# Multi-leg one loop amplitudes from tree amplitudes

KET LHC-D Workshop Zurich, June 3-4,2008

Fully automated algorithm of polynomial complexity for evaluating one loop amplitudes in the Standard Model and Beyond

**R. K. Ellis, W. Giele, Z.K, arXiv.0708.2398 (EGK)**

**W. Giele, Z.K., K. Melnikov,arXiv0801.2237 (GKM)**

**W. Giele, G. Zanderighi, arXiv08005.2152 (GZ)**

# Calculating amplitudes of multi-leg processes in perturbation theory, standard approach

- **First estimates: MC's based on Born amplitudes**
- **More quantitative evaluations require NLO (QCD and EW ) corrections**
- **Standard Feynman-diagram approach are problematic:**

  **Stronger than factorial growth in number of external particles**
  **N-gluon scattering: CPU grows as $N^{(N-3)}$ (E-algorithm)**

- **Solution: use of recursion relations** ( Berends, Giele; Britto, Cachazo, Feng) :
  **CPU time has polynomial growth in the number of the external legs**
  **$N^{\alpha}$ (P- algorithm)**
- **Tree-level general purpose codes: ALPGEN, HELAC (P), MADGRAPH (E)**

- **Next-to-leading order:** **most calculations use E-algorithms**
- **Standard Feynman-graph based method using computer codes like**

  **e.g: QGRAF , FORM , Tensor Reduction** (Passarino,Veltman)

- **Many diagram (E-algorithm),**
- **Many terms from tensor reduction (new additional E-algorithm)**
  **but: very systematic and efficient simplifications in analytic treatments**

- **Semi-analytic, standard methods pushed to their limits**

  K. Ellis, Giele, Zanderighi **6g one-loop amplitudes**

  Denner Dittmaier $e^+e^- \to \mu^+\mu^- \tau^+\tau^-$ **QED NLO**

  Dittmaier, Uwer, Weinzierl $p+p \to t+t$ **jet NLO**

- **Completely numerical methods use Feynman diagram but** avoid tensor reduction
  Nagy, Soper; Lazapoulos, Melnikov, Petriello; Anastasiou, Beerli, Daleo

# The Unitarity Method: successful P-algorithm at NLO

*Brief history*

**S-matrix theory :** **two particle scattering amplitude is given in terms of its imaginary part** (Landau, ...1950's)

**Perturbative gauge theories at NLO** (Bern, Dixon, Dunbar, Kosower,1994) **:**

**i) Decomposition of one-loop amplitudes in terms of finite number of well defined scalar integral functions**
*(*Passarino, Veltman*)*

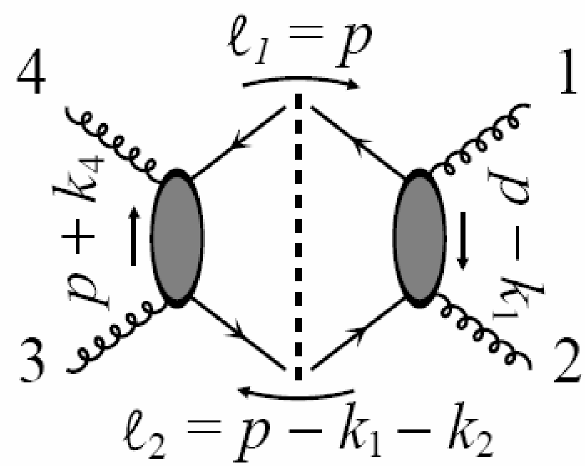
**ii) imaginary part of one-loop amplitudes is given in terms of products gauge invariant tree amplitudes**

Bern, Dixon, Kosower **(**1994-1998) : **construct gauge theory one-loop amplitudes from tree amplitudes.**

