RG-improved fully differential predictions for top-pair production

Andrew Papanastasiou*

LHC Physics Discussion: top

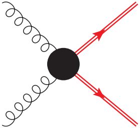
20th October 2014, DESY

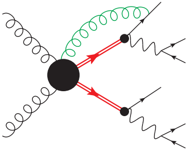
*In collaboration with: **Alessandro Broggio** and **Adrian Signer** (PSI) [1407.2532]

Differential top-pair production

- measurements of differential cross-sections → test theory predictions

LHC:
- deal with reconstructed quantities since top quarks decay, $t \to W^+ b$
- cut-based analyses with leptons-jets-$\not{E}_T$ in final state (non-inclusive)





Stable tops:

$\sim d\sigma^{\mathrm{NNLO}}$ [Czakon,Fiedler,Mitov - soon]

✓ $\sigma^{\mathrm{NNLO+NNLL}}_{\mathrm{total}}$ [Bärnreuther,Czakon,Fiedler,Mitov]

✓ $\frac{d\sigma^{\mathrm{NLO+NNLL}}}{dM_{tt}\, d\cos\theta}$, $\frac{d\sigma^{\mathrm{NLO+NNLL}}}{dp_T\, dy}$

[Kidonakis,Laenen,Moch,Vogt,...; DiffTop: Guzzi,Lipka,Moch Ahrens,Ferroglia,Neubert,Pecjak,Yang]

Unstable tops:

✓ $d\sigma^{\mathrm{NLO}}$: production and decay in NWA [Bernreuther et al.; Melnikov, Schulze; Campbell, Ellis]

✓ $d\sigma^{\mathrm{NLO}}$: offshell [Bevilacqua et al; Denner et al; Falgari et al.; Frederix; Cascioli et al., Heinrich et al.]

Is it possible to improve fixed-order, fully differential NLO predictions for unstable top-quarks, without computing $W^+W^-b\bar{b}$ @ NNLO?

Improvement of $t\bar{t}$ via RGEs

RG improvement:

- RGEs provide access to a class of contributions from higher orders in perturbation theory using knowledge of lower orders

  → terms proportional to singular distributions, P_n, and some constants

- these can capture the important contributions from higher orders (?)
- RGEs → approx-NNLO predictions for (stable) $t\bar{t}$

  → $M_{t\bar{t}}$ & p_T distributions known in both QCD & SCET (PIM & 1PI)

Idea:

- 'improve' the weights of events using approx.-NNLO corrections
- use these to study arbitrary experimentally relevant distributions (i.e. not just $p_T(t)$, $M(t\bar{t})$ ...)

Outcomes:

- generalize approx-NNLO corrections of [Ahrens et al (SCET)] for use in a fully-differential parton-level Monte Carlo (including decay of tops)
  - study universality of soft-gluon approximations/resummations
  - hope to obtain important NNLO corrections of $pp \to t\bar{t} \to W^+ b W^- \bar{b}$ (in production subprocess)

Pair-invariant mass (PIM) & one-particle inclusive (1PI) kinematics

PIM: $h_1(P_1) + h_2(P_2) \to (t + \bar{t})(p_t + p_{\bar{t}}) + X(p_X)$

$$\frac{d\sigma}{dM_{t\bar{t}}d\cos\theta} \sim \sum_{i,j} \int_\tau^1 \frac{dz}{z} \int_{\tau/z}^1 \frac{dx}{x} f_{i/h_1}(x,\mu_F) f_{j/h_2}(\tau/(zx),\mu_F) \left(\mathrm{Tr}\left[\mathbf{H_{ij}} \cdot \mathbf{S_{ij}^{PIM}}\right] + \mathcal{O}(1-z) \right)$$

- soft-gluon limit: $z = M_{t\bar{t}}/\hat{s} = (p_t + p_{\bar{t}})^2/\hat{s} \to 1$
- plus-distributions: $P_n(z) = \left[\frac{\log^n(1-z)}{1-z}\right]_+ \in \mathbf{S_{ij}^{PIM}}$

