Universität Hamburg

Current bounds on neutrino oscillation parameters and future prospects with medium baseline reactor oscillations

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Outline

- **Introduction to neutrino oscillations**

- **Oscillation experiments**

- **Current bounds on oscillation parameters: global analysis**

(*arXiv:1312.2878, Phys. Rev. D (2014) – with G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo*)

  - effect of global analysis on δ
  - effect of global analysis on θ_{23} octant
  - future prospects

- **A method to measure mass hierarchy: MBL reactor experiments**

(*arXiv:1309.1638, Phys. Rev. D 89 (2014) - with E. Lisi, A.Marrone*)

  - theoretical calculations (probability, cross section,…)
  - statistical analysis
  - energy scale errors

- **Conclusions**

Neutrino oscillations

In the 3ν framework, the mixing matrix U is:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \qquad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^* \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

3 MIXING ANGLES
$\theta_{12}, \theta_{13}, \theta_{23}$

1 PHASE δ
CPV FOR $\delta \neq 0, \pi$

Oscillations depend on mass differences. Assuming $m_2 > m_1$:

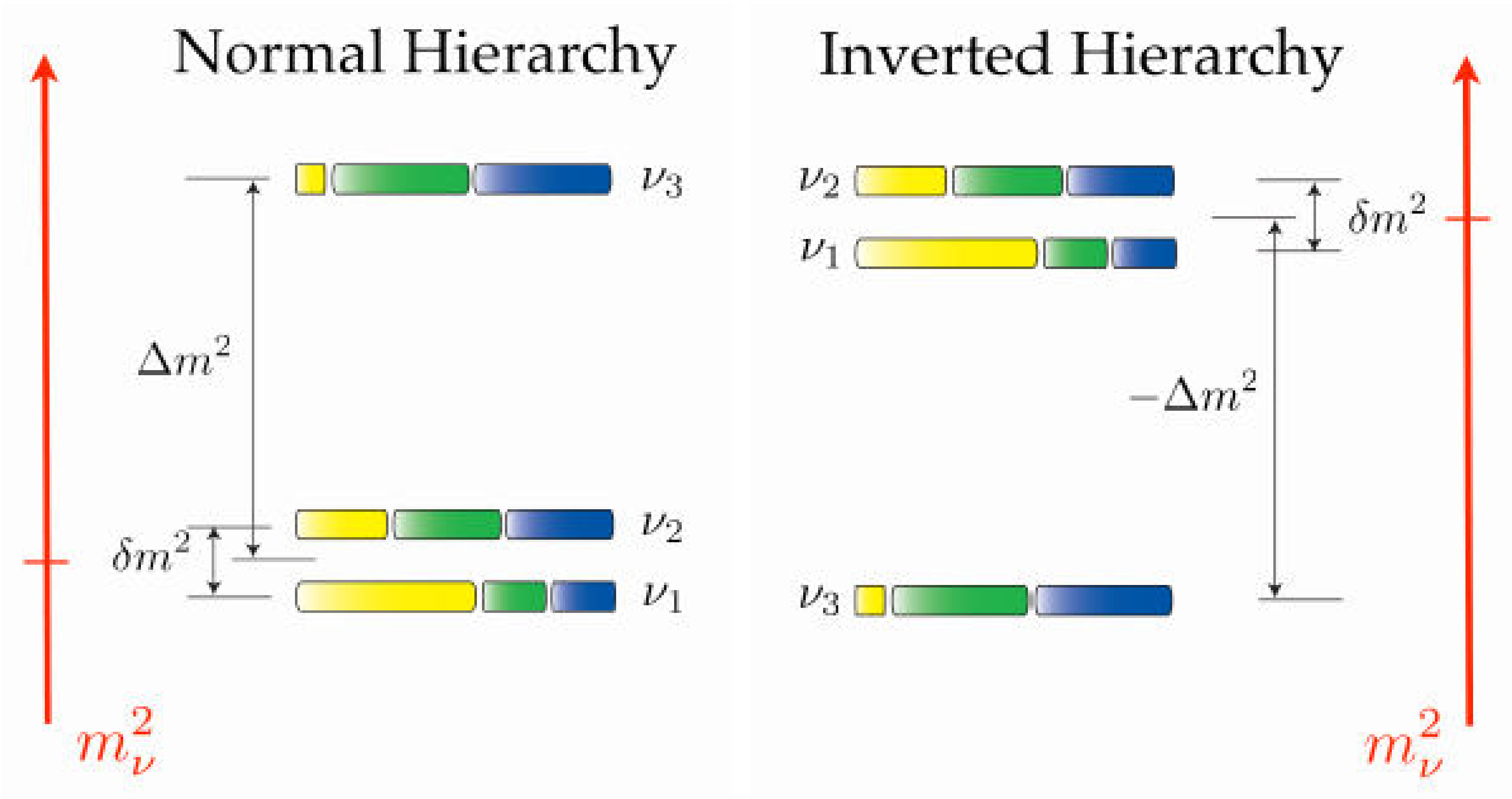
$$\delta m^2 = m_2^2 - \mathrm{m}_1^2 > 0$$

"SOLAR"

$$\Delta m^2 = m_3^2 - \frac{(m_2^2 + m_1^2)}{2}$$

"ATMOSPHERIC"

Mass hierarchy



Neutrino oscillation experiments

They compare the observed events spectrum with those expected for different combinations of oscillation parameters values.

$$P^{2\nu}_{\nu_\alpha\to\nu_\beta}(L,E)=\sin^2 2\theta \sin^2\frac{\Delta m^2 L}{4E}\,,\alpha\neq\beta$$

Can be classified according to:

Neutrino source | ***L/E*** | **Channel $\nu_\alpha\to\nu_\beta$**

Each class of experiments is sensitive only to some of the parameters

LBL Accelerators (T2K, MINOS)	$\rightarrow$	$\Delta m^2, \theta_{23}, \theta_{13}, \delta$
Solar	$\rightarrow$	$\theta_{12}, \delta m^2, \theta_{13}$
SBL Reactors (DChooz,RENO,DB)	$\rightarrow$	$\theta_{13}, \Delta m^2$
LBL Reactors (KamLAND)	$\rightarrow$	$\delta m^2, \theta_{12}, \theta_{13}$
Atmospheric (Super-K)	$\rightarrow$	$\theta_{23}, \Delta m^2, \delta$

Global analysis

$(\Delta m^2, \delta m^2, \theta_{13}, \theta_{23}, \theta_{12})$ are known with good accuracy ($\sim 10\%$), mainly dominated by a single class of experiments.

UNKNOWN PARAMETERS

θ_{23} octant
$(\theta_{23} > \frac{\pi}{4}, \theta_{23} < \frac{\pi}{4})$

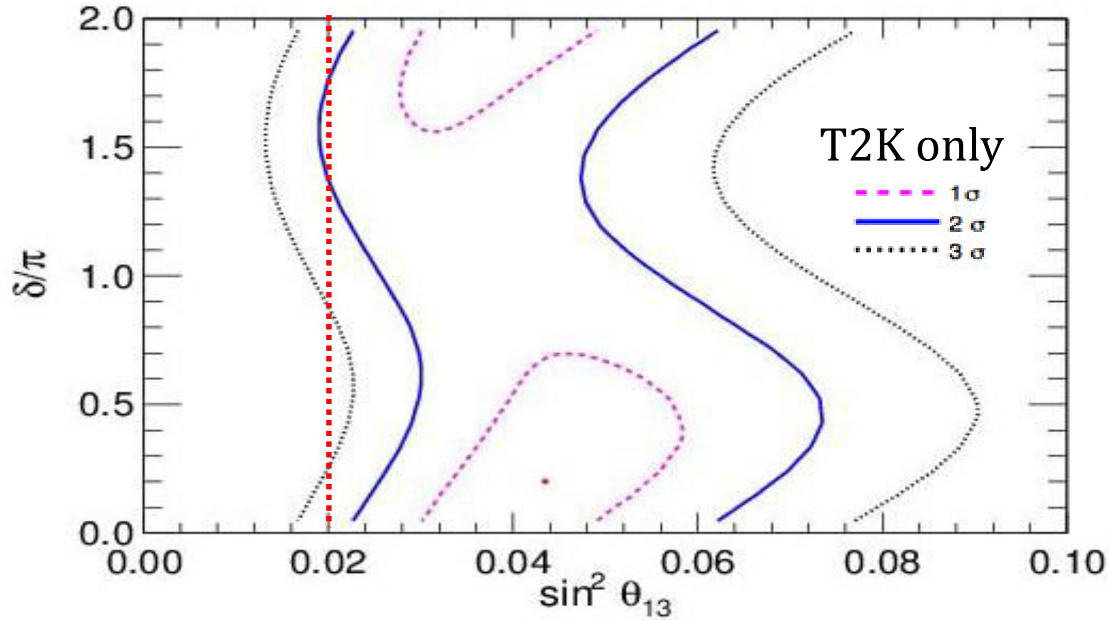
δ

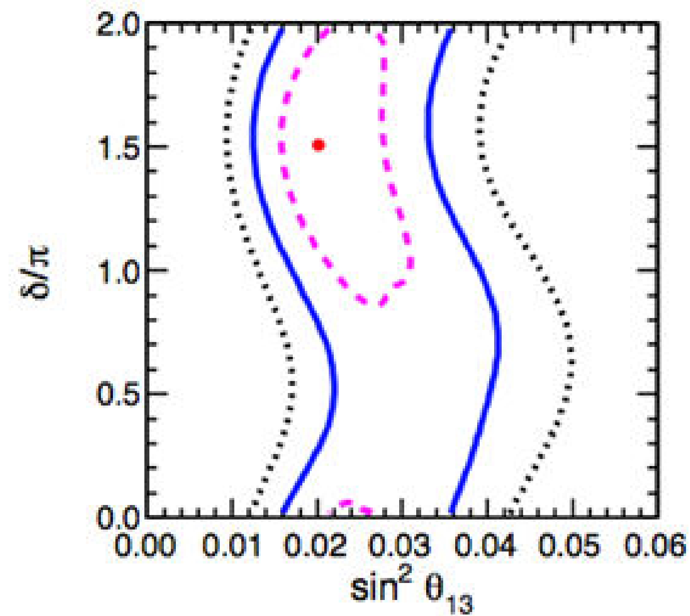
Mass hierarchy

GLOBAL ANALYSIS

Combine data from all the experiments. Maybe useful to get the most restrictive bounds and may provide guidance for the unknown parameters, taking advantage of parameters correlations. Example: hint $\theta_{13} \neq 0$ (Fogli *et al.* arXiv:08062649 (2008)).