# Unitarity based on-shell method

$$T^\dagger - T = -2iT^\dagger T$$

$\text{Im } T^{1\text{-loop}} = \sum c_j \text{ Im } I_j$



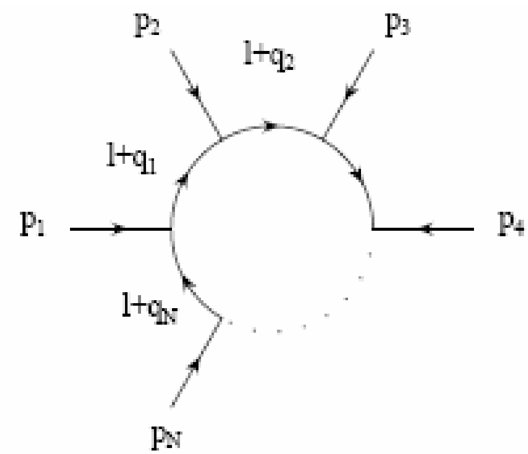

$$-i\,\text{Disc}\ A_4(1,2,3,4)\Big|_{s-\text{cut}} = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\delta^{(+)}(\ell_1^2 - m^2)\, 2\pi\delta^{(+)}(\ell_2^2 - m^2) \times A_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2)\, A_4^{\text{tree}}(-\ell_2, 3, 4, \ell_1)\,,$$

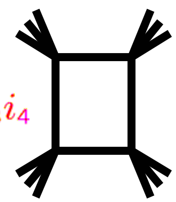
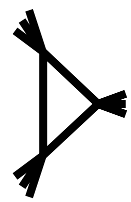
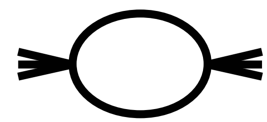
# One-loop N-point amplitudes in terms of master integrals

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N) = \int [d\,l]\ \mathcal{A}(p_1,p_2,\ldots,p_N;l)$$

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2 \cdots d_N}$$

$$d_i = (l+q_i)^2 - m_i^2 = (l - q_0 + \sum_{j=1}^{i} p_i)^2 - m_i^2$$




$$\mathcal{A}_N(\{p_i\}) = \sum d_{i_1 i_2 i_3 i_4}$$  $$+\sum c_{i_1 i_2 i_3}$$  $$+\sum b_{i_1 i_2}$$ 

+ Tadpoles + Rational part

- Calculate the discontinuites of the left-hand side using unitarity
- Calculate the discontinuites of the right-hand side and read out the coefficients
- Rational part: ε dependence of the coefficients $b_{ij}$, FDHS

# Initial attempts

***DIFFICULTIES:***

i) Tensor integrals in the imaginary part with Passarino-Veltman reduction

ii) The discontinuities are treated in four dimensions (no rational parts)

iii) Only double cuts have been applied.

***RESULTS:***

i) BDK theorem: SUSY gauge theories no rational parts
important applications to N=1,N=4 SYM

ii) Impressive QCD result: $e^+ + e^-$ annihilation to four jets in NLO (1998)

(analytic result, only four-dimensional state on cut lines, spinor helicity formalism, set of magic tricks, rational part is fixed from soft and collinear limits, even triple cuts has been used as a tricky procedure, SUSY identities etc. )

# Attempts for more systematic treatments

***LIMITED INTEREST (1998-2004), even though prospect for progress was there***

i) D-dimensional integrals can be reconstructed fully from the imaginary part ( van Neerven, 1986)
ii) To get the rational part treat the cut lines in D-dimension (Bern-Morgan,1996)
iii) Application where we cut massive fermion lines (Chalmers, Bern, 1996)

***NEW INSPIRATION FROM TWISTOR FORMULATION*** (Witten, 2003, Santa Barbara Workshop 2004)

i) Generalized unitarity (Britto, Cachazo, Feng) , complex four momenta
ii) New tree level recursion relations (Britto,Cachazo, Feng, Witten)
iii) New loop level recursion relations for rational parts (Bern, Dixon, Koswer, …)
iv) Reduction with spinorial integration (Britto, Cachazo, Feng, Mastrolia).

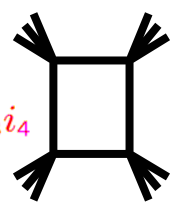
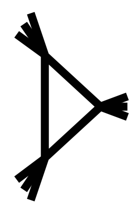
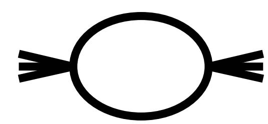
***RECENT DEVELOPMENTS***

i) New algebraic reduction (parametric integration) (Ossola, Papadopoulos, Pittau ,2006)
ii) D-dimensional unitarity and reduction with spinorial integration (Anastasiou, Britto, Feng, Kunszt, Mastrolia,2006)
iii) Numerical implementation of unitarity method for cut-constructible part of the 6gluon amplitude (Ellis, Giele, Kunszt)

