

# Soft-collinear factorization in $B$ decays

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## Outline

- Introduction and overview
- NLO phenomenology
- Factorization and NNLO
- Power-suppressed effects
- Summary and further results



# SCET in Heavy Quark Physics

- HQET

$B \rightarrow D\bar{\nu}$ ,  $B \rightarrow \pi\bar{\nu}$ , inclusive  $B$  decays  
[small  $\pi$  energy]

Heavy quark interacting with  
small-momentum ("soft")  
stuff

- SCET

Particles or jets with energy  $\mathcal{O}(m_B)$



- $B \rightarrow \pi\bar{\nu}$  [large  $\pi$  energy]
  - $B \rightarrow \gamma\bar{\nu}$
  - $B \rightarrow M\gamma^{(*)}, M_1M_2$
  - $B \rightarrow X_u(\text{jet})\bar{\nu}, X_s(\text{jet})\gamma$
- Exclusive or semi-inclusive processes

- ↪ isolate strong coupling physics from calculable weak coupling physics  
[ $\alpha_s(m_b), \alpha_s(\sqrt{m_b\Lambda}) \ll 1$ ]
- ↪ sum logs with renormalization group

# Overview

## I NLO Phenomenology

- Charmless  $B$  decays, radiative/electroweak decays
- CP violation

## II The next order

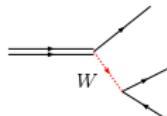
- 1-loop spectator scattering, concepts and calculations
- 2-loop vertex kernels

## III Developments in factorization

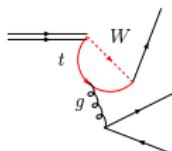
- All orders
- Power corrections
- Further processes

G. Bell, S. Bosch, F. Campanario, T. Feldmann, S. Jäger, T. Huber, X.Q. Li, B. Pecjak, J. Rohrer, J. Rohrwild, D. Seidel, O. Tarasov, P. Urban, L. Vernazza, Y. Wang, D. Yang

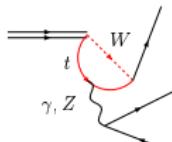
# EW scale effects and the effective Lagrangian



tree



QCD penguin



EW penguin

+ any new heavy  
particles ( $Z'$ , gluino  
KK states, ...)

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \left( C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i=\text{pen}} C_i \mathcal{O}_{i,\text{pen}} \right)$$

$$\mathcal{A}(\bar{B} \rightarrow f) = \sum_i [ \lambda_{\text{CKM}} \times C \times \langle f | \mathcal{O} | \bar{B} \rangle_{\text{QCD+QED}} ]_i$$

- Effective local interactions with a variety of flavour, colour and spin structures ( $D = d, s$ )

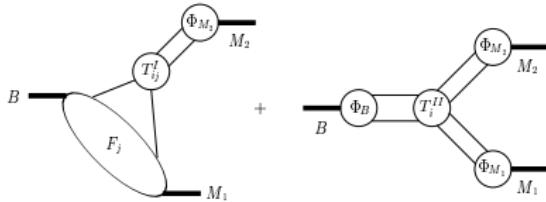
$$b \rightarrow D\gamma, b \rightarrow Dg, \quad b \rightarrow q_1 q_2 \bar{q}_3, b \rightarrow D\ell^+ \ell^-$$

- Many transitions only at the loop-level, all FCNCs  
Loop-induced transition can be dominant, e.g.  $b \rightarrow su\bar{u}$

$$V_{ub} V_{us}^* \sim 8 \cdot 10^{-4} \quad V_{tb} V_{ts}^* \sim 4 \cdot 10^{-2}$$

- $\langle f | \mathcal{O} | \bar{B} \rangle$  – “Hadronic matrix element” depends on spin and parity of final state, and whether reached only through annihilation. CP violation depends on rescattering phases.

# Charmless B decays at leading power [MB, Buchalla, Neubert, Sachrajda, 1999]



Form factor term +  
Spectator scattering

$$T, C, P^{c,u}, \dots \sim \langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\text{eff}}} =$$

$$\sum_{\text{terms}} \mathbf{C}(\mu_h) \times \left\{ F_{B \rightarrow M_1} \times \underbrace{T^{\text{I}}(\mu_h, \mu_s)}_{1+\alpha_s+...} \star f_{M_2} \Phi_{M_2}(\mu_s) \right.$$

$$+ f_B \Phi_B(\mu_s) \star \underbrace{T^{\text{II}}(\mu_h, \mu_s)}_{\alpha_s+...} \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

$$+ 1/m_b\text{-suppressed terms}$$

Direct CP asymmetries [ $B \rightarrow f$  vs.  $\bar{B} \rightarrow \bar{f}$ ]  
 $A_{\text{CP}} \propto \alpha_s, \Lambda_{\text{QCD}}/m_b!$

Similar factorization formula for  
 $B \rightarrow (M, \gamma)(\gamma, \ell^+ \ell^-, \ell \nu)$