1PI: $h_1(P_1) + h_2(P_2) \to t(p_t) + (\bar{t} + X)(p_{\bar{t}} + p_X)$

$$\frac{d\sigma}{dp_T dy} \sim \sum_{i,j} \int_{x_1^{\min}}^1 \frac{dx_1}{x_1} \int_{x_2^{\min}}^1 \frac{dx_2}{x_2} f_{i/h_1}(x_1,\mu_F) f_{j/h_1}(x_2,\mu_F) \left(\mathrm{Tr}\left[\mathbf{H_{ij}} \cdot \mathbf{S_{ij}^{1PI}}\right] + \mathcal{O}(s_4) \right)$$

- soft-gluon limit: $s_4 = (p_{\bar{t}} + p_X)^2 - m_t^2 \to 0$
- plus-distributions: $P_n(s_4) = \left[\frac{1}{s_4}\log^n\left(\frac{s_4}{m_t^2}\right)\right]_+ \in \mathbf{S_{ij}^{1PI}}$

[QCD: Kidonakis,Laenen,Moch,Smith,Sterman,Vogt ... '97-] [SCET: Ahrens, Ferroglia, Neubert, Pecjak, Yang '10, '11]

Obtaining approximate (N)NLO contributions

1. Hard function, $\mathbf{H_{ij}}$ (new):
   - computed 1-loop modified hard function [using helicity amplitudes of Badger et al '10]
   - tops are decayed (fixed-order, in NWA), semi-leptonically ($t \to l^+ \nu_l b$)
   - production/decay amplitudes sowed together using spinor-helicity methods

2. Soft functions: $\mathbf{S_{ij}}$ & RGEs
   - 1-loop $\mathbf{S_{ij}^{PIM}}$ and $\mathbf{S_{ij}^{1PI}}$ and structure of RGEs [Ferroglia et al. '09, Ahrens et al '10, '11] unchanged by decay
   - obtain approximate N(N)LO contributions by setting $\mu_h = \mu_s = \mu_F$ equal and expanding solutions to α_s (α_s^2) $\to$ coefficients of plus-distributions

$$\left[\mathbf{H_{ij}} \cdot \mathbf{S_{ij}^{PIM}}\right]^{(2)} \sim \sum_{m=0}^{3} D_m^{(2)}(M_{t\bar{t}}, \cos\theta, \mu_F) P_m + C_0^{(2)}(M_{t\bar{t}}, \cos\theta, \mu_F)\delta + R^{(2)}$$

- $\mathbf{H_{ij}^{(1)}} \cdot \mathbf{S_{ij}^{(1)}}$: include fully
- $\mathbf{H_{ij}^{(0)}} \cdot \mathbf{S_{ij}^{(2)}}$: include partially
- $\mathbf{H_{ij}^{(2)}} \cdot \mathbf{S_{ij}^{(0)}}$: don't include

(first two:) obtain $D_{0,1,2,3}^{(2)}$ completely

(last two:) obtain $C_0^{(2)}$ only partially

Monte Carlo integration

Inclusively:

$$\mathbf{H_{ij}} \cdot \mathbf{S_{ij}} \sim \sum_{m=0}^{2n-1} D_m^n(M_{t\bar{t}}, \cos\theta, \mu_F) P_n + C_0^n(M_{t\bar{t}}, \cos\theta, \mu_F)\delta + R^n$$

Restore explicit dependence on outgoing particle momenta, $\{p_i\}$:

$$\rightarrow \sum_{m=0}^{2n-1} D_{m,ij}^n(\{p_i\}, \mu_F) P_n + C_0^n(\{p_i\}, \mu_F)\delta + R^n$$

Monte-Carlo phase-space integrator:

- generate phase-space (momentum-configurations) $\{p_i\}$
- evaluate approximate contributions using $\{p_i\} \rightarrow$ weights
- bin weights according to observables constructed from final-state momenta
- phase-space numerical:
  - choose to make same approximations in phase-space as in [Ahrens et al.]
  - different implementations would differ by formally sub-leading terms

