

Parton showers and the dipole formalism

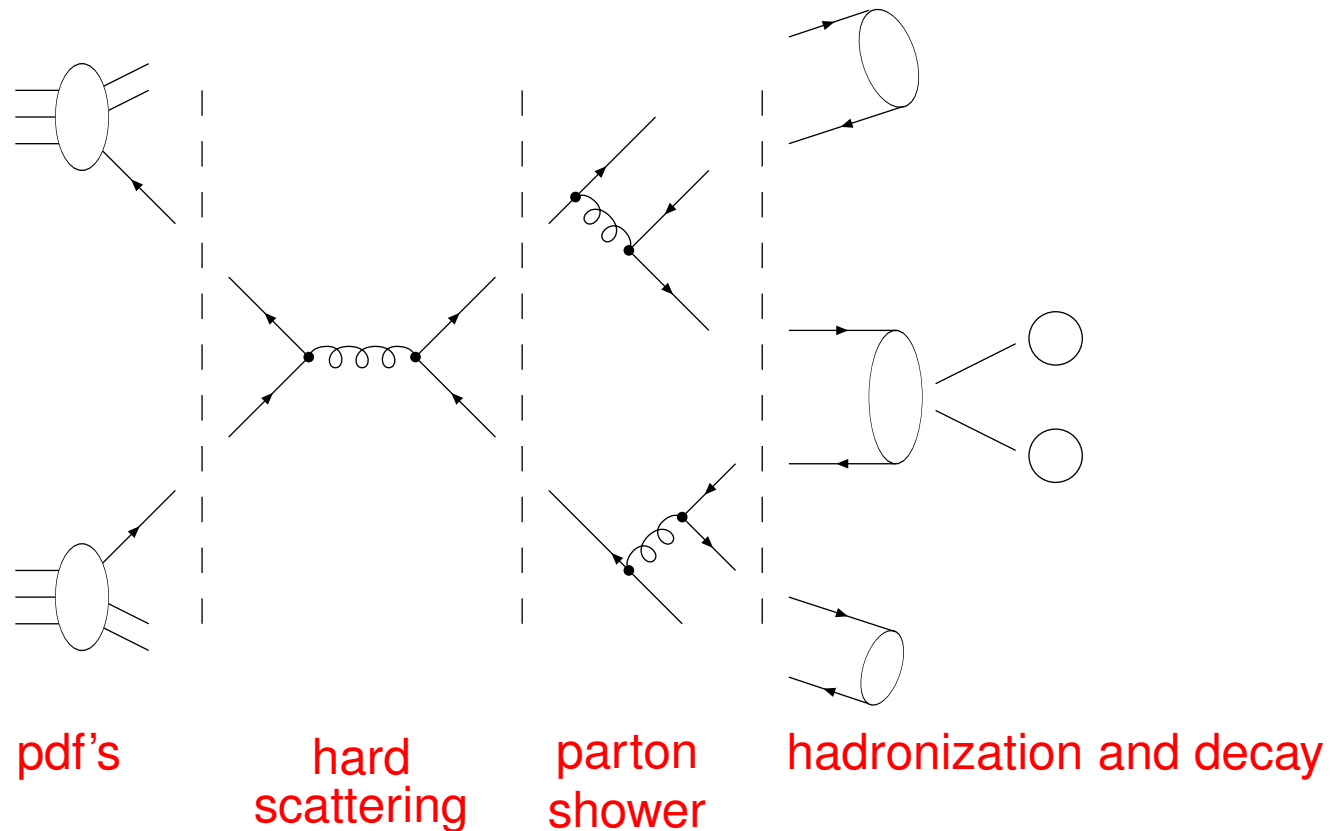
Stefan Weinzierl

Universität Mainz

in collaboration with [M. Dinsdale](#) and [M. Ternick](#)

- Introduction:** **Event generators and perturbative calculations**
- I.:** **Parton showers**
- II.:** **Showers from dipoles**
- III.:** **Numerical results**

Event generators



Underlying event:

Interactions of the proton remnants.