# Decomposing one-loop N-point amplitudes in terms of master integrals (cont.)

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N) = \sum_{1\le i_1<i_2<i_3<i_4\le N} d_{i_1i_2i_3i_4}(p_1,p_2,\ldots,p_N) I_{i_1i_2i_3i_4}$$
$$+ \sum_{1\le i_1<i_2<i_3\le N} c_{i_1i_2i_3}(p_1,p_2,\ldots,p_N) I_{i_1i_2i_3}$$
$$+ \sum_{1\le i_1<i_2\le N} b_{i_1i_2}(p_1,p_2,\ldots,p_N) I_{i_1i_2}$$
$$+ \sum_{1\le i_1\le N} a_{i_1}(p_1,p_2,\ldots,p_N) I_{i_1}$$

$$I_{i_1\cdots i_M} = \int [d\,l]\,\frac{1}{d_{i_1}\cdots d_{i_M}}$$

$\mathcal{A}_N(\{p_i\}) = \sum d_{i_1i_2i_3i_4}$  $+\sum c_{i_1i_2i_3}$  $+\sum b_{i_1i_2}$ 

**FDHS scheme: coefficients d and c are independent from ε**

**Rational part is generated by the order ε part of $b_{ij}$**

# Numerical implementation of the unitarity method in four dimension (EGK,unitarity.f)

Ossola, Papadopoulos, Pittau: there is a systematic way of algebraic reduction at the integrand level. The numerator can be decomposed as linear combination of 4-,3-,2,-1 denominator factors

EGK (2007): follow OPP but use the van Neerven Vermaseren basis and multi-pole expansion of rational functions and use as input only tree amplitudes

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2 \cdots d_N} =$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\overline{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\overline{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\overline{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1\le i_1\le N} \frac{\overline{a}_{i_1}(l)}{d_{i_1}}$$

Denominator factors : $d_i=(l + q_i)^2 - m_i^2$

The residues can be decomposed into finite number of Lorenz-structure in the loop momenta

# Parametrizing the loop momenta

The loop momenta can be decomposed in terms of suitable fixed basis vectors.

**we use:** **dual momenta $v_i$** **$p_i v_j = \delta_{ij}$**
**and orthogonal** **unit vectors $n_i$**

Decomposition of the loop-momentum

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2}(d_i - d_{i-1})\, v_i^\mu + \sum_{i=1}^{D_T} \alpha_i\, n_i^\mu \ ,$$

$$V_R^\mu = -\frac{1}{2} \sum_{i=1}^{D_P} \left( (q_i^2 - m_i^2) - (q_{i-1}^2 - m_{i-1}^2) \right) v_i^\mu$$

# Solving the unitarity conditions

**Contributions with four cut propagators $d_i=d_j=d_k=d_l=0$ two solutions**

$$l^\mu = V_4^\mu + \alpha_1\, n_1^\mu$$

$$l_\pm^\mu = V_4^\mu \pm i\,\sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

**Complex valued loop momenta**

**Triangle, infinite # of solutions (on a circle)**

$$l^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu$$

$$l_{\alpha_1\alpha_2}^\mu = V_3^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu;\;\; \alpha_1^2 + \alpha_2^2 = -(V_3^2 - m_k^2)$$

**Bubble, infinite # of solutions (on a "sphere")**

$$l^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu$$

$$l_{\alpha_1\alpha_2\alpha_3}^\mu = V_2^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu + \alpha_3\, n_3^\mu;\;\; \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = -(V_2^2 - m_j^2)\ .$$

# l-dependence of the residues in 4 dimension

**Box residue, 2 structures:**

$$\bar{d}_{ijkl}(l) \equiv \bar{d}_{ijkl}(n_1 \cdot l)\,, \quad (n_1 \cdot l)^2 \sim n_1^2 = 1\,, \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$$

**Triangle residues, 7 structures:**

$$\bar{c}_{ijk}(l) = c_{ijk}^{(0)} + c_{ijk}^{(1)} s_1 + c_{ijk}^{(2)} s_2 + c_{ijk}^{(3)} (s_1^2 - s_2^2) + s_1 s_2 (c_{ijk}^{(4)} + c_{ijk}^{(5)} s_1 + c_{ijk}^{(6)} s_2)$$

**Bubble residues, 9 structures**

$$\bar{b}_{ij}(l) = b_{ij}^{(0)} + b_{ij}^{(1)} s_1 + b_{ij}^{(2)} s_2 + b_{ij}^{(3)} s_3 + \ldots$$

**Non-vanishing contributions come from d_{ijkl},c_{ijk}^{(0)} and b_{ij}^{(0)}.**
**The other terms are called spurious.**