# Charmless NLO phenomenology

## QCD factorization for $B \rightarrow PP$ and $B \rightarrow PV$ decays

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### Abstract

A comprehensive study of exclusive hadronic  $B$ -meson decays into final states containing two pseudoscalar mesons ( $PP$ ) or a pseudoscalar and a vector meson ( $PV$ ) is presented. The decay amplitudes are calculated at leading power in  $\Lambda_{\text{QCD}}/m_B$  and at next-to-leading order in  $\alpha_s$  using the QCD factorization approach. The calculation of the relevant hard-scattering kernels is completed. Important classes of power corrections, including “chirally-enhanced” terms and weak annihilation contributions, are estimated and included in the phenomenological analysis. Predictions are presented for the branching ratios of the complete set of the 96 decays of  $B^-$ ,  $\bar{B}^0$ , and  $\bar{B}_s$  mesons into  $PP$  and  $PV$  final states, and for most of the corresponding CP asymmetries. Several decays and observables

## Branching fractions, polarisation and asymmetries of $B \rightarrow VV$ decays

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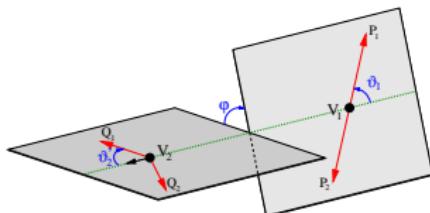
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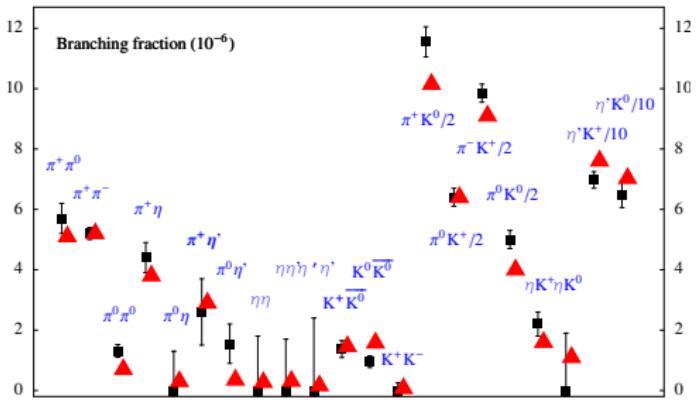
### Abstract

We calculate the hard-scattering kernels relevant to the negative-helicity decay amplitude in  $B$  decays to two vector mesons in the framework of QCD factorisation. We then perform a comprehensive analysis of the 34  $B \rightarrow VV$  decays, including  $B_s$  decays and the complete set of polarisation observables. We

$$\begin{aligned} B &= B_d, B_u, B_s \\ P &= \pi^0, \pi^\pm, K^0 \bar{K}^0, K^\pm, \bar{K}^\pm, \eta, \eta' \\ V &= \rho^0, \rho^\pm, K^{*0} \bar{K}^{*0}, K^{*\pm}, \bar{K}^{*\pm}, \omega, \Phi \end{aligned}$$

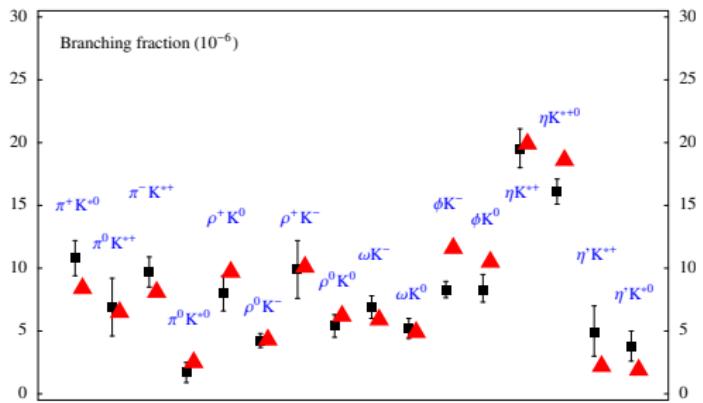
- 96 PP or PV final states
- Br,  $A_{\text{CP}}$ , S
- 34 VV final states
- Br,  $A_{\text{CP}}$ , S, polarisation fractions, angular distributions

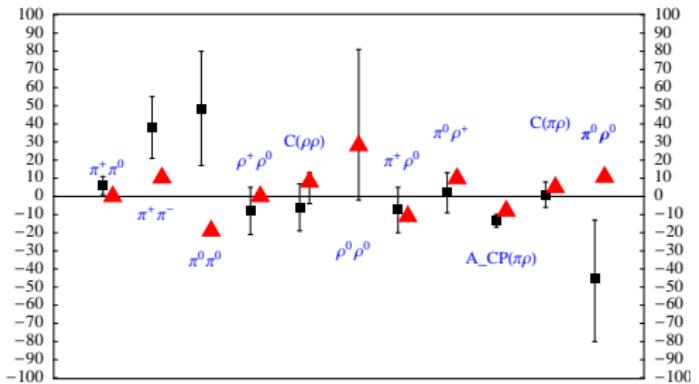




Branching fractions of tree- and penguin-dominated  $B \rightarrow PP$  decays (upper) and penguin-dominated  $\Delta S = 1$  pseudoscalar-vector (PV) final states.