Multiple interactions:

more than one pair of partons undergo hard scattering

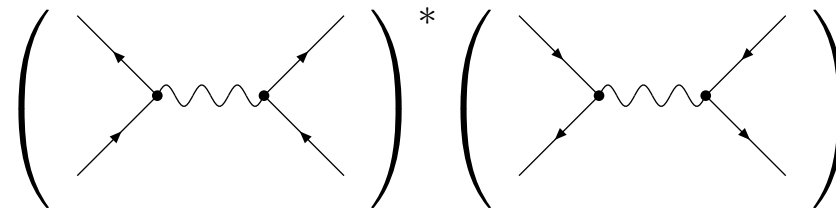
Pile-up events:

more than one hadron-hadron scattering within a bunch crossing

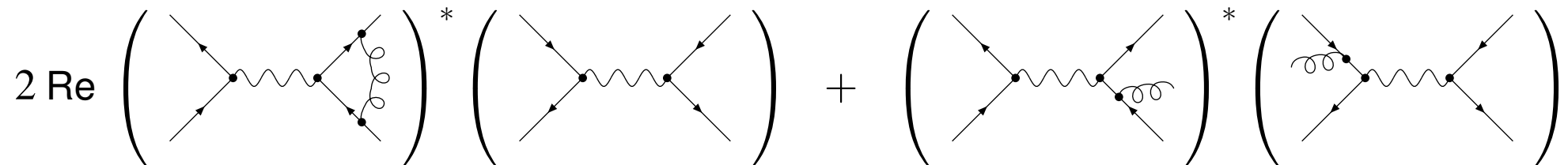
Exact perturbative calculations

Leading order (LO) and next-to-leading order (NLO):

At leading order only **Born amplitudes** contribute:



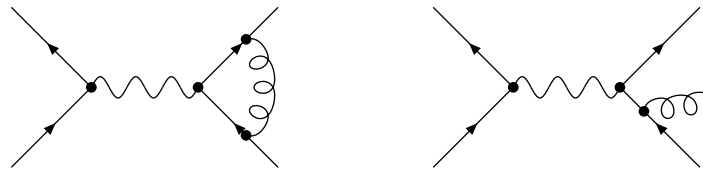
At next-to-leading order: **One-loop amplitudes** and Born amplitudes with an additional parton.



Infrared divergences and the Kinoshita-Lee-Nauenberg theorem

In addition to ultraviolet divergences, **loop integrals** can **have infrared divergences**.

For each IR divergence there is a **corresponding divergence with the opposite sign** in the real emission amplitude, when particles becomes **soft** or **collinear** (e.g. unresolved).



The **Kinoshita-Lee-Nauenberg** theorem: Any observable, summed over all states degenerate according to some resolution criteria, will be finite.

General methods at NLO

Fully differential **NLO Monte Carlo programs** need a general method to handle the cancelation of infrared divergencies.

- **Phase space slicing**

- e^+e^- : W. Giele and N. Glover, (1992)
- **initial hadrons**: W. Giele, N. Glover and D.A. Kosower, (1993)
- **massive partons, fragmentation**: S. Keller and E. Laenen, (1999)

- **Subtraction method**

- **residue approach**: S. Frixione, Z. Kunzst and A. Signer, (1995)
- **dipole formalism**: S. Catani and M. Seymour, (1996)
- **massive partons**: L. Phaf and S.W. (2001), S. Catani, S. Dittmaier, M. Seymour and Z. Trócsányi, (2002)

The dipole formalism

The dipole formalism is based on the subtraction method. The NLO cross section is rewritten as

$$\begin{aligned}\sigma^{NLO} &= \int_{n+1} d\sigma^R + \int_n d\sigma^V \\ &= \int_{n+1} (d\sigma^R - d\sigma^A) + \int_n \left(d\sigma^V + \int_1 d\sigma^A \right)\end{aligned}$$

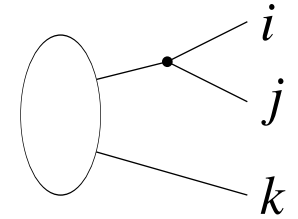
The approximation $d\sigma^A$ has to fulfill the following requirements:

- $d\sigma^A$ must be a proper approximation of $d\sigma^R$ such as to have the **same pointwise singular behaviour in D dimensions** as $d\sigma^R$ itself. Thus, $d\sigma^A$ acts as a local counterterm for $d\sigma^R$ and one can safely perform the limit $\varepsilon \rightarrow 0$.
- **Analytic integrability in D dimensions** over the one-parton subspace leading to soft and collinear divergences.