# The residues of the poles can be obtained algebraically.

The residue is taken at special loop momentum defined by the unitarity conditions.

$$\text{Res}_{ij\cdots k}\left[F(l)\right] \equiv \left[d_i(l)d_j(l)\cdots d_k(l)F(l)\right]\Big|_{l=l_{ij\cdots k}} .$$

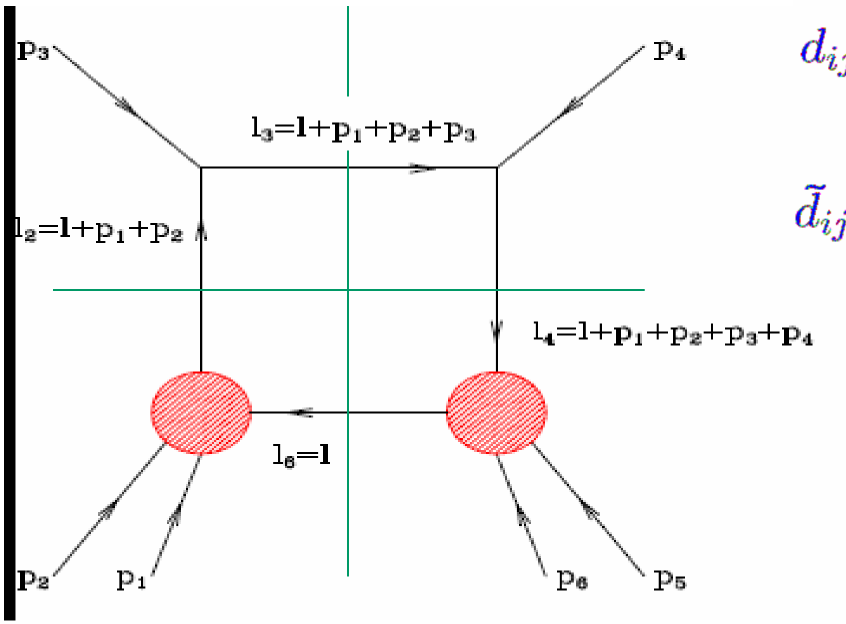
$$\bar{d}_{ijkl}(l) = \text{Res}_{ijkl}\big(\mathcal{A}_N(l)\big)$$ $d_i=d_j=d_k=d_l=0$

$$\bar{c}_{ijk}(l) = \text{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=d_k=0$

$$\bar{b}_{ij}(l) = \text{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j}\frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=0$

# Calculating the box residue

$$\bar{d}_{ijkl}(l) \equiv \bar{d}_{ijkl}(n_1 \cdot l)\,, \quad (n_1 \cdot l)^2 \sim n_1^2 = 1\,, \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$$




$$d_{ijkl} = \frac{\mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l^+)\right) + \mathrm{Cut}_{ijkl}\left(\mathcal{A}_N(l^-)\right)}{2}$$

$$\tilde{d}_{ijkl} = \frac{\mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l^+)\right) - \mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l^-)\right)}{2i\sqrt{V_4^2 - m_l^2}}$$

**the residue of the amplitude factorizes to the product of tree amplitudes**

$$\mathrm{Rest}_{2346}\left(\mathcal{A}_6(l^\pm)\right) = \mathcal{M}_4^{(0)}(l_6^\pm; p_1, p_2; -l_2^\pm) \times \mathcal{M}_3^{(0)}(l_2^\pm; p_3; -l_3^\pm)\mathcal{M}_3^{(0)}(l_3^\pm; p_4; -l_4^\pm)$$
$$\times \mathcal{M}_4^{(0)}(l_4^\pm; p_5, p_6; -l_6^\pm)$$

# Numerical Implementation

**Check the singular parts**:

$$m^{(1)}(1,2,\ldots,n) \sim \left(-\frac{n}{\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{11}{3} + \sum_{i=1}^{n} \log\left(\frac{s_{i,i+1}}{\mu^2}\right)\right)\right) \times m^{(0)}(1,2,\ldots,n) + \mathcal{O}(1)$$

**Compare with known analytic and numerical results.**

**Compare  CPU time with those of the traditional method in case of  6g, 5g ,… amplitudes**

# Numerical Implementation (cont.)

## Comparison of CPU times

**100000 points are generated away from soft and collinear region.**
**Cuts on transverse momenta, rapidity and separation of the outgoing gluons**

