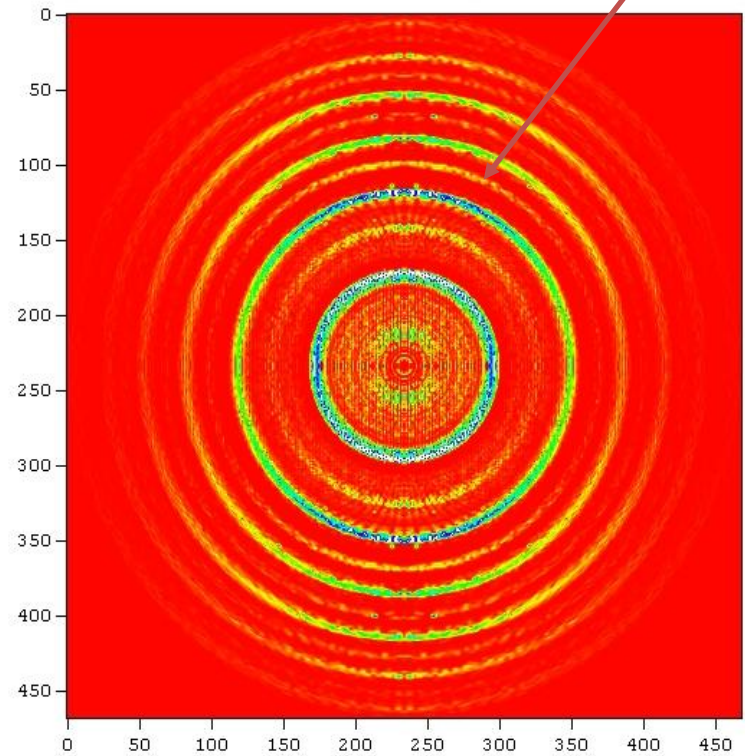


Inverted Photoelectron Image

15th Harmonic

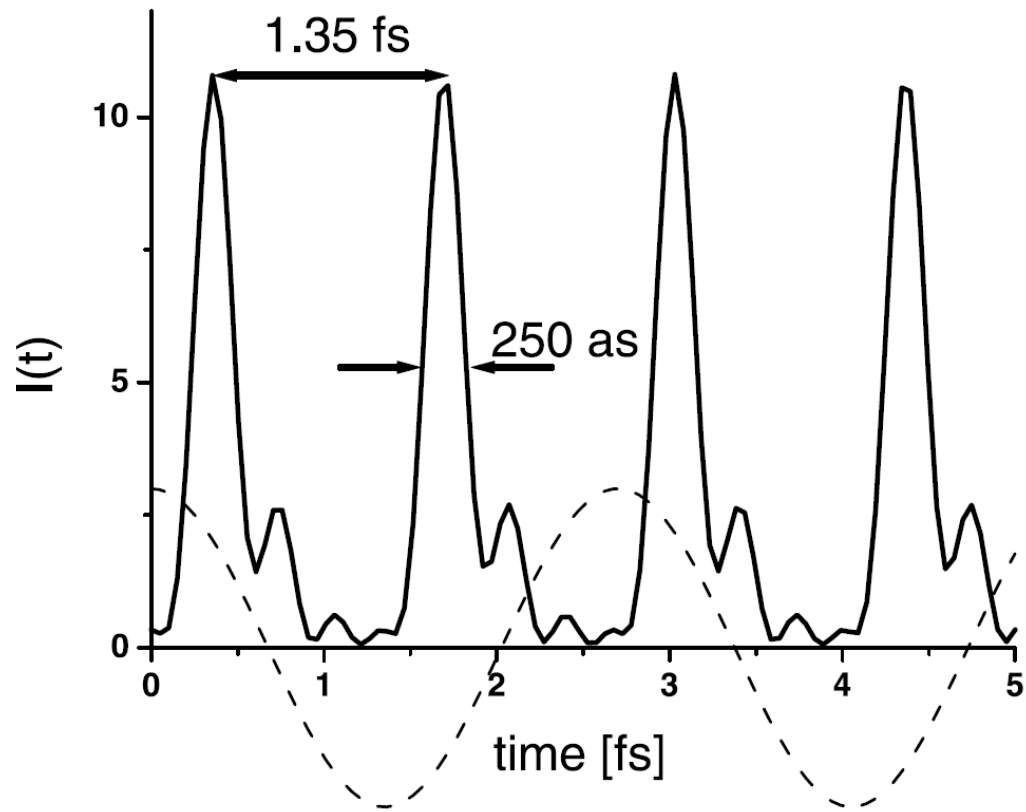
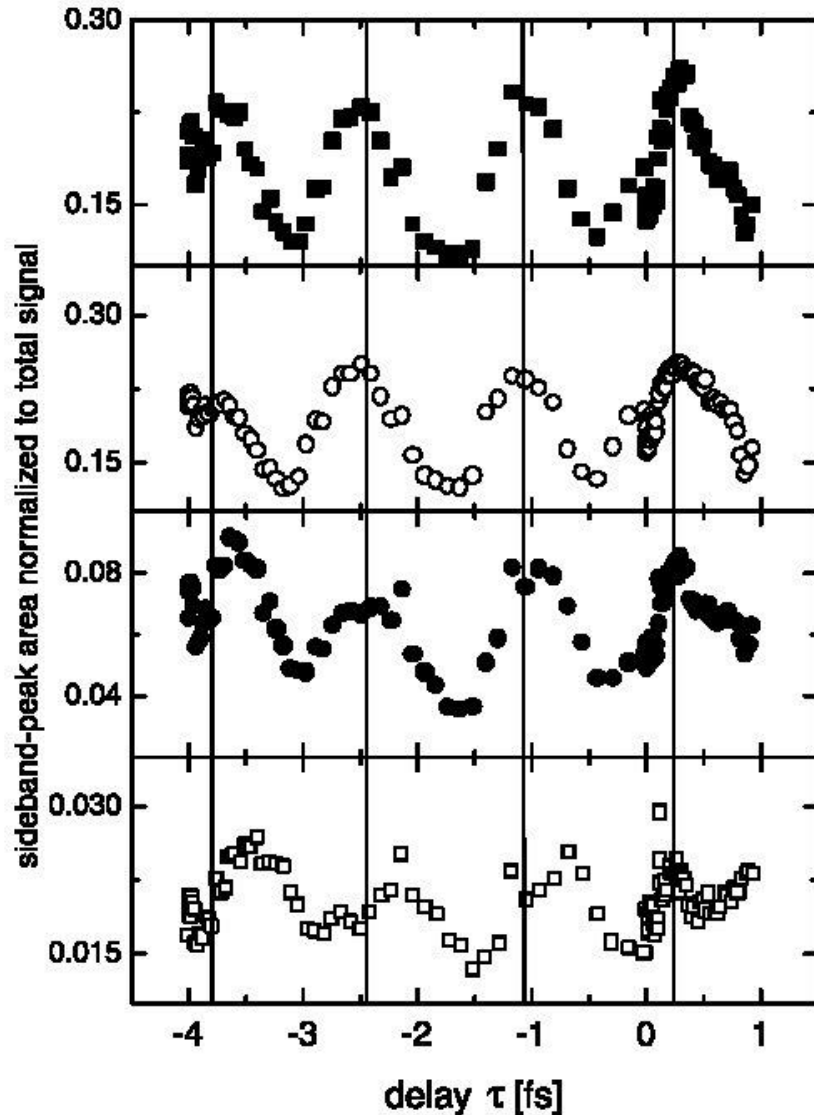


