Effective Lagrangian Approach

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Hamburg Workshop on Higgs Physics
Preparing for Higgs Boson Studies with Future LHC Data

DESY Hamburg, October 22, 2014

- Introduction
- Frameworks for Higgs-coupling measurements
- Effective Field Theory approach
- Simplified Effective Field Theory approach
- Higher orders in Effective Field Theory

Introduction

Status:

- discovery of new boson in 2012 spectacular success of LHC
- consistent with Higgs boson of Standard Model (SM)
  within present experimental sensitivities
- no new physics seen at LHC so far

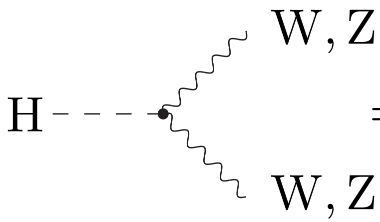
goals of upcoming LHC run at $13\,\mathrm{TeV}$

- search for physics beyond the SM
  at higher energies and higher luminosity
- more precise investigations of SM processes
- precise studies of Higgs boson

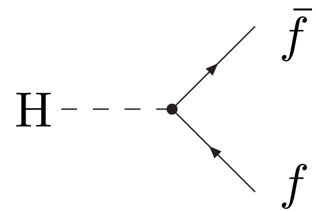
key question: Is the new boson the Higgs boson of the Standard Model?
$\Rightarrow$ need to measure its interactions with SM particles, i.e. its couplings

Fundamental Higgs-boson couplings in the SM

Couplings of single Higgs boson to SM particles proportional to their masses



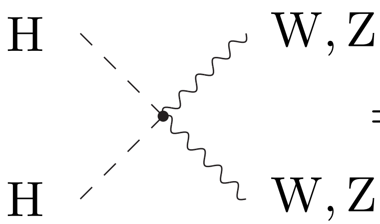
$$= \mathrm{i}e g_{\mu\nu} \left\{ \frac{M_\mathrm{W}}{s_\mathrm{w}}, \frac{M_\mathrm{Z}}{s_\mathrm{w} c_\mathrm{w}} \right\}$$



$$= -\mathrm{i}e \frac{1}{2 s_\mathrm{w}} \frac{m_f}{M_\mathrm{W}}$$

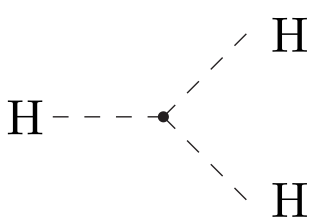
important for Higgs production and decay at LHC

couplings involving two Higgs bosons

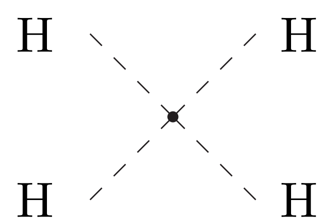


$$= \mathrm{i}e^2 g_{\mu\nu} \frac{1}{2 s_\mathrm{w}^2} \left\{ 1, \frac{1}{c_\mathrm{w}^2} \right\}$$

Higgs-boson self-couplings



$$= -\mathrm{i}e \frac{3}{2 s_\mathrm{w}} \frac{M_\mathrm{H}^2}{M_\mathrm{W}}$$

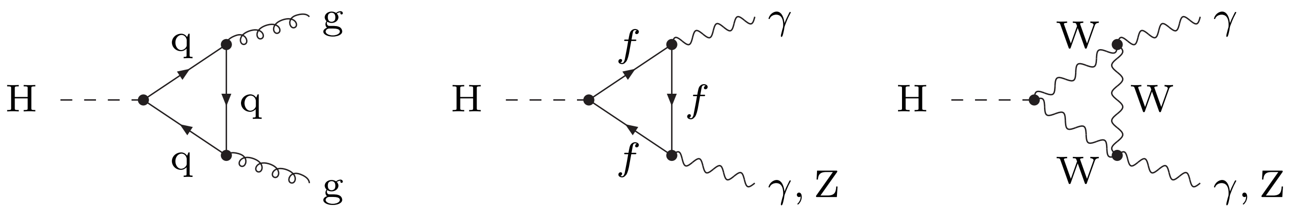


$$= -\mathrm{i}e^2 \frac{3}{4 s_\mathrm{w}^2} \frac{M_\mathrm{H}^2}{M_\mathrm{W}^2}$$

couplings are uniquely fixed by SM parameters e, M_W, M_Z, M_H, m_f

with $c_\mathrm{w} = M_\mathrm{W}/M_\mathrm{Z}$, $s_\mathrm{w} = \sqrt{1 - c_\mathrm{w}^2}$

Loop-induced Higgs-boson couplings



- very important for Higgs production and decay at LHC
- only particles in loop with mass $m \gtrsim M_{\mathrm{H}}$ contribute appreciably (t, W)
- contributions depend on elementary Higgs-boson couplings
  (mainly to HWW and Htt)
- couplings are uniquely fixed in SM

Measurement of Higgs-boson couplings

Within SM

- Higgs-boson couplings are not free parameters
  $\Rightarrow$ cannot be varied independently
- Higgs-boson couplings cannot be fitted
- only compatibility of measured values with SM can be tested

any variation of Higgs couplings goes beyond SM !

ad-hoc variation $\Rightarrow$ no consistent quantum field theory

- violation of gauge invariance $\Rightarrow$ results become gauge-dependent
- loss of renormalisability $\Rightarrow$ no consistent higher-order calculations
- perturbative unitarity is not guaranteed

$\Rightarrow$ measurement of Higgs couplings, in particular interpretation of possible deviations from SM, requires consistent framework beyond SM

Frameworks for Higgs-coupling measurements

Frameworks to measure Higgs-boson couplings

- Specific models with more free parameters
  - allow consistent fits of independent parameters (couplings)
  - allow consistent calculations of higher orders (if renormalisable)
  - analysis must be done model by model
  - model with good coverage of SM-like Higgs sector with free couplings: general Yukawa-aligned 2-Higgs-Doublet Model López-Val, Plehn, Rauch ’13
- interim framework for Higgs-boson coupling analysis HXSWG ’13
  also called: κ framework, scalar-coupling-deviations framework
- form-factor approach
- Effective Field Theory approach