The subtraction terms

The approximation term $d\sigma^A$ is given as a sum over dipoles:

$$d\sigma^A = \sum_{\text{pairs } i,j} \sum_{k \neq i,j} \mathcal{D}_{ij,k}.$$



Each dipole contribution has the following form:

$$\mathcal{D}_{ij,k} = -\frac{1}{2p_i \cdot p_j} \mathcal{A}_n^{(0)*} (p_1, \dots, \tilde{p}_{(ij)}, \dots, \tilde{p}_k, \dots) \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} V_{ij,k} \mathcal{A}_n^{(0)} (p_1, \dots, \tilde{p}_{(ij)}, \dots, \tilde{p}_k, \dots).$$

- Colour correlations through $\mathbf{T}_k \cdot \mathbf{T}_{ij}$.
- Spin correlations through $V_{ij,k}$.

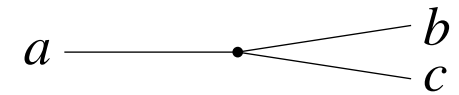
The dipoles have the correct soft and collinear limit.

The physical origin of the correlations

- In the **soft limit**, amplitudes **factorize completely in spin space**, but **colour correlations** remain.
- In the **collinear limit**, amplitudes **factorize completely in colour space**, but **spin correlations** remain.
Complete factorization after average over azimuthal angle.

Basics of shower algorithm

Starting point: **Collinear factorization**.



Probability for particle a to split into particles b and c :

$$d\mathcal{P}_a = \sum_{b,c} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) dt dz, \quad t = \ln \left(\frac{Q^2}{\Lambda^2} \right)$$

Splitting kernels:

$$P_{q \rightarrow qg}(z) = C_F \frac{1+z^2}{1-z},$$

$$P_{g \rightarrow gg}(z) = C_A \frac{(1-z(1-z))^2}{z(1-z)},$$

$$P_{g \rightarrow q\bar{q}}(z) = T_R (z^2 + (1-z)^2).$$

$P_{q \rightarrow qg}$ has a **soft singularity** for $z \rightarrow 1$, $P_{g \rightarrow gg}$ has a **soft singularity** for $z \rightarrow 1$ and $z \rightarrow 0$.

The Sudakov factor

Probability that a branching occurs during a small range of t :

$$dI(t) = dt \int_{z_-(t)}^{z_+(t)} dz \sum_{b,c} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z),$$

Sudakov factor: Probability that no branching occurs between t_0 and t_1 :

$$\Delta(t_1, t_0) = \exp \left(- \int_{t_0}^{t_1} dt \int_{z_-(t)}^{z_+(t)} dz \sum_{b,c} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) \right)$$

A typical shower algorithm

- Choose the **next scale** t according to the Sudakov factor.
- Choose the **momentum fraction** z according to $P_{a \rightarrow bc}(z)$.
- Choose the **azimuthal angle** uniform or according to spin-dependent splitting functions.
- **Insert the new particle.**
- If $t > t_{min}$ goto first step, otherwise stop.

Angular ordering

Amplitude for the emission of a soft gluon from a $q\bar{q}$ -antenna:

$$d\sigma_g = d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} d\cos\theta \frac{1 - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})}$$

$$\frac{1 - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})} = W_q + W_{\bar{q}}, \quad W_q = \frac{1}{2} \left[\frac{\cos\theta_{g\bar{q}} - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})} + \frac{1}{(1 - \cos\theta_{qg})} \right],$$

$$W_{\bar{q}} = \frac{1}{2} \left[\frac{\cos\theta_{qg} - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})} + \frac{1}{(1 - \cos\theta_{g\bar{q}})} \right].$$

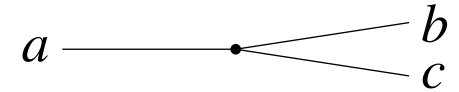
$$\int \frac{d\phi}{2\pi} W_q = \begin{cases} \frac{1}{1 - \cos\theta_{qg}}, & \text{if } \theta_{qg} < \theta_{q\bar{q}}, \\ 0 & \text{otherwise} \end{cases}$$

Angular ordering: No emission if $\theta_{qg} > \theta_{q\bar{q}}$!