**EGZ: 9s per ordered amplitude on 2.8GHz Pentium processor**
**EGK: 0.01s per ordered amplitude on 2.8GHz Pentium processor**

| | ev.time | # of cuts |
|---|---|---|
| 4 gluon: | 0.0009s | 6 |
| 5 gluon: | 0.0035s | 20 |
| 6 gluon : | 0.0107s | 44 |

**Computer time: scales with $\approx$ n^4 (# of cuts) not as n!**

# Numerical Unitarity Method in D-dimension with arbitrary spin and mass (GKM)

**Two sources of D-dependence**

**i) spin-polarization states live in $D_s$ .**

**ii) loop momentum component live in D. ($D_s$>D)**

$$\mathcal{A}_{(D,D_s)}(\{p_i\},\{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\},\{J_i\};l)}{d_1 d_2 \cdots d_N}.$$

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b},$$

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^{D} l_i^2$$

# Two key features

**1. Dependence on $D_s$ is linear**

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

Enough to choose two integer values $D_s = D_1$ and $D_s = D_2$ to reconstruct the full $D_s$ dependence. Suitable for numerical implementation.

$D_s=4-2\varepsilon$ 't Hooft Veltman scheme, $D_s=4$ FDHS

**2. Only one extra momentum component. It enters quadratically**

$N(l) = N(l_4, \mu^2)$ **where** $\mu^2 = -l_5^2 - \ldots - l_D^2$

In D-dimension for one loop integrals maximum 5 constraint:
we need to consider also pentagon integrals and pentagon cuts.

# Reduction in D-dimensions

**The parametrization of the N-particle amplitude**

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}}$$

$$+ \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

**Parametrization of the residues**

**Pentuple residue:** $\overline{e}^{(D_s)}_{ijkmn}(l) = e^{(D_s,(0))}_{ijkmn}$

**Box residue:** $\overline{d}^{\mathrm{FDH}}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn} s_1 + (d^{(2)}_{ijkn} + d^{(3)}_{ijkn} s_1) s_e^2 + d^{(4)}_{ijkn} s_e^4$

**Three extra structures for triple, three for double and zero for single cuts**

# Four new master integrals

**Four of the $s_e^2$ dependent master integrals are not spurious**

$$\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = -\frac{D-4}{2} I^{D+2}_{i_1 i_2 i_3 i_4}, \quad \ldots$$

**We obtain the full D-dependence of the amplitude**

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5} \; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$

$$+ \sum_{[i_1|i_4]} \left( d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} - \frac{D-4}{2} d^{(2)}_{i_1 i_2 i_3 i_4} \; I^{(D+2)}_{i_1 i_2 i_3 i_4} + \frac{(D-4)(D-2)}{4} d^{(4)}_{i_1 i_2 i_3 i_4} \; I^{(D+4)}_{i_1 i_2 i_3 i_4} \right)$$

**+ similar terms for triangle, bubble and tadpole contributions.**

**As ε -> 0 the new master integrals can be decomposed in the old basis and generate ε dependent bubble coefficients !**

# One-loop amplitudes up to terms of order ε

**One loop amplitudes as sum of cut-constructible and rational parts:**

$$\mathcal{A}_N = \mathcal{A}_N^{CC} + R_N .$$

**The cut constructible part is as before (EGK):**

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} \tilde{d}^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} \; I^{(4-2\epsilon)}_{i_1 i_2} + \sum_{i_1=1}^{N} a^{(0)}_{i_1} \; I^{(4-2\epsilon)}_{i_1} ,$$

**The rational part is new (GKM):**

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{3} - \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left( \frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m^2_{i_1} + m^2_{i_2}}{2} \right) b^{(9)}_{i_1 i_2} .$$

# 4g,5g,6g scattering amplitudes in QCD

- **One type of color ordered subamplitude for each helicity.**
- **We choose $D_1=5$ and $D_2=6$.**

$$\mathcal{A}^{\text{FDH}} = 2\mathcal{A}_{(D,D_s=5)} - \mathcal{A}_{(D,D_s=6)}$$

- **For the computation of the residues we have to consider four and five dimensional loop-momenta on the cuts, embedded into five- and six-dimensional space-time and fulfilling the unitarity constraints;**
- **we do not use supersymmetry;**
- **tree amplitudes are calculated with Berends-Giele recursion relations in $D_s=5$ and $D_s=6$ dimensions;**
- **Individual coefficients have been obtained by projection (sum over specially chosen loop momenta on the cut).**