Interim framework for Higgs coupling analysis

Basic assumptions

HXSWG '13

- zero-width approximation: $\sigma = \sigma(ii \to \mathrm{H})\Gamma(\mathrm{H} \to ff)/\Gamma_\mathrm{H}$
- tensor structure of Higgs couplings kept as in SM, $J^{\mathrm{CP}} = 0^{++}$
- only SM coupling strengths are modified (rescaled)
- scale factors
  - for fundamental Higgs couplings: $\kappa_\mathrm{W}, \kappa_\mathrm{Z}, \kappa_f$ $(\kappa_\mathrm{t}, \kappa_\mathrm{b}, \kappa_\tau)$
  - for loop-induced couplings: $\kappa_\mathrm{g}, \kappa_\gamma, \kappa_{\gamma\mathrm{Z}}$

implementation:

- full SM corrections can be included by scaling (parts of) SM predictions $\Rightarrow$ limit $\kappa = 1$ reproduces best SM prediction
- typically only subsets of couplings are scaled
- loop-induced decay widths are quadratic polynomials of fundamental couplings
- an invisible decay width can be included

Interim framework for Higgs coupling analysis II

Virtues:

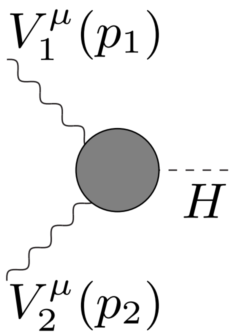
- simple effective parametrisation of deviations from SM
- allows to fit coupling strengths
- easy to implement based on existing SM calculations
  $\Rightarrow$ e.g. implementation in eHDECAY exists Contino et al. '14
- allows to include dominant perturbative corrections of SM
  (dominant QCD corrections factorise)
- allows consistency checks of SM

drawbacks:

- electroweak corrections can only be included effectively
- based on total rates, disregards information from angular distributions
- possible deviations have no direct interpretation with quantum field theory

Form-factor approach

Example: generalised Feynman rule for $\mathrm{H}VV$ vertex
$(V_1V_2 = WW, ZZ, Z\gamma, \gamma\gamma)$



$$= \mathrm{i}a^{(1)}_{HV_1V_2} g^{\mu\nu} + \mathrm{i}a^{(2)}_{HV_1V_2}\left[p_1^\nu p_2^\mu - (p_1p_2)g^{\mu\nu}\right] + \mathrm{i}a^{(3)}_{HV_1V_2}\epsilon^{\mu\nu\rho\sigma}p_{1,\rho}p_{2,\sigma}$$

- structure fixed by Lorentz invariance and transversality ($p_i^\mu V_{i,\mu} = 0$)
- form factors $a^{(i)}_{HV_1V_2} = a^{(i)}_{HV_1V_2}(p_\mathrm{H}^2, p_1^2, p_2^2)$ general functions of p_H^2, p_1^2, p_2^2
- independent form factors for each vertex
- parity-conserving form factors $a^{(1)}_{HV_1V_2}$, $a^{(2)}_{HV_1V_2}$
- parity-violating form factor $a^{(3)}_{HV_1V_2}$
- SM values: $a^{(1)}_{HV_1V_2} = \text{const.}, \quad a^{(2,3)}_{HV_1V_2} = 0$
- interim framework: $a^{(1)}_{HV_1V_2} = \text{free const.}, \quad a^{(2,3)}_{HV_1V_2} \equiv 0$

Form-factor approach II

Virtues:

- (almost) completely general and model independent

drawbacks:

- parametrisation of form factors necessary in practice
- gauge invariance violated $\Rightarrow$ no consistent higher-order predictions
- no correlations between different processes

Effective Field Theory

Virtues

- rather model independent
- respects symmetries of SM
- allows for consistent calculation of perturbative corrections
- comes with power counting
- allows for global fits of parameters (correlations, e.g. to LEP results)

drawbacks

- requires decoupling of New Physics (basic assumption!)
- based on specific low-energy Lagrangian (SM with Higgs doublet)
- depends on many free parameters

example: $\mathrm{H}VV$ vertex

$a^{(1,2,3)}_{HV_1V_2} = \text{free const.},$ correlated with constants in other vertices

Effective Field Theory

Idea of effective Field Theory

Starting point

- SM describes physics well up to $\sim 8\,\mathrm{TeV}$
- no new particle found so far

reasonable assumptions for general parametrisation of New Physics

- SM extension is a quantum field theory $\Rightarrow$ unitarity, renormalisatbility
- gauge symmetry of SM $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$ holds
- SM recovered in low-energy limit
  $\Rightarrow$ SM degrees of freedom should be incorporated
  including Higgs doublet
- new physics decouples if corresponding scale gets large

properties satisfied by Effective Field Theories Weinberg '79; Georgi '93

Effective Field Theory of SM

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda^2}\sum_k \alpha_k \mathcal{O}_k$$

- $\mathcal{L}_{\mathrm{SM}}$: SM Lagrangian (dimension ≤ 4)
- Λ: scale of new physics ($\Lambda \gg v$)
- α_k: dimensionless Wilson coefficients
- $\mathcal{O}_k$: $d=6$ operators constructed from SM fields
  expected to generate leading effects of New Physics

remarks

- single $d=5$ operator for one fermion generation $\left(\frac{1}{\Lambda^1}\alpha^{(d=5)}\mathcal{O}^{(d=5)}\right)$
  violates L, generates Majorana-neutrino masses Weinberg '79
- all $d=5$ and $d=7$ operators violate B and/or L Degrande et al. '12
- $d=8$ operators suppressed by $1/\Lambda^4$
- validity of EFT assumes $E \ll \Lambda$

Dimension-6 operators

- Must consider all $d = 6$ operators that can be constructed from SM fields
  pioneering paper: Buchmüller, Wyler '86
- number of operators can be reduced by integration by parts and equations of motion
  $\Rightarrow$ minimal complete set of operators
- discrete symmetries allow further reduction of operators:
  - B and L conservation (excludes 5 operators for one generation)
  - flavour symmetries
  - CP symmetry
- assuming B and L conservation: number of independent effective $d = 6$ operators
  - for one generation: 59 (compared to 14 in SM)
  - for three generations: 2499 Alonso et al. '14
- no unique basis, different variants in use
  - HISZ basis: no fermionic operators Hagiwara, Ishihara, Szalapski, Zeppenfeld '93
  - GIMR basis: first minimal complete basis Grzadkowski, Iskrzyński, Misiak, Rosiek '10
  - SILH basis: complete Giudice, Grojean, Pomarol, Rattazzi '07;
    Elias-Miro, Espinosa, Masso, Pomarol '13

