Electroweak Radiative Corrections at High Energies

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SFB/TR9 project A1, 2000-2014

J. H. Kuhn, A. A. Penin and V. A. Smirnov, Eur. Phys. J. C **17** (2000) 97

J. H. Kuhn, S. Moch, A. A. Penin and V. A. Smirnov, Nucl. Phys. B **616** (2001) 286

B. Feucht, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Phys. Rev. Lett. **93** (2004) 101802

A. A. Penin, Phys. Rev. Lett. **95** (2005) 010408

B. Jantzen, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Nucl. Phys. B **731** (2005) 188

J. H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Nucl. Phys. B **727** (2005) 368

J. H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Nucl. Phys. B **797** (2008) 27

J. H. Kuhn, F. Metzler and A. A. Penin, Nucl. Phys. B **795** (2008) 277

R. Bonciani, A. Ferroglia and A. A. Penin, Phys. Rev. Lett. **100**, 131601 (2008)

J. H. Kuhn, F. Metzler, A. A. Penin and S. Uccirati, JHEP **1106** (2011) 143

A. A. Penin and G. Ryan, JHEP **1111** (2011) 081

A. Bierweiler, T. Kasprzik, J. H. Kuhn and S. Uccirati, JHEP **1211** (2012) 093

A. Bierweiler, T. Kasprzik and J. H. Kuhn, JHEP **1312** (2013) 071

S. Gieseke, T. Kasprzik and J. H. Kuhn, arXiv:1401.3964 [hep-ph].

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S. Gieseke, T. Kasprzik and J. H. Kuhn, arXiv:1401.3964 [hep-ph].

No more refs in the talk! (see the papers for an excessive list)!

Topics discussed

- Electroweak corrections at high energy
  - *Sudakov logarithms*

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  - *vector boson production*
- Bhabha scattering
  - *two-loop photonic corrections*
  - *two-loop heavy-flavor corrections*

Why high-order electroweak corrections?

- Massive W, Z bosons ⇨ exclusive reactions
- Sudakov double logs: $\ln^2(s/M_{Z,W}^2)$ per loop
- LHC, ILC: $\ln^2(s/M_{Z,W}^2) \sim 25$
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  - *40% in one loop*
  - *10% in two loops*
- Two loops crucial for:
  - *gauge boson production (search for anomalous couplings)*
  - *Drell-Yan (luminosity LHC)*
  - *Bhabha scattering (luminosity ILC)*

How to compute?

- Many scales: $M_Z\,,\ M_W\,,\ M_H\,,\ m_t\,,\ \lambda,\ m_f$

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Two types of large logs

Electroweak	*QED*
$\ln(s/M^2)$	$\ln(s/\lambda^2)$

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QED $\ln(s/\lambda^2)$

hard evolution equation

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Two types of large logs

Electroweak

$$\ln(s/M^2)$$

QED

$$\ln(s/\lambda^2)$$

hard evolution equation *infrared evolution equation*

Hard evolution

Form factor

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Reduced amplitude

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}$$

Amplitude decomposition ⇨ $\mathcal{A} = -\frac{ig^2(Q^2)}{Q^2}\mathcal{F}^2\tilde{\mathcal{A}}$

Hard evolution

$\boxed{\text{Solution}}$

$$\mathcal{F} \quad = \quad F_0(\alpha(M^2))\,\exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

$$\tilde{\mathcal{A}} \quad = \quad \tilde{\mathcal{A}}_0(\alpha(M^2))\mathrm{Pexp}\left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\chi(\alpha(x))\right]$$

Anomalous dimensions ⇨ $\gamma, \quad \zeta, \quad \chi$ *massless approximation*

Initial conditions ⇨ $\xi, \quad F_0, \quad \mathcal{A}_0$ *expansion in* M^2/s

Two-loop corrections

$\boxed{\text{Two-loop log}^{4,3,2}}$

$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

Two-loop corrections

Two-loop $\log^{4,3,2}$

$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

Two-loop linear log

$\chi^{(2)}$ ⇨ *two-loop massless amplitudes*

$\zeta^{(2)} + \xi^{(2)}$ ⇨ *two-loop massive form factor*

$SU(2) \times U(1)$ model

Factorization of infrared singularities

$$\mathcal{A}(\mathrm{in}\,\overline{\mathrm{in}} \to \mathrm{out}\,\overline{\mathrm{out}}) = \exp\left[-\frac{\alpha_e}{4\pi}(Q_{in}^2 + Q_{out}^2)\ln^2\left(\frac{s}{\lambda^2}\right) + \ldots\right]\bar{\mathcal{A}}(M^2/s) + \mathcal{O}(\lambda/M)$$

$SU(2) \times U(1)$ **model**

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NNLL approximation ➪ no nontrivial dependence on λ/M

- *Compute in symmetric phase with* $\lambda = M$
- *Factorize* $\exp\left[-\frac{\alpha_e}{4\pi}(Q_{in}^2 + Q_{out}^2)\ln^2\left(\frac{s}{M^2}\right) + \dots\right]$

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N^{3}LL approximation ➪ $\xi^{(2)}(\lambda/M)$

Four-fermion processes

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

Massive $SU(2)$ *model,* $M_H = M$*, six left-handed massless doublets*

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Massive $SU(2)$ *model,* $M_H = M$*, six left-handed massless doublets*

$$\left[\frac{\delta\sigma}{\sigma}\right]_{\uparrow\downarrow} = \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(-\frac{799}{9} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right) \right.$$
$$\left. + \left(\frac{51613}{216} - \frac{815}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\ln\left(\frac{s}{M^2}\right)\right]$$

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\approx & \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[{\color{red}4.50}\ln^4\left(\frac{s}{M^2}\right) - {\color{blue}20.83}\ln^3\left(\frac{s}{M^2}\right) + {\color{green}32.95}\ln^2\left(\frac{s}{M^2}\right) - {\color{purple}227.25}\ln\left(\frac{s}{M^2}\right)\right]
\end{aligned}
$$

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$$\left. + \left(\frac{34441}{216} - \frac{1247}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\ln\left(\frac{s}{M^2}\right)\right]$$
$$\approx \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[{\color{red}4.50}\ln^4\left(\frac{s}{M^2}\right) - {\color{blue}74.83}\ln^3\left(\frac{s}{M^2}\right) + {\color{green}391.45}\ln^2\left(\frac{s}{M^2}\right) - {\color{purple}543.62}\ln\left(\frac{s}{M^2}\right)\right]$$