# Numerical Evaluation using Maple

1. Comparison with
   i) known analytic results (Bern,Kosower, Britto; Feng, Mastrolia)
   ii) known semi-numerical results (IBP) (Ellis, Giele, Zanderighi)

2. Additional computer time due to D-dimensional treatment is insignificant (< factor 2).

| $\lambda_1, \lambda_2, \ldots, \lambda_6$ | $\Delta^{\rm cut}$ | $\Delta^{\rm rat}$ | $\Delta$ |
|---|---|---|---|
| - - + + + + | -19.481065+78.147162 $i$ | 28.508591-74.507275 $i$ | 9.027526+3.639887 $i$ |
| - + - + + + | -241.10930+27.176200 $i$ | 250.27357-25.695269 $i$ | 9.164272+1.480930 $i$ |
| - + + - + + | 5.4801516-12.433657 $i$ | 0.19703574+0.25452928 $i$ | 5.677187-12.179127 $i$ |
| - - - + + + | 15.478408-2.7380153 $i$ | 2.2486654+1.0766607 $i$ | 17.727073-1.661354 $i$ |
| - - + - + + | -339.15056-328.58047 $i$ | 348.65907+336.44983 $i$ | 9.508509+7.869351 $i$ |
| - + - + - + | 31.947346+507.44665 $i$ | -17.430910-510.42171 $i$ | 14.516436-2.975062 $i$ |

TABLE III: Finite parts of singular six-gluon scattering amplitudes for various gluon helicities.

# Comment on application to massive quarks

1. $D_s$ can be chosen to be any arbitrary EVEN integer values
2. For massive particles we have more master integrals and tadpole contributions have to be also evaluated
3. Special treatment of the external self enery corrections