Basis for $d = 6$ operators

Grzadkowski et al. ’10

Φ^6 and $\Phi^4 D^2$	$\psi^2\Phi^3$	X^3
$\mathcal{O}_\Phi = (\Phi^\dagger\Phi)^3$	$\mathcal{O}_{\mathrm{e}\Phi} = (\Phi^\dagger\Phi)(\bar{\mathrm{l}}\,\Gamma_\mathrm{e}\,\mathrm{e}\Phi)$	$\mathcal{O}_G = f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{\Phi\Box} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$	$\mathcal{O}_{\mathrm{u}\Phi} = (\Phi^\dagger\Phi)(\bar{\mathrm{q}}\,\Gamma_\mathrm{u}\,\mathrm{u}\widetilde{\Phi})$	$\mathcal{O}_{\widetilde{G}} = f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{\Phi D} = (\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$	$\mathcal{O}_{\mathrm{d}\Phi} = (\Phi^\dagger\Phi)(\bar{\mathrm{q}}\,\Gamma_\mathrm{d}\,\mathrm{d}\Phi)$	$\mathcal{O}_\mathrm{W} = \varepsilon^{IJK} \mathrm{W}_\mu^{I\nu} \mathrm{W}_\nu^{J\rho} \mathrm{W}_\rho^{K\mu}$
		$\mathcal{O}_{\widetilde{\mathrm{W}}} = \varepsilon^{IJK} \widetilde{\mathrm{W}}_\mu^{I\nu} \mathrm{W}_\nu^{J\rho} \mathrm{W}_\rho^{K\mu}$
$X^2\Phi^2$	$\psi^2 X\Phi$	$\psi^2\Phi^2 D$
$\mathcal{O}_{\Phi G} = (\Phi^\dagger\Phi) G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\mathrm{u}G} = (\bar{\mathrm{q}}\sigma^{\mu\nu} \frac{\lambda^A}{2} \Gamma_\mathrm{u}\,\mathrm{u}\widetilde{\Phi}) G_{\mu\nu}^A$	$\mathcal{O}_{\Phi\mathrm{l}}^{(1)} = (\Phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu \Phi)(\bar{\mathrm{l}}\gamma^\mu \mathrm{l})$
$\mathcal{O}_{\Phi\widetilde{G}} = (\Phi^\dagger\Phi) \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\mathrm{d}G} = (\bar{\mathrm{q}}\sigma^{\mu\nu} \frac{\lambda^A}{2} \Gamma_\mathrm{d}\,\mathrm{d}\Phi) G_{\mu\nu}^A$	$\mathcal{O}_{\Phi\mathrm{l}}^{(3)} = (\Phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu^I \Phi)(\bar{\mathrm{l}}\gamma^\mu \tau^I \mathrm{l})$
$\mathcal{O}_{\Phi\mathrm{W}} = (\Phi^\dagger\Phi) \mathrm{W}_{\mu\nu}^I \mathrm{W}^{I\mu\nu}$	$\mathcal{O}_{\mathrm{eW}} = (\bar{\mathrm{l}}\sigma^{\mu\nu}\Gamma_\mathrm{e}\,\mathrm{e}\tau^I\Phi) \mathrm{W}_{\mu\nu}^I$	$\mathcal{O}_{\Phi\mathrm{e}} = (\Phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu \Phi)(\bar{\mathrm{e}}\gamma^\mu \mathrm{e})$
$\mathcal{O}_{\Phi\widetilde{\mathrm{W}}} = (\Phi^\dagger\Phi) \widetilde{\mathrm{W}}_{\mu\nu}^I \mathrm{W}^{I\mu\nu}$	$\mathcal{O}_{\mathrm{uW}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_\mathrm{u}\,\mathrm{u}\tau^I\widetilde{\Phi}) \mathrm{W}_{\mu\nu}^I$	$\mathcal{O}_{\Phi\mathrm{q}}^{(1)} = (\Phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu \Phi)(\bar{\mathrm{q}}\gamma^\mu \mathrm{q})$
$\mathcal{O}_{\Phi\mathrm{B}} = (\Phi^\dagger\Phi) \mathrm{B}_{\mu\nu} \mathrm{B}^{\mu\nu}$	$\mathcal{O}_{\mathrm{dW}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_\mathrm{d}\,\mathrm{d}\tau^I\Phi) \mathrm{W}_{\mu\nu}^I$	$\mathcal{O}_{\Phi\mathrm{q}}^{(3)} = (\Phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu^I \Phi)(\bar{\mathrm{q}}\gamma^\mu \tau^I \mathrm{q})$
$\mathcal{O}_{\Phi\widetilde{\mathrm{B}}} = (\Phi^\dagger\Phi) \widetilde{\mathrm{B}}_{\mu\nu} \mathrm{B}^{\mu\nu}$	$\mathcal{O}_{\mathrm{eB}} = (\bar{\mathrm{l}}\sigma^{\mu\nu}\Gamma_\mathrm{e}\,\mathrm{e}\Phi) \mathrm{B}_{\mu\nu}$	$\mathcal{O}_{\Phi\mathrm{u}} = (\Phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu \Phi)(\bar{\mathrm{u}}\gamma^\mu \mathrm{u})$
$\mathcal{O}_{\Phi\mathrm{WB}} = (\Phi^\dagger\tau^I\Phi) \mathrm{W}_{\mu\nu}^I \mathrm{B}^{\mu\nu}$	$\mathcal{O}_{\mathrm{uB}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_\mathrm{u}\,\mathrm{u}\widetilde{\Phi}) \mathrm{B}_{\mu\nu}$	$\mathcal{O}_{\Phi\mathrm{d}} = (\Phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu \Phi)(\bar{\mathrm{d}}\gamma^\mu \mathrm{d})$
$\mathcal{O}_{\Phi\widetilde{\mathrm{W}}\mathrm{B}} = (\Phi^\dagger\tau^I\Phi) \widetilde{\mathrm{W}}_{\mu\nu}^I \mathrm{B}^{\mu\nu}$	$\mathcal{O}_{\mathrm{dB}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_\mathrm{d}\,\mathrm{d}\Phi) \mathrm{B}_{\mu\nu}$	$\mathcal{O}_{\Phi\mathrm{ud}} = \mathrm{i}(\widetilde{\Phi}^\dagger D_\mu \Phi)(\bar{\mathrm{u}}\gamma^\mu \Gamma_\mathrm{ud}\,\mathrm{d})$