$SU(2) \times U(1)$ model

$\boxed{\text{Two-loop } \log^{4,3,2}}$

- *Symmetric phase calculation,* $M_W = M_Z$
- *Naïve factorization of QED logs*
- $M_W \neq M_Z$ *through one-loop result* ⇨ *5% effect*

J. H. Kuhn, A. A. Penin and V. A. Smirnov, Eur. Phys. J. C **17** (2000) 97

J. H. Kuhn, S. Moch, A. A. Penin and V. A. Smirnov, Nucl. Phys. B **616** (2001) 286

$SU(2) \times U(1)$ model

Two-loop linear log

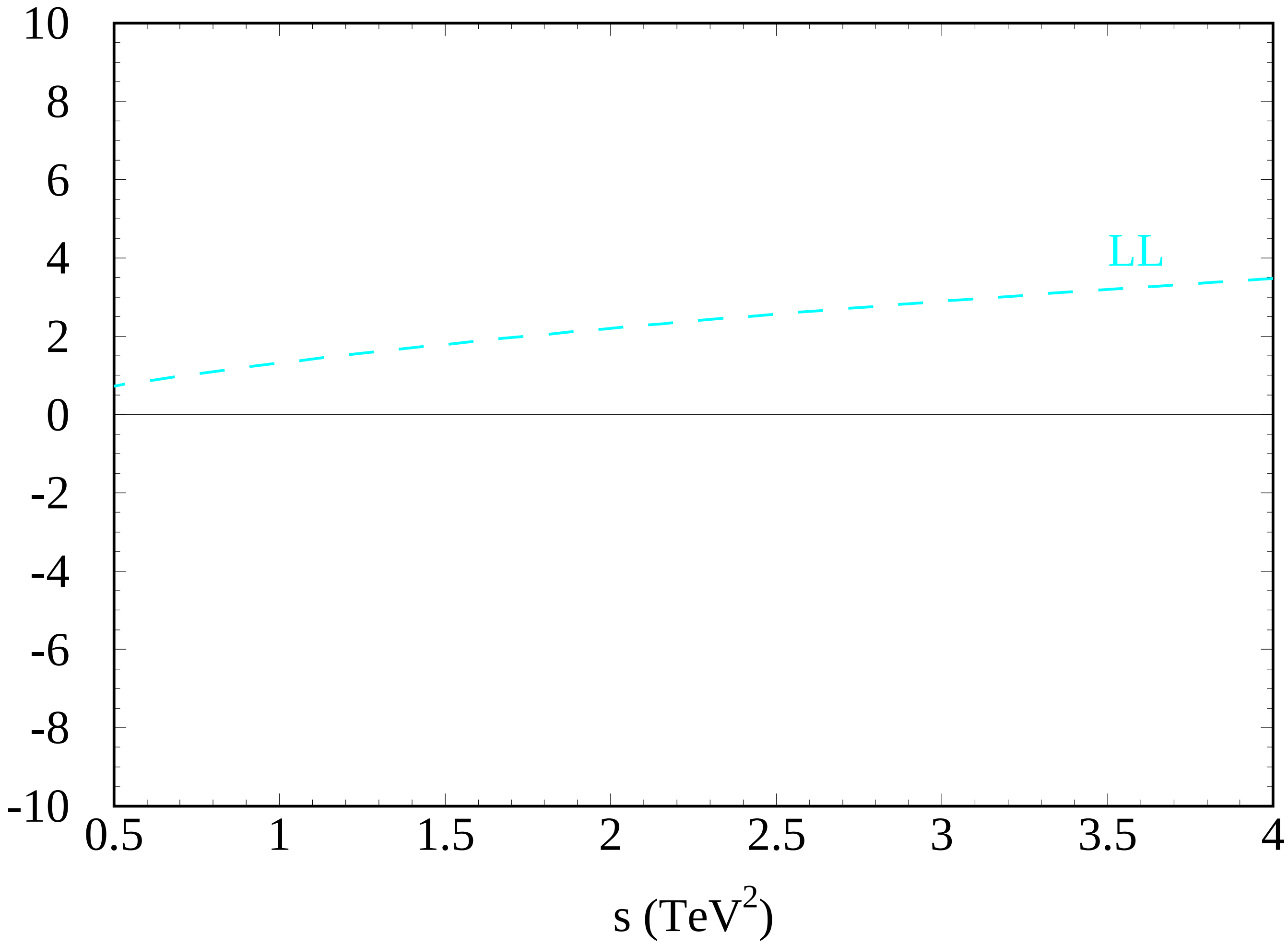
- *Approximation: Higgs boson of zero hypercharge,* $M_H = M_W = M_Z$
- *No mixing* $\xi^{(2)}_{\lambda=0} = \xi^{(2)}_{\lambda=M}$ ⇨ *naïve factorization of infrared logs*
- *Mixing effects are suppressed by* $\sin^2\theta_W$ ⇨ *20% error,* $M_H \neq M_W$ *effect is negligible*

