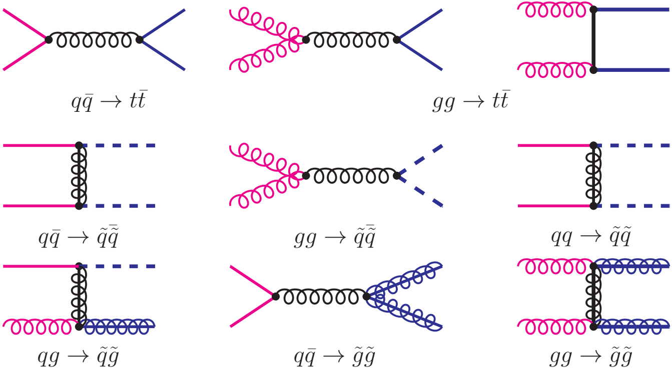
Threshold resummation of heavy (s)particles pair production at hadron colliders

Jan Piclum

based on:
M. Beneke, P. Falgari, C. Schwinn, Nucl. Phys. B 828 (2010) 69
M. Beneke, P. Falgari, C. Schwinn, Nucl. Phys. B 842 (2011) 414
M. Beneke, P. Falgari, S. Klein, C. Schwinn, Nucl. Phys. B 855 (2012) 695
M. Beneke, P. Falgari, S. Klein, JP, C. Schwinn, M. Ubiali, F. Yan, JHEP 07 (2012) 195
M. Beneke, P. Falgari, JP, C. Schwinn, C. Wever, in preparation

Total Cross Section for $pp \to HH'X$

$$\sigma_{HH'}(s) = \sum_{i,j} \int_{4M^2}^{s} \mathrm{d}\hat{s}\, \mathcal{L}_{ij}(s, \hat{s}, \mu_f)\, \hat{\sigma}_{ij}(\hat{s}, \mu_f, \mu_r)$$



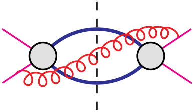
heavy (s)particles $\Rightarrow$ production close to threshold

Dominant Terms

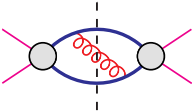
threshold limit: $\beta = \sqrt{1 - 4M^2/\hat{s}} \to 0$, $M = (m_H + m_{H'})/2$

Sterman 1987; Laenen et al. 1991; Catani et al.; Berger, Contopanagos; Kidonakis et al. 1996; Bonciani et al. 1998

Soft corrections:



$\sim \alpha_s \ln^2 \beta$



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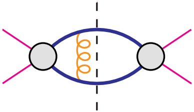
Coulomb corrections:



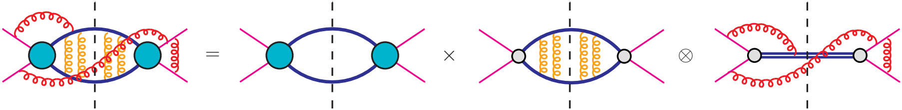
$\sim \alpha_s/\beta$

$\alpha_s/\beta \sim 1$, $\alpha_s \ln \beta \sim 1 \rightsquigarrow$ resum terms to all orders

Resummation in Momentum Space

Soft and Coulomb resummation: Beneke, Falgari, Schwinn 2009, 2010

$$\hat{\sigma}_{ij} = \sum_R H_{ij}^R \int \mathrm{d}\omega \, J_R(M\beta^2 - \omega/2) \, W^R(\omega)$$



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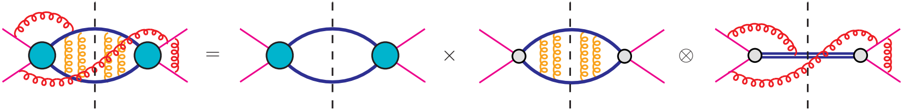

- factorisation formula is derived in EFT framework:
    - SCET for soft and collinear modes
    - pNRQCD for potential and soft modes
- factorisation of soft and Coulomb interaction is non-trivial
- soft function can be diagonalised by choice of colour basis

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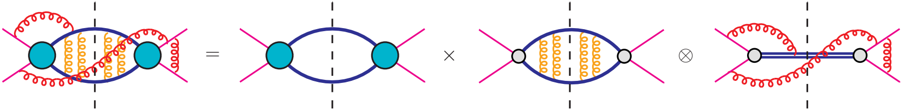