$+$ 25 four-fermion operators

Alternative basis for $d = 6$ operators

Φ^6 and $\Phi^4 D^2$	$\psi^2\Phi^3$	X^3
$\mathcal{O}'_6 = (\Phi^\dagger\Phi)^3$	$\mathcal{O}'_{\mathrm{e}\Phi} = (\Phi^\dagger\Phi)(\bar{\mathrm{l}}\,\Gamma_{\mathrm{e}}\,\mathrm{e}\Phi)$	$\mathcal{O}'_G = f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}'_\Phi = \partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$	$\mathcal{O}'_{\mathrm{u}\Phi} = (\Phi^\dagger\Phi)(\bar{\mathrm{q}}\,\Gamma_{\mathrm{u}}\,\mathrm{u}\widetilde{\Phi})$	$\mathcal{O}'_{\widetilde{G}} = f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}'_{\mathrm{T}} = (\Phi^\dagger \overleftrightarrow{D}_\mu\Phi)(\Phi^\dagger\overleftrightarrow{D}^\mu\Phi)$	$\mathcal{O}'_{\mathrm{d}\Phi} = (\Phi^\dagger\Phi)(\bar{\mathrm{q}}\,\Gamma_d\, d\Phi)$	$\mathcal{O}'_{\mathrm{W}} = \varepsilon^{IJK}\mathrm{W}_\mu^{I\nu}\mathrm{W}_\nu^{J\rho}\mathrm{W}_\rho^{K\mu}$
Giudice et al. '07, Contino et al. '13		$\mathcal{O}'_{\widetilde{\mathrm{W}}} = \varepsilon^{IJK}\widetilde{\mathrm{W}}_\mu^{I\nu}\mathrm{W}_\nu^{J\rho}\mathrm{W}_\rho^{K\mu}$
$X^2\Phi^2$	$\psi^2 X\Phi$	$\psi^2\Phi^2 D$
$\mathcal{O}'_{\mathrm{D}W} = \left(\Phi^\dagger\tau^I\mathrm{i}\overleftrightarrow{D^\mu}\Phi\right)(D^\nu \mathrm{W}_{\mu\nu})^I$	$\mathcal{O}'_{\mathrm{u}G} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\frac{\lambda^A}{2}\Gamma_{\mathrm{u}}\,\mathrm{u}\widetilde{\Phi})G^A_{\mu\nu}$	$\mathcal{O}'^{(1)}_{\Phi\mathrm{l}} = (\Phi^\dagger\mathrm{i}\overleftrightarrow{D}_\mu\Phi)(\bar{\mathrm{l}}\gamma^\mu\mathrm{l})$
$\mathcal{O}'_{DB} = \left(\Phi^\dagger\mathrm{i}\overleftrightarrow{D^\mu}\Phi\right)(\partial^\nu \mathrm{B}_{\mu\nu})$	$\mathcal{O}'_{\mathrm{d}G} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\frac{\lambda^A}{2}\Gamma_{\mathrm{d}}\,\mathrm{d}\Phi)G^A_{\mu\nu}$	$\mathcal{O}'^{(3)}_{\Phi\mathrm{l}} = (\Phi^\dagger\mathrm{i}\overleftrightarrow{D}^I_\mu\Phi)(\bar{\mathrm{l}}\gamma^\mu\tau^I\mathrm{l})$
$\mathcal{O}'_{D\Phi\mathrm{W}} = \mathrm{i}(D^\mu\Phi)^\dagger\tau^I(D^\nu\Phi)\mathrm{W}^I_{\mu\nu}$	$\mathcal{O}'_{\mathrm{eW}} = (\bar{\mathrm{l}}\sigma^{\mu\nu}\Gamma_{\mathrm{e}}\,\mathrm{e}\tau^I\Phi)\mathrm{W}^I_{\mu\nu}$	$\mathcal{O}'_{\Phi\mathrm{e}} = (\Phi^\dagger\mathrm{i}\overleftrightarrow{D}_\mu\Phi)(\bar{\mathrm{e}}\gamma^\mu\mathrm{e})$
$\mathcal{O}'_{D\Phi\widetilde{\mathrm{W}}} = \mathrm{i}(D^\mu\Phi)^\dagger\tau^I(D^\nu\Phi)\widetilde{\mathrm{W}}^I_{\mu\nu}$	$\mathcal{O}'_{\mathrm{uW}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_{\mathrm{u}}\,\mathrm{u}\tau^I\widetilde{\Phi})\mathrm{W}^I_{\mu\nu}$	$\mathcal{O}'^{(1)}_{\Phi\mathrm{q}} = (\Phi^\dagger\mathrm{i}\overleftrightarrow{D}_\mu\Phi)(\bar{\mathrm{q}}\gamma^\mu\mathrm{q})$
$\mathcal{O}'_{D\Phi\mathrm{B}} = \mathrm{i}(D^\mu\Phi)^\dagger(D^\nu\Phi)\mathrm{B}_{\mu\nu}$	$\mathcal{O}'_{\mathrm{dW}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_{\mathrm{d}}\,\mathrm{d}\tau^I\Phi)\mathrm{W}^I_{\mu\nu}$	$\mathcal{O}'^{(3)}_{\Phi\mathrm{q}} = (\Phi^\dagger\mathrm{i}\overleftrightarrow{D}^I_\mu\Phi)(\bar{\mathrm{q}}\gamma^\mu\tau^I\mathrm{q})$
$\mathcal{O}'_{D\Phi\widetilde{\mathrm{B}}} = \mathrm{i}(D^\mu\Phi)^\dagger(D^\nu\Phi)\widetilde{\mathrm{B}}_{\mu\nu}$	$\mathcal{O}'_{\mathrm{eB}} = (\bar{l}\sigma^{\mu\nu}\Gamma_{\mathrm{e}}\,\mathrm{e}\Phi)B_{\mu\nu}$	$\mathcal{O}'_{\Phi\mathrm{u}} = (\Phi^\dagger\mathrm{i}\overleftrightarrow{D}_\mu\Phi)(\bar{\mathrm{u}}\gamma^\mu\mathrm{u})$
$\mathcal{O}'_{\Phi\mathrm{B}} = (\Phi^\dagger\Phi)B_{\mu\nu}\mathrm{B}^{\mu\nu}$	$\mathcal{O}'_{\mathrm{uB}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_{\mathrm{u}}\,\mathrm{u}\widetilde{\Phi})\mathrm{B}_{\mu\nu}$	$\mathcal{O}'_{\Phi\mathrm{d}} = (\Phi^\dagger\mathrm{i}\overleftrightarrow{D}_\mu\Phi)(\bar{\mathrm{d}}\gamma^\mu\mathrm{d})$
$\mathcal{O}'_{\Phi\widetilde{\mathrm{B}}} = (\Phi^\dagger\Phi)\mathrm{B}_{\mu\nu}\widetilde{\mathrm{B}}^{\mu\nu}$	$\mathcal{O}'_{\mathrm{dB}} = (\bar{\mathrm{q}}\sigma^{\mu\nu}\Gamma_{\mathrm{d}}\,\mathrm{d}\Phi)\mathrm{B}_{\mu\nu}$	$\mathcal{O}'_{\Phi\mathrm{ud}} = \mathrm{i}(\widetilde{\Phi}^\dagger D_\mu\Phi)(\bar{\mathrm{u}}\gamma^\mu\Gamma_{\mathrm{ud}}\mathrm{d})$
$\mathcal{O}'_{\Phi G} = \Phi^\dagger\Phi G^A_{\mu\nu}G^{A\mu\nu}$		
$\mathcal{O}'_{\Phi\widetilde{G}} = \Phi^\dagger\Phi G^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$		$+$(25-2) four-fermion operators