Example: δ behaviour

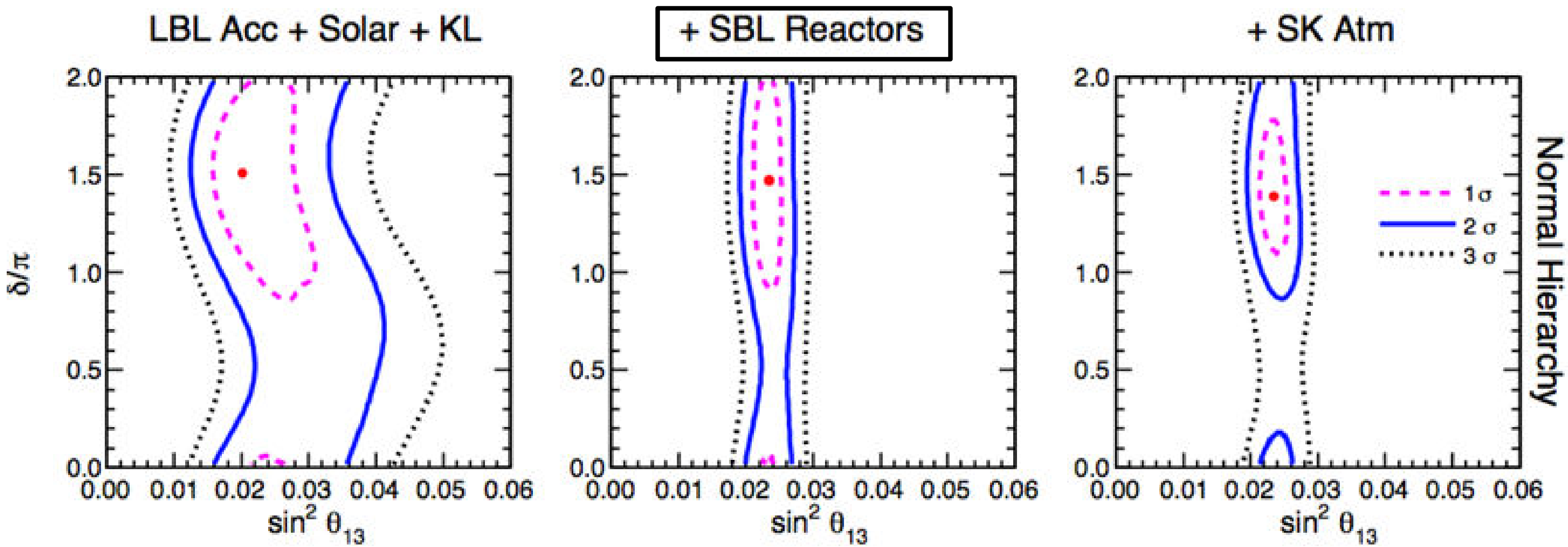




T2K prefers $\sin^2\theta_{13} \sim 0.04$, whereas Solar + KL ($\delta$ independent) and MINOS both give $\sin^2\theta_{13} \sim 0.02$. For this value of θ_{13} T2K has a preference for $\delta \sim 1.5\pi$

Example: δ behaviour

- SBL Reactors gives the most stringent bound of θ_{13} around $\sin^2\theta_{13} \sim 0.023$ and they are independent from δ. For this value of θ_{13} we still are in the part of the plot where $\delta \sim 1.5\pi$ is favored. Adding this contribution to the previous combination we just reduce the limits on θ_{13} without changing δ best fit.



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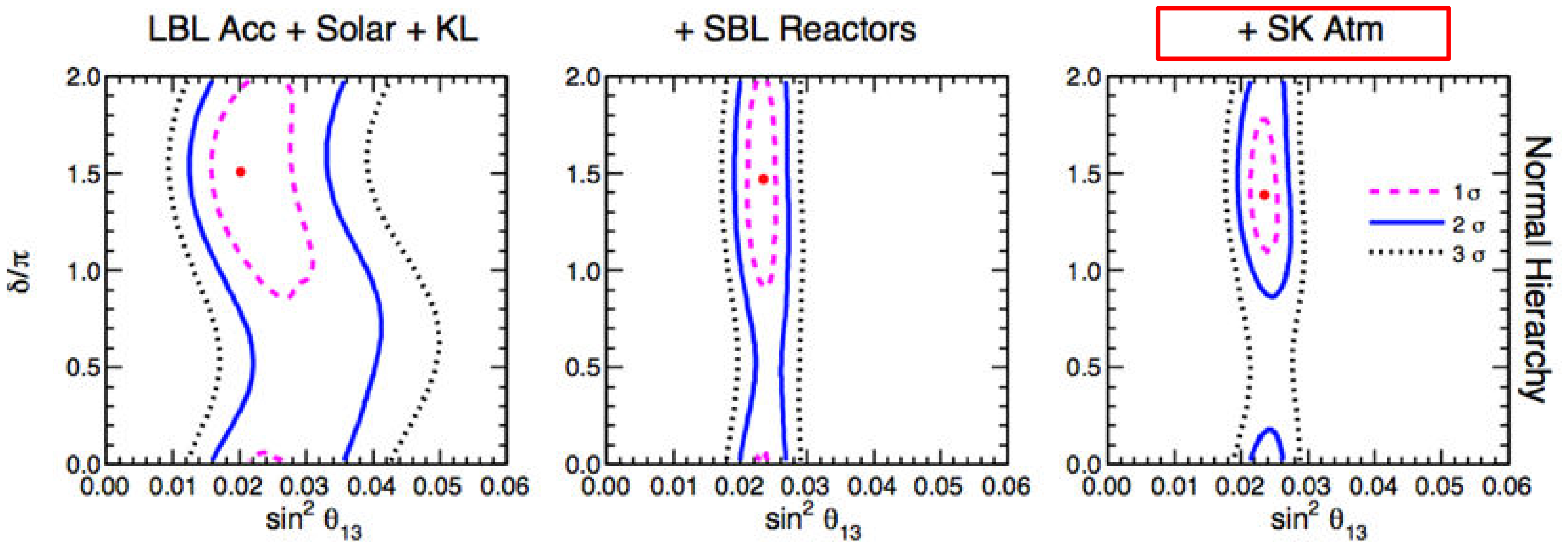
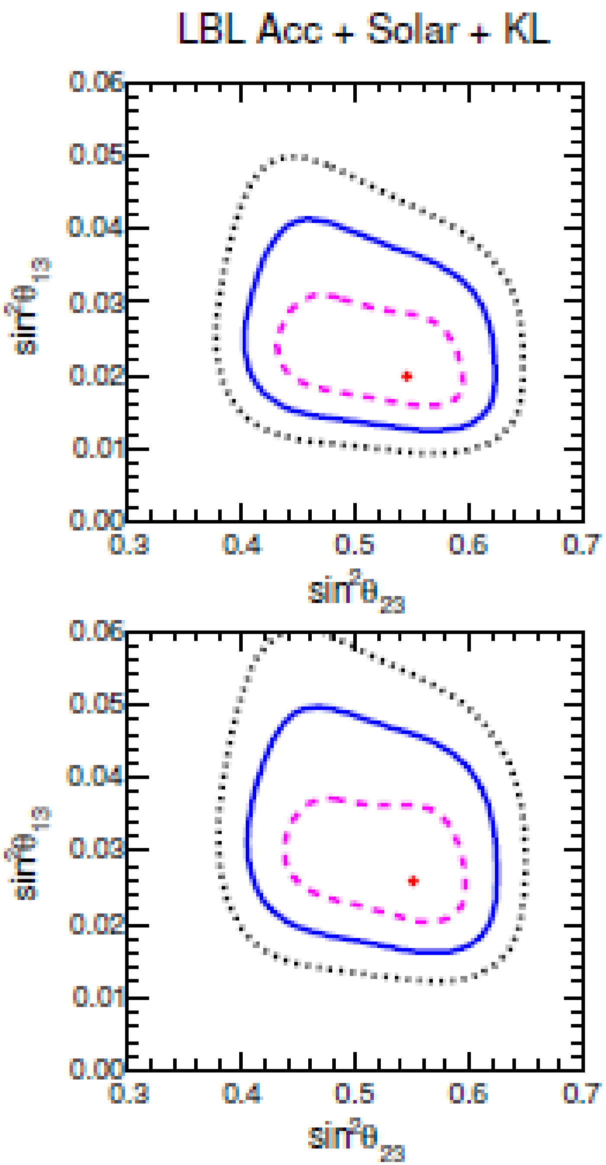
- Atmospheric data prefer $\frac{\delta}{\pi} \sim 1$. Combining we have $\frac{\delta}{\pi} \sim 1.4$ and a stronger preference for $\sin\delta < 0$