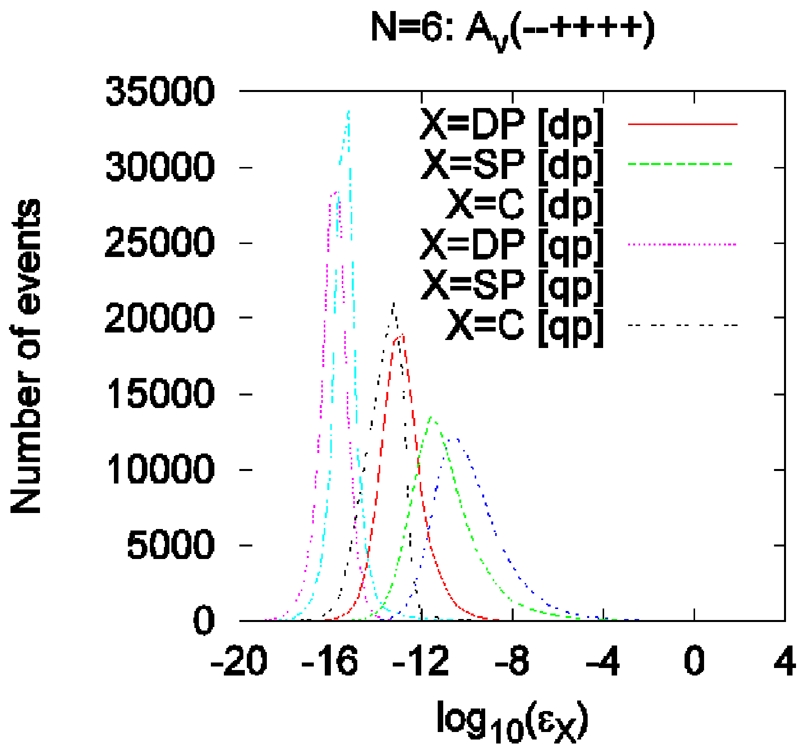
# Recent application: ROCKET

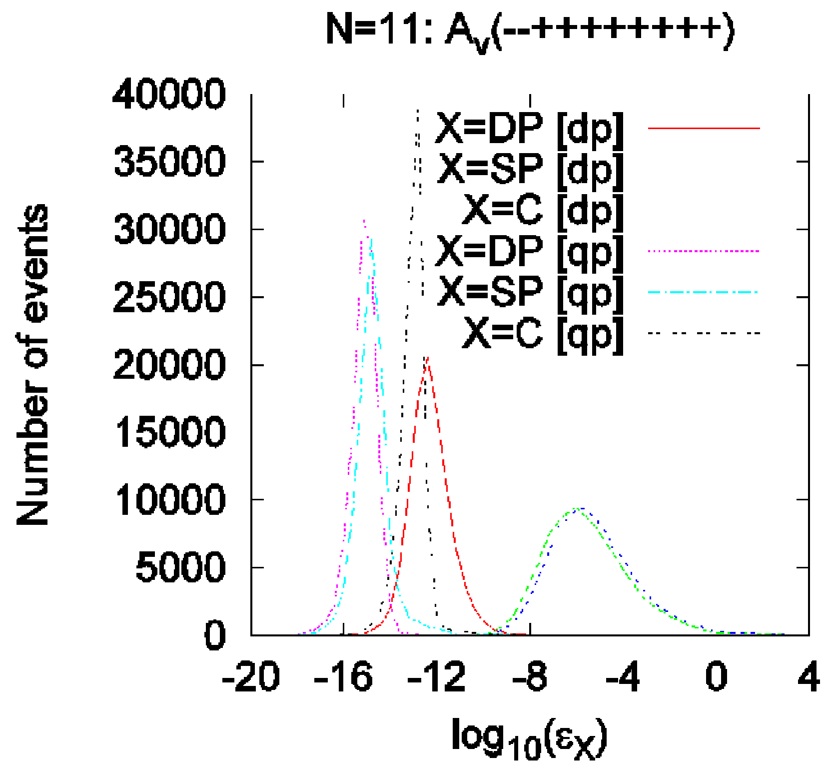
( Giele and Zanderighi )

The polynomial algorithm of GKM for the calculation of full one-loop amplitudes has been recently implemented in the fortran code Rocket.

- **Computer time: scales with # of cuts ( $n^4$ ) and not as n! confirms that time scale growth $n^4$ found earlier up to 20 gluons.**
- **Using quadrupole precision excellent accuracy is obtained**

# Some numerical results of (GZ), MHV amplitudes




$$S = \log_{10}\left(\left|\frac{m^{(1)}_{\text{unitarity}} - m^{(1)}_{\text{analytic}}}{m^{(1)}_{\text{analytic}}}\right|\right) .$$

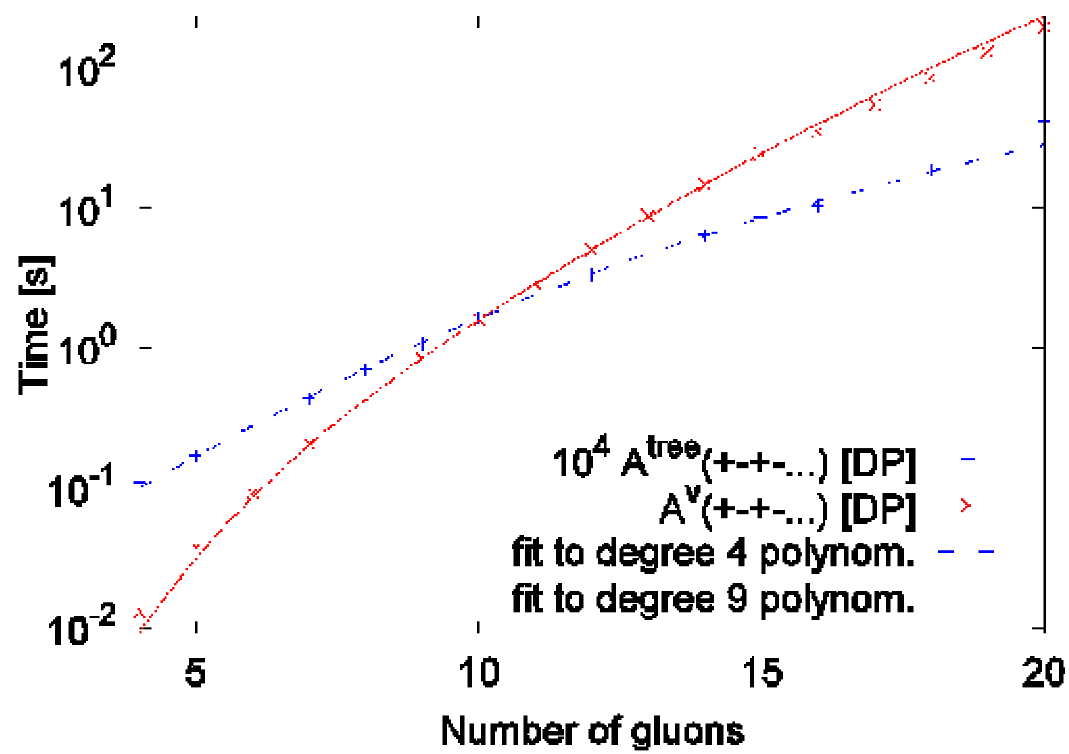
# Time in seconds needed to calculate tree and loop amplitudes