Relations between $d = 6$ bases

Operator relations:

$$\frac{g}{2}\mathcal{O}'_{\mathrm{D}W} - \frac{g'}{2}\mathcal{O}'_{\mathrm{D}B} + g'\mathcal{O}'_{D\Phi\mathrm{B}} - g\mathcal{O}'_{D\Phi\mathrm{W}} - \frac{g'^2}{4}\mathcal{O}'_{\Phi\mathrm{B}} = -\frac{g^2}{4}\mathcal{O}_{\Phi\mathrm{W}}$$

$$\frac{g'}{2}\mathcal{O}'_{\mathrm{D}B} - g'\mathcal{O}'_{D\Phi\mathrm{B}} + \frac{g'^2}{4}\mathcal{O}'_{\Phi\mathrm{B}} = -\frac{gg'}{4}\mathcal{O}_{\Phi\mathrm{WB}}$$

and similar relations for $\mathcal{O}_{\Phi\widetilde{\mathrm{W}}}$ and $\mathcal{O}_{\Phi\widetilde{\mathrm{W}}\mathrm{B}}$

identities to eliminate redundant 4-fermion operators $\mathcal{O}'^{(1)}_{\Phi\mathrm{l}}$ and $\mathcal{O}'^{(3)}_{\Phi\mathrm{l}}$

$$\mathcal{O}'^{(1)}_{\Phi\mathrm{l}} - \frac{1}{3}\mathcal{O}'^{(1)}_{\Phi\mathrm{q}} + 2\mathcal{O}'_{\Phi\mathrm{e}} - \frac{4}{3}\mathcal{O}'_{\Phi\mathrm{u}} + \frac{2}{3}\mathcal{O}'_{\Phi\mathrm{d}} = -\mathcal{O}'_{T} + \frac{2}{g'}\mathcal{O}'_{\mathrm{D}B}$$

$$2(\mathcal{O}'_{\mathrm{u}\Phi} + \mathcal{O}'_{\mathrm{d}\Phi} + \mathcal{O}'_{\mathrm{e}\Phi} + \mathrm{h.c.}) + \mathcal{O}'^{(3)}_{\Phi\mathrm{q}} + \mathcal{O}'^{(3)}_{\Phi\mathrm{l}} = 3\mathcal{O}'_{\Phi} - 4\lambda\mathcal{O}'_{6} + 4m^2(\Phi^\dagger\Phi)^2 - \frac{2}{g}\mathcal{O}'_{\mathrm{D}W}$$

⇒ relations between Wilson coefficients

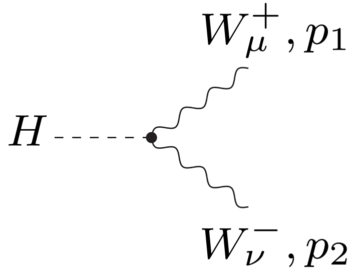
Hierarchies of $d = 6$ operators

- weakly interacting theories:
  - operators involving field strengths result only from loops of heavy degrees of freedom
    $\Rightarrow$ suppressed by additional loop factor $1/16\pi^2$
    $\Rightarrow$ $d = 8$ operators are equally important for $\frac{v^2}{\Lambda^2} \gtrsim \frac{1}{16\pi^2}$
    or $\Lambda \lesssim 4\pi v \approx 3\,\mathrm{TeV}$ Passarino '12
  - other operators can be generated by tree diagrams
- strongly interacting theories:
  $\Rightarrow$ different hierarchies Giudice, Grojean, Pomarol, Rattazzi '07;
  Elias-Miro, Espinosa, Masso, Pomarol '13
  - operators involving extra covariant derivatives or gauge fields scale as g^2v^2/Λ^2, g = SM gauge coupling
  - other operators
    scale as $g_*^2v^2/\Lambda^2$, g_* = generic BSM gauge coupling

  $\Rightarrow$ for $g_* \sim 4\pi$, latter more important

Feynman rules for HWW vertex

HWW coupling: $\quad g = \frac{e}{s_{\mathrm{W}}} \qquad \frac{1}{\sqrt{2}G_\mu} = v^2[1 + \mathcal{O}(\alpha_i)]$



$$= \mathrm{i} g M_{\mathrm{W}} g_{\mu\nu} \left[1 + \frac{1}{\sqrt{2}G_\mu \Lambda^2}\left(\alpha_{\phi W} + \alpha_{\phi\Box} - \frac{1}{4}\alpha_{\phi D}\right)\right]$$

$$+ \mathrm{i}\frac{2g}{M_{\mathrm{W}}}\frac{1}{\sqrt{2}G_\mu \Lambda^2}\left[\alpha_{\phi W}(p_{2\mu}p_{1\nu} - p_1 p_2 g_{\mu\nu}) + \alpha_{\phi\widetilde{W}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right]$$

$$a^{(1)}_{HW^+W^-} = g M_{\mathrm{W}}\left[1 + \frac{1}{\sqrt{2}G_\mu \Lambda^2}\left(\alpha_{\phi W} + \alpha_{\phi\Box} - \frac{1}{4}\alpha_{\phi D}\right)\right]$$

$$a^{(2)}_{HW^+W^-} = \frac{2g}{M_{\mathrm{W}}}\frac{1}{\sqrt{2}G_\mu \Lambda^2}\alpha_{\phi W}, \qquad a^{(3)}_{HW^+W^-} = \frac{2g}{M_{\mathrm{W}}}\frac{1}{\sqrt{2}G_\mu \Lambda^2}\alpha_{\phi\widetilde{W}}$$