Example: octant behaviour

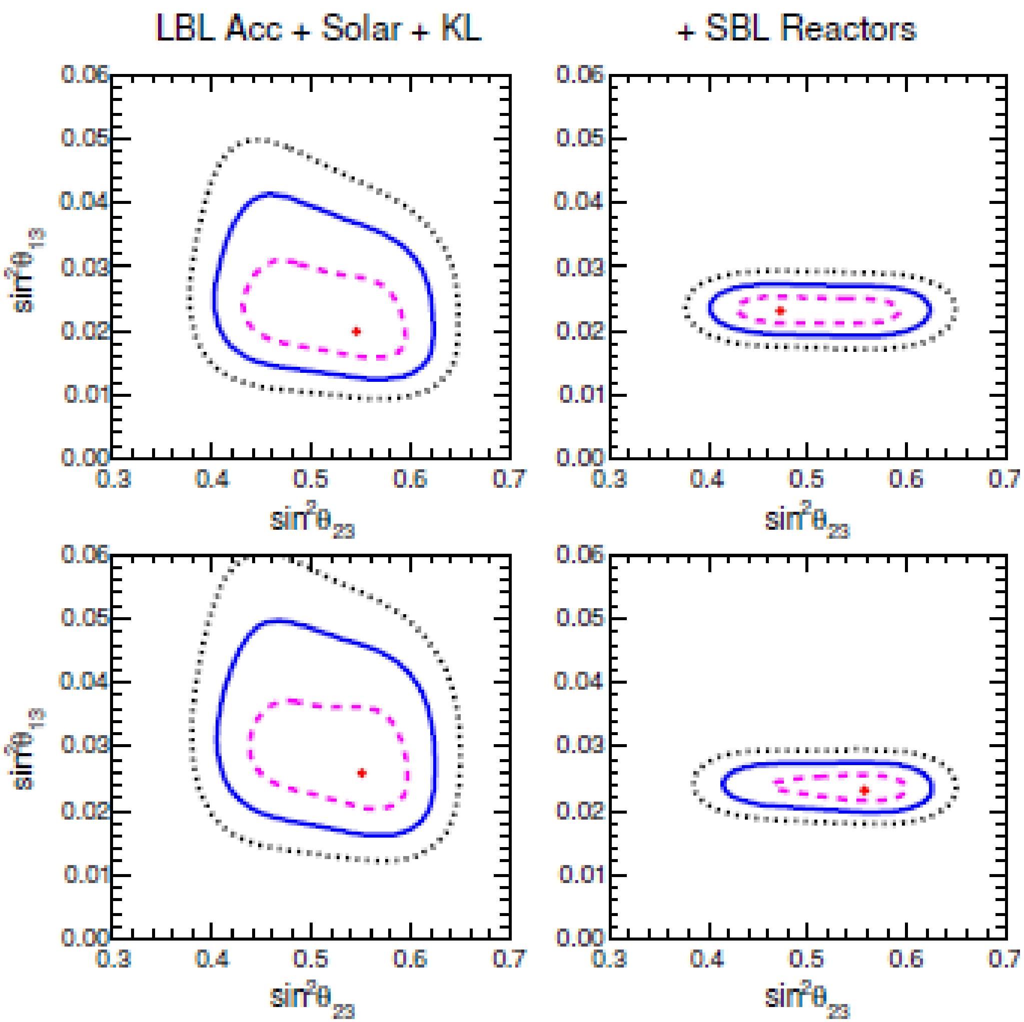


- MINOS prefers non-maximal θ_{23}, but cannot distinguish between the octants, whereas T2K prefers ~maximal mixing. T2K + MINOS give two degenerate minima.

- $(\theta_{23}, \theta_{13})$ are anticorrelated

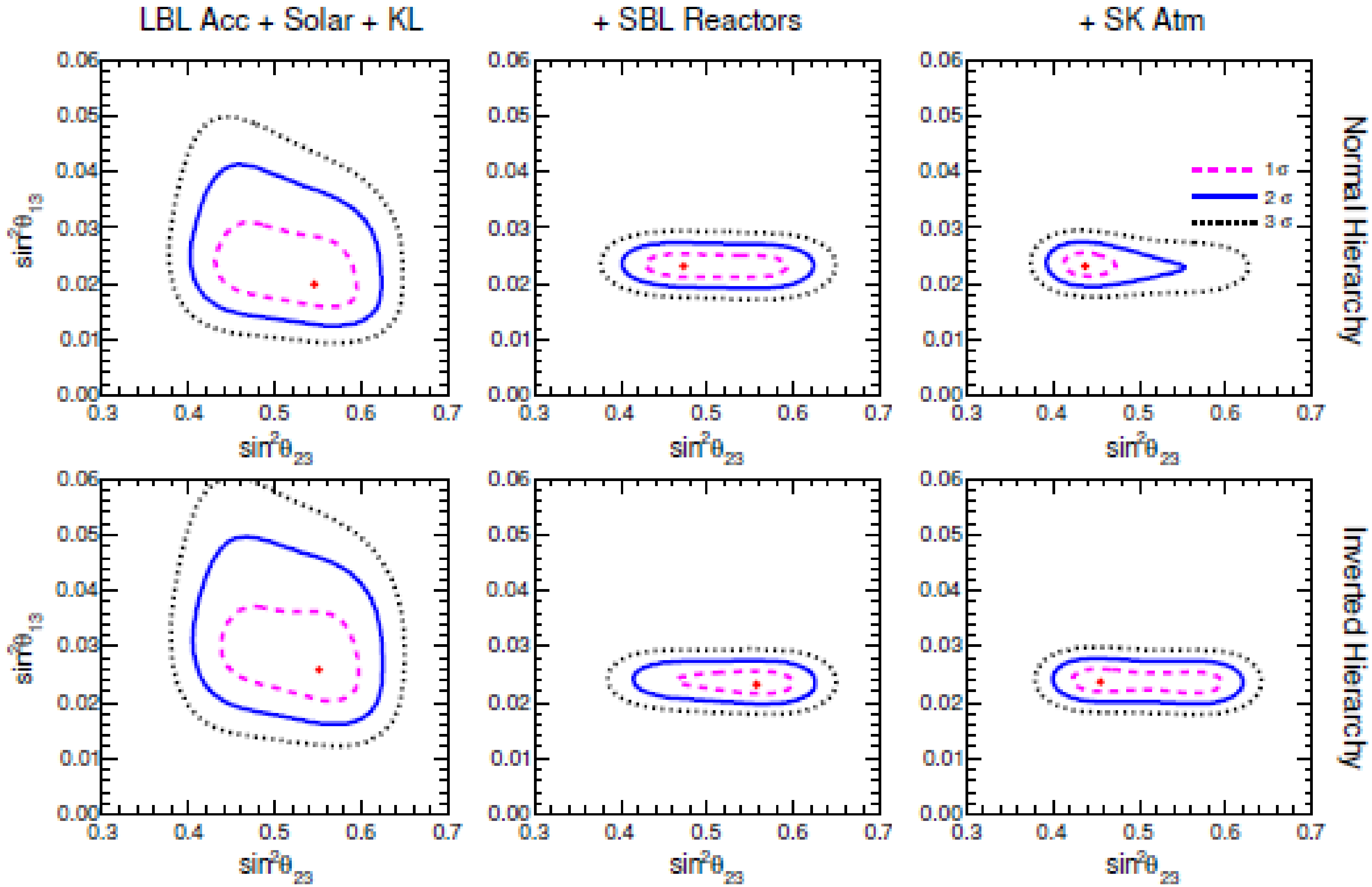
- Solar + KL have their best fit for $\sin^2\theta_{13} \sim 0.02$, smaller than the one of T2K+MINOS. In the combination this data eliminate the degeneracy and there is a small preference for the second octant.