Time for large N: $\tau(\text{tree}) \approx N^4$ $\tau(\text{loop}) \approx N^9$

**Evaluation times:**

factor ≈ 30 increase
from doubleP to QP

2.33GHz Xeon proc
6gluon ≈ 90ms

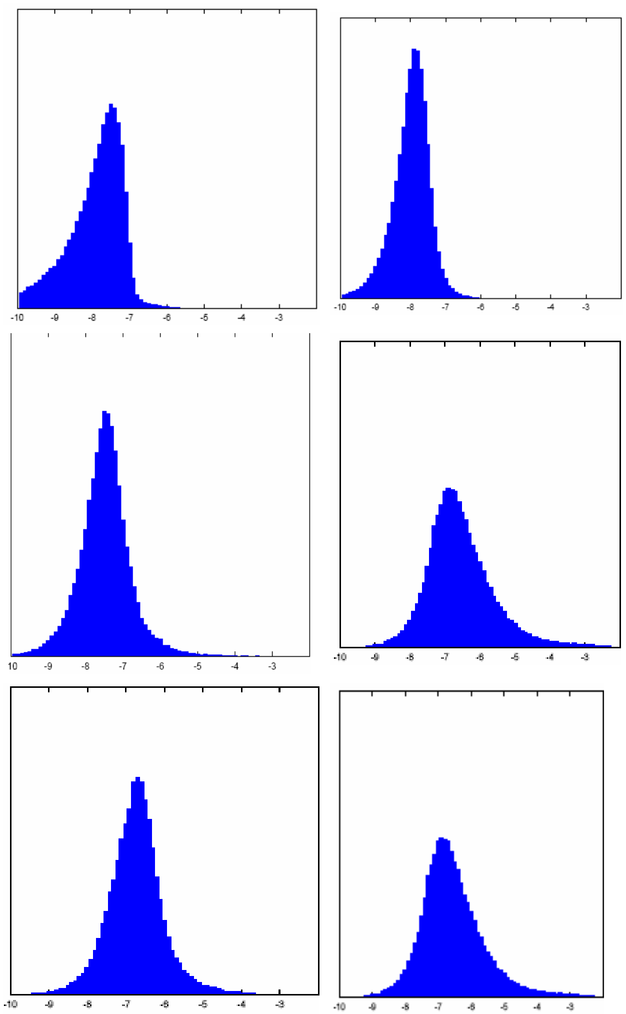


# Concluding remarks

- **EGK, GMK have developed novel method for calculating the full one-loop scattering amplitudes including the rational parts.**
- **The method is based on unitarity cuts in higher-dimensional space-time.**
- **The algorithm has polynomial complexity and suitable for efficient numerical evaluation one-loop amplitudes**
- **Important step towards automated computation of NLO cross-section of multi-leg processes.**
- **Applicable to any multi-particle processes. The virtual particles involved can have arbtirary spin 0,1/2,1 and arbitrary masses.**
- **One can use it to calculate the NLO corrections for such complicated process as PP $\rightarrow$ tt + 2,3 jets and PP $\rightarrow$ V + 3,4,5 jets.**
- **The computer time depends on the speed of calculating tree amplitudes!**

# Relative errors for 100000 ordered amplitudes

4-gluon 6-gluon



Horizontal axis: $S = \log_{10}\left(\left|\frac{m^{(1)}_{\text{unitarity}} - m^{(1)}_{\text{analytic}}}{m^{(1)}_{\text{analytic}}}\right|\right)$ .

Range of S: (-10, -2)

Vertical axis: number of events

Majority of events agree with rel. precision $10^{-6}$ or better

# Numerical instabilities

Presence of Gram-determinants in the box, triangle and bubble coefficients.

In the solution of the unitarity constraints we have maximum power: -2

# 4g,5g,6g scattering amplitudes in QCD

- **We obtain a tirivial derivation of the known result**

$$\mathcal{A}^{\mathrm{HV}} = \mathcal{A}^{\mathrm{FDH}} - \frac{c_\Gamma}{3}\mathcal{A}_{\mathrm{tree}} \ .$$

- **Polarization vectors in five and six dimension for loop momenta embedded in four and five dimensions.**