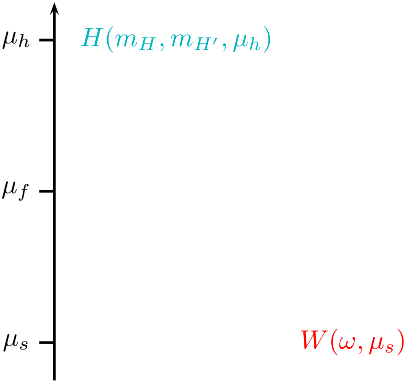
$$\hat{\sigma} \quad \propto \quad \hat{\sigma}^{(0)} \sum_k \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{LL}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{NLL}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{NNLL}} + \ldots]$$

$$\times \{1\,(\text{LL, NLL})\,;\alpha_s, \beta\,(\text{NNLL})\,;\ldots\}$$

Resummation of Soft Logarithms

Becher, Neubert, Pecjak 2006; Becher, Neubert, Xu 2007;
Beneke, Falgari, Schwinn 2009;
Czakon, Mitov, Sterman 2009

- typical scales:
  $\mu_h \sim 2M$, $\mu_s \sim M\beta^2$
- hard and soft function obey RGEs
- solve RGEs in momentum space



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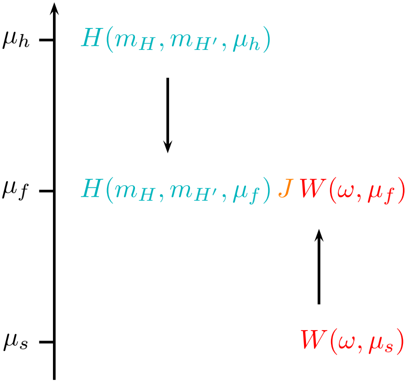
- evolve H from μ_h to μ_f
- evolve W from μ_s to μ_f

$\Rightarrow$ resums logarithms of β



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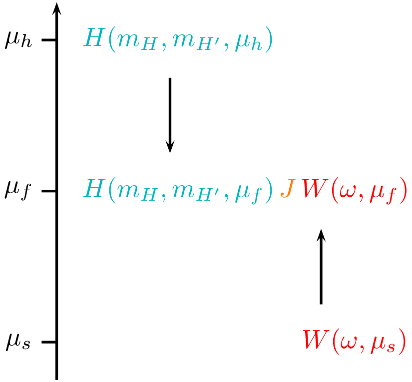

NNLL:

- 1-loop hard function — top: Czakon, Mitov 2008; SUSY: Kauth et al. 2011; Langenfeld et al. 2012; Beenakker et al. 2011, 2013
- 1-loop soft function — Beneke, Falgari, Schwinn 2009
- 2-loop anomalous dimensions — Becher, Neubert 2009; Ferroglia et al. 2009; Beneke, Falgari, Schwinn 2009; Czakon, Mitov, Sterman 2009; Moch, Vermaseren, Vogt 2004
- 3-loop cusp anomalous dimension — Moch, Vermaseren, Vogt 2005

Resummation of Coulomb corrections from non-relativistic Greens function

Fadin, Khoze 1987; Peskin, Strassler 1990; ...

$$\left[-\frac{\vec{\nabla}^2}{2m_{\mathrm{red}}} + D_R \frac{\alpha_s}{r}\right] G^{(0)}_{C,R}(\vec{r}, \vec{r}\,', E) = \delta(\vec{r} - \vec{r}\,')$$

$$J_R(E) = 2\,\mathrm{Im}\, G^{(0)}_{C,R}(0,0;E)\,, \quad E = \sqrt{\hat{s}} - 2M$$

- includes bound states below threshold ($E < 0$)
- depends on Coulomb scale: $\mu_C = \max\{2\alpha_s(\mu_C) m_{\mathrm{red}} |D_R|,\, 2\sqrt{2m_{\mathrm{red}} M}\beta\}$

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corrections at NNLL:

- NLO Coulomb potential $\rightsquigarrow \mathcal{O}(\alpha_s^2/\beta)$
- NNLO non-Coulomb potential and kinetic energy $\rightsquigarrow \mathcal{O}(\alpha_s^2 \ln\beta)$
- Coulomb Green function in s-channel exchange $\rightsquigarrow \mathcal{O}(\alpha_s^2 \ln\beta)$

Bärnreuther, Czakon, Fiedler 2013

NNLL Coulomb function

$$J_R^{\mathsf{NNLL}}(E) = 2\,\mathrm{Im}\left[G^{(0)}_{C,R}(0,0;E)\,\Delta_{\mathrm{nC}}(E) + G^{(1)}_{C,R}(0,0;E)\right]$$