(Triangles: theory NLO(S) [MB, Neubert, 2003]; Squares: BaBar/Belle data until 2007)

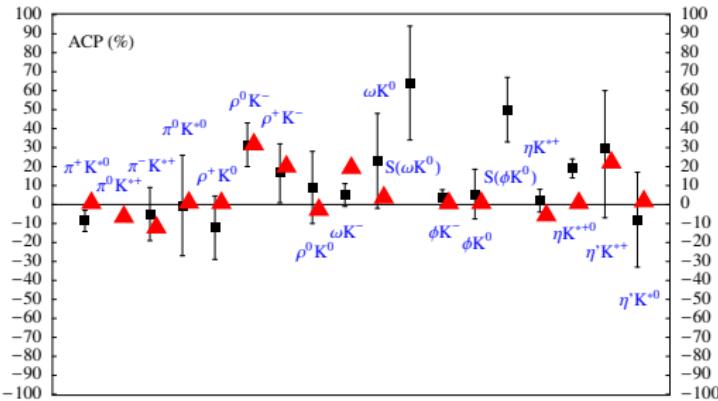




Comparison of direct CP violation in  $\Delta D = 1$  (upper plot) and PV  $\Delta S = 1$  decays (lower plot).

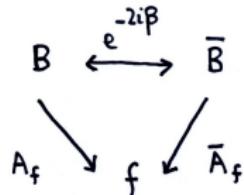
(Triangles: theory [MB, Neubert, 2003;

MB, Rohrer, Yang, 2006])



Note: Identically zero at LO

CKM angle  $\gamma$  from time-dependent CP violation in  $b \rightarrow d$  transitions



$$\frac{\Gamma(B(t) \rightarrow f) - \Gamma(\bar{B}(t) \rightarrow f)}{\Gamma(B(t) \rightarrow f) + \Gamma(\bar{B}(t) \rightarrow f)} = S_f \sin \Delta M_B t - C_f \cos \Delta M_B t$$

$$- \frac{2 \operatorname{Im} (e^{-2i\beta} \bar{A}_f / A_f)}{1 + |\bar{A}_f / A_f|^2}$$

$B \rightarrow J/\psi K_S$

$$\bar{A}_f = A_f \Rightarrow S_f = \sin(2\beta)$$



$B \rightarrow \pi\pi, \pi\eta, \eta\eta$

$$A_f = e^{i\gamma} + P_T \text{ "penguin"}$$

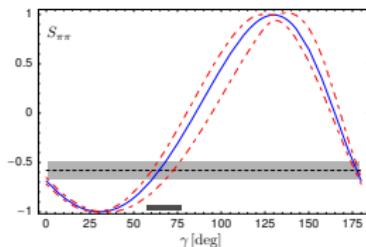


$$\Rightarrow S_f = -\sin 2(\beta + \gamma) + O(P_T)$$

longer for  $\pi\pi$  than for  $\pi\eta, \eta\eta$

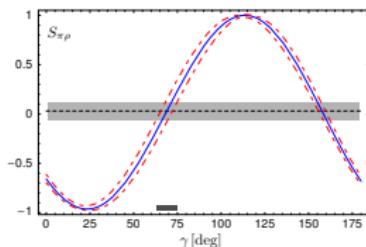
[ $\pi\eta$  is actually more complicated, since  $f$  is not a CP eigenstate]

# $\gamma$ determination from time-dependent CP asymmetry



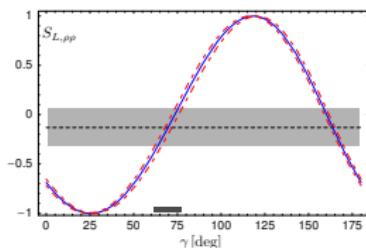
$$S_{\pi\pi} = -0.58 \pm 0.09$$
$$\Rightarrow \quad \gamma = (65^{+12}_{-8})^\circ$$

Mutually consistent



$$S_{\pi\rho} = 0.03 \pm 0.09$$
$$\Rightarrow \quad \gamma = (69^{+6}_{-6})^\circ$$
$$\gamma = (68 \pm 4)^\circ$$

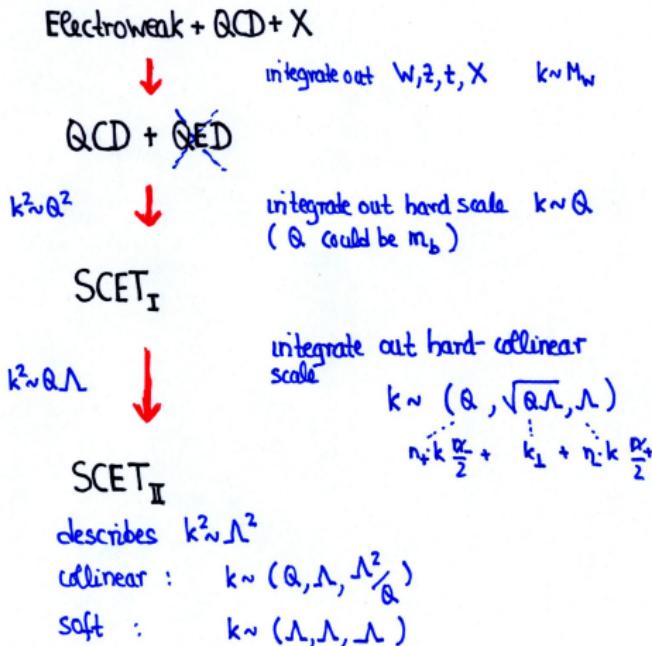
and consistent with the global unitarity triangle fit (CKMfitter, 2014):



$$S_{\rho\rho} = -0.13 \pm 0.19$$
$$\Rightarrow \quad \gamma = (69^{+8}_{-8})^\circ$$
$$\gamma = (66^{+1.3}_{-2.5})^\circ$$