Momentum conservation

1. Momentum conservation:

$$p_a = p_b + p_c$$



2. Momenta are on-shell, for massless particles:

$$p_a^2 = p_b^2 = p_c^2 = 0.$$

3. Momenta are real.

For $1 \rightarrow 2$ splittings it is not possible to satisfy all three requirements.

Recent developments

- Rewriting and improvement of [Pythia](#), [Herwig](#) and [Ariadne](#), [Sherpa](#) as a new event generator
Sjöstrand, Skands; Gieseke, Stephens, Webber; Lönnblad, Krauss, Kuhn, Schälicke, Soff;
- [Uncertainties](#) of parton showers
Gieseke; Stephens, van Hameren; Bauer, Tackmann
- Parton showers in the framework of [SCET](#)
Bauer, Schwartz, Tackmann, Thaler
- [Matching](#) of parton showers [with fixed-order tree level matrix elements](#)
Catani, Krauss, Kuhn, Webber; Mangano, Moretti, Pittau; Mrenna and Richardson;
- [Matching](#) of parton showers [with NLO](#): → next page

Recent developments

- **Matching** of parton showers **with NLO**

Frixione, Gieseke, Laenen, Latunde-Dada, Motylinski, Nason, Oleari, Ridolfi, Webber; Krämer, Mrenna, Soper; Odaka, Kurihara; Giele, Kosower, Skands;

- **Parton shower** based on the **dipole formalism**

- * **Foundation:** Nagy, Soper, '05, '06

- * **For electron-positron annihilation very similar to Ariadne.**

- * **Implementation:** Schumann, Krauss '07 and Dinsdale, Ternick, SW. '07

Parton shower based on the dipole formalism

$2 \rightarrow 3$ splittings: An emitter-spectator pair radiates off an additional particle.
Can satisfy momentum conservation and on-shell conditions.

Splitting kernels of the Sudakov factors are given by the dipole splitting functions.
Correct behaviour in the collinear and the soft limit.

No conceptual distinction between initial- and final-state shower.

Natural choice to combine with NLO

Technical details

- 4 cases for emitter-spectator-pair: final-final, final-initial, initial-final, initial-initial.
- Only the singular terms of the dipole splitting functions are unique.
- Freedom to choose the finite terms.
- For a parton shower we would like to have a probabilistic interpretation:
The splitting functions have to be non-negative everywhere.
 - Adjust finite terms
 - Rearrange terms between $\mathcal{D}_{ij,k}$ and $\mathcal{D}_{kj,i}$