B. Feucht, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Phys. Rev. Lett. **93** (2004) 101802

B. Jantzen, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Nucl. Phys. B **731** (2005) 188

A. A. Penin and G. Ryan, JHEP **1111** (2011) 081 (application to Bhabha scattering)

Two-loop corrections to $\sigma/\sigma_{\mathrm{Born}}(d\bar{d} \to l\bar{l})$



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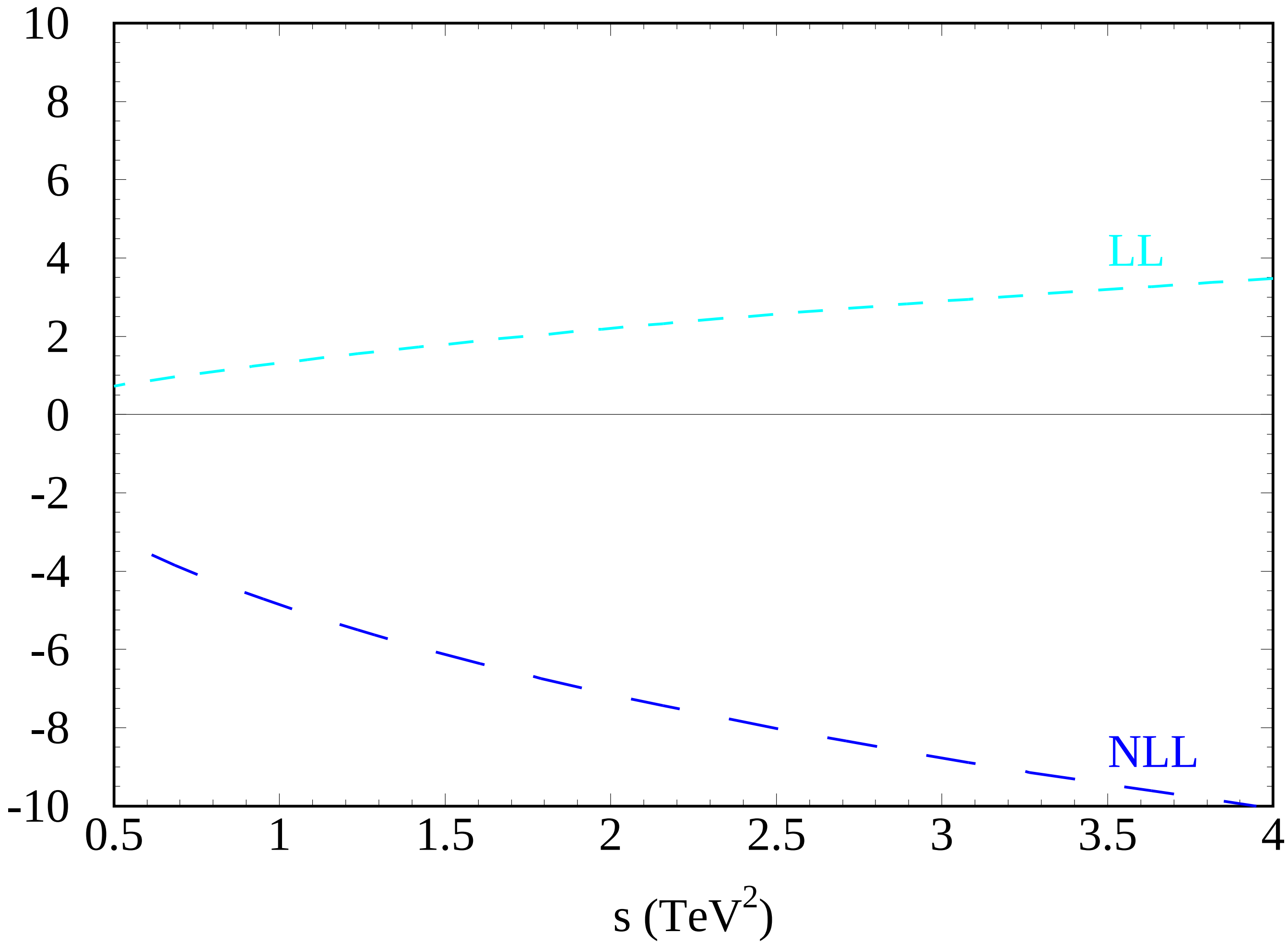

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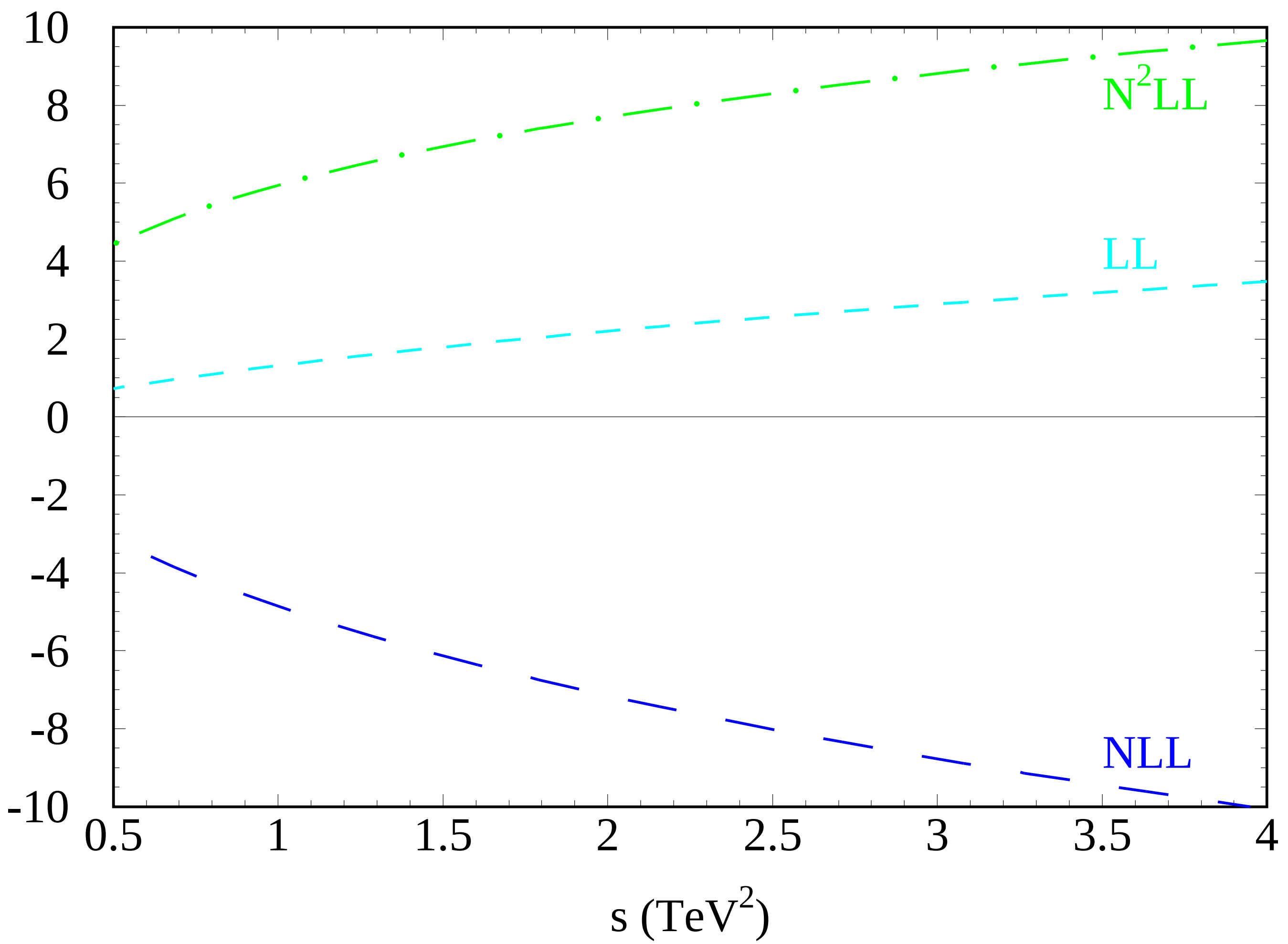