RG-improved predictions

notation: 'n' = approx-N

Study nLO and nNLO compared with standard NLO

$$d\sigma^{\text{nLO}}_{\text{full}} = (\Gamma_t^{\text{NLO}})^{-2} \Big\{ \Big(d\sigma^{(0)}_{t\bar{t}} + d\tilde{\sigma}^{(1)}_{t\bar{t}} \Big) \otimes d\Gamma^{(0)}_{t\to l^+\nu_l b} \otimes d\Gamma^{(0)}_{\bar{t}\to l^-\bar{\nu}_l\bar{b}} + d\sigma^{(0)}_{t\bar{t}} \otimes d\Gamma^{(1)}_{t\to l^+\nu_l b} \otimes d\Gamma^{(0)}_{\bar{t}\to l^-\bar{\nu}_l\bar{b}} + d\sigma^{(0)}_{t\bar{t}} \otimes d\Gamma^{(0)}_{t\to l^+\nu_l b} \otimes d\Gamma^{(1)}_{\bar{t}\to l^-\bar{\nu}_l\bar{b}} \Big\}$$

$$d\sigma^{\text{NLO}}_{\text{full}} = (\Gamma_t^{\text{NLO}})^{-2} \Big\{ \Big(d\sigma^{(0)}_{t\bar{t}} + d\sigma^{(1)}_{t\bar{t}} \Big) \otimes d\Gamma^{(0)}_{t\to l^+\nu_l b} \otimes d\Gamma^{(0)}_{\bar{t}\to l^-\bar{\nu}_l\bar{b}} + \text{decay corrections as for } d\sigma^{\text{nLO}}_{\text{full}} \Big\}$$

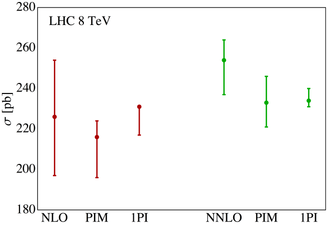
$$d\sigma^{\text{nNLO}}_{\text{full}} = (\Gamma_t^{\text{NLO}})^{-2} \Big\{ \Big(d\sigma^{(0)}_{t\bar{t}} + d\sigma^{(1)}_{t\bar{t}} + d\tilde{\sigma}^{(2)}_{t\bar{t}} \Big) \otimes d\Gamma^{(0)}_{t\to l^+\nu_l b} \otimes d\Gamma^{(0)}_{\bar{t}\to l^-\bar{\nu}_l\bar{b}} + \text{decay corrections as for } d\sigma^{\text{nLO}}_{\text{full}} \Big\}$$

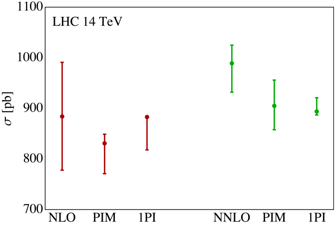
- production part succesively improved: nLO → NLO → nNLO
- top decay@NLO (except for checks/validation, where decay@LO)

Validation: inclusive cross-section

- compare $\sigma^{\mathrm{n(N)LO}}$ with $\sigma^{\mathrm{(N)NLO}}$ [top++; Czakon,Mitov]
- separately for PIM & 1PI

$\mu_F = \mu_R \in \{m_t/2, m_t, 2m_t\}$
MSTW08 PDFs

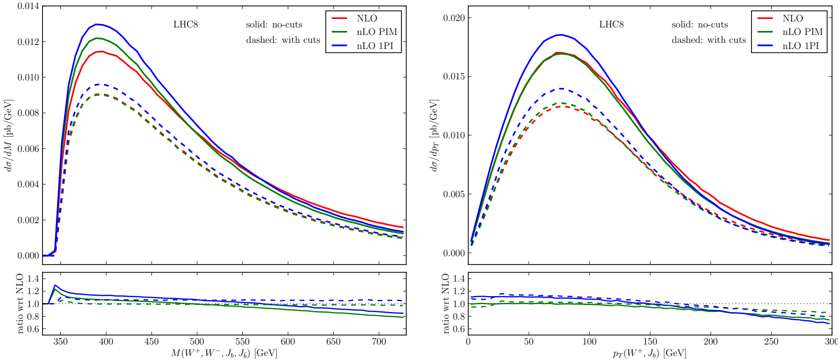




- n(N)LO approximations reasonable, but central values a bit low for nNLO
- (incomplete) overlap of uncertainty bands (worse for Tevatron)
- Recall: at NNLO we do not include any parts of $\mathbf{H}_{\mathbf{ij}}^{(2)} \cdot \mathbf{S}_{\mathbf{ij}}^{(0)}$ and some parts of $\mathbf{H}_{\mathbf{ij}}^{(0)} \cdot \mathbf{S}_{\mathbf{ij}}^{(2)}$ (whereas at NLO we have all relevant pieces)