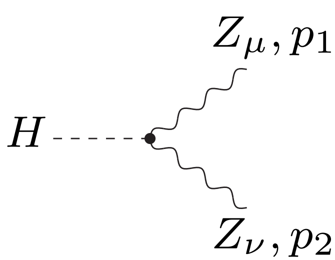
- $\alpha_{\phi\Box}$ parametrises strength of HWW coupling
- $\alpha_{\phi D}$ parametrises difference of HWW and HZZ couplings
- $\alpha_{\phi W}$ parametrises strength of new CP-conserving tensor structure
- $\alpha_{\phi\widetilde{W}}$ parametrises strength of new CP-violating tensor structure

form factors expressed in terms of Wilson coefficients

Feynman rules for HVV vertex

HZZ coupling:

$$\alpha_{ZZ} = c_{\mathrm{w}}^2 \alpha_{\phi W} + s_{\mathrm{w}}^2 \alpha_{\phi B} + c_{\mathrm{w}} s_{\mathrm{w}} \alpha_{\phi WB}$$
$$\alpha_{Z\widetilde{Z}} = c_{\mathrm{w}}^2 \alpha_{\phi \widetilde{W}} + s_{\mathrm{w}}^2 \alpha_{\phi \widetilde{B}} + c_{\mathrm{w}} s_{\mathrm{w}} \alpha_{\phi W\widetilde{B}}$$



$$= \mathrm{i} g \frac{M_{\mathrm{Z}}}{c_{\mathrm{w}}} g_{\mu\nu} \left[1 + \frac{1}{\sqrt{2} G_\mu \Lambda^2} \left(\alpha_{\phi W} + \alpha_{\phi \Box} + \frac{1}{4} \alpha_{\phi D} \right) \right]$$
$$+ \mathrm{i} \frac{2g}{M_{\mathrm{W}}} \frac{1}{\sqrt{2} G_\mu \Lambda^2} \left[\alpha_{ZZ} (p_{2\mu} p_{1\nu} - p_1 p_2 g_{\mu\nu}) + \alpha_{Z\widetilde{Z}} \varepsilon_{\mu\nu\rho\sigma} p_1^\rho p_2^\sigma \right]$$

first term absent for HAZ and HAA vertex

$$a^{(1)}_{HZZ} = g \frac{M_{\mathrm{Z}}}{c_{\mathrm{w}}} \left[1 + \frac{1}{\sqrt{2} G_\mu \Lambda^2} \left(\alpha_{\phi W} + \alpha_{\phi \Box} + \frac{1}{4} \alpha_{\phi D} \right) \right], \qquad a^{(1)}_{HAZ} = 0, \qquad a^{(1)}_{HAA} = 0$$

$$a^{(2)}_{HV'V} = \frac{2g}{M_{\mathrm{W}}} \frac{1}{\sqrt{2} G_\mu \Lambda^2} \alpha_{V'V}, \qquad a^{(3)}_{HV'V} = \frac{2g}{M_{\mathrm{W}}} \frac{1}{\sqrt{2} G_\mu \Lambda^2} \alpha_{V'\widetilde{V}}, \qquad V'V = ZZ, AZ, AA$$

$$\alpha_{AA} = s_{\mathrm{w}}^2 \alpha_{\phi W} + c_{\mathrm{w}}^2 \alpha_{\phi B} - c_{\mathrm{w}} s_{\mathrm{w}} \alpha_{\phi WB}$$
$$\alpha_{AZ} = s_{\mathrm{w}} c_{\mathrm{w}} (\alpha_{\phi W} - \alpha_{\phi B}) + \frac{(c_{\mathrm{w}}^2 - s_{\mathrm{w}}^2)}{2} \alpha_{\phi WB}$$

different couplings parametrised by the same Wilson coefficients $\Rightarrow$ correlations

Input-parameter dependence

Input M_Z, M_W, G_μ $\Rightarrow$

$$g = 2M_\mathrm{W}\sqrt{\sqrt{2}G_\mu}\left(1 - \frac{1}{\sqrt{2}G_\mu\Lambda^2}\left(\alpha_{\phi W} + \alpha^{(3)}_{\phi\mu}\right)\right)$$

$\Rightarrow$ $\alpha_{\phi W}$ in rescaled SM coupling replaced by $-\alpha^{(3)}_{\phi\mu}$
(effective operator contributing to μ decay)

$$a^{(1)}_{HW^+W^-} = gM_\mathrm{W}\left[1 + \frac{1}{\sqrt{2}G_\mu\Lambda^2}\left(\alpha_{\phi W} + \alpha_{\phi\Box} - \frac{1}{4}\alpha_{\phi D}\right)\right]$$

$$\to \quad a^{(1)}_{HW^+W^-} = 2M_\mathrm{W}\sqrt{\sqrt{2}G_\mu}\left[1 + \frac{1}{\sqrt{2}G_\mu\Lambda^2}\left(-\alpha^{(3)}_{\phi\mu} + \alpha_{\phi\Box} - \frac{1}{4}\alpha_{\phi D}\right)\right]$$

expression for g depends on input parameter set and EFT basis!

Simplified Effective Field Theory

Coupling analysis in EFT

- Optimal approach: global fit of all Wilson coefficients using all available experimental observables
- to many independent parameters
  $\Rightarrow$ need simplification or staged fitting procedure
- large reduction by flavour symmetries, but possibly not enough

Simplified EFT approach Elias-Miro, Espinosa, Masso, Pomarol '13, Pomarol, Riva '14

- exploit constraints from precision experiments
- if LHC cannot probe a Wilson coefficient beyond existing bounds
  $\Rightarrow$ omit it from EFT Lagrangian
  need basis independent definition consistent with equations of motion!
  Trott '14

Example of simplified EFT approach

Elias-Miro, Espinosa, Masso, Pomarol '13, Pomarol, Riva '14

Assumptions

- minimal flavour violation
  $\Rightarrow$ fermionic dipole operators $\propto$ Yukawa couplings, i.e. negligible
  (top treated separately)
- neglect CP-odd operators
  (no interference with SM contributions $\Rightarrow$ appear only quadratically)
- consider only operators that can affect Higgs physics at tree level

