

ATLAS

Hadron
Calorimeters

Forward
Calorimeters

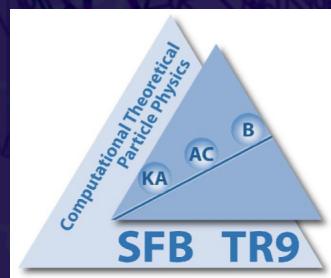
S.C. Solenoid

S.C. Air Core
Toroids

New ideas on parton showers and their implementation

M. Czakon

RWTH Aachen University



Muon Shieldings

EM Calorimeters

Inner Detector

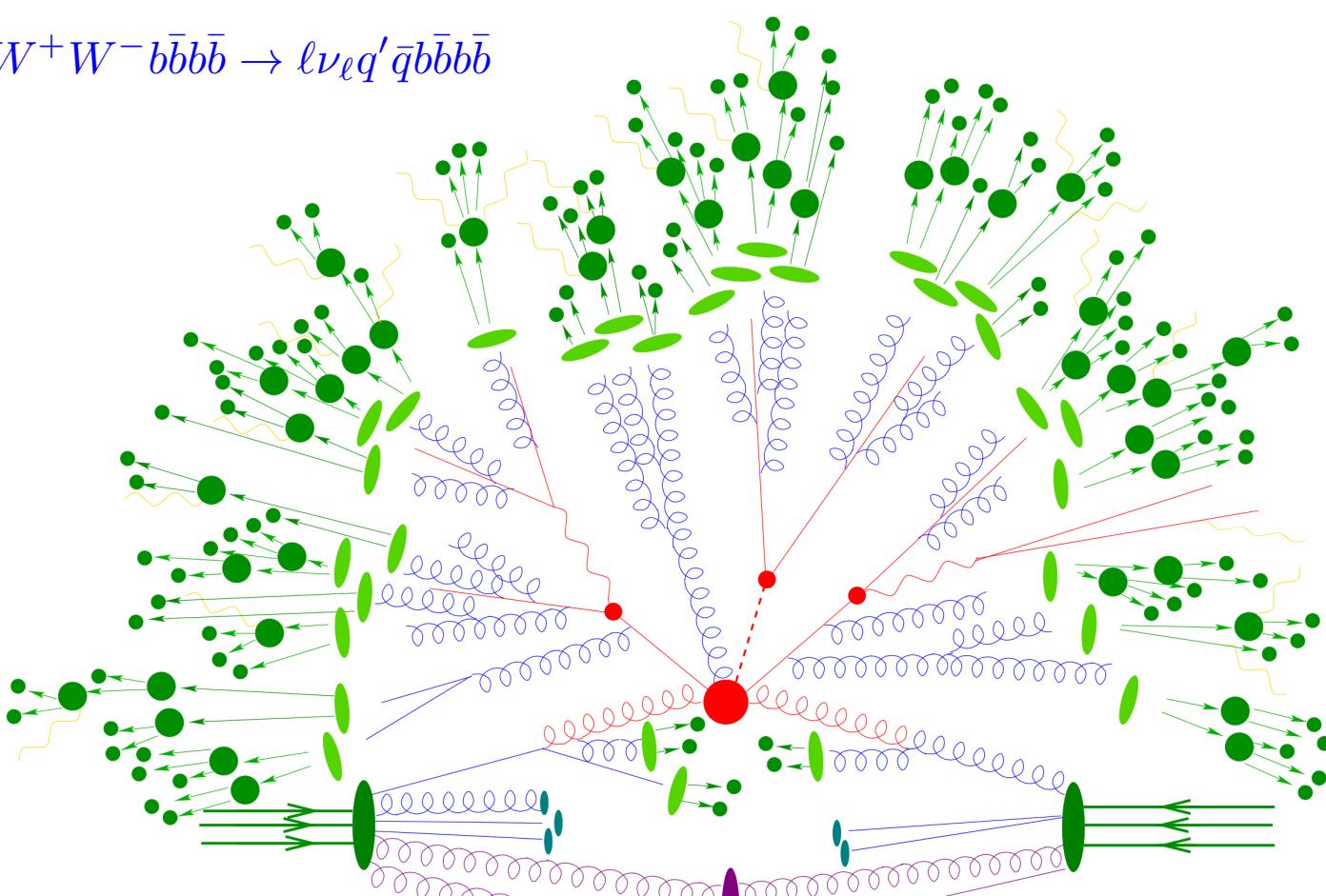
Heisenberg-
Programm

Deutsche
Forschungsgemeinschaft

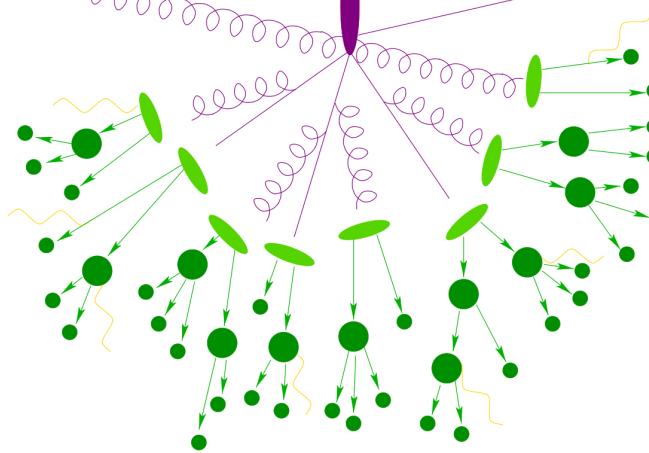


- PIs
 - 1. MC
 - 2. Michael Krämer
- Postdocs within the SFB
 - 1. Giuseppe Bevilacqua
 - 2. Michael Kubocz
 - 3. Heribertus Hartanto
- External Phd Student
 - 1. Manfred Kraus (DFG Grant M. Worek)
- Main external collaborator
 - 1. Małgorzata Worek

$pp \rightarrow t\bar{t}H \rightarrow W^+W^-b\bar{b}b\bar{b} \rightarrow \ell\nu_\ell q'\bar{q}b\bar{b}b\bar{b}$



*Hard interaction
Parton showers
Underlying Events
Hadronization
Decay of hadron
QED Bremsstrahlung*