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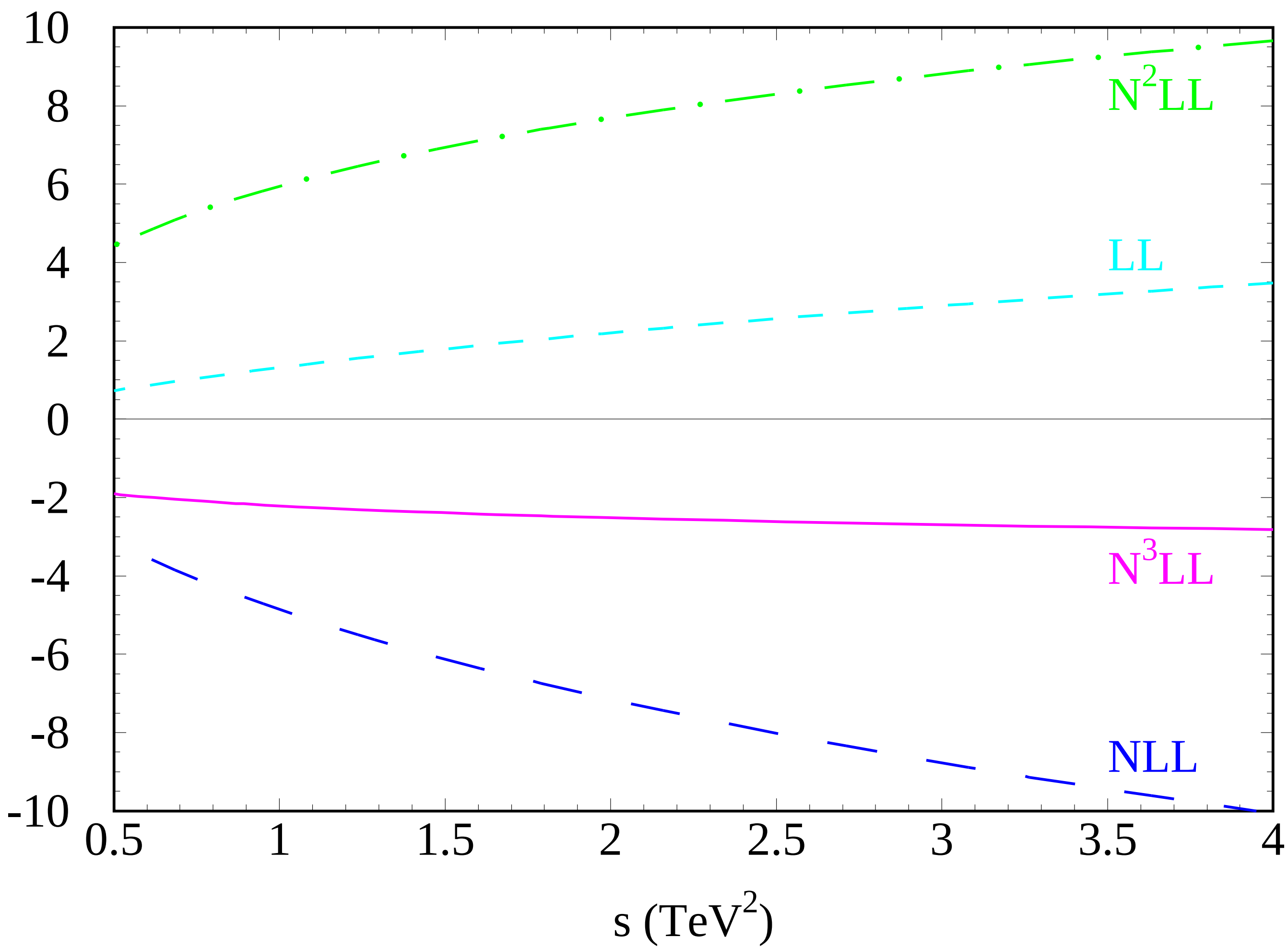

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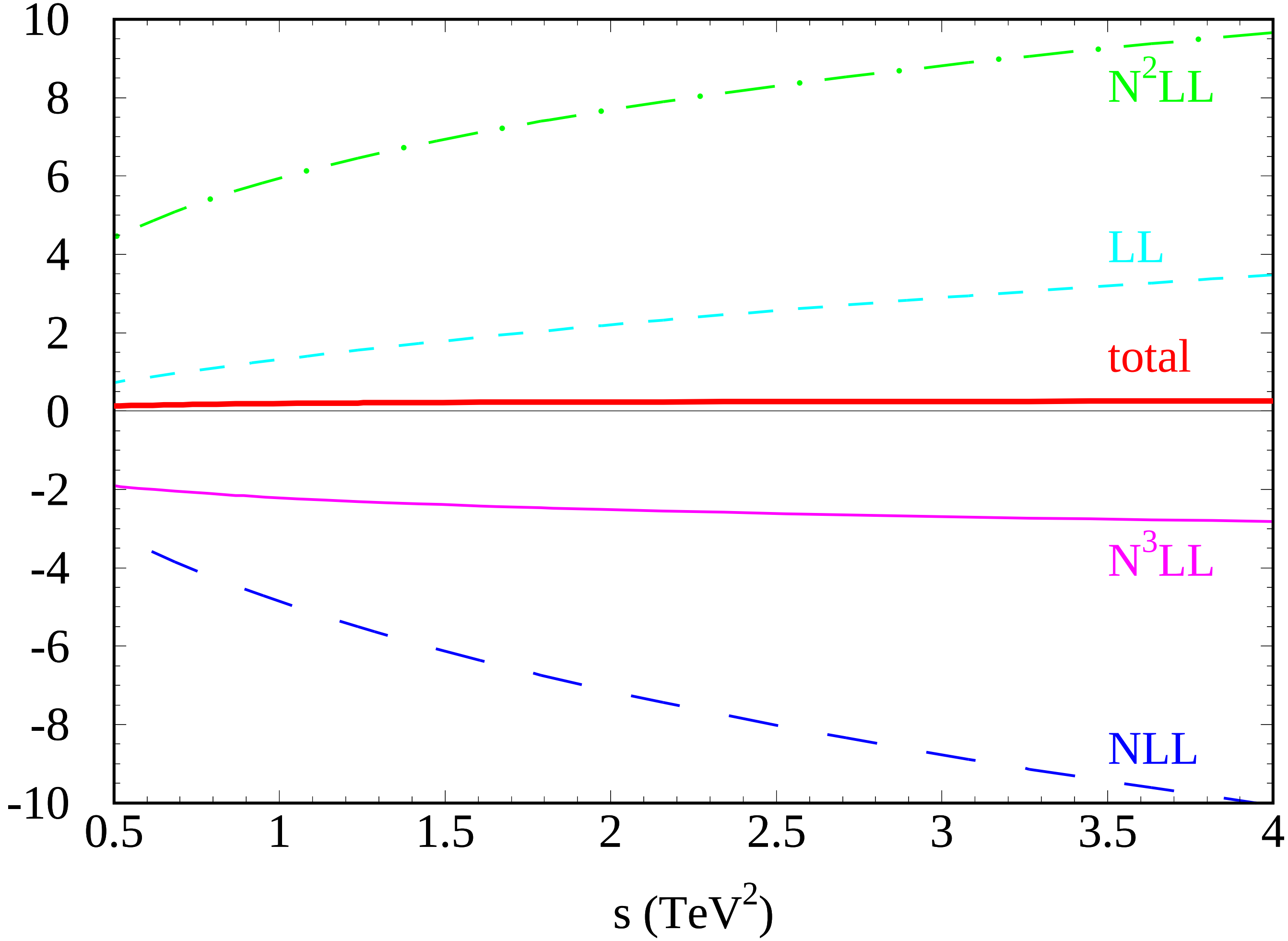