Technical details

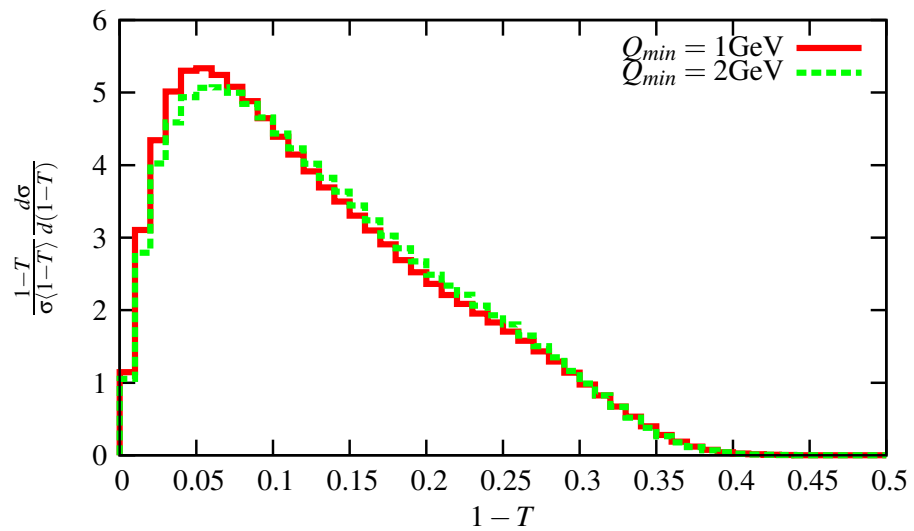
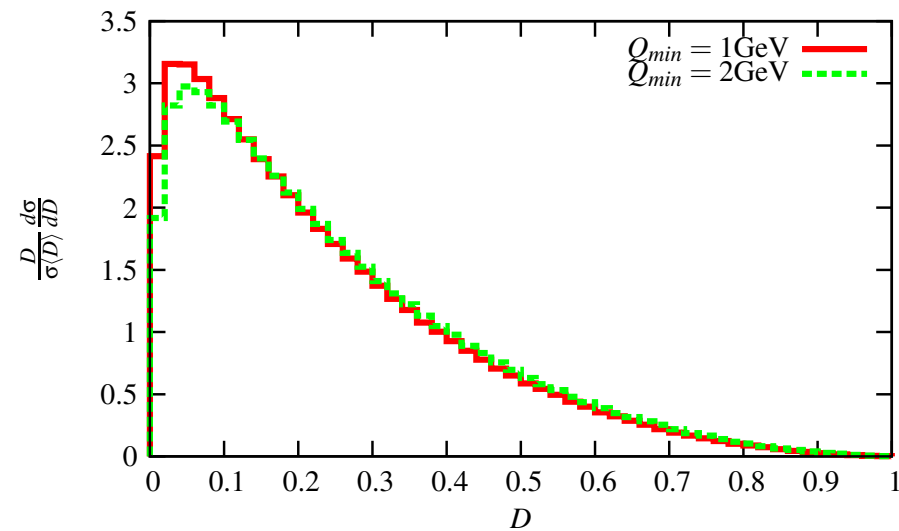
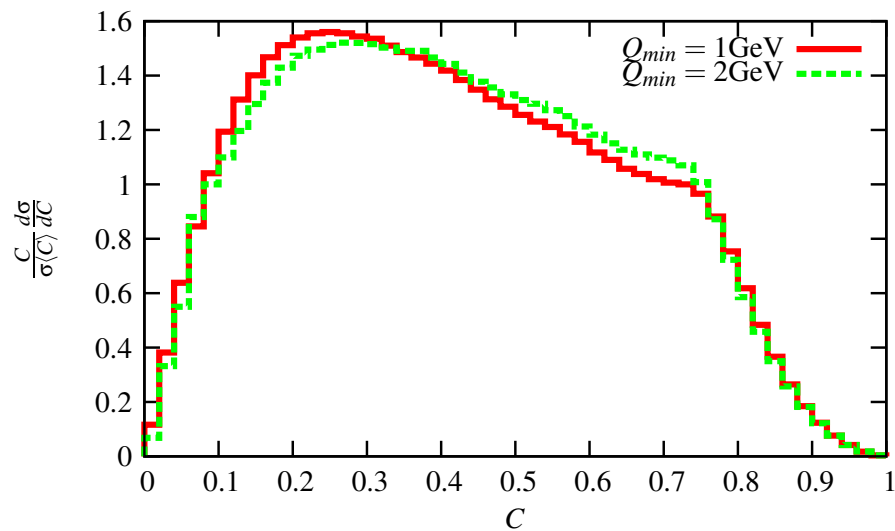
Sudakov factor:

$$\Delta_{ij,k}(t_1, t_2) = \exp \left(- \int_{t_2}^{t_1} dt C_{\tilde{i}, \tilde{k}} \int d\phi_{unres} \delta(t - T_{\tilde{i}, \tilde{k}}) \mathcal{P}_{ij,k} \right), \quad t = \ln \frac{-k_{\perp}^2}{Q^2}.$$

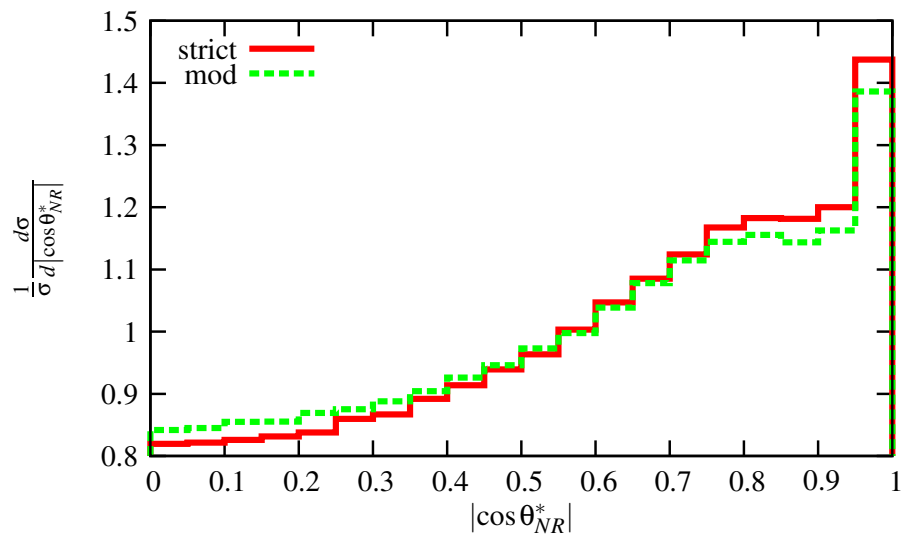
Dipole phase space:

$$\int d\phi_{unres} = \frac{(p_{\tilde{i}} + p_{\tilde{k}})^2}{16\pi^2} \int_0^1 d\kappa \int_{z_-(\kappa)}^{z_+(\kappa)} dz \frac{1}{4z(1-z)} \left(1 - \frac{\kappa}{4z(1-z)} \right), \quad \kappa = 4 \frac{(-k_{\perp}^2)}{(p_{\tilde{i}} + p_{\tilde{k}})^2}.$$

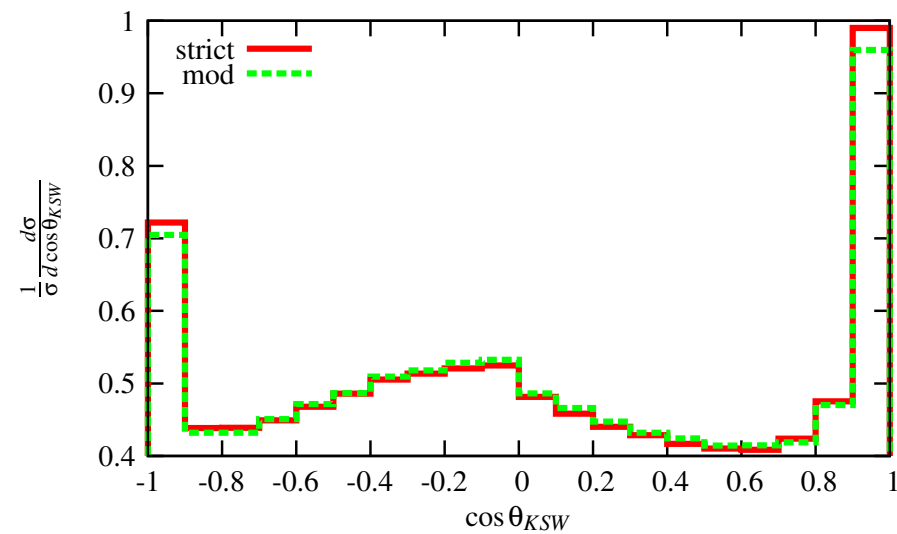
Numerical results: Electron-positron annihilation