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NLO Coulomb potential:

$$\delta\tilde{V}_{\mathbf{NLO}} = \frac{D_R \alpha_s^2}{\vec{q}^{\,2}}\left(a_1 - \beta_0 \ln\frac{\vec{q}^{\,2}}{\mu^2}\right) \quad \rightsquigarrow \quad G^{(1)}_{C,R}(0,0;E)$$

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NNLO non-Coulomb potential:

$$\begin{aligned}\delta\tilde{V}_{\mathbf{NNLO}} \quad &= \quad \frac{4\pi D_R\alpha_s}{\vec{q}^{\,2}}\left[\frac{\pi\alpha_s|\vec{q}|}{8m_{\mathrm{red}}}\left(\frac{D_R m_{\mathrm{red}}}{M} + C_A\right) + \frac{\vec{p}^{\,2}}{m_H m_{H'}}\right. \\ &\qquad - \frac{\vec{q}^{\,2}}{8m_H^2 m_{H'}^2}\left(2m_H m_{H'} + m_H^2 c_2(m_{H'}) + m_{H'}^2 c_2(m_H)\right) \\ &\qquad \left. + \frac{1}{16 m_H m_{H'}}[\sigma^i,\sigma^j]q^j \otimes [\sigma^i,\sigma^k]q^k + \dots\right] \\ \rightsquigarrow \Delta_{\mathrm{nC}}(E) \quad &= \quad 1 + \alpha_s^2 \ln\beta\left[-2D_R^2\,(1+v_{\mathrm{spin}}) + D_R C_A\right]\theta(E)\end{aligned}$$

$$v_{\mathrm{spin}}(\tilde{q}\bar{\tilde{q}},\tilde{q}\tilde{q}) = \frac{-m_{\mathrm{red}}}{2M}\,,\ v_{\mathrm{spin}}(\tilde{q}\tilde{g}) = \frac{1}{2}\left(\frac{m_{\tilde{g}}^2}{(m_{\tilde{q}}+m_{\tilde{g}})^2} - 1\right)\,,\ v_{\mathrm{spin}}(\tilde{g}\tilde{g},S=0) = 0\,,\ v_{\mathrm{spin}}(\tilde{g}\tilde{g},S=1) = -\frac{2}{3}$$

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$$\rightsquigarrow \Delta_{\mathrm{nC}}(E) \quad = \quad 1 + \alpha_s^2 \ln\beta\left[-2D_R^2\,(1+v_{\mathrm{spin}}) + D_R C_A + \frac{4m_{\mathrm{red}}}{M^2}\,D_R\,v^R_{\mathrm{ann}}\right]\theta(E)$$

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Matching to Fixed Order Cross Section

- threshold approximation is good for $4M^2 \gtrsim 0.2\,s$ Becher, Neubert, Xu 2007
- match to fixed order result to improve behaviour at large β

$$\hat{\sigma}^{\mathsf{NNLL+NNLO_{(app)}}} = \left(\hat{\sigma}^{\mathsf{NNLL}} - \hat{\sigma}^{\mathsf{NNLL}}\Big|_{\mathsf{NNLO}}\right) + \hat{\sigma}^{\mathsf{NNLO_{(app)}}}$$

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top: match to full NNLO result [Czakon, Fiedler, Mitov 2013]

SUSY: $\sigma^{\text{NNLO}_{\text{app}}} = \sigma^{\text{NLO}}[\text{PROSPINO}] + \sigma^{\text{NNLO}}[\mathcal{O}(\alpha^2), \text{ soft+Coulomb}]$

[PROSPINO: Beenakker, Höpker, Spira 1996]

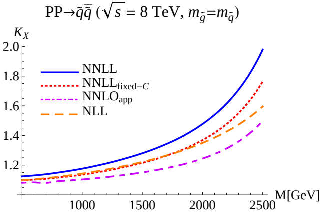
Top Results

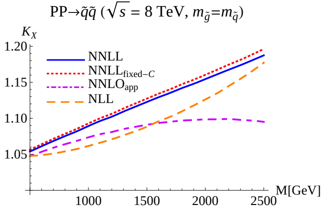
$\sigma_{t\bar{t}}$ [pb]	Tevatron	LHC @ 7 TeV	LHC @ 8 TeV
NLO	$6.68\,^{+0.36}_{-0.75}\,^{+0.23}_{-0.22}$	$158.1\,^{+19.5}_{-21.2}\,^{+6.8}_{-6.2}$	$226.2\,^{+27.8}_{-29.7}\,^{+9.2}_{-8.3}$
NNLO	$7.01\,^{+0.27}_{-0.37}\,^{+0.29}_{-0.24}$	$167.1\,^{+6.7}_{-10.7}\,^{+7.7}_{-7.1}$	$239.1\,^{+9.3}_{-14.8}\,^{+10.3}_{-9.6}$
NNLL+NNLO	$7.15\,^{+0.24}_{-0.10}\,^{+0.30}_{-0.25}$	$168.5\,^{+6.3}_{-7.5}\,^{+7.7}_{-7.0}$	$241.0\,^{+8.7}_{-11.1}\,^{+10.5}_{-9.7}$
experiment	7.60 ± 0.41	173 ± 10	242.4 ± 9.5