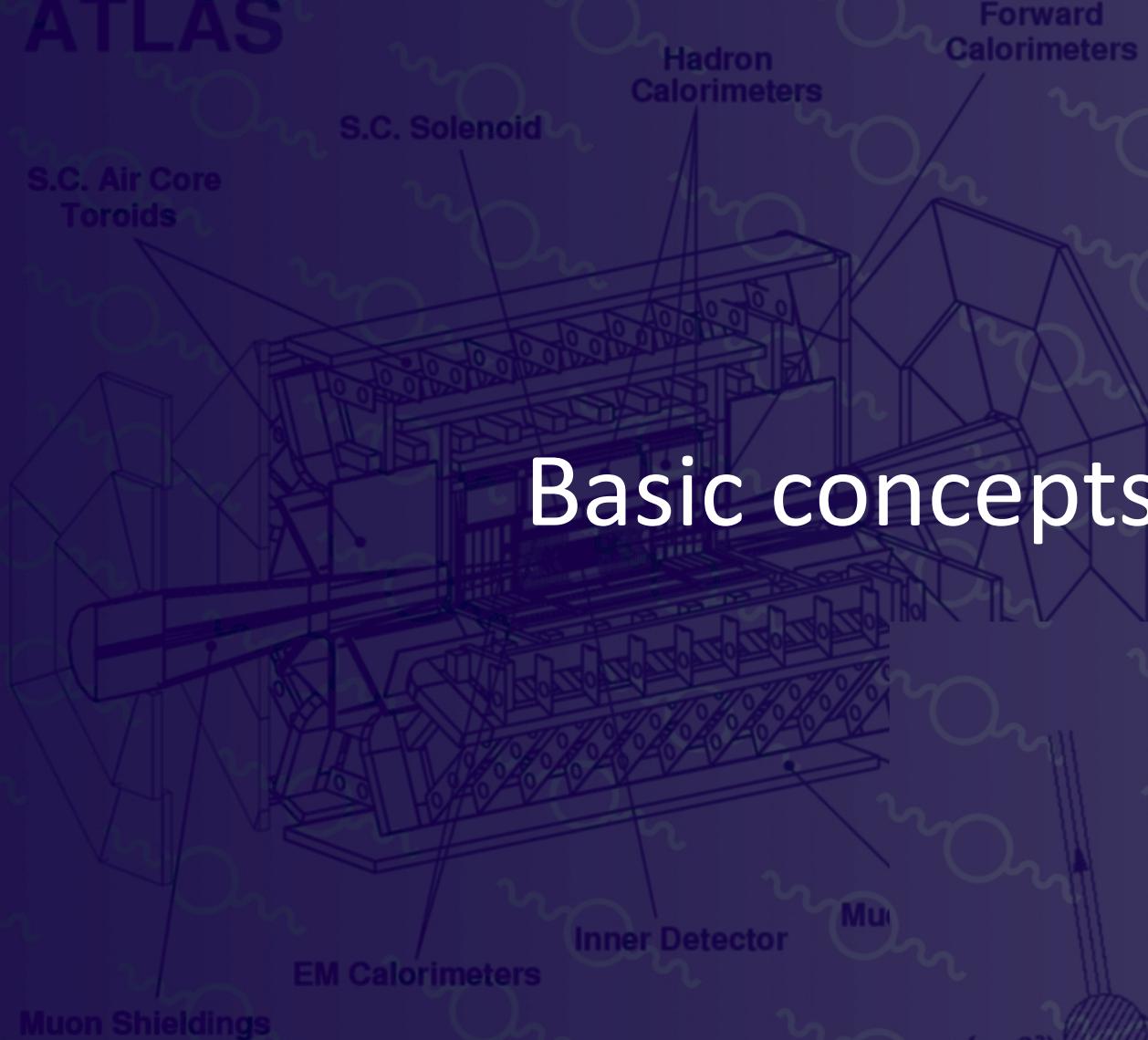


SHERPA

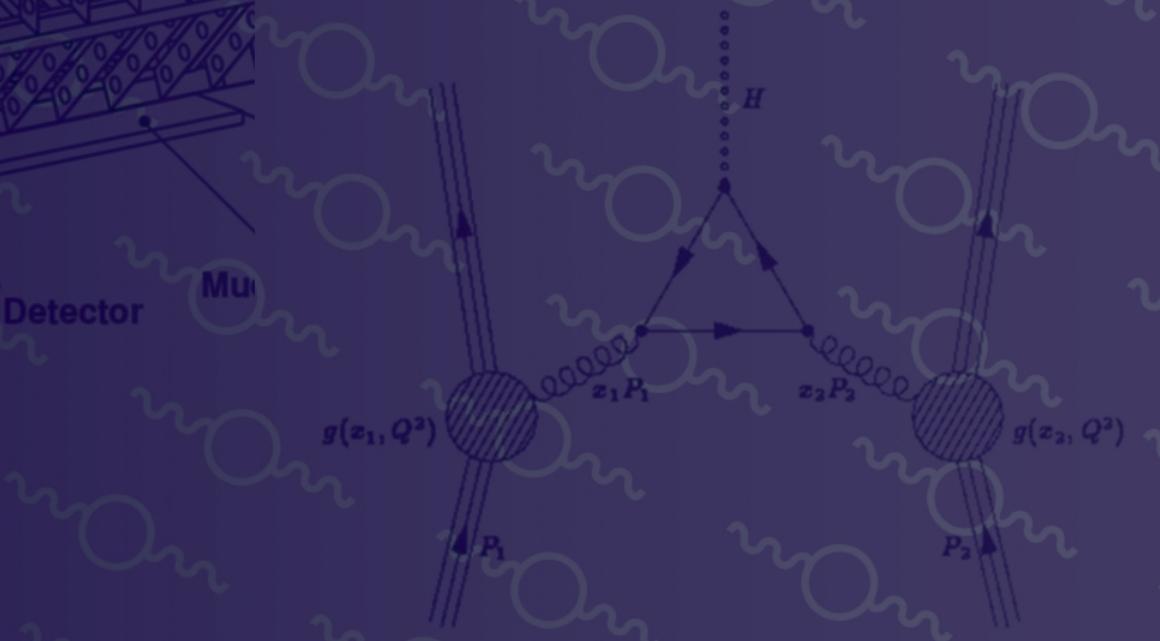
Nagy-Soper parton shower

- Idea introduced in Nagy, Soper '07 "Parton showers with quantum interference"
+ several subsequent papers
- Publicly available software DEDUCTOR Nagy, Soper '14
- Our project
 - ✓ Understand ideas behind the new concept
 - ✓ Use it to define a subtraction scheme
 - ✓ Implement the subtraction scheme in HELAC-DIPOLES
 - ✓ Devise a matching procedure at NLO
 - ✓ Implement the matching procedure in HELAC-NLO
 - ❑ Perform phenomenological studies
- Topics
 1. Basic concepts
 2. Subtraction scheme
 3. Matching
 4. Results for $t\bar{t} + \text{jet}$