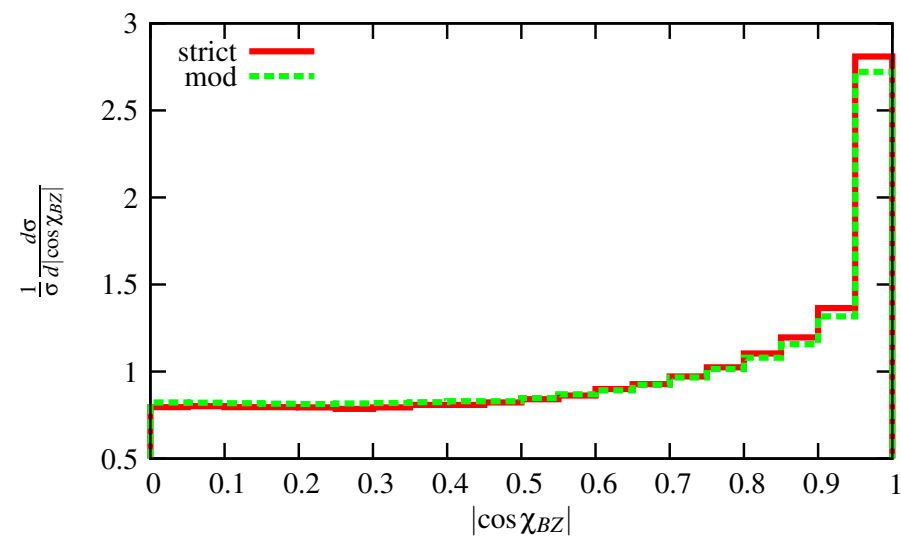
Numerical results: Four-jet angles



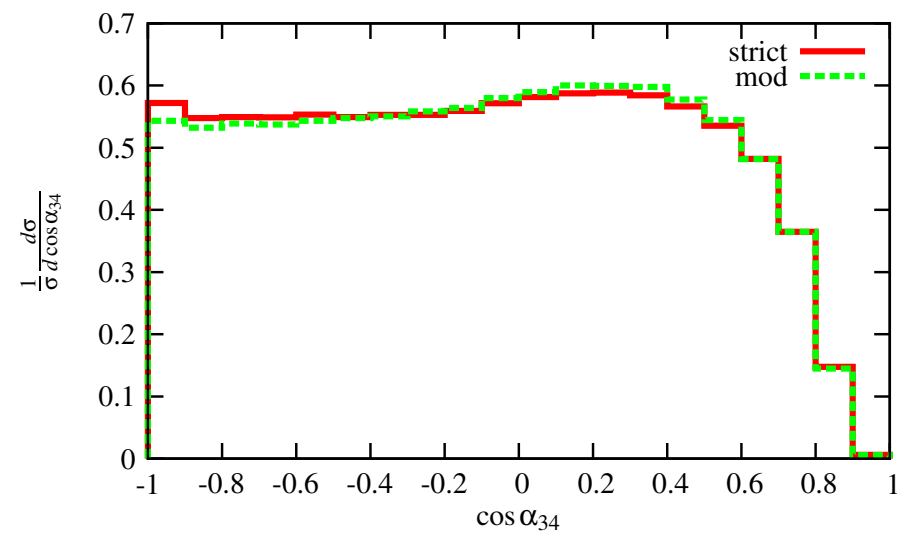
modified Nachtmann-Reiter angle



Körner-Schierholz-Willrodt angle

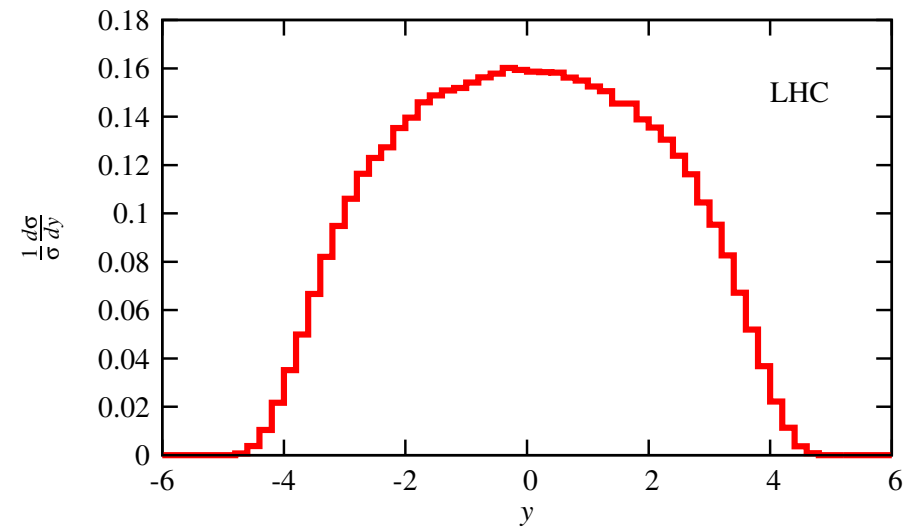
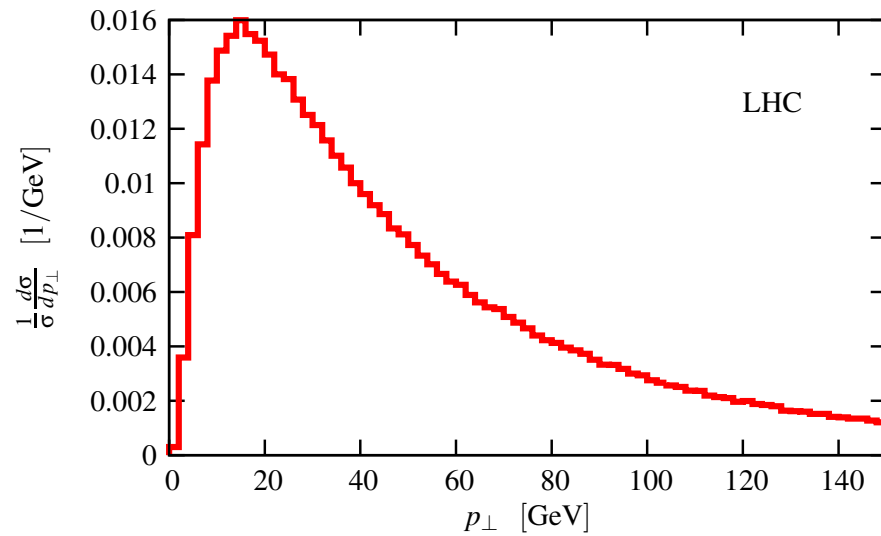
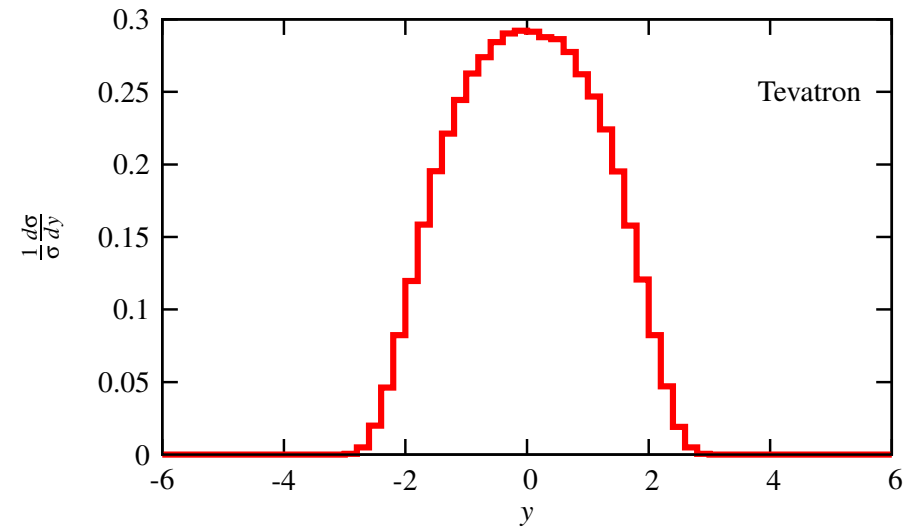
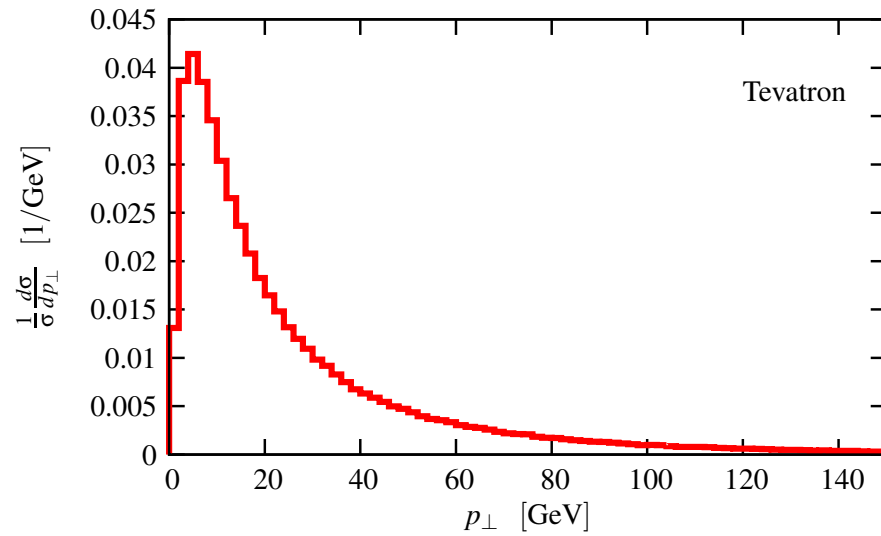


Bengtsson-Zerwas angle



angle α_{34}

Z/γ^* -production at the Tevatron and at the LHC



Summary

Implementation of a new parton shower algorithm based on the dipole formalism.

Transverse momentum as evolution variable.

Momentum conservation and “angular ordering” are inherent.

Initial- and final-state partons are treated on the same footing.

Natural choice to combine with NLO.