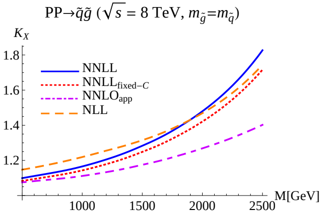
obtained with `TOPIXS 2.0` using m_t=173.3 GeV, MSTW 2008 NLO/NNLO, $\alpha_s(M_Z) = 0.1171$
NNLO uses results by Bärnreuther, Czakon, Mitov, Fiedler
experiment: Tevatron combination; LHC combination; ATLAS 1406.5375 using m_t=172.5 GeV

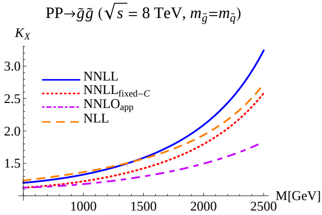
- good agreement with experiment and
  Mellin space result by Cacciari, Czakon, Mangano, Mitov, Nason
- resummation improves theory uncertainty
- theory uncertainty $\sim \pm$ 3–5%
- PDF+α_s uncertainty $\sim \pm$ 4–5%
- calculation has been implemented in public program `TOPIXS`

SUSY Results: K Factors







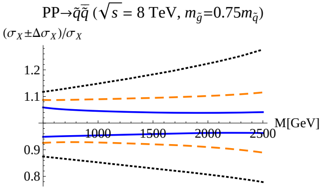


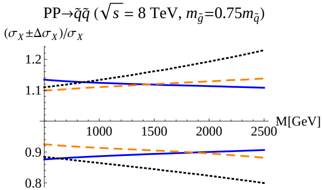
$$K_X = \frac{\sigma^X}{\sigma^{\textbf{NLO}}}$$

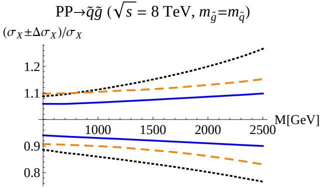
$\rightsquigarrow$ large NNLL corrections:

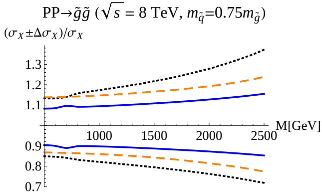
- 10–200% of NLO
- 0–40% on top of NLL
- 0–150% beyond NNLO
- Coulomb resummation can be large effect

soft resummation in Mellin space → talk by Eric Laenen









theoretical uncertainties:

- scale variation: μ_f, μ_h, μ_C
- estimate of NNLO constant
- β_{cut} for running soft scale
- $E = M\beta^2$ vs. $E = \sqrt{\hat{s}} - 2M$

reduced uncertainty at NNLL:

- NLO: $\pm(20$–$30)\%$
- NLL: $\pm(10$–$20)\%$
- NNLL: $\pm(5$–$14)\%$

Summary

- formalism for combined resummation of soft and Coulomb gluons in momentum space has been developed
- total cross section for top-pair production has been computed at NNLL+NNLO
- total cross section for pair production of gluinos and squarks has been computed at NNLL+NNLO_{app}
- results for top-pair production have been published in the public program `TOPIXS`
- public program for sparticle pairs is in preparation

Fixed vs. Running Soft Scale

fixed soft scale: Becher, Neubert, Xu 2007

- minimises relative fixed-order 1-loop soft correction to $\sigma_{HH'}$
- resums logarithms in hadronic cross section
- does not predict partonic cross section

alternative method: logarithmic derivative of parton luminosity
Sterman, Zeng 2013

running soft scale: Beneke, Falgari, Klein, Schwinn 2011

- divide β integration into two regions
- $\beta < \beta_{\text{cut}}$: small ambiguities, $\mu_s = M\beta_{\text{cut}}^2$
- $\beta > \beta_{\text{cut}}$: no large logarithms, $\mu_s = M\beta^2$
- for $\tilde{g}\tilde{g}$: $\beta_{\text{cut}} = 0.50$–$0.39$ (LHC7), $\beta_{\text{cut}} = 0.52$–$0.40$ (LHC14)
Falgari, Schwinn, Wever 2012