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Basic concepts



- Cross section for a given measurement function

$$\begin{aligned}\sigma[F] &= \sum_m \frac{1}{m!} \int [d\{p, f\}_m] \langle M(\{p, f\}_m) | F(\{p, f\}_m) | M(\{p, f\}_m) \rangle \frac{f_a(\eta_a, \mu_F^2) f_b(\eta_b, \mu_F^2)}{4n_c(a)n_c(b) \times \text{flux}} \\ &= \sum_m \frac{1}{m!} \int [d\{p, f\}_m] Tr[\rho(\{p, f\}_m) F(\{p, f\}_m)]\end{aligned}$$

- Density matrix for fixed multiplicity

$$\rho(\{p, f\}_m) = |M(\{p, f\}_m)\rangle \langle M(\{p, f\}_m)| = \sum_{s,c} \sum_{s',c'} |\{s, c\}_m\rangle \rho(\{p, f, s', c', s, c\}_m) \langle \{s', c'\}_m|$$

- Explicit expression for cross section

$$\sigma[F] = (F|\rho) = \sum_m \frac{1}{m!} \int [d\{p, f, s', c', s, c\}_m] F(\{p, f\}_m) \langle s'_m | s_m \rangle \langle c'_m | c_m \rangle \rho(\{p, f, s', c', s, c\}_m)$$

- Defined in terms of a time variable

$$\sigma[F] = (F|\rho(t)) = (F|U(t, t_0)|\rho(t_0))$$

- Unitary

$$\sigma_T = (1|\rho(t)) = (1|\rho(t_0)) = (1|\rho)$$

- Evolution equation

$$\frac{dU(t, t_0)}{dt} = [H_I(t) - V(t)] U(t, t_0)$$

real radiation virtual corrections

• Explicit solution

$$V(t) = V_E(t) + V_S(t)$$

$$U(t, t_0) = N(t, t_0) + \int_{t_0}^t d\tau U(t, \tau) [H_I(\tau) - V_S(\tau)] N(\tau, t_0)$$

$$N(t, t_0) = T \exp \left(- \int_{t_0}^t d\tau V_E(\tau) \right) \quad \text{Sudakov form factor}$$

- Real splittings derived from a soft-collinear approximation to matrix element

$$\rho(\{\hat{p}, \hat{f}\}_{m+1}) \sim \sum_{l,k} t_l^\dagger(f_l \rightarrow \hat{f}_l + \hat{f}_{m+1}) V_l^\dagger(\{\hat{p}, \hat{f}\}_{m+1}) \rho(\{p, f\}_m) V_k(\{\hat{p}, \hat{f}\}_{m+1}) t_k(f_k \rightarrow \hat{f}_k + \hat{f}_{m+1})$$

$$|\rho_{m+1}\rangle = \sum_l \mathcal{S}_l |\rho_m\rangle$$

$$H_I(t) = \sum_l \mathcal{S}_l \delta\left(t - \log\left(\frac{\Lambda_l^2}{Q_0^2}\right)\right)$$

- Virtual corrections defined by unitarity (up to ambiguities)

$$0 = (1|H_I(t) - V(t)|U(t, t_0)|\rho)$$

$$(1|H_I(t) - V_S(t) = (1|V_E(t)$$

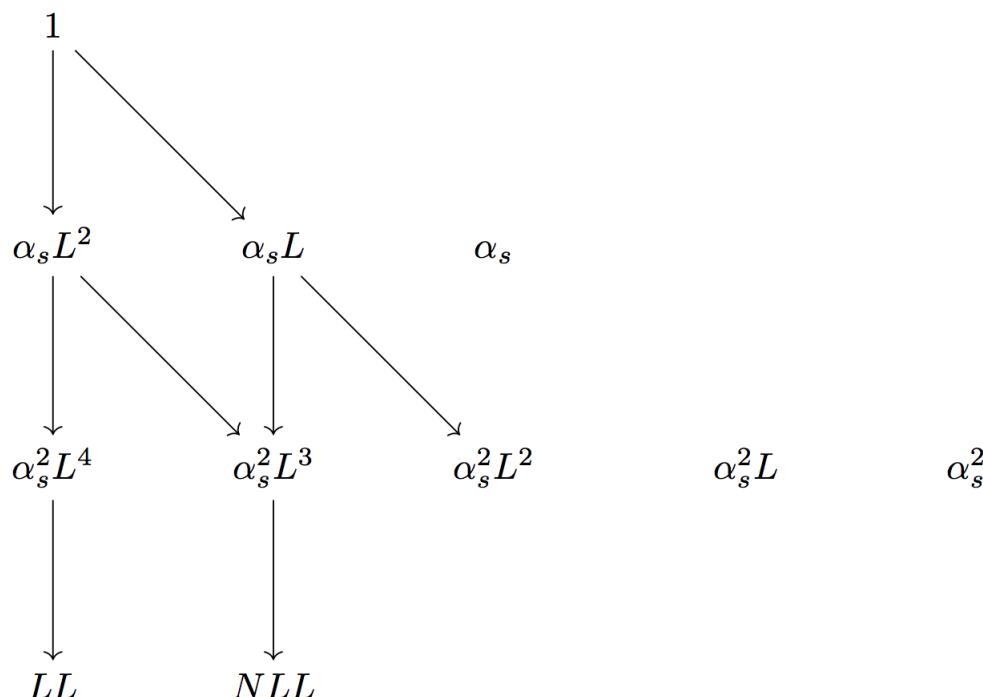
- Exponentiate color diagonal corrections, treat color off-diagonal corrections perturbatively (depends on the representation)

- Defined by virtuality

$$e^{-t} = \frac{\Lambda_l^2}{Q_0^2}$$

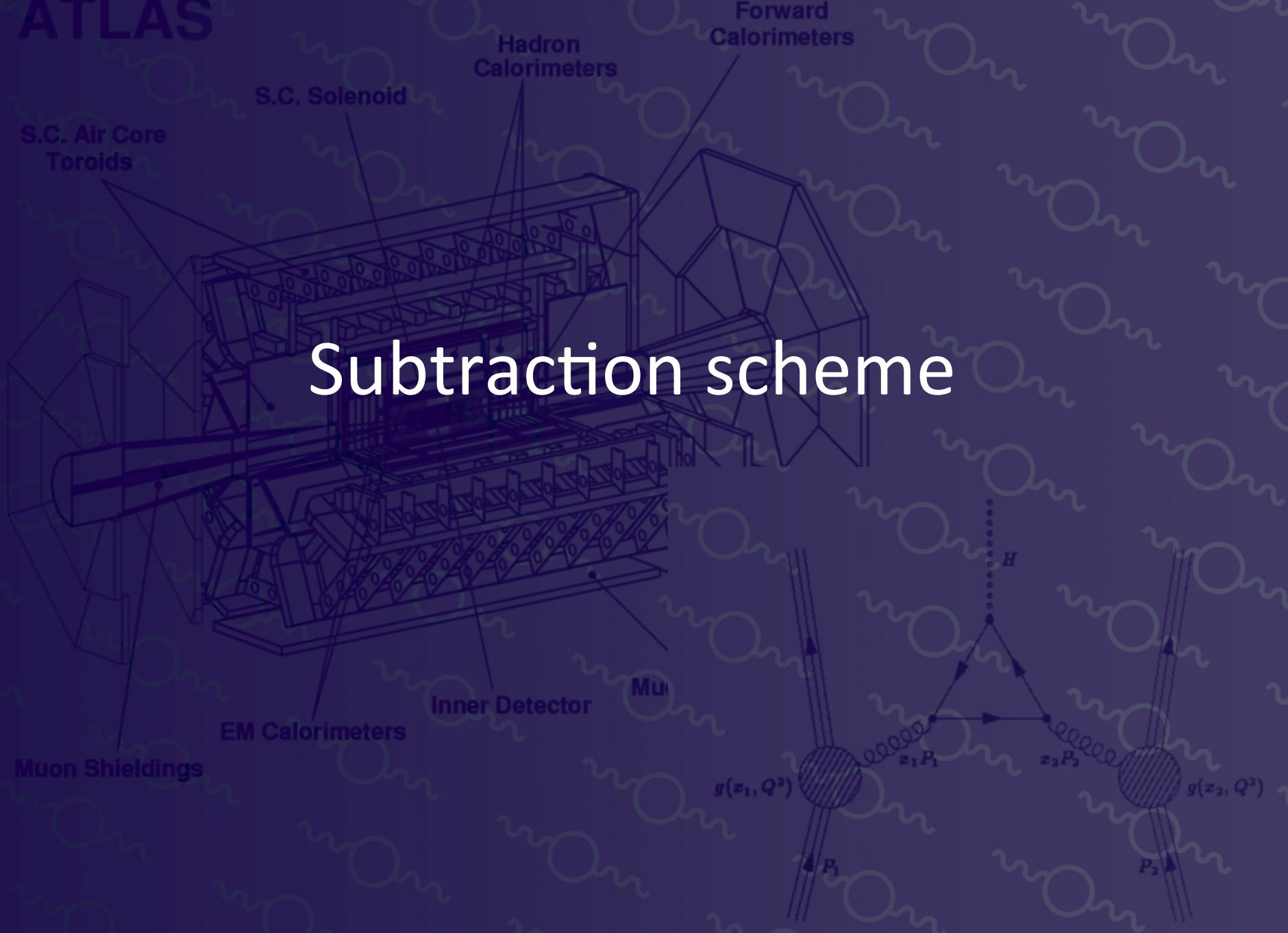
$$\Lambda_l^2 = \frac{|(\hat{p}_l \pm \hat{p}_{m+1})^2 - m^2(l)|}{2p_l \cdot Q_0} Q_0^2$$

- Logarithmic accuracy



1. Definition of time
 2. Kinematic mappings
 3. Splitting functions
 4. Treatment of color
 5. Evolution of PDFs
- Approximations
 1. LC+ (leading color + ...)
 2. Spin averaged

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Subtraction scheme

1. Use the mappings and soft/splitting functions from the shower
2. Determine integrated subtraction terms numerically
3. Implement in HELAC-DIPOLES (publicly available)
Bevilacqua, MC, Kubocz, Worek '13
4. Test while performing non-trivial phenomenological studies

Bevilacqua, MC, Krämer, Kubocz, Worek '13

mass effects in bBbB production at the LHC

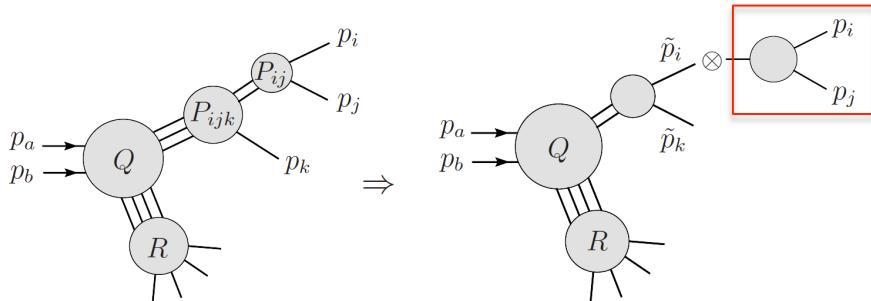
CS vs. NS

CATANI-SEYmour

$$\{p_i, p_j\} \rightarrow \tilde{p}_i ; \quad \{p_k, R, Q\} \rightarrow \{\tilde{p}_k, R, Q\}$$

$$p_i + p_j + p_k = \tilde{p}_i + \tilde{p}_k$$

- ✧ Easier dipole integration ✓
- ✧ Cubic growth of # of subtraction terms with multiplicity ✗

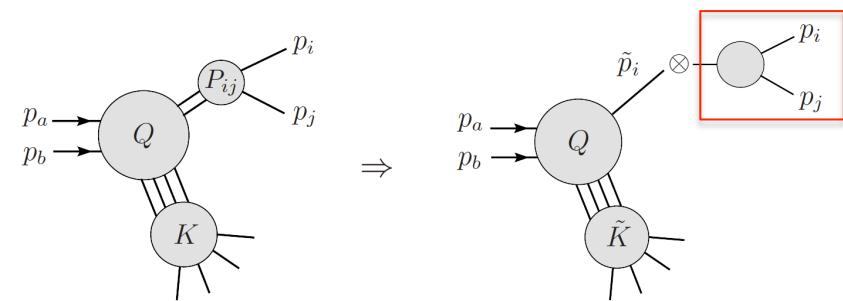


Nagy-Soper

$$\{p_i, p_j\} \rightarrow \tilde{p}_i ; \quad \{K, Q\} \rightarrow \{\tilde{K}, Q\}$$

$$p_i + p_j + K = \tilde{p}_i + \tilde{K}$$

- ✧ More complex dipole integration ✗
- ✧ Quadratic growth of # of subtraction terms with multiplicity ✓



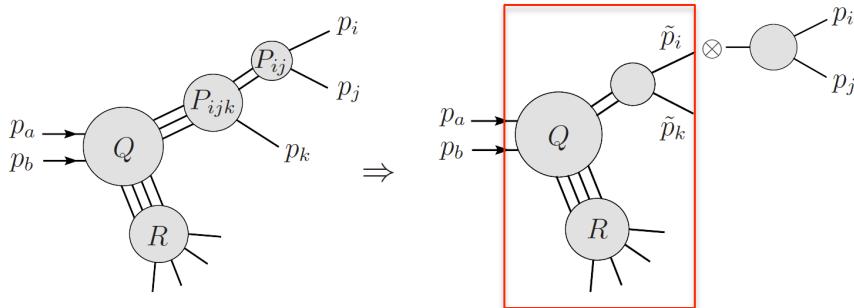
- ❑ Splitting functions have equal singular limits, but different non-singular parts
- ❑ Different number of mappings from $(m+1)$ to m -parton kinematics
- ❑ Different dipole phase space factorization and kinematics

CS vs. NS

CATANI-SEYmour

$$\begin{aligned} \{\mathbf{p}_i, \mathbf{p}_j\} &\rightarrow \tilde{\mathbf{p}}_i ; \quad \{\mathbf{p}_k, \mathbf{R}, \mathbf{Q}\} \rightarrow \{\tilde{\mathbf{p}}_k, \mathbf{R}, \mathbf{Q}\} \\ \mathbf{p}_i + \mathbf{p}_j + \mathbf{p}_k &= \tilde{\mathbf{p}}_i + \tilde{\mathbf{p}}_k \end{aligned}$$

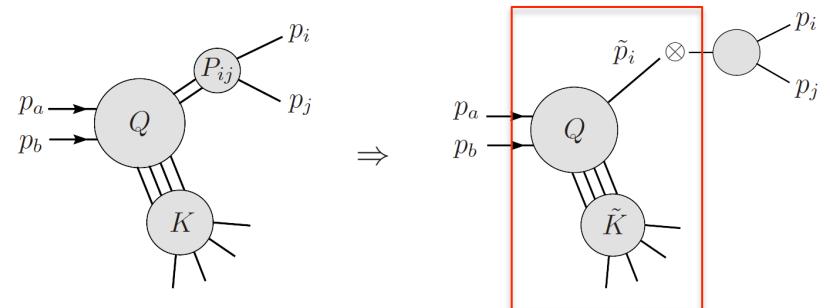
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NAGY-SOPER

$$\begin{aligned} \{\mathbf{p}_i, \mathbf{p}_j\} &\rightarrow \tilde{\mathbf{p}}_i ; \quad \{\mathbf{K}, \mathbf{Q}\} \rightarrow \{\tilde{\mathbf{K}}, \mathbf{Q}\} \\ \mathbf{p}_i + \mathbf{p}_j + \mathbf{K} &= \tilde{\mathbf{p}}_i + \tilde{\mathbf{K}} \end{aligned}$$

- ✧ More complex dipole integration ✗
- ✧ Quadratic growth of # of subtraction terms with multiplicity ✓



- ◻ Splitting functions have equal singular limits, but different finite parts
- ◻ Different number of mappings from $(m+1)$ to m -parton kinematics
- ◻ Different dipole phase space factorization and kinematics

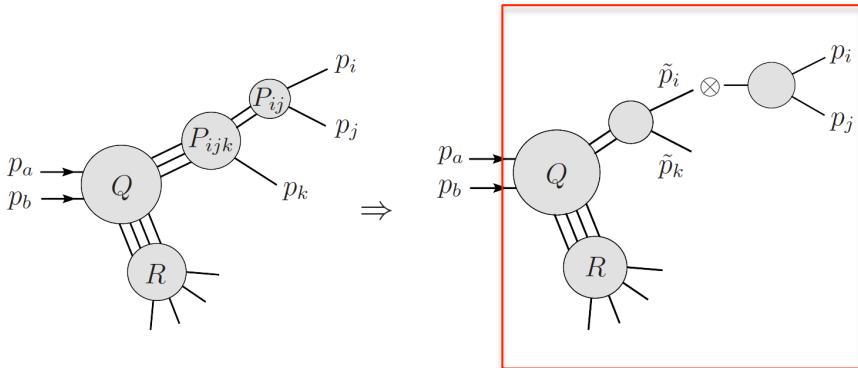
CS vs. NS

CATANI-SEYmour

$$\{\mathbf{p}_i, \mathbf{p}_j\} \rightarrow \tilde{\mathbf{p}}_i ; \quad \{\mathbf{p}_k, \mathbf{R}, \mathbf{Q}\} \rightarrow \{\tilde{\mathbf{p}}_k, \mathbf{R}, \mathbf{Q}\}$$

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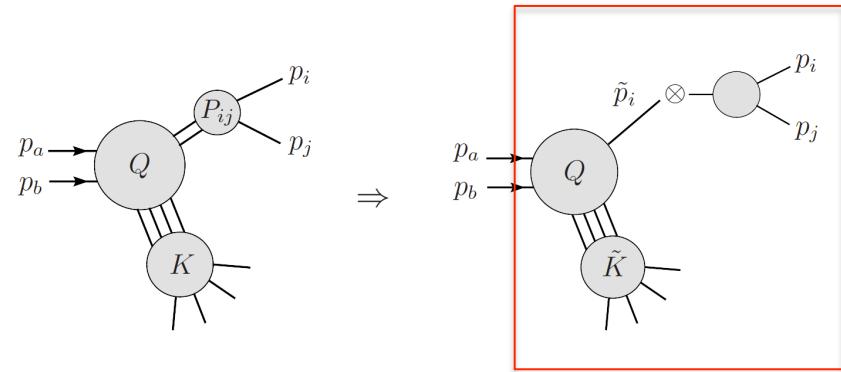


Nagy-Soper

$$\{\mathbf{p}_i, \mathbf{p}_j\} \rightarrow \tilde{\mathbf{p}}_i ; \quad \{\mathbf{K}, \mathbf{Q}\} \rightarrow \{\tilde{\mathbf{K}}, \mathbf{Q}\}$$

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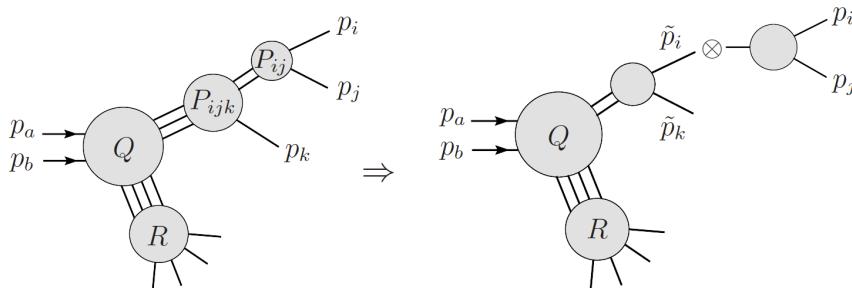
CS vs. NS

CATANI-SEYmour

$$\{\mathbf{p}_i, \mathbf{p}_j\} \rightarrow \tilde{\mathbf{p}}_i ; \quad \{\mathbf{p}_k, \mathbf{R}, \mathbf{Q}\} \rightarrow \{\tilde{\mathbf{p}}_k, \mathbf{R}, \mathbf{Q}\}$$

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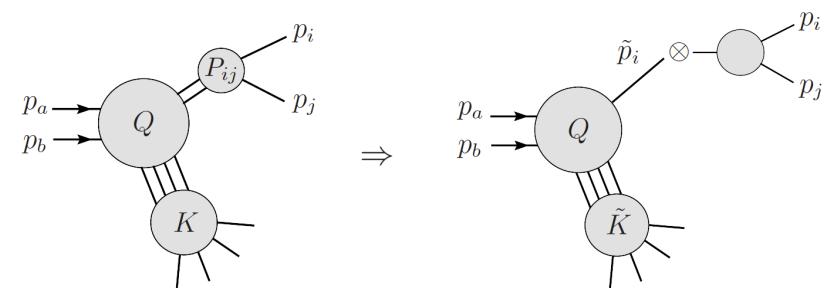


NAGY-SOPER

$$\{\mathbf{p}_i, \mathbf{p}_j\} \rightarrow \tilde{\mathbf{p}}_i ; \quad \{\mathbf{K}, \mathbf{Q}\} \rightarrow \{\tilde{\mathbf{K}}, \mathbf{Q}\}$$

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