Sideband 14

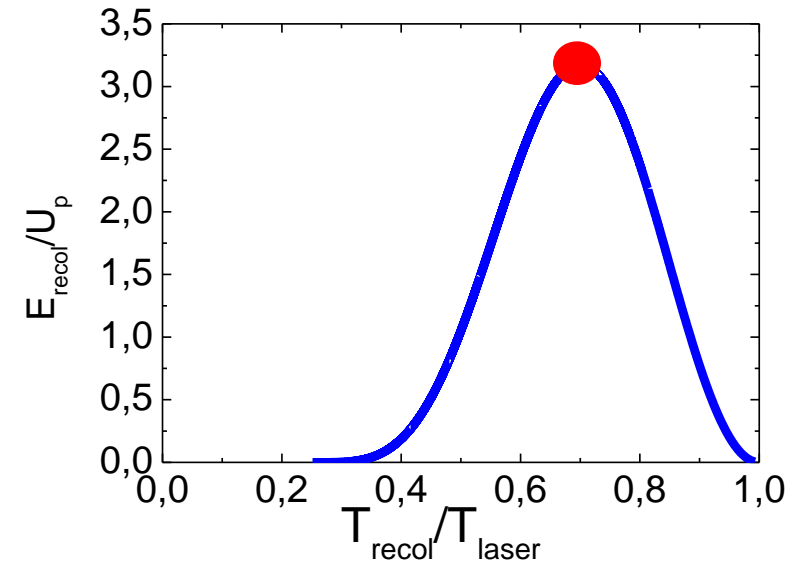


Sideband intensity

$$S_f(\Delta t) \sim A_f \cos[2\omega_{IR}\Delta t + \varphi_{q-1} - \varphi_{q+1} - \Delta\varphi_{atomic}^f]$$



Isolated Attosecond Pulse (IAP) from an Attosecond Pulse Train (APT)



Method 1: use cut-off harmonics