Example: octant behaviour



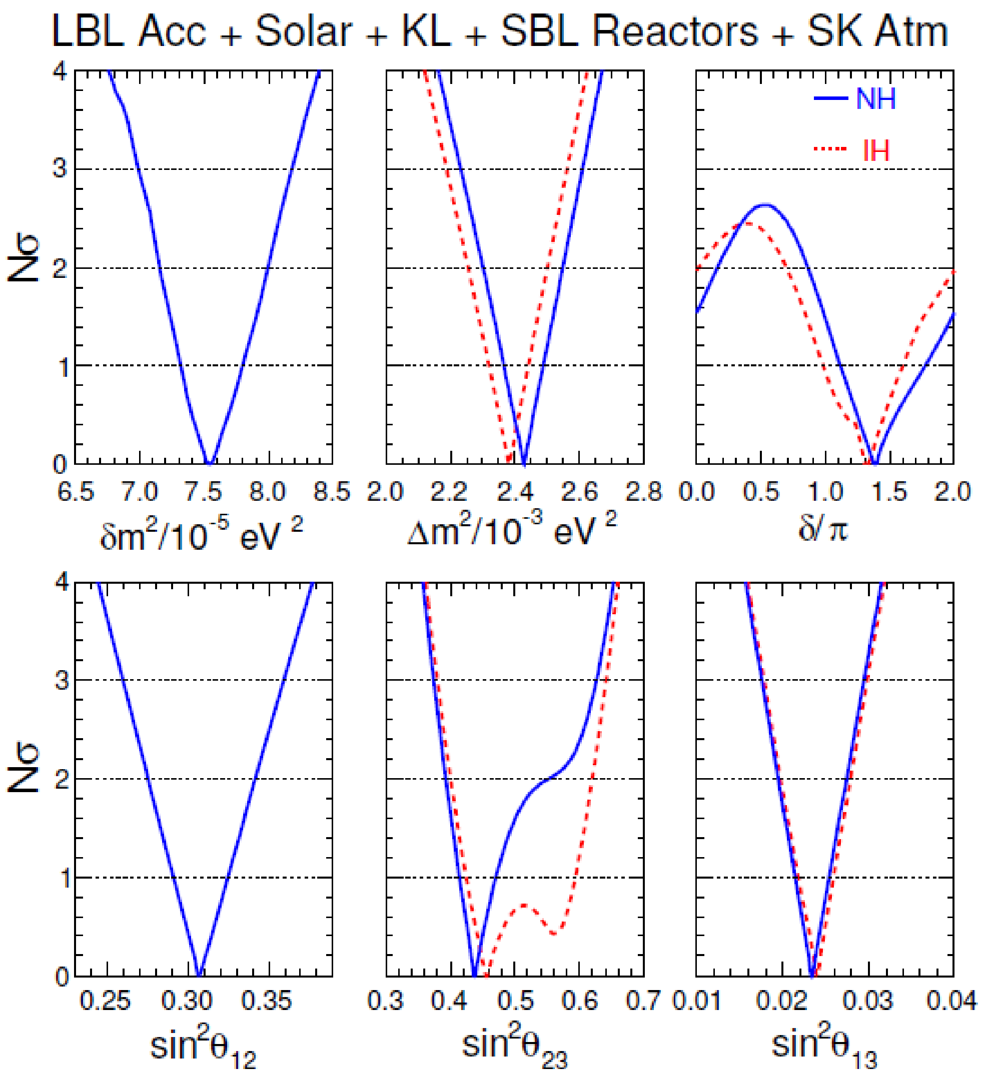
SBL Reactors data give $\sin^2\theta_{13} \sim 0.023$, higher than the one obtained for LBL Acc+Solar+ KL. Adding this data, taking into account the anticorrelation, there is a flip of octant from the second one to the first (just for normal hierarchy).

Example: octant behaviour



Atmospheric data alone prefer the first octant at over 2σ for NH and 1.5σ for IH. In combination with other experiments we still have preference for first octant, but with less significance.

Global analysis: summary



Preference for CP violating $\delta \sim 1.4\pi$ and $\sin\delta < 0$

Preference for non-maximal θ_{23} and for first octant. Weaker in inverted hierarchy

$$\chi^2_{\min}(NH) - \chi^2_{\min}(IH) = -0.3$$

no hierarchy information yet

Measuring mass hierarchy

LBL accelerator experiments are in principle capable of performing this measurement, taking advantage of matter effects, but it is strongly dependent from the value of δ (also θ_{23}). In the best conditions T2K+NOνA can reach 2σ confidence level.

Need a method independent from $\boldsymbol{\delta}$ or $\boldsymbol{\theta_{23}}$

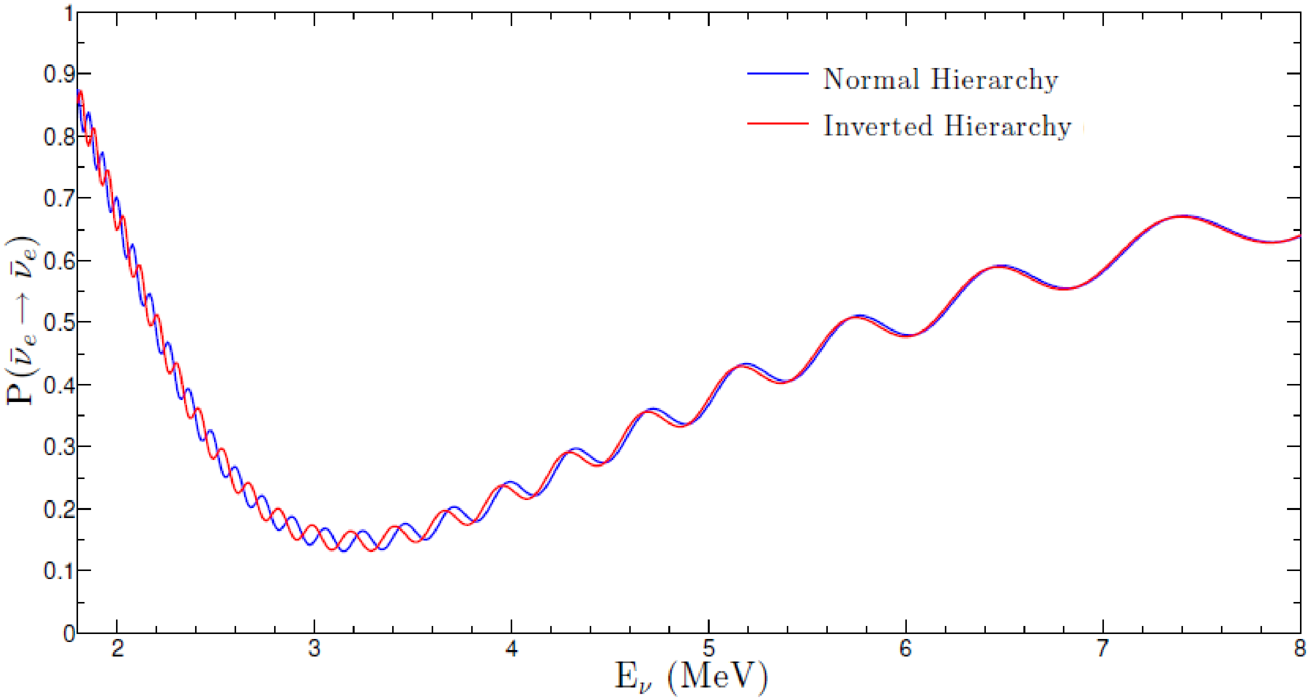
↓

MEDIUM BASELINE REACTOR EXPERIMENT (JUNO)

Medium baseline reactor oscillations

Ten years ago it was proposed that medium baseline reactor experiments (with baseline ~ 50 km) can probe neutrino mass hierarchy, through the study of the channel $\bar{\nu}_e \rightarrow \bar{\nu}_e$.

They are sensitive to short wavelenght oscillations, governed by $(\theta_{13}, \Delta m^2)$, to long wavelength oscillations, governed by $(\theta_{12}, \delta m^2)$, and to their tiny interference, which depends on mass hierarchy.



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REQUIREMENTS

EXPERIMENTAL:
High statistics, high resolution

THEORETICAL:
Very accurate theoretical calculations

PHENOMENOLOGICAL:
Refined stat/syst anlyses

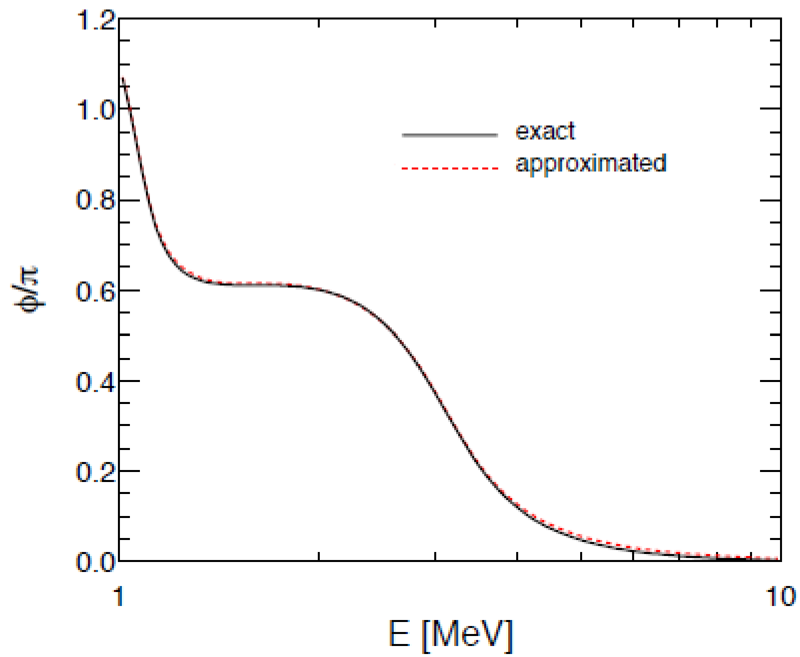
Approximate oscillation probability

$$P(\overline{\nu}_e \to \overline{\nu}_e) \simeq c_{13}^4 P_{\text{mat}}^{2\nu} + s_{13}^4 + 2 s_{13}^2 c_{13}^2 \sqrt{P_{\text{mat}}^{2\nu}}\, \omega \cos\left(\frac{\Delta m_{ee}^2 L}{2E} + \alpha\varphi\right)$$

$$\alpha = \begin{cases} +1 & \text{(Normal hierarchy)} \\ -1 & \text{(Inverted hierarchy)} \end{cases}$$

$$\varphi \simeq 2 s_{12}^2 \delta \left(1 - \frac{\sin\delta}{2\delta\sqrt{P_{\text{vac}}^{2\nu}}}\right)$$

$$\delta = \frac{\delta m^2 L}{2E}, \quad P_{\text{vac}}^{2\nu} = 1 - 4 s_{12}^2 c_{12}^2 \sin^2\delta$$