Gauge boson production

Gauge boson production

- Specific of longitudinal polarization
  - *equivalence theorem:* $\sigma(W_L^+W_L^-) \Rightarrow \sigma(\phi^+\phi^-)$
  - *Yukawa enhanced logs* $\propto (m_t^2/M_W^2)$

Results

- ILC
  - WW *to NNLL*

    J. H. Kuhn, F. Metzler and A. A. Penin, Nucl. Phys. B **795** (2008) 277

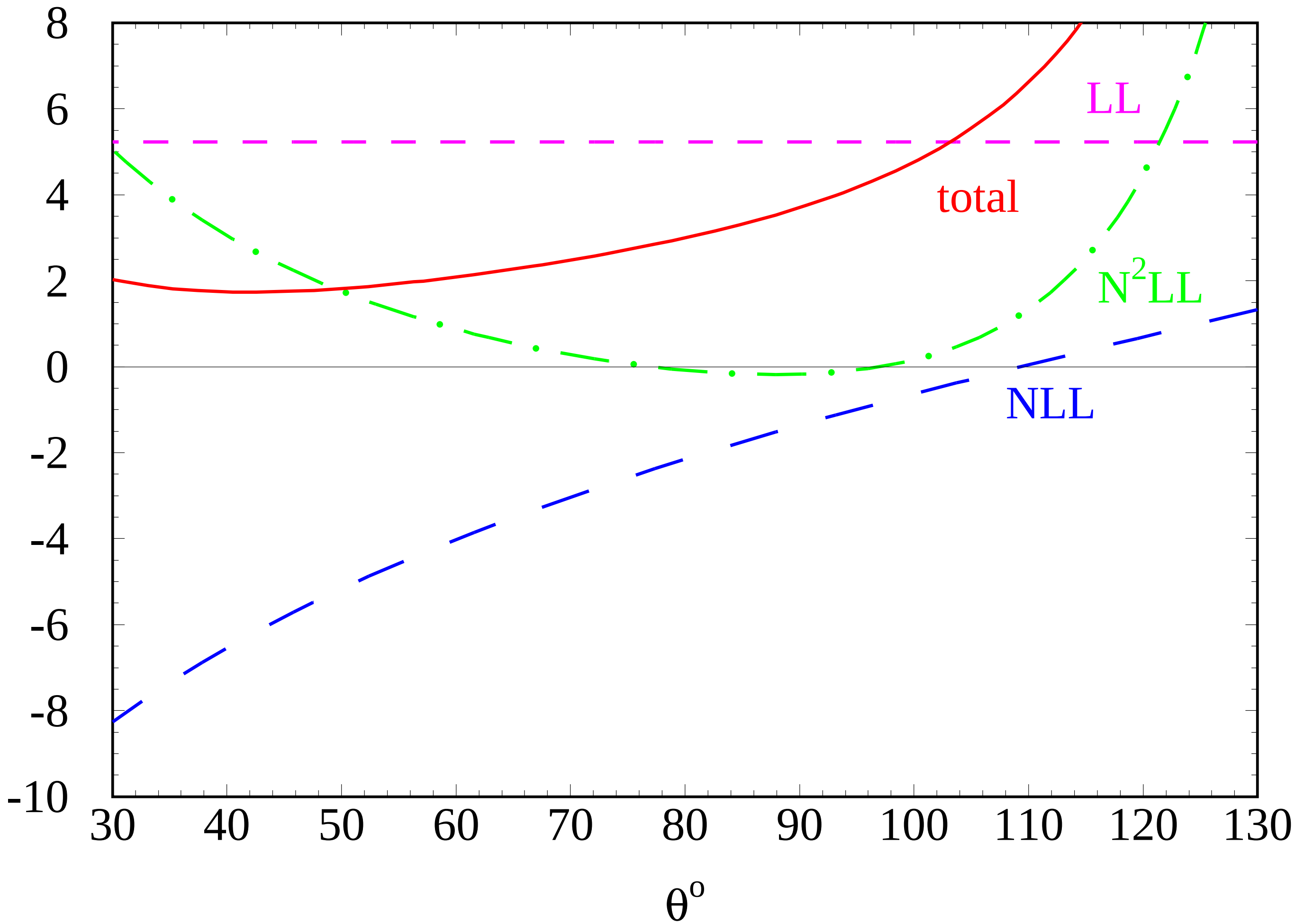
- LHC
  - Zg *to NLL*

    J. H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Nucl. Phys. B **727** (2005) 368
  - Wg *to NLL*

    J. H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Nucl. Phys. B **797** (2008) 27
  - WW *to NNLL*

    J. H. Kuhn, F. Metzler, A. A. Penin and S. Uccirati, JHEP **1106** (2011) 143

Two-loop corrections to $\mathrm{d}\sigma/\mathrm{d}\sigma_{\mathrm{Born}}(e^+e^- \to W_T^+W_T^-)$



$$s = 1\ \mathrm{TeV}^2$$

Application to LHC

❶ One-loop

- *full gauge boson mass dependence*
- *QED endpoint subtraction plus real radiaion*