Validation: differential cross-section & 'universality' decay@LO

- compare nLO-PIM, nLO-1PI with NLO for p_T and $M_{t\bar{t}}$ distributions
- 'tops' reconstructed: $p(t) = p(J_b) + p(l^+) + p(\nu_l)$
- solid = no cuts, dashed = with generic cuts



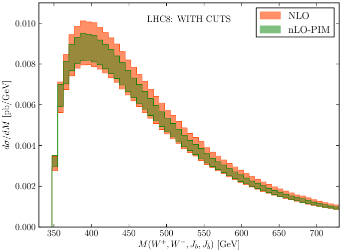
- 'wrong' approximation gives reasonable results (PIM for p_T & 1PI for M_{tt})
- PIM does "better" than 1PI (even for p_T!?)
- approximations seem to improve with cuts

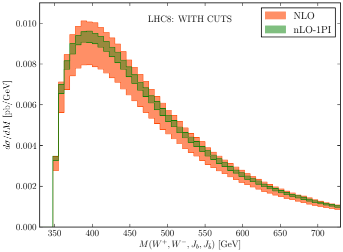
} generic feature

Validation: uncertainties

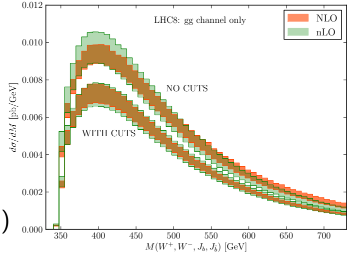
decay@LO

- compare scale uncertainty bands of nLO-PIM and nLO-1PI with NLO

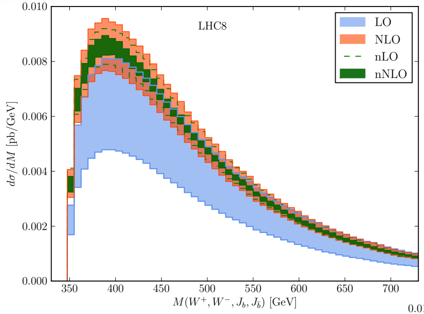




- PIM or 1PI separately clearly underestimate NLO scale-uncertainties
- channel-by-channel envelope of scale-uncert and {PIM, 1PI} overestimates uncertainty
- overestimation to some extent compensates for missing channels (qg not included in nLO)



Generic LHC8 cut-based analysis: $m_{t\bar{t}}$ and $p_T(t)$ decay@NLO



Cut-based analysis:

$p_T(J_{b/\bar{b}}) > 15$ GeV

$E_T(l^\pm) > 15$ GeV, $\not{E}_T > 20$ GeV

$M(W^+, W^-, J_b, J_{\bar{b}}) > 350$ GeV

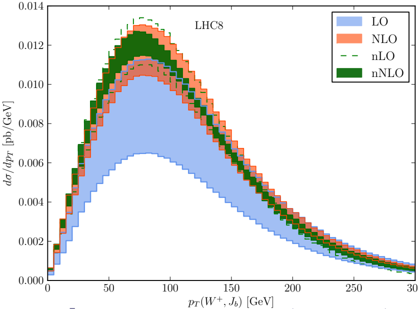
k_t-clustering algorithm, $R = 0.7$

Assess quality of approximation by comparing nLO to full NLO

✓ nLO bands are close to those of full NLO

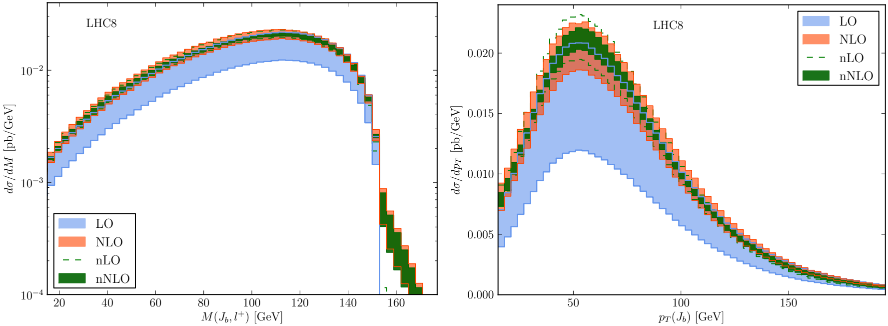
✓ Good perturbative behaviour

Uncertainty bands roughly halved



Generic LHC8 cut-based analysis: other observables decay@NLO

Monte Carlo framework allows us to look at other observables constructed from final-state lepton and jet momenta: e.g. $M(J_b, l^+)$ and $p_T(J_b)$



- obviously care is required and sensible observables must be picked.
- better prediction: fully-differential $W^+W^-b\bar{b}$ @NNLO (unavailable)
- reasonable uncertainty estimate provided

Conclusions & Outlook

Conclusions:

- generalised PIM and 1PI approx-(N)NLO kernels, adding top decay (NWA)
- fully-differential parton-level Monte Carlo code with nNLO $t\bar{t}$ production and NLO top decay $(t \to l^+ \nu_l b)$
- features of resummation seem robust under exchange of PIM $\leftrightarrow$ 1PI (!)
- empirical evidence suggests nLO approximates NLO rather well (for arbitrary observables & non-inclusive setups)
- for realistic theory error: must take envelope {PIM, 1PI}

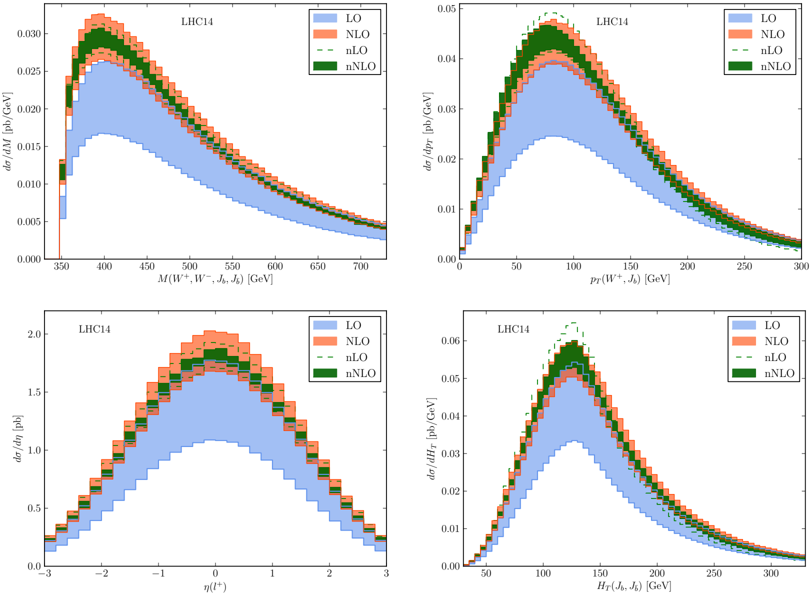
Outlook/Improvements:

- comparison with $d\sigma^{\text{NNLO}}$ [Czakon,Fiedler,Mitov - soon] (understand sub-leading)
- production incomplete: currently miss $\mathbf{H}_{\mathbf{ij}}^{(2)} \cdot \mathbf{S}_{\mathbf{ij}}^{(0)}$ and finite parts of $\mathbf{H}_{\mathbf{ij}}^{(0)} \cdot \mathbf{S}_{\mathbf{ij}}^{(2)}$ (need 2-loop virtuals with spin-info, 2-loop soft-functions)
- at the moment have a mismatch in corrections to production/decay $\to$ add NNLO decay correction [Gao, Li, Zhu '12, Brucherseifer, Caola, Melnikov '13]
- relax on-shell assumption using ET framework [Falgari, AP, Signer '13]

Backup slides

Generic LHC14 cut-based analysis

decay@NLO



Validation: differential: $M(J_b, l^+)$, $p_T(J_b)$ decay@LO