α = FREE PARAMETER

We «connect» continuously the two hypotheses:
normal hierarchy ($\alpha = +1$) and inverted hierarchy ($\alpha = -1$)

Approximate oscillation probability

$$\mathbf{P}(\overline{\nu}_e \to \overline{\nu}_e) \simeq c_{13}^4 P_{\text{mat}}^{2\nu} + s_{13}^4 + 2 s_{13}^2 c_{13}^2 \sqrt{P_{\text{mat}}^{2\nu}}\, \omega \cos\left(\frac{\Delta m_{ee}^2 L}{2E} + \alpha\varphi\right)$$

$$P_{\text{mat}}^{2\nu} = 1 - 4 \tilde{s}_{12}^2 \tilde{c}_{12}^2 \sin^2 \tilde{\delta}$$

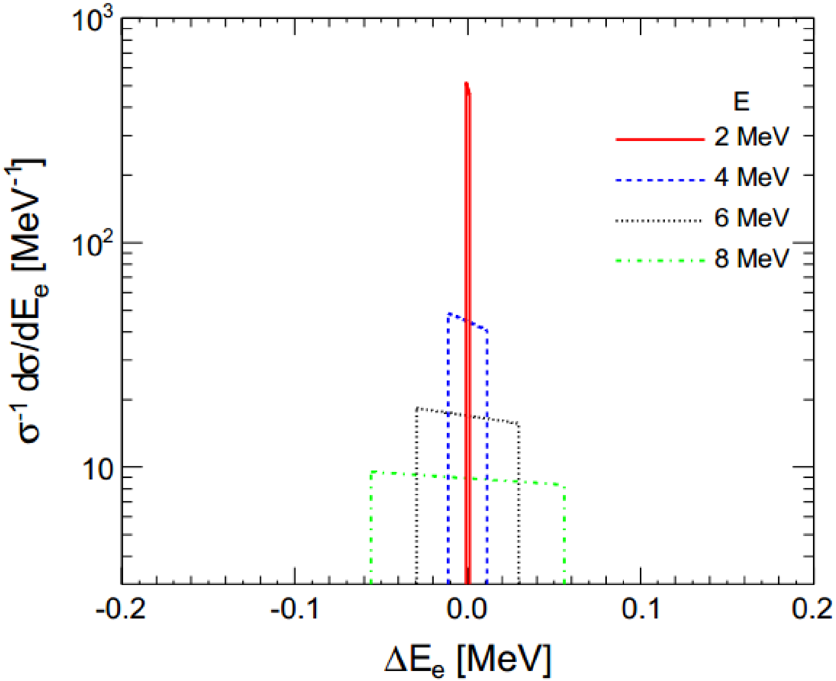
$(\theta_{12}, \delta m^2) \to (\tilde{\theta}_{12}, \delta \tilde{m}^2)$ with a fractional correction of 8×10^{-3}, which is of the same order of magnitude of the fit accuracy. The corrections are negligible for $(\theta_{13}, \Delta m^2)$.

ω is a damping factor taking into account the presence of multiple reactors. For a JUNO-like configuration there are 10 reactors with a baseline (flux-averaged) of L=52.474 km. In this case the amplitude of the hierarchy sensitive cosine is reduced of 28% at low energy.

Differences with the exact probability can be neglected in the analysis ($<2 \times 10^{-3}$)

Nucleon recoil

INVERSE β DECAY: $\bar{\nu}_e + p \rightarrow e^+ + n$



In the high energy part of the spectrum there is a $O(E/m_p)$ correction to the relation $E - E_e \simeq m_n - m_p$ (recoiless approx). This correction is of the same order of energy resolution $\frac{\Delta E}{E} \sim O(1\%) \Rightarrow E_e \in [E_1, E_2]$

Reasonable approximation: Top-hat function within kinematical limits

$$\frac{1}{\sigma(E)}\frac{d\sigma(E,E_e)}{dE_e} \simeq \frac{1}{E_2 - E_1}$$

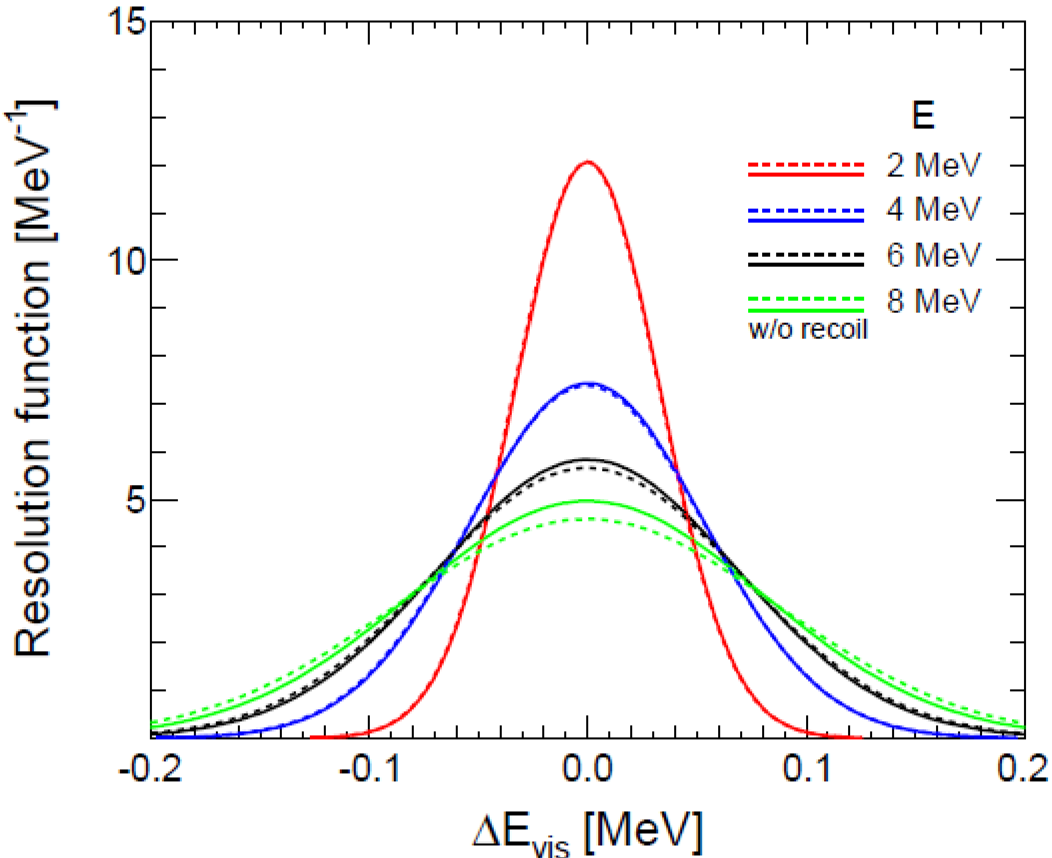
Energy res. function (gaussian)

Analytical integral

$$\int_{m_e}^{\infty} dE_e \frac{d\sigma(E,E_e)}{dE_e} \boxed{r(E_e + m_e, E_{vis})} \simeq \frac{\sigma(E)}{E_2 - E_1} \boxed{\int_{E_1}^{E_2} dE_e\, r(E_e + m_e, E_{vis})}$$