A. Bierweiler, T. Kasprzik, J. H. Kuhn and S. Uccirati, JHEP **1211** (2012) 093

A. Bierweiler, T. Kasprzik and J. H. Kuhn, JHEP **1312** (2013) 071

❷ Two-loop

- *Sudakov*

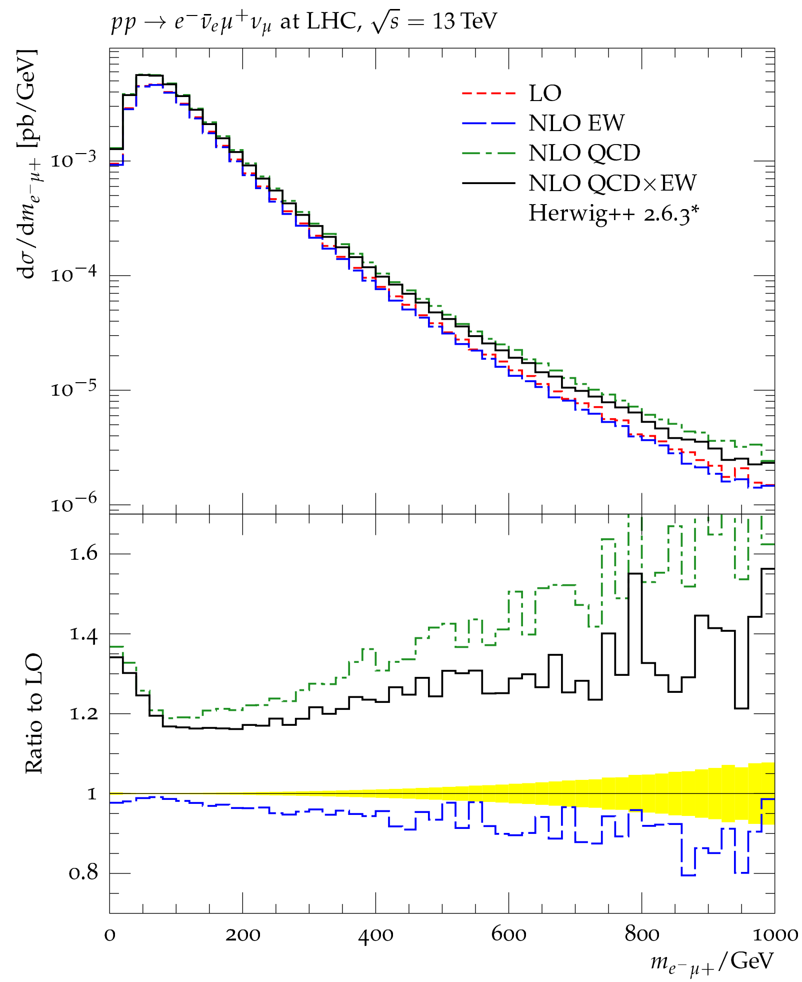
J. H. Kuhn, F. Metzler, A. A. Penin and S. Uccirati, JHEP **1106** (2011) 143

❸ Event generator implementation

- $\hat{s},\ \hat{t}$ *dependent K-factors*

S. Gieseke, T. Kasprzik and J. H. Kuhn, arXiv:1401.3964 [hep-ph]

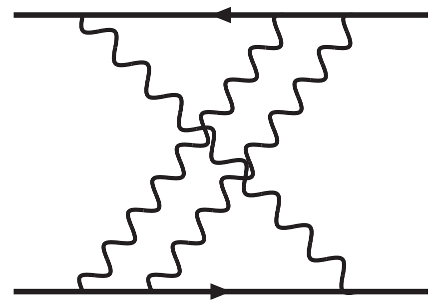
$pp \to W^+W^-$ with HERWIG++



Bhabha scattering

Photonic corrections

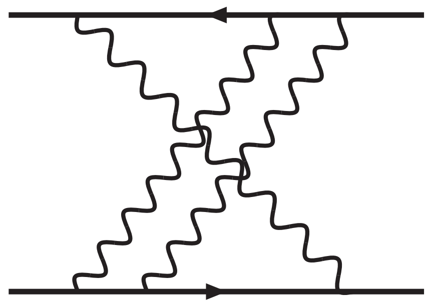
A. A. Penin, Phys. Rev. Lett. **95** (2005) 010408



2 loops + 4 legs + 3 scales $s,\ t,\ m_e^2$ ⇨ *no chance?*

Photonic corrections

A. A. Penin, Phys. Rev. Lett. **95** (2005) 010408



2 loops + 4 legs + 3 scales $s,\ t,\ m_e^2$ ➪ *no chance?*

Leading order in m_e^2/s ➪ *Infrared matching*

$$\boxed{m_e, \lambda \Rightarrow d = 4 - 2\epsilon}$$

Infrared matching

1. For a given amplitude $\mathcal{A}$ construct an auxilary amplitude $\bar{\mathcal{A}}$ with the same structure of IR singularities

2. Compute the matching term for $\lambda,\ m_e = 0$

$$\delta\mathcal{A} = \left[\mathcal{A}(\epsilon) - \bar{\mathcal{A}}(\epsilon)\right]_{\epsilon\to 0}$$

3. Compute the auxilary amplitude $\bar{\mathcal{A}}$ for $\lambda,\ m_e \to 0$

4. The amplitude $\mathcal{A}$ in the limit $\lambda,\ m_e \to 0$ is given by

$$\mathcal{A}(\lambda, m_e)\Big|_{\lambda,\ m_e\to 0} = \bar{\mathcal{A}}(\lambda, m_e)\Big|_{\lambda,\ m_e\to 0} + \delta\mathcal{A}$$

How to construct $\bar{\mathcal{A}}$?

- Factorization, exponentiation, nonrenormalization

↪ *the auxilary amplitude*

$$\boxed{\bar{\mathcal{A}}^{(2)} = \frac{1}{2}\left(\mathcal{A}^{(1)}\right)^2 + 2\left[\mathcal{F}^{(2)} - \frac{1}{2}\left(\mathcal{F}^{(1)}\right)^2\right]}$$

Result *(page 1 of 2)*

$$
\begin{aligned}
\delta_0^{(2)} = {} & 8\mathcal{L}_\varepsilon^2 + \left(1-x+x^2\right)^{-2} \Big[\left(\frac{4}{3} - \frac{8}{3}x - x^2 + \frac{10}{3}x^3 - \frac{8}{3}x^4\right)\pi^2 + \left(-12+16x-18x^2+6x^3\right)\ln(x) \\
& + \left(2x+2x^3\right)\ln(1-x) + \left(-3x+x^2+3x^3-4x^4\right)\ln^2(x) + \left(-8+16x-14x^2+4x^3\right)\ln(x) \\
& \times \ln(1-x) + \left(4-10x+14x^2-10x^3+4x^4\right)\ln^2(1-x) + \left(1-x+x^2\right)^2 \big(16 + 8\mathrm{Li}_2(x) \\
& -8\mathrm{Li}_2(1-x)\big)\Big]\mathcal{L}_\varepsilon + \frac{27}{2} - 2\pi^2\ln(2) + \left(1-x+x^2\right)^{-2} \Bigg(\left(\frac{83}{24} - \frac{125}{24}x + \frac{13}{4}x^2 + \frac{19}{24}x^3 - \frac{25}{24}x^4\right) \\
& \times\pi^2 + \left(-9 + \frac{43}{2}x - 34x^2 + 22x^3 - 9x^4\right)\zeta(3) + \left(-\frac{11}{90} - \frac{5}{24}x + \frac{29}{180}x^2 + \frac{23}{180}x^3 - \frac{49}{480}x^4\right)\pi^4 \\
& + \Big[-\frac{93}{8} + \frac{231}{16}x - \frac{279}{16}x^2 + \frac{93}{16}x^3 + \left(-\frac{3}{2} + \frac{13}{4}x - \frac{7}{12}x^2 - \frac{11}{8}x^3\right)\pi^2 + \big(12 - 12x + 8x^2 \\
& -x^3\big)\zeta(3)\Big]\ln(x) + \left[\frac{9}{2} - \frac{43}{8}x + \frac{17}{8}x^2 + \frac{29}{8}x^3 - \frac{9}{2}x^4 + \left(\frac{x}{4} + \frac{x^2}{2} + \frac{5}{24}x^3 + \frac{19}{48}x^4\right)\pi^2\right]\ln^2(x) \\
& + \left(\frac{67}{24}x - \frac{5}{4}x^2 - \frac{2}{3}x^3\right)\ln^3(x) + \left(\frac{7}{48}x + \frac{5}{96}x^2 - \frac{x^3}{12} + \frac{43}{96}x^4\right)\ln^4(x) + \Big\{3x + 3x^3 + \Big(\frac{7}{6}x \\
& -\frac{73}{24}x^2 + \frac{15}{8}x^3\Big)\pi^2 + \left(-6+6x-x^2-4x^3\right)\zeta(3) + \Big[-8 + \frac{21}{2}x - \frac{45}{4}x^2 + x^4 + \Big(1 - \frac{x}{6} + \frac{x^2}{12} \\
& -\frac{x^3}{3} - \frac{x^4}{8}\Big)\pi^2\Big]\ln(x) + \left(6 - 11x + \frac{35}{4}x^2 - \frac{15}{8}x^3\right)\ln^2(x) + \left(\frac{2}{3} + \frac{x}{12} - \frac{x^3}{3} + \frac{5}{24}x^4\right)\ln^3(x)\Big\} \\
& \times\ln(1-x) + \Big[\frac{7}{2} - 6x + \frac{45}{4}x^2 - 6x^3 + \frac{7}{2}x^4 + \left(-\frac{17}{24} + \frac{7}{6}x - \frac{25}{24}x^2 - \frac{13}{48}x^4\right)\pi^2 + \Big(-3 + \frac{23}{4}x \\
& -\frac{23}{4}x^2 + \frac{9}{8}x^3\Big)\ln(x) + \left(\frac{7}{2} - \frac{41}{8}x + \frac{31}{8}x^2 + \frac{3}{8}x^3 - \frac{13}{16}x^4\right)\ln^2(x)\Big]\ln^2(1-x) + \Big[\frac{3}{8}x + \frac{1}{6}x^2 \\
& +\frac{3}{8}x^3 + \left(-4 + \frac{29}{6}x - \frac{49}{12}x^2 + \frac{5}{6}x^3 + \frac{7}{8}x^4\right)\ln(x)\Big]\ln^3(1-x) + \Big(\frac{1}{32} - \frac{3}{4}x + \frac{71}{48}x^2 - \frac{29}{24}x^3 \\
& +\frac{9}{32}x^4\Big)\ln^4(1-x) + \Big\{8 - 16x + 24x^2 - 16x^3 + 8x^4 + \left(\frac{7}{3} - 3x + \frac{3}{4}x^2 + \frac{5}{6}x^3 - \frac{2}{3}x^4\right)\pi^2
\end{aligned}
$$

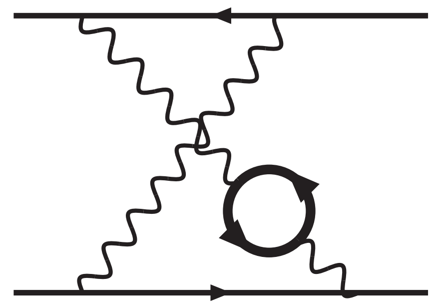
Result *(page 2 of 2)*

$$
\begin{aligned}
&+\left[-6+\frac{11}{2}x-4x^2+x^3+\left(2-\frac{11}{4}x+\frac{7}{4}x^2+\frac{x^3}{4}-x^4\right)\ln(x)\right]\ln(x)+\left[\frac{3}{2}x-\frac{x^2}{4}+x^3\right.\\
&+\left(-4+9x-\frac{15}{2}x^2+2x^3\right)\ln(x)+\left(-1-\frac{7}{2}x+\frac{25}{4}x^2-5x^3+2x^4\right)\ln(1-x)\Big]\ln(1-x)+\Big(2\\
&-4x+6x^2-4x^3+2x^4\Big)\,\mathrm{Li}_2(x)\Big\}\mathrm{Li}_2\,(x)+\Big\{-8+16x-24x^2+16x^3-8x^4+\Big[-\frac{2}{3}+\frac{4}{3}x\\
&+\frac{x^2}{2}-\frac{5}{3}x^3+\frac{2}{3}x^4\Big]\pi^2+\left[6-8x+9x^2-3x^3+\left(\frac{3}{2}x-\frac{x^2}{2}-\frac{3}{2}x^3+2x^4\right)\ln(x)\right]\ln(x)+\Big[-x\\
&-\frac{x^2}{4}-\frac{x^3}{2}+\left(10-14x+9x^2\right)\ln(x)+\left(-8+11x-\frac{31}{4}x^2+\frac{x^3}{2}+x^4\right)\ln(1-x)\Big]\ln(1-x)\\
&+\left(-4+8x-12x^2+8x^3-4x^4\right)\mathrm{Li}_2(x)+\left(2-4x+6x^2-4x^3+2x^4\right)\mathrm{Li}_2(1-x)\Big\}\mathrm{Li}_2\,(1-x)\\
&+\left[\frac{5}{2}x-5x^2+2x^3+\left(-4-x+x^2+2x^3-2x^4\right)\ln(x)+(6-6x+x^2+4x^3)\ln(1-x)\right]\mathrm{Li}_3\,(x)\\
&+\left[\frac{x}{2}-\frac{x^3}{2}+(-6+5x+3x^2-5x^3)\ln(x)+\left(6-10x+10x^3-6x^4\right)\ln(1-x)\right]\mathrm{Li}_3\,(1-x)\\
&+\left(-2+\frac{17}{2}x-\frac{17}{2}x^3+2x^4\right)\mathrm{Li}_4\,(x)+\left(7x-\frac{9}{2}x^2-4x^3+6x^4\right)\mathrm{Li}_4\,(1-x)+\Big(-6+4x\\
&+\frac{9}{2}x^2-7x^3\Big)\,\mathrm{Li}_4\left(-\frac{x}{1-x}\right)\Bigg),
\end{aligned}
$$

$$
\mathcal{L}_\varepsilon=[1-\ln\left(x/(1-x)\right)]\ln\left(\mathcal{E}_{cut}/\mathcal{E}\right).
$$

Heavy flavor corrections

R. Bonciani, A. Ferroglia and A. A. Penin, Phys. Rev. Lett. **100**, 131601 (2008)



2 loops + 4 legs + 4 scales $s,\ t,\ m_e^2,\ m_f^2$ ⇨ *no chance?*

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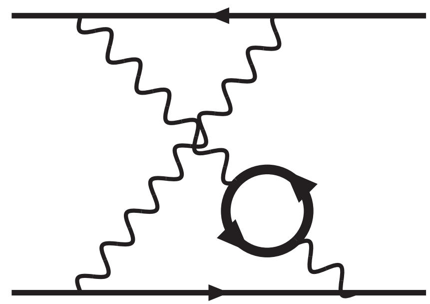

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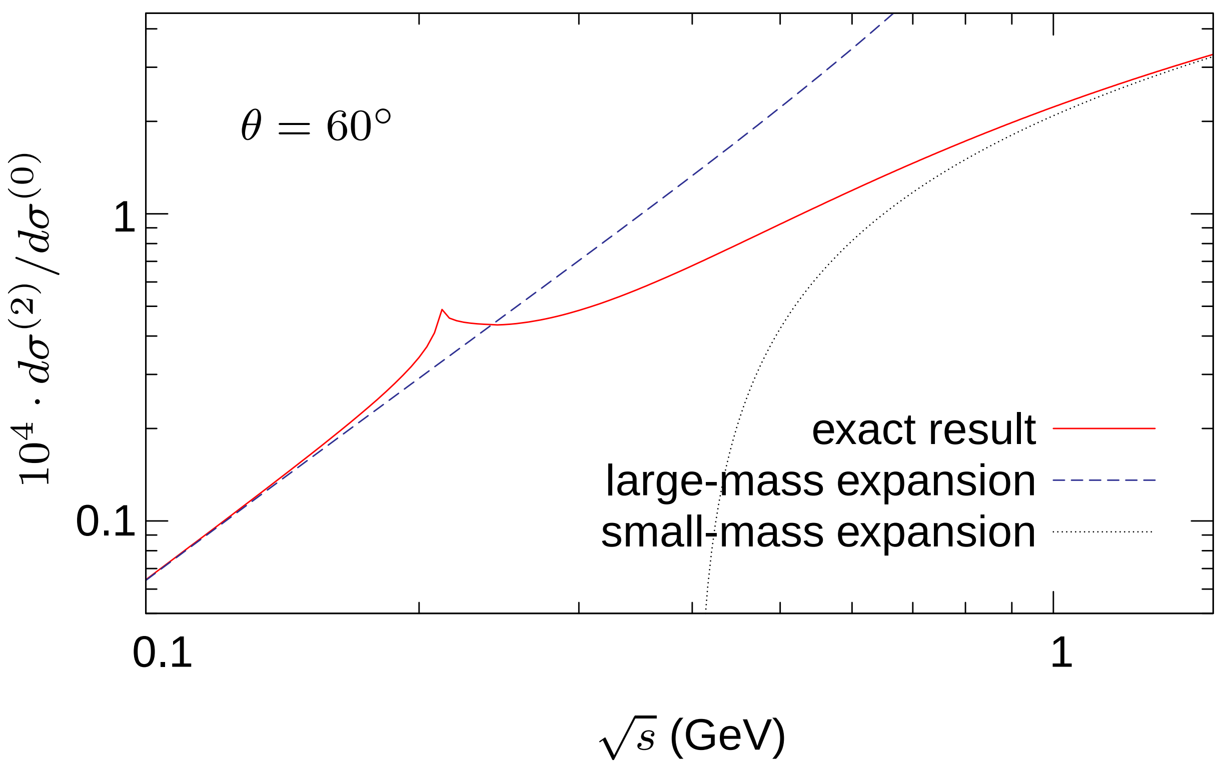
Factorization of collinear divergences:

the sum of 2PI diagrams is finite in the limit $m_e \to 0$

↪ Analytic result in terms of GPLs

Result



$\theta = 60^\circ$
$10^4 \cdot d\sigma^{(2)}/d\sigma^{(0)}$
1
0.1
exact result
large-mass expansion
small-mass expansion
0.1
1
$\sqrt{s}$ (GeV)

Summary

- Enhanced two-loop electroweak corrections for LHC/ILC
  - *effective method of calculations*
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- New insight in infrared structure of perturbative QED
  - ↪ *two-loop Bhabha scatering*