$\Rightarrow$ 18 out of 59 operators left

- 7 Wilson coefficients constrained by EW precision measurements (0.1%)
- 3 Wilson coefficients constrained by anomalous gauge couplings (1%)

$\Rightarrow$ 8 Wilson coefficients can be independently constraint by Higgs physics

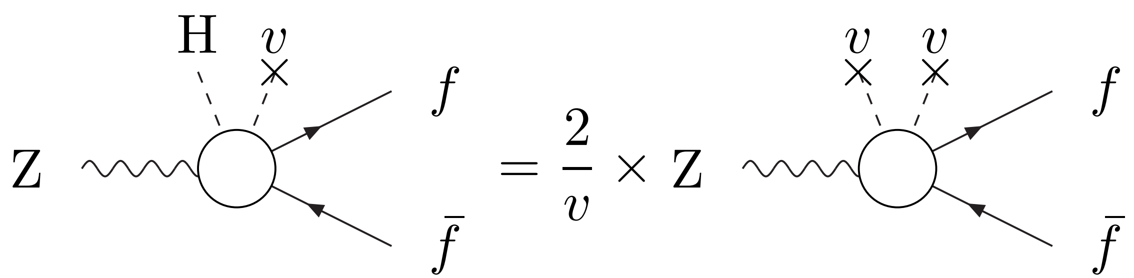
Operators for Higgs physics

Elias-Miro, Espinosa, Pomarol, Masso ’13

SM Higgs is excitation around vacuum: $\Phi = (v + H)/\sqrt{2}$

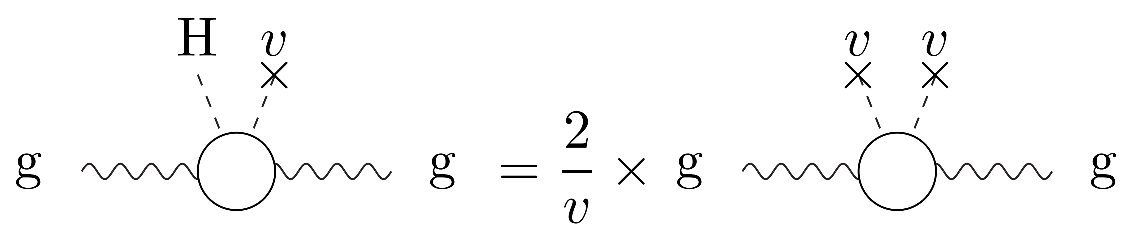
$\Rightarrow$ BSM effects in Higgs physics tested already in other experiments?

example 1: $d = 6$ operator $\mathcal{O}^{(1)}_{\Phi f} = (\Phi^\dagger \mathrm{i} \overleftrightarrow{D}_\mu \Phi)(\bar{f}\gamma^\mu f)$



$\Rightarrow$ effects in $\mathrm{H} \to \mathrm{Z} f \bar{f}$ related to $\mathrm{Z} \to f \bar{f} \Rightarrow$ constrained by LEP data

example 2: $\mathcal{O}_{\Phi G} = (\Phi^\dagger \Phi) G^A_{\mu\nu} G^{A\mu\nu}$



visible in Higgs physics
affects $\mathrm{gg} \to \mathrm{H}$

redefines only SM parameters
$\Rightarrow$ no experimental constraint

Operators affecting exclusively Higgs physics

Pomarol, Riva '13; Elias-Miró, Espinosa, Masso, Pomarol '13; Gupta, Pomarol, Riva '14;

How many Wilson coefficients cannot be tested outside Higgs physics?
answer: as many as parameters in the SM!
others constrained by experiments without Higgs

8 relevant parameters for Higgs physics			constraints from LHC
g_s	$(\Phi^\dagger\Phi)G^A_{\mu\nu}G^{A\mu\nu}$	$\Rightarrow$ ggH coupling	constrained at ‰ level
g_1	$(\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu}$	$\Rightarrow$ $\mathrm{H}\gamma\gamma$ coupling	constrained at ‰ level
g_2	$(\Phi^\dagger\Phi)W^I_{\mu\nu}W^{I\mu\nu}$	$\Rightarrow$ $\mathrm{H}\gamma\mathrm{Z}$ coupling	to be constrained
M_W	$(\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$	$\Rightarrow$ $\mathrm{H}VV$ coupling	constrained
M_H	$(\Phi^\dagger\Phi)^3$	$\Rightarrow$ H^3 coupling	to be constrained
m_f	$(\Phi^\dagger\Phi)(\bar{f}\,\Gamma_f\,f\Phi)$	$\Rightarrow$ $\mathrm{H}ff$ coupling	constrained

$f = \mathrm{t}, \mathrm{b}, \tau$

(N.B.: bounds are for $\Lambda = M_\mathrm{W}$, e.g. for $\kappa_\mathrm{gg}\frac{M_\mathrm{W}^2}{\Lambda^2}$)

Higher orders in Effective Field Theory

Calculation of higher perturbative orders

Effective Field Theory allows consistent calculation of higher orders

- power counting $\Rightarrow$ consistent separation of orders
  simultaneous expansion in $\alpha/4\pi$ and v^2/Λ^2
- Wilson coefficients need renormalisation $\Rightarrow$ running
- insertion of multiple higher-dimensional operators in loops requires counterterms of even higher dimension (power counting)
  - insertion of one $d = 6$ operators in a loop requires only counterterms from $d = 6$ operators ($\propto 1/\Lambda^2$)
  - insertion of two $d = 6$ operators in a loop requires counterterms from $d = 8$ operators ($\propto 1/\Lambda^4$)

Practical recipes

Effects of $d = 6$ operators expected to be small
$\Rightarrow$ consider only leading contributions
= terms linear in $d = 6$ Wilson coefficients

assumption: suppression of $d = 6$ operators $\lesssim$ loop suppression $\left(\frac{v^2}{\Lambda^2} \lesssim \frac{g^2}{16\pi^2}\right)$

tree-level-induced processes:
recipe for matrix element: $\mathcal{M} = \mathcal{M}_0^{\mathrm{SM}} + \mathcal{M}_1^{\mathrm{SM}} + \mathcal{M}_0^{d=6}$
recipe for cross section: $\sigma \propto |\mathcal{M}_0^{\mathrm{SM}}|^2 + 2\,\mathrm{Re}\,(\mathcal{M}_1^{\mathrm{SM}} + \mathcal{M}_0^{d=6})\mathcal{M}_0^{\mathrm{SM},*}$