Nucleon recoil ⇒ Correction to energy resolution function

Nucleon recoil



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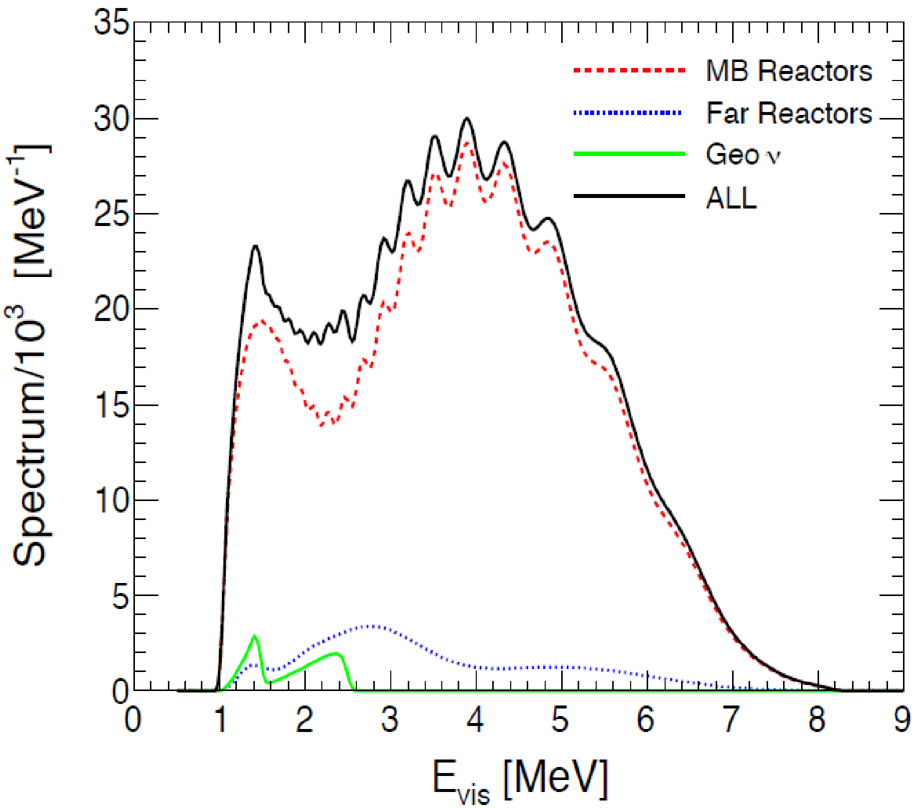
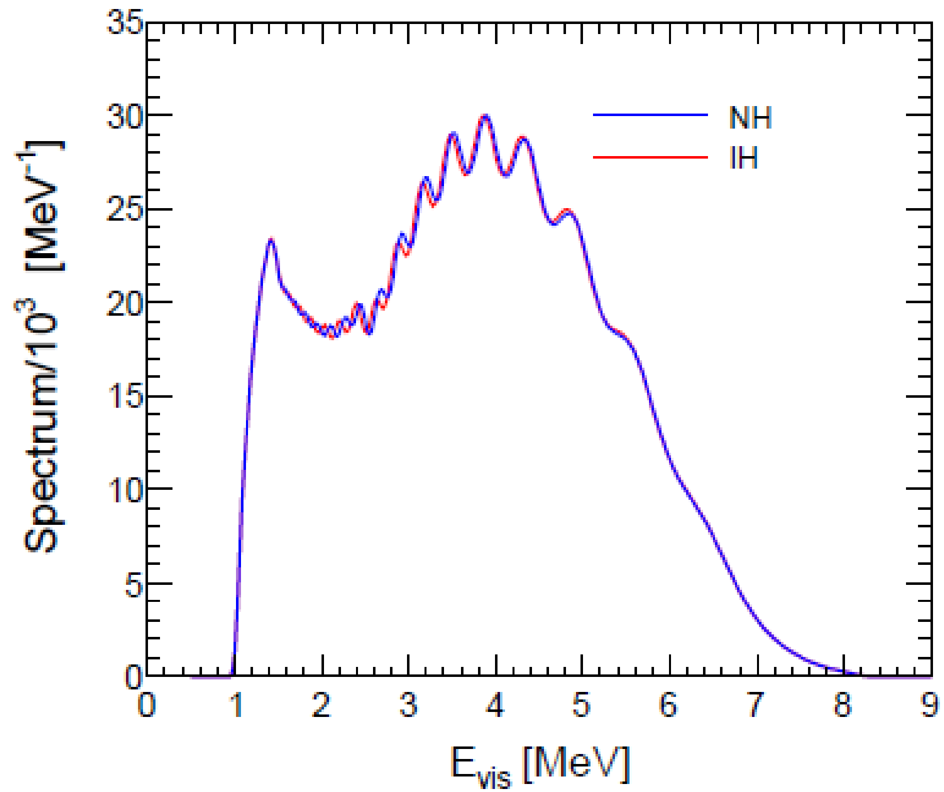
Nucleon recoil⇒Correction to energy resolution function

Example of spectrum for JUNO-like setup

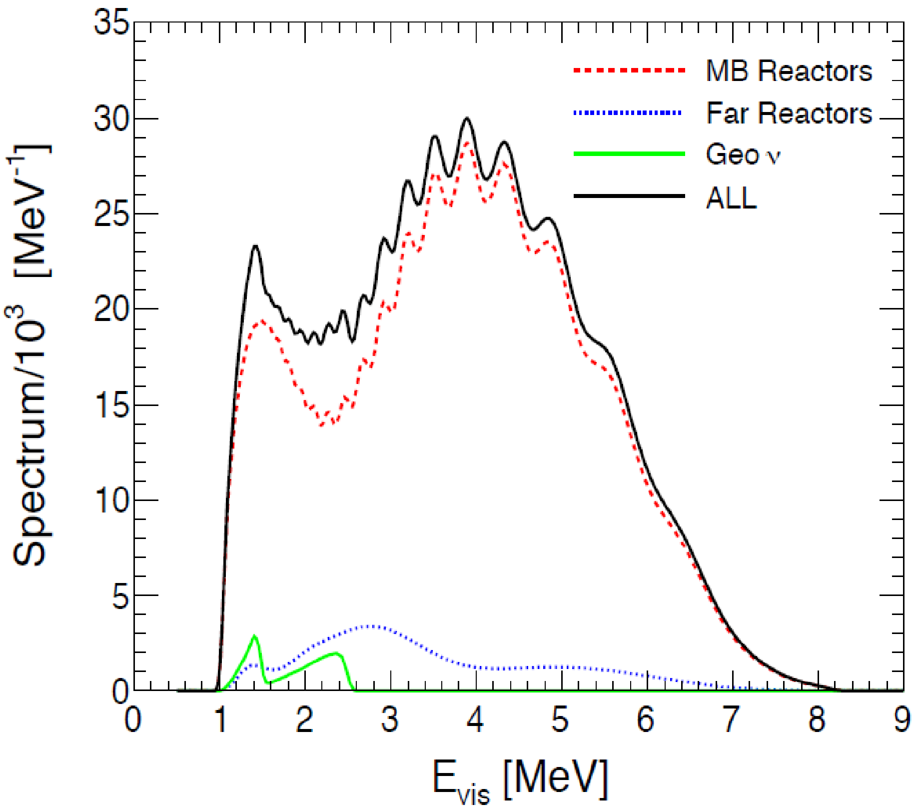


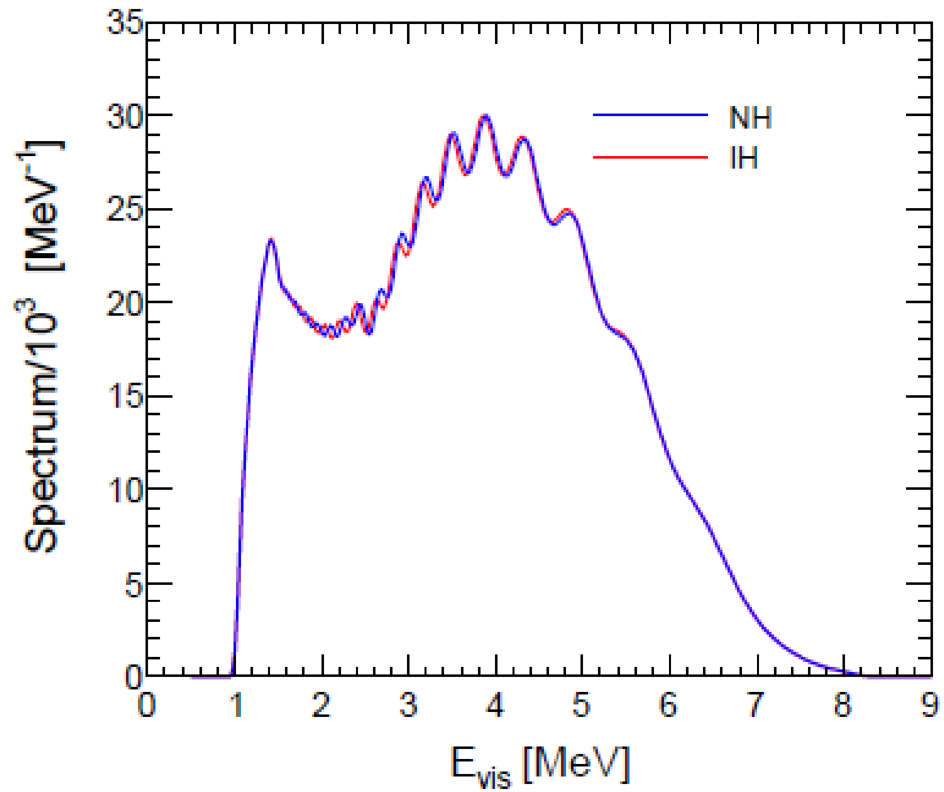


STATISTICS
5 years

- 10^5 events with oscillations from MBL reactors
- 1.65×10^4 unoscillated events from LBL reactors
- 3.5×10^3 unoscillated events from geo$-\nu$

Example of spectrum for JUNO-like setup





SYSTEMATICS

$$\Phi_{\text{reactor}} \rightarrow \Phi_{\text{reactor}}(1 + f_R)$$

$$\Phi_{\text{geo}} \rightarrow \Phi_{\text{geo}}(1 + f_U)$$

$$\Phi_{\text{geo}} \rightarrow \Phi_{\text{geo}}(1 + f_{Th})$$

Statistical analysis

The true spectrum of events $S^*(E_{\text{vis}})$ is calculated for global analysis best fit values of oscillation parameters and for a fixed hierarchy. Then $S^*(E_{\text{vis}})$ is compared to a family of spectra $\mathrm{S}(E_{\text{vis}})$ obtained by varying 8 parameters.

$$\chi^2 = \chi^2_{\text{stat}} + \chi^2_{\text{par}} + \chi^2_{\text{sys}}$$

$$\chi^2 = \chi^2\left(\boxed{\delta m^2, \Delta m^2_{ee}, \theta_{12}, \theta_{13}}, \alpha, \boxed{f_{\mathrm{R}}, f_{\mathrm{U}}, f_{\mathrm{Th}}}\right)$$

all parameters floating

$$\chi^2_{\text{par}} = \sum_{i=1}^{4} \left(\frac{p_i - \bar{p}_i}{\sigma_i}\right)^2 \qquad \chi^2_{\text{sys}} = \sum_i \left(\frac{f_i - 1}{s_i}\right)^2$$

σ_{i} are the 1σ errors taken from global analysis

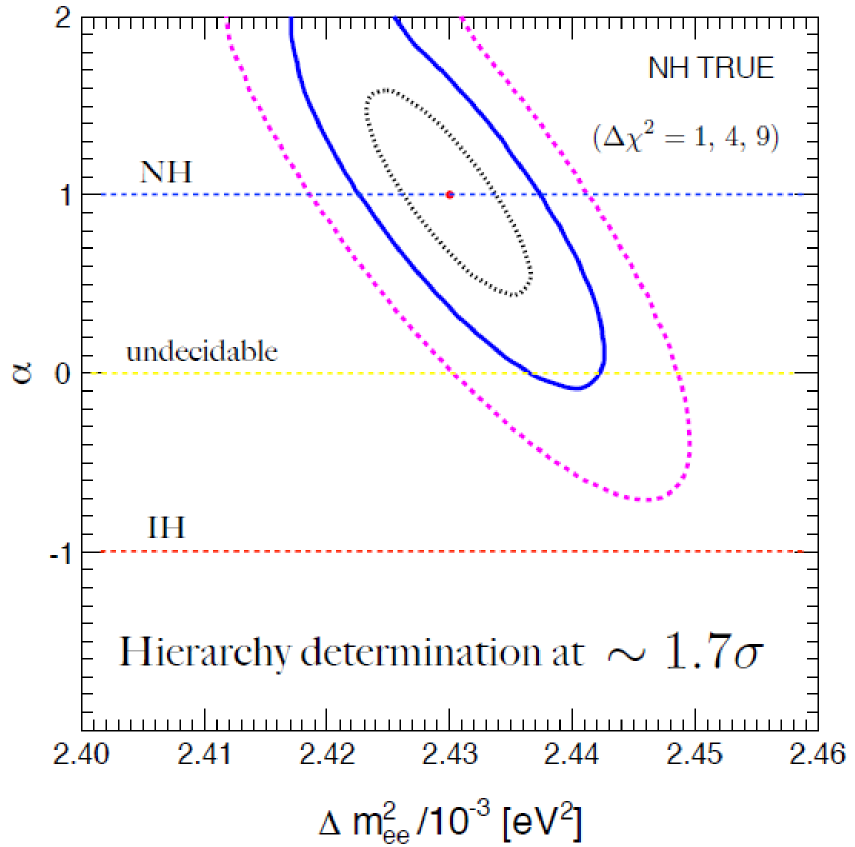
$s_{\mathrm{U,Th}} = 0.2$
$s_{\mathrm{R}} = 0.03$

Statistical analysis

$$\chi^2_{\text{stat}} = \int_{0\,\text{MeV}}^{9\,\text{MeV}} dE_{\text{vis}} \left(\frac{S^*(E_{\text{vis}}) - S(E_{\text{vis}})}{\sqrt{S^*(E_{\text{vis}})}} \right)^2$$

RESULTS FOR «TRUE NH»:

- Distance from $\alpha = 1$ and $\alpha = -1$ $\sim 3.4\sigma$
- Distance from $\alpha = 0$ and $\alpha = -1$ $\sim 1.7\sigma$



Hierarchy sensitivity

Our definition of hierarchy sensitivity is

$$\sqrt{\boldsymbol{\chi^2_{\min}(\alpha = -1) - \chi^2_{\min}(\alpha = 0)}} \sim \mathbf{1.7\sigma}$$

1) α is a continuous parameter. Using this parameter we solve the statistical issues connected to the applicability of χ^2 to discrete hypotheses tests. We recover the same result obtained by other methods:

$$0.5\sqrt{\boldsymbol{\chi^2_{\min}}(\boldsymbol{\alpha} = -\mathbf{1}) - \boldsymbol{\chi^2_{\min}}(\boldsymbol{\alpha} = 1)}$$

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$$0.5\sqrt{\boldsymbol{\chi^2_{\min}(\alpha=-1)-\chi^2_{\min}(\alpha}=1)}$$

2) Measuring mass hierarchy means finding the sign of an extra phase (φ). In a realistic situation an experiment can find no evidence of an extra phase ($\alpha=0$) and in this case hierarchy determination is already compromised.

Accuracy on parameters

Parameter	% error (prior)	% error after fit (NH true) all data	all − far	all − geo	% after fit (IH true) all data	all − far	all − geo
α	∞	59.2	59.0	57.0	56.2	55.3	54.0
Δm^2_{ee}	2.0	0.26	0.25	0.26	0.26	0.25	0.25
δm^2	3.2	0.22	0.21	0.16	0.21	0.21	0.16
s^2_{12}	5.5	0.49	0.47	0.39	0.49	0.46	0.42
s^2_{13}	10.3	6.95	6.88	6.95	6.84	6.77	6.84
f_R	3.0	0.66	0.66	0.64	0.65	0.65	0.64
f_{Th}	20.0	15.3	14.6	—	15.5	15.4	—
f_{U}	20.0	13.3	13.3	—	13.3	13.3	—

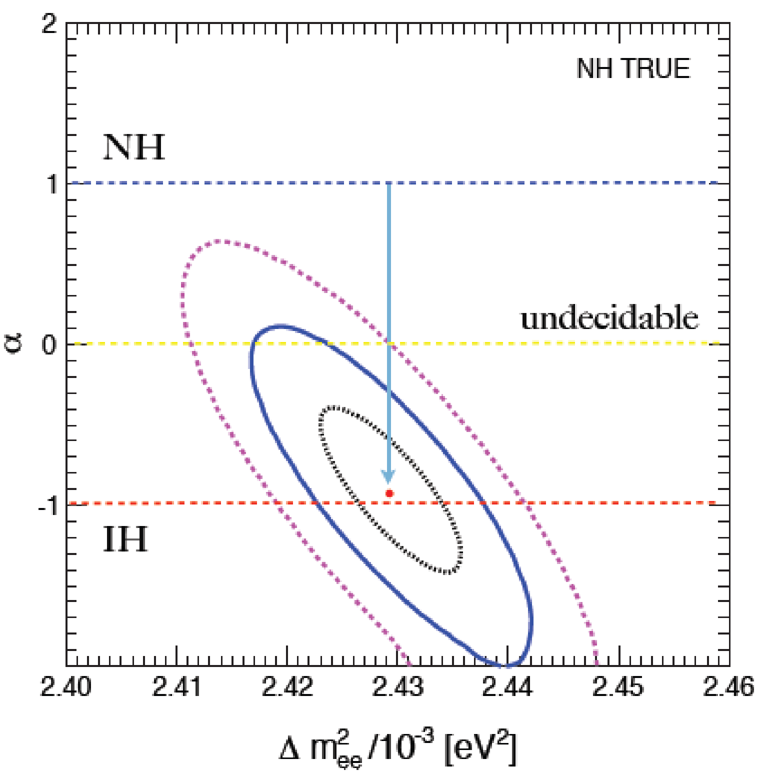
- Accuracy is improved of a factor ~ 10 for $(\Delta m^2_{ee}, \delta m^2, s^2_{12}, f_R)$, but this change is less significant for the other parameters.
- Far reactors do not affect precision on parameters, while geoν decrease precision on $(\delta m^2, s^2_{12})$, whose observable oscillations are in the same part of the spectrum where there are geoν.

Non linear $E \to E'$ transformation

Non linear $E \to E'$ transformations can mimic the wrong hierarchy at a slightly different Δm^2, if the following equation is satisfied:

$$\boxed{\frac{\Delta m^2_{ee} L}{2E} \pm \varphi(E) = \frac{\Delta m^{2\,\prime}_{ee} L}{2E'} \mp \varphi(E')}$$

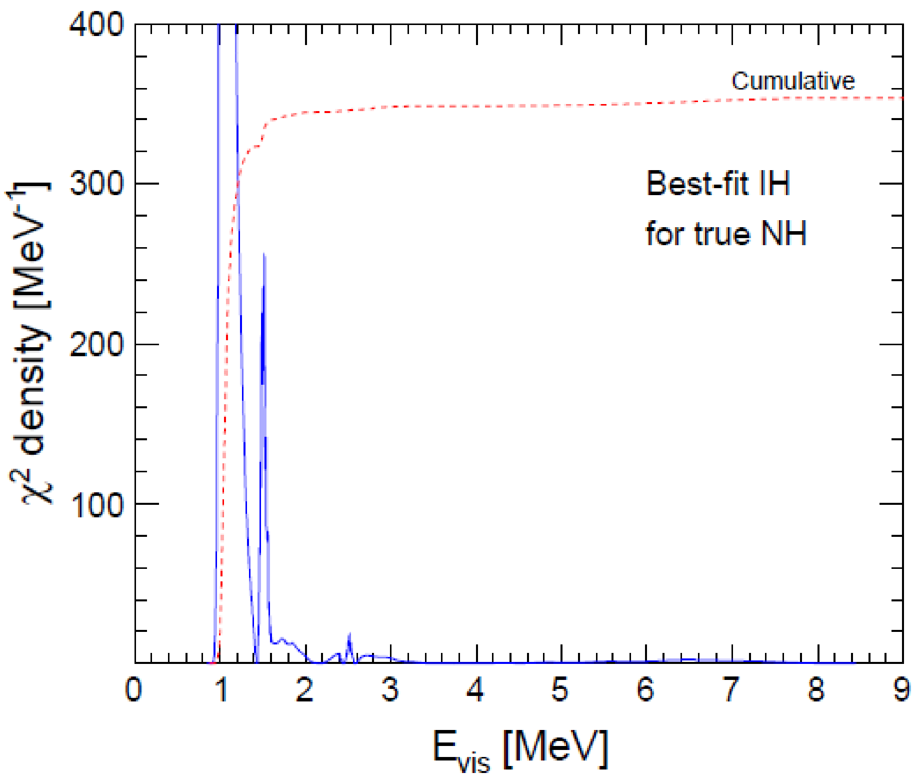
There is an infinite class of such transformations. For instance consider the case $E \to E'$ with $E = E'$at ∞.

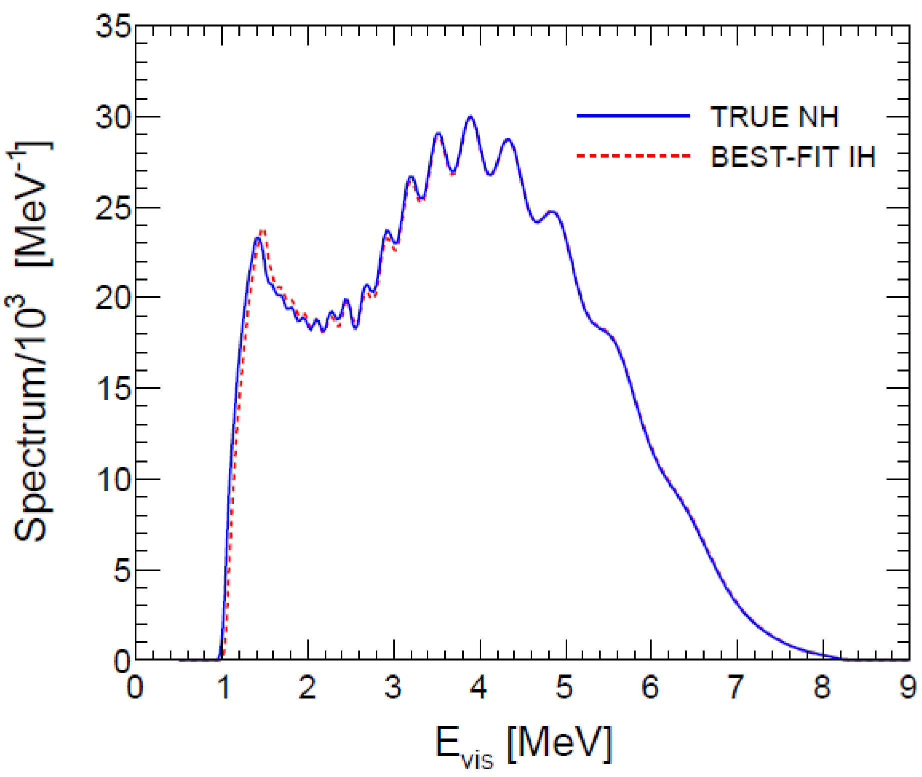


Now the best fit is at $\alpha = -1$, despite having assumed normal hierarchy as the true one. Normal hierarchy is excluded at $>3\sigma$

Non linear $E \to E'$ transformation

However the χ^2 is very high O(100) , because this transformation of E create a mismatch of spectra at low energy.



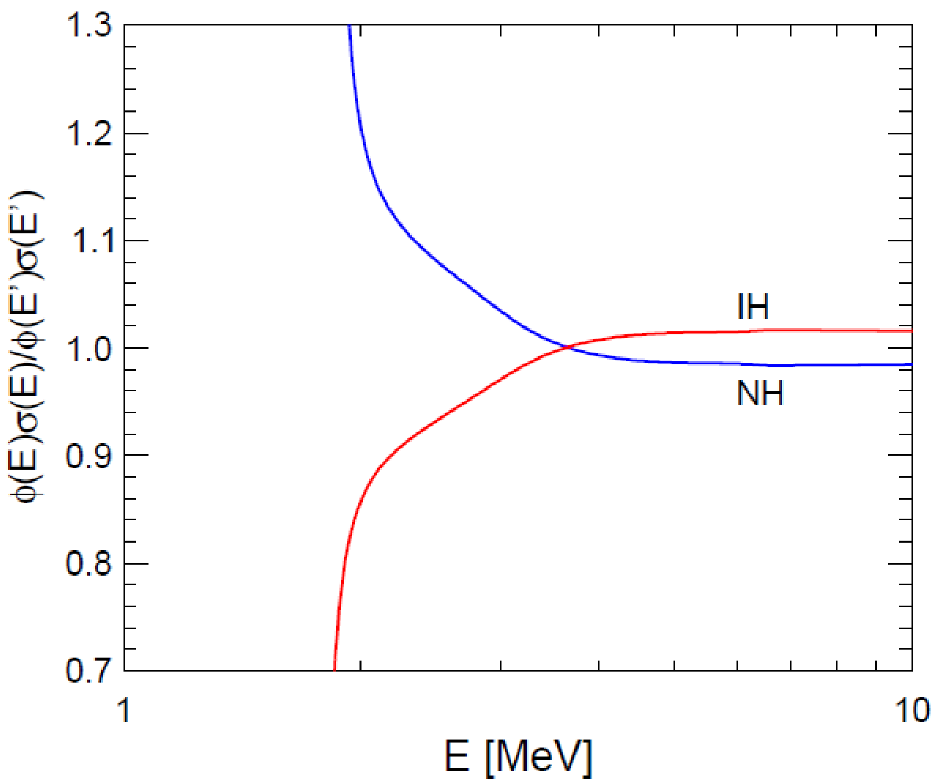


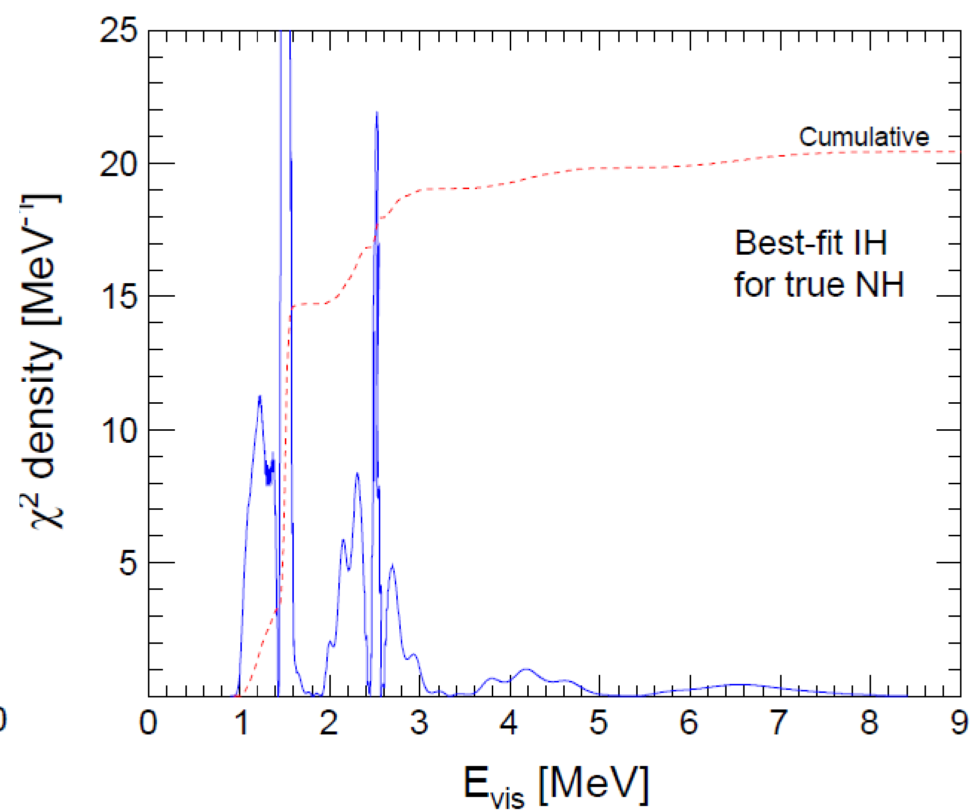
Non linear $E \rightarrow E'$ transformation

If the shape errors in $\Phi(E)\sigma(E)$ are of the same order of the deviations $\Phi(E)\sigma(E) \rightarrow \Phi(E')\sigma(E')$, caused by the transformation $E \rightarrow \mathrm{E}'$, then the mismatch at low energy can be almost undone by a factor

$$f(E) = \frac{\Phi(E)\sigma(E)}{\Phi(E')\sigma(E')}$$

As a result we have $\chi^2 \sim O(10)$.





Conclusions

Medium baseline reactor experiments can probe neutrino mass hierarchy up to $2\boldsymbol{\sigma}$, but high precision is required on both experimental and theoretical side. In this context we have shown:

How to include analitically the recoil effects of the nucleon

An analytical approximation of the oscillation probabilities including matter effects and multiple reactors

That non linear $E \rightarrow E'$ transformations, togheter with spectral uncertainties, may mimic wrong hierarchy.
However ,this issue deserves further studies.

That it is possible to condense hierarchy information in a continuous parameter α, (+1=NH, -1=IH). This solves the issues related to the statistical interpretation of the data analysis .The distance between $\boldsymbol{\alpha = +1}$ and $\boldsymbol{\alpha = -1}$ is $3\boldsymbol{\sigma}$ in JUNO-like experiments.