# Systematics of factorization

## Fields & Scales



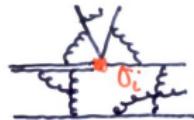
### Fields

	light quark	gluon	heavy quark
hard-collinear	$\xi_{hc}$	$A_{hc}$	-
collinear	$\xi_c$	$A_c$	-
soft	$q_s$	$A_s$	$h_v$

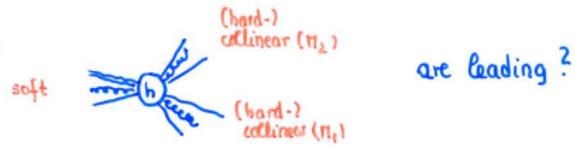
If energetic particles move in different directions  $n_i$  need collinear fields for every direction

# Integrating out the scale $m_b$ [QCD $\rightarrow$ SCET<sub>I</sub>]

(BBNS; Chay, Kim; MB, Feldmann, Bauer et al.)



Which hard subgraphs



Result:

$$(\bar{u}b)(\bar{d}u) \longrightarrow [\bar{X}_{(b,u)}^{(0)} X_{(d,u)}^{(0)}] + \left( C^I \cdot [\bar{g}(s_{\mu\nu}) h_{\mu\nu}] + C^{II} \cdot [\bar{g}(s_{\mu\nu}) A_{1\mu\nu} h_{\mu\nu}] \right)$$

$\vdots$

$\Phi_{M_2}$

$\vdots$

$\bar{g}(0) [F_{(0)}^{B\bar{u}}]$

$\bar{x} \quad x$

$\nabla$

$\bar{s}$

Correction to  
naive fact.

New effect: spectator  
Scattering

$\vdots$

$\Xi(\tau; 0)$

$\bar{x} \quad x$

$\nabla$

$\bar{s}$

- $M_2$  factorizes at scales  $\mu < m_b$
- Strong phases in perturbative coefficient functions  $C^{I,II}$  ONLY
- Leaves out  $1/m_b$  corrections (see below)

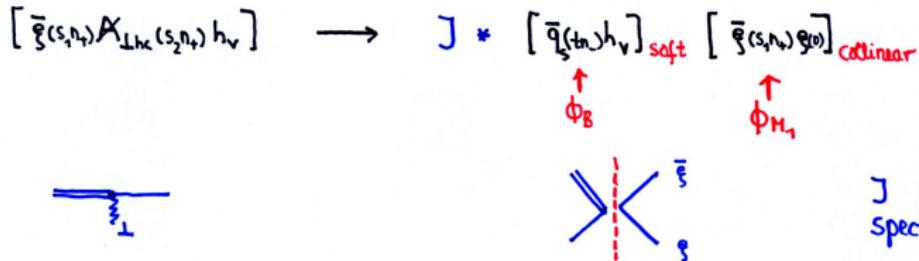


$\text{SCET}_I \rightarrow \text{SCET}_{II}$  (integrate out scale  $\sqrt{m_B \Lambda}$ )

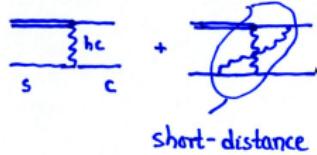
(MB, Feldmann;  
Neubert, Lange)

$[\bar{q}(s_1 n_+) h_v]$  is an example, where naive  $\text{SCET}_{II}$  factorization is wrong

- keep this in  $\text{SCET}_I$  (or use  $F^{B M_1(0)} \sim \langle M_1 | \bar{u} b | \bar{B} \rangle$  in QCD)



$J$  contains the hard-collinear spectator interactions



Power counting implies that convolution integrals converge, and that only four-quark operators appear  $\simeq$  product of 2-particle light-cone distribution amplitudes

Checked by 1-loop calculations  
(MB, Kiyo, Yang; Hill et al; MB, Yang)

## Status of radiative calculations (non-leptonic)

$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\text{eff}}} = \sum_{\text{terms}} C(\mu_h) \times \left\{ F_{B \rightarrow M_1} \times \underbrace{T^I(\mu_h, \mu_s)}_{1+\alpha_s+...} \star f_{M_2} \Phi_{M_2}(\mu_s) \right.$$

$$+ f_B \Phi_B(\mu_s) \star \left[ \underbrace{H^{II}(\mu_h, \mu_I)}_{1+\dots} \star \underbrace{J^{II}(\mu_I, \mu_s)}_{\alpha_s+...} \right] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

+  $1/m_b$ -suppressed terms

Status	2-loop vertex corrections ( $T_i^I$ )	1-loop spectator scattering ( $T_i^{II}$ )
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 in progress	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

from G. Bell [FPCP 2010/CKM2014]

Other non-leptonic and  $B \rightarrow (M, \gamma)(\gamma, \ell^+ \ell^-, \ell \nu)$  at NLO (= partly 2-loop).

# Operator matching for spectator scattering

$\text{QCD} \rightarrow \text{SCET}_I$  matching ( $\tilde{T}^I(\hat{t})$  and  $\tilde{H}^{II}(\hat{t}, \hat{s})$  are short-distance)

$$Q = \int dt \tilde{T}^I(\hat{t}) O^I(t) + \int dt d\hat{s} \tilde{H}^{II}(\hat{t}, \hat{s}) O^{II}(t, s) \quad \text{where} \quad H^{II}(u, v) = \int dt d\hat{s} e^{i(u\hat{t} + (1-v)\hat{s})} \tilde{H}^{II}(\hat{t}, \hat{s}).$$

$$O^I(t) = (\bar{x} W_{c2})(m_-) \frac{\not{h}_-}{2} (1 - \gamma_5) (W_{c2}^\dagger \chi) [\bar{q} \not{h}_+ (1 - \gamma_5) b]$$

$$O^{II}(t, s) = \frac{1}{m_b} \left[ (\bar{x} W_{c2})(m_-) \frac{\not{h}_-}{2} (1 - \gamma_5) (W_{c2}^\dagger \chi) \right] \left[ (\bar{\xi} W_{c1}) \frac{\not{h}_+}{2} [W_{c1}^\dagger i \not{D}_{\perp c1} W_{c1}] (sn_+) (1 + \gamma_5) h_v \right]$$

Hadronic matrix elements (light-cone distribution amplitude and generalized light-cone form factor)

$$\langle M_2 | (\bar{x} W_{c2})(m_-) \frac{\not{h}_-}{2} (1 - \gamma_5) (W_{c2}^\dagger \chi) | 0 \rangle = \frac{i f_{M_2} m_B}{2} \int_0^1 du e^{iu\hat{t}} \phi_{M_2}(u)$$

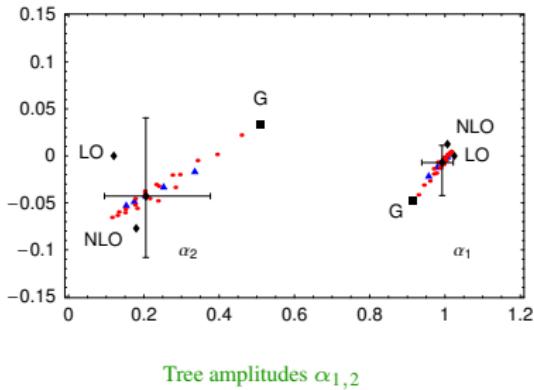
$$\langle M_1 | (\bar{\xi} W_{c1}) \frac{\not{h}_+}{2} [W_{c1}^\dagger i \not{D}_{\perp c1} W_{c1}] (sn_+) (1 + \gamma_5) h_v | \bar{B} \rangle = -m_b m_B \int_0^1 d\tau e^{i\tau\hat{s}} \Xi_{M_1}(\tau)$$

$$\langle M_1 M_2 | Q | \bar{B} \rangle = i m_B^2 \left\{ f_+^{BM_1}(0) \int_0^1 du T^I(u) f_{M_2} \phi_{M_2}(u) - \frac{1}{2} \int_0^1 du dz H^{II}(u, z) \Xi_{M_1}(1-z) f_{M_2} \phi_{M_2}(u) \right\}$$

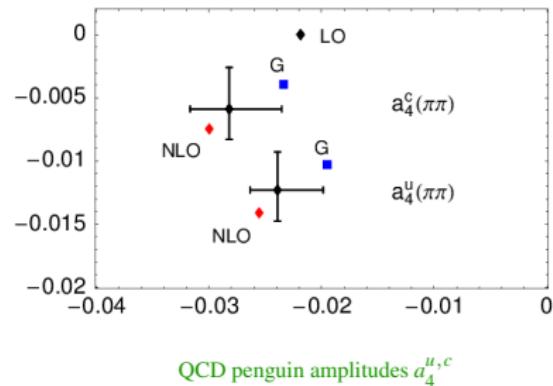
$\Xi_{M_1}(\tau)$  factorizes further [ $\text{SCET}_I \rightarrow \text{SCET}_{II}$  matching]

$$\Xi_{M_1}(\tau) = \frac{m_B}{4m_b} \int_0^\infty d\omega \int_0^1 dv J_{||}(\tau; v, \omega) \hat{f}_B \phi_{B+}(\omega) f_{M_1} \phi_{M_1}(v)$$

# Amplitudes and $\pi\pi$ branching fractions with NLO spectator scattering



Tree amplitudes  $\alpha_{1,2}$



QCD penguin amplitudes  $a_4^{u,c}$

- $\Gamma[\pi^0\pi^0]/\Gamma[\pi^+\pi^-]$  can be enhanced from  $1/50$  to  $1/7$ .

$10^6 \text{ Br}_{\text{Av}}$	Theory (NLO <sub>Sp</sub> )	Exp.
$\pi^-\pi^0$	$5.5^{+0.3}_{-0.3} (\text{CKM})^{+0.5}_{-0.4} (\text{hadr.})^{+0.9}_{-0.8} (\text{pow.})$	$5.7 \pm 0.5$
$\pi^+\pi^-$	$5.0^{+0.8}_{-0.9} (\text{CKM})^{+0.3}_{-0.5} (\text{hadr.})^{+1.0}_{-0.5} (\text{pow.})$	$5.2 \pm 0.2$
$\pi^0\pi^0$	$0.73^{+0.27}_{-0.24} (\text{CKM})^{+0.52}_{-0.21} (\text{hadr.})^{+0.35}_{-0.25} (\text{pow.})$	$1.31 \pm 0.21$

# All-order factorization, power corrections and “endpoint divergences”

$$\langle M_1 M_2 | Q | \bar{B} \rangle = i m_B^2 \left\{ f_+^{BM_1}(0) \int_0^1 du T^I(u) f_{M_2} \phi_{M_2}(u) - \frac{1}{2} \int_0^1 du dz H^{\text{II}}(u, z) \Xi_{M_1}(1-z) f_{M_2} \phi_{M_2}(u) \right\}$$

$$\Xi_{M_1}(\tau) = \frac{m_B}{4m_b} \int_0^\infty d\omega \int_0^1 dv J_{||}(\tau; v, \omega) \hat{f}_B \phi_{B+}(\omega) f_{M_1} \phi_{M_1}(v)$$

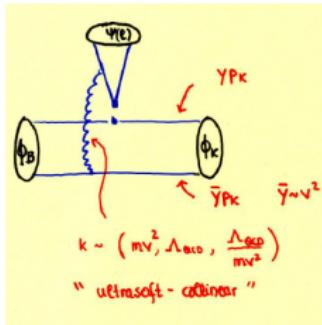
Why not

$$f_+^{BM_1}(0) = \int_0^\infty d\omega \int_0^1 dv K(v, \omega) \hat{f}_B \phi_{B+}(\omega) f_{M_1} \phi_{M_1}(v) ? \quad K(v, \omega) \supset \int_0^1 \frac{du}{u^2} \Phi(u) = \infty$$

(Brodsky, Szczepaniak, 1990)

- All-order factorization (convergence) for  $\Xi_{M_1}(\tau)$  [MB, Feldmann, 2003]
- Basic problem with collinear/SCET factorization in higher orders in  $\Lambda/m_b$ : operator interpretation of end-point contributions and their estimation.  
 [Known as rapidity factorization in collider physics.]  
 Every amplitude receives corrections  $\Lambda/m_b$ , which cannot be calculated in general.
- Some insight from non-relativistic systems [Bell, Feldmann, 2007] especially  $B \rightarrow \chi_{cJ} K$  [MB, Vernazza, 2008]

# Sketch of spectator-scattering $B \rightarrow \chi_{cJ} K$ [MB, Vernazza, 2008]



$$A(B \rightarrow \chi_c K)_{\text{spect}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \cdot 2C_1 \cdot \frac{\pi \alpha_s G_F}{N_c^2} \langle \phi(n)|H(n) \rangle + \frac{1}{\lambda_B} \cdot M_B$$

$$\int_0^1 dy f_k \phi_k(y) \left\{ \left[ \frac{e^{B[n]y}}{y} + \frac{B[n]}{y^2} \right] \Theta(1-\mu-y)$$

hard / P-wave colour singlet

$$+ B[n] \frac{1}{(b + \sqrt{-(\bar{y}+a)})^4} \Theta(\bar{y}-(1-\mu)) \right\}$$

ultrasoft / S-wave colour octet

Endpoint div. in hard spectator-scattering

$b = \frac{\sigma}{m_B \sqrt{1-z}}$

$a = \frac{4 m_c E_B}{m_B^2 (1-z)}$

NEW

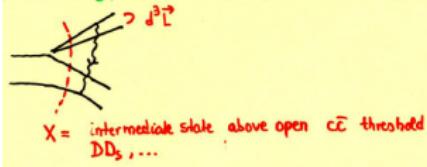
(Song et al., 2002; Meng et al., 2005)

$$\int_0^1 dy \phi_k(y) \left\{ \right\} = e^{B[n]} \int_0^1 dy \frac{\phi_k(y)}{y} + B[n] \int_0^1 dy \frac{\phi_k(y) + \bar{y} \phi'_k(y)}{\bar{y}^2} + B[n] \phi_k'(y) \ln \mu$$

$$- B[n] \phi_k'(y) \left\{ \ln \mu + \ln \frac{m_B^2 (1-z)}{\bar{y}^2} - i\pi - 2 \ln(1+A) + 1 + \frac{2}{3} \frac{4+A}{(1+A)^2} \right\}$$

"large log"  $\ln \frac{m_B^2}{m_c^2 v^2}$  : endpoint log

$$A = \sqrt{-\frac{4(E_B + i\epsilon)}{B^2/m_c}} = \mathcal{O}(1)$$

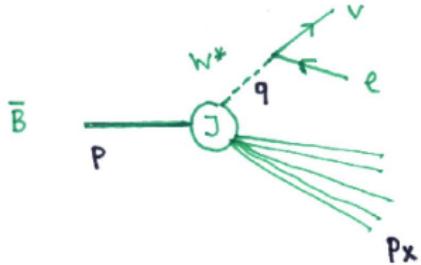


Large rescattering phase from endpoint contribution, none from hard scattering.

# All-order factorization, and power corrections

- Cannot be extended to relativistic case.
- All-order factorization and NNLO computation of part of the power-suppressed contributions to the (scalar) QCD penguin amplitude.
- $B \rightarrow \gamma \ell \nu$  – phenomenology and factorization at order  $1/m_b$  [MB, Rohrwild, + Kirilin, 2011 –]
- $B \rightarrow X_u \ell \nu$  for  $m_X^2 \ll m_b^2$  [MB, Campanario, Mannel, Pecjak, 2004]

## Semi-inclusive semi-leptonic decay



$$\frac{d\Gamma}{dq^2 dE_e} = G_F^2 |V_{ub}|^2 * \text{kinematical factors} * J_{\mu} T$$

$$T \equiv i \int d^4x e^{-iq \cdot x} \langle \bar{B}(p) | T(J_{\mu}^{(+)}, J_{(0)}^{\mu}) | \bar{B}(p) \rangle$$

"hadronic tensor"

$$J_{\mu} = \bar{u} \gamma_{\mu} (1 - \gamma_5) b$$

$$T^{\mu\nu} = \tilde{H}_{jj'}(\hat{s}_1, \dots, \hat{s}_n) \otimes T_{jj'}^{\text{eff},\mu\nu}(\hat{s}_1, \dots, \hat{s}_n)$$

$$\begin{aligned} \langle \bar{B} | T^{\text{eff}}(\hat{s}_1, \dots, \hat{s}_n) | \bar{B} \rangle &= i \int d^4x d^4y \dots e^{i(m_b v - q)x} \langle \bar{B} | \bar{h}_v [\text{soft fields}] h_v | \bar{B} \rangle(x_-, y_-, \dots) \\ &\times \langle 0 | [\text{collinear fields}] | 0 \rangle(\hat{s}_1, \dots, \hat{s}_n; x, y, \dots) \end{aligned}$$

$$T = \sum H(u_1, \dots u_i) \otimes \mathcal{J}(u_1, \dots, u_i; \omega_1, \dots, \omega_n) \otimes S(\omega_1, \dots, \omega_n)$$


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$$\mathcal{L}_{\xi}^{(1)} = \bar{\xi} x_{\perp}^{\mu} n_{-}^{\nu} W_c g F_{\mu\nu}^s W_c^{\dagger} \frac{\not{q}_{+}}{2} \xi,$$

$$\mathcal{L}_{1\xi}^{(2)} = \frac{1}{2} \bar{\xi} n_{-} x n_{+}^{\mu} n_{-}^{\nu} W_c g F_{\mu\nu}^s W_c^{\dagger} \frac{\not{q}_{+}}{2} \xi,$$

$$\mathcal{L}_{2\xi}^{(2)} = \frac{1}{2} \bar{\xi} x_{\perp}^{\mu} x_{\perp\rho} n_{-}^{\nu} W_c [D_{\perp s}^{\rho}, g F_{\mu\nu}^s] W_c^{\dagger} \frac{\not{q}_{+}}{2} \xi,$$

$$\begin{aligned} \mathcal{L}_{3\xi}^{(2)} &= \frac{1}{2} \bar{\xi} i \not{D}_{\perp c} \frac{1}{in_{+} D_c} x_{\perp}^{\mu} \gamma_{\perp}^{\nu} W_c g F_{\mu\nu}^s W_c^{\dagger} \frac{\not{q}_{+}}{2} \xi \\ &+ \frac{1}{2} \bar{\xi} x_{\perp}^{\mu} \gamma_{\perp}^{\nu} W_c g F_{\mu\nu}^s W_c^{\dagger} \frac{1}{in_{+} D_c} i \not{D}_{\perp c} \frac{\not{q}_{+}}{2} \xi, \end{aligned}$$

$$\mathcal{L}_{\xi q}^{(1)} = \bar{q}_s W_c^{\dagger} i \not{D}_{\perp c} \xi - \bar{\xi} i \not{\overline{D}}_{\perp c} W_c q_s,$$

$$J_1^{(1)} = (\bar{\xi} W_c)_s \Gamma_j(x_{\perp} D_s h_v)$$

$$J_2^{(1)} = (\bar{\xi} W_c)_s i \not{\overline{\partial}}_{\perp}^{\mu} \Gamma_j h_v$$

$$J_3^{(1)} = (\bar{\xi} W_c)_{s1} [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s2} \Gamma_j h_v$$

$$J_1^{(2)} = \frac{1}{2} (\bar{\xi} W_c)_s \Gamma_j(n_- x n_+ D_s h_v)$$

$$J_2^{(2)} = \frac{1}{2} (\bar{\xi} W_c)_s \Gamma_j(x_{\perp\mu} x_{\perp\nu} D_s^{\mu} D_s^{\nu} h_v)$$

$$J_3^{(2)} = (\bar{\xi} W_c)_s \Gamma_j(i D_s^{\mu} h_v)$$

$$J_5^{(2)} = (\bar{\xi} W_c)_s i \not{\overline{D}}_{\perp}^{\mu} \Gamma_j h_v$$

$$J_6^{(2)} = (\bar{\xi} W_c)_s i \not{\overline{\partial}}_{\perp}^{\mu} i \not{\overline{\partial}}_{\perp}^{\nu} \Gamma_j h_v$$

$$J_7^{(2)} = (\bar{\xi} W_c)_{s1} [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s2} \Gamma_j(x_{\perp} D_s h_v)$$

$$J_8^{(2)} = (\bar{\xi} W_c)_{s1} i \not{\overline{\partial}}_{\perp}^{\mu} [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s2} \Gamma_j h_v$$

$$J_{10}^{(2)} = (\bar{\xi} W_c)_{s1} [W_c^{\dagger} in_{-} D W_c]_{s2} \Gamma_j h_v$$

$$J_{11}^{(2)} = (\bar{\xi} W_c)_{s1} [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s2}, [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s3}] \Gamma_j h_v$$

$$J_{12}^{(2)} = (\bar{\xi} W_c)_{s1} \{ [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s2}, [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s3} \} \Gamma_j h_v$$

$$J_{13}^{(2)} = (\bar{\xi} W_c)_{s1} \text{tr} [[W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s2}, [W_c^{\dagger} i D_{\perp c}^{\mu} W_c]_{s3}] \Gamma_j h_v$$

$$J_{14}^{(2)} = [[(\bar{\xi} W_c)_{s1} \Gamma_j h_v] \left[ (\bar{\xi} W_c)_{s2} \frac{\not{q}_{+}}{2} \Gamma_{j'} (W_c^{\dagger} \xi)_{s3} \right]]$$

$$J_{15}^{(2)} = \left[ [[(\bar{\xi} W_c)_{s1} \Gamma_j T^A h_v] \left[ [[(\bar{\xi} W_c)_{s2} \frac{\not{q}_{+}}{2} \Gamma_{j'} T^A (W_c^{\dagger} \xi)_{s3} \right]] \right]$$

## Tree-level results

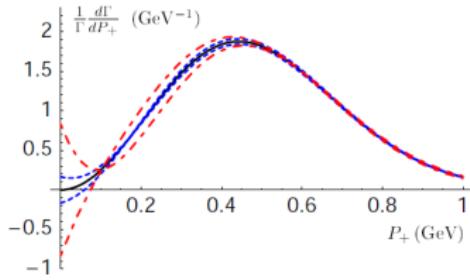
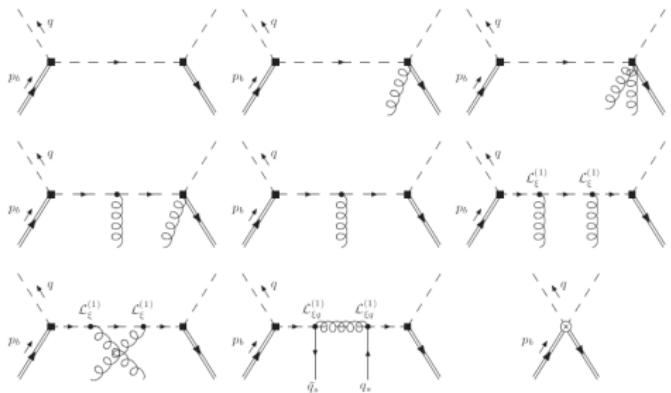


Figure 2: Distortion of the  $P_+$  spectrum in  $B^- \rightarrow X_u \ell \nu$  decay by four-quark contributions assuming the model (82). The solid central curve is  $\hat{S}(P_+)$ . The dashed curves correspond to  $\lambda = 100 \text{ MeV}$  (long dashes) and  $\lambda = 500 \text{ MeV}$  (short dashes). Each pair is for  $\varepsilon = \pm 0.1$ .

## Summary/Outlook

I

Mature theory at leading power (SCET).

Similarities and difference from collider physics. Soft initial state. Power-suppressed interactions relevant at LP.

$$\mathcal{L}_{\text{eik}} = \bar{\xi} i n_- D_s \frac{\not{q}_+}{2} \xi \quad \mathcal{L}_{\xi q}^{(1)} = \bar{q}_s W_c^\dagger i \not{D}_{\perp c} \xi - \bar{\xi} i \not{D}_{\perp c} W_c q_s$$

II

NNLO computation for charmless decays (nearly) completed

Soon ready for a major improvement of QCDF predictions (excluding polarisation):

- NLO → NNLO
- Improved input parameters

Belle-II start-up 2016, B2TiP effort started.

Still many unmeasured, but predicted observables.

III

LHCb: Electroweak penguin decay  $B \rightarrow K^{(*)} \ell \ell$

NLO [2001/2004] → NNLO (3-loop  $b \rightarrow s \ell^+ \ell^-$  and 2-loop spectator-scattering)?