- SM contributions in LO and NLO (and NNLO)
- LO contributions involving one $d = 6$ operator
- terms “SM-NLO$\times(d = 6)$" and “$(d = 6)^2$" neglected $\Rightarrow$ linear polynomial in α_i
- (parts of) QCD corrections can be factorised
  and multiplied with EW corrections and contributions of $d = 6$ operators

implemented in different codes: eHDECAY, HAWK, VBFNLO
MadGraph5_aMC@NLO

Practical recipes

loop-induced processes:
recipe for matrix element: $\mathcal{M} = \mathcal{M}_1^{\mathrm{SM}} + \mathcal{M}_2^{\mathrm{SM}} + \mathcal{M}_0^{d=6} + \mathcal{M}_1^{d=6}$

- SM contributions in LO (1-loop) and NLO (2-loop) (and NNLO (3-loop))
- tree-level contributions involving one $d = 6$ operator
  typically suppressed (loop-generated), e.g. in $\mathrm{H} \to \mathrm{gg}$, $\mathrm{H} \to \gamma\gamma$
- 1-loop contributions involving one $d = 6$ operator (inserted in loop)
  $\Rightarrow$ need renormalisation of $d = 6$ operators
- $d = 8$ operators neglible, for $\frac{v^2}{\Lambda^2} \ll \frac{g^2}{16\pi^2}$, or loop-generated
- large QCD corrections can be factorized

Passarino '12 defines admissible d=6 operators in loop calculations

- do not alter UV power counting of SM diagrams or
- result in multiplicative modification of finite sets of SM diagrams

$\Rightarrow$ no renormalisation of d=6 operators needed, NLO corrections factorize
only subset of $d = 6$ operators allowed!

Implementation in eHDECAY

example: $\mathrm{H} \to \gamma\gamma$, non-linear parametrisation

Contino et al. '14

$$\Gamma(\gamma\gamma)\big|_{NL} = \frac{G_F \alpha_{em}^2 m_h^3}{128\sqrt{2}\pi^3} \left| \sum_{q=t,b,c} \frac{4}{3} c_q \, 3Q_q^2 \, A_{1/2}^{NLO}(\tau_q) + \frac{4}{3} c_\tau Q_\tau^2 A_{1/2}(\tau_\tau) + c_W A_1(\tau_W) + \frac{4\pi}{\alpha_{em}} c_{\gamma\gamma} \right|^2$$

$A_{1/2}^{NLO}(\tau_q)$ SM form factor from fermion loops including QCD corrections

$A_1(\tau_W)$ SM form factor from W-boson loops

c_q, c_τ, c_W scale factors for tree-level couplings
Wilson coefficients

$c_{\gamma\gamma}$ effective $\mathrm{H}\gamma\gamma$ coupling
Wilson coefficient

- EW corrections neglected
- contributions from angular-dependent effective couplings missing
- corresponds to interim framework
- $\Gamma(\gamma\gamma)\big|_{NL}$ quadratic polynomial in Wilson coefficients

Implementation in eHDECAY

example: $\mathrm{H} \to \gamma\gamma$, SILH parametrisation — Contino et al. ’14

$$\Gamma(\gamma\gamma)\big|_{SILH} = \frac{G_F \alpha_{em}^2 m_h^3}{128\sqrt{2}\pi^3} \Bigg\{ |A_{NLO}^{SM}(\gamma\gamma)|^2 + 2\,Re\Big(A_{LO}^{SM*}(\gamma\gamma)\, A_{ew}^{SM}(\gamma\gamma)\Big) + 2\,Re\Big[A_{NLO}^{SM*}(\gamma\gamma)\Big(\Delta A(\gamma\gamma) + \frac{32\pi \sin^2\theta_W\, \bar{c}_\gamma}{\alpha_{em}}\Big)\Big] \Bigg\}$$

SM amplitude (LO, NLO QCD)

$$A_X^{SM}(\gamma\gamma) = \sum_{q=t,b,c} \frac{4}{3}\, 3Q_q^2\, A_{1/2}^X(\tau_q) + \frac{4}{3} Q_\tau^2\, A_{1/2}(\tau_\tau) + A_1(\tau_W)\,, \qquad X = LO, NLO$$

EW corrections in SM: $A_{ew}^{SM}(\gamma\gamma)$

EFT contribution (Wilson coefficients: $\bar{c}_H, \bar{c}_q, \bar{c}_\tau, \bar{c}_W$)

$$\Delta A(\gamma\gamma) = -\sum_{q=t,b,c} \frac{4}{3}\left(\frac{\bar{c}_H}{2} + \bar{c}_q\right) 3Q_q^2\, A_{1/2}^{NLO}(\tau_q) - \left(\frac{\bar{c}_H}{2} + \bar{c}_\tau\right)\frac{4}{3} Q_\tau^2\, A_{1/2}(\tau_\tau) - \left(\frac{\bar{c}_H}{2} - 2\bar{c}_W\right) A_1(\tau_W)$$

- contributions from angular-dependent effective couplings missing
- $\Gamma(\gamma\gamma)\big|_{SILH}$ linear polynomial in Wilson coefficients

RG evolution of Wilson coefficients

Wilson coefficients are scale dependent
governed by renormalisation group (RG) equations

running needed to match experimental results at low energy with theory predictions at high energy

power counting $\Rightarrow$ logarithmic singularities of $d=6$ operators result only from diagrams with one $d=6$ operator insertion
$\Rightarrow$ RG equation linear and homogeneous in α_i

at leading order in $\alpha_{\rm SM} = \alpha, \alpha/s_{\rm w}^2, \alpha_{\rm s}$:

$$\alpha_i(\mu) = \left(\delta_{ij} + \gamma_{ij}^{(0)} \frac{\alpha_{\rm SM}(\mu)}{4\pi} \log\left(\frac{\mu}{M}\right)\right) \alpha_i(M)$$

$\gamma_{ij}^{(0)}$: LO coefficients of anomalous dimension matrix

general analysis of anomalous dimensions available: Alonso et al. ’14
(for 2499 $d=6$ operators)

Conclusions

- Higgs-coupling measurements need consistent framework beyond SM
- Effective Field Theory provides general framework
  if scale of New Physics large compared to EW scale
- tasks for the (near) future ($\Rightarrow$ HXSWG)
  - agree on basis of $d = 6$ operators
    or a few bases with translation tables
  - agree on suitable subsets of operators (for initial fits)
  - calculate NLO corrections in EFT framework
  - implement in appropriate tools
  - perform coupling fits
- remember to use other frameworks as well!
  - specific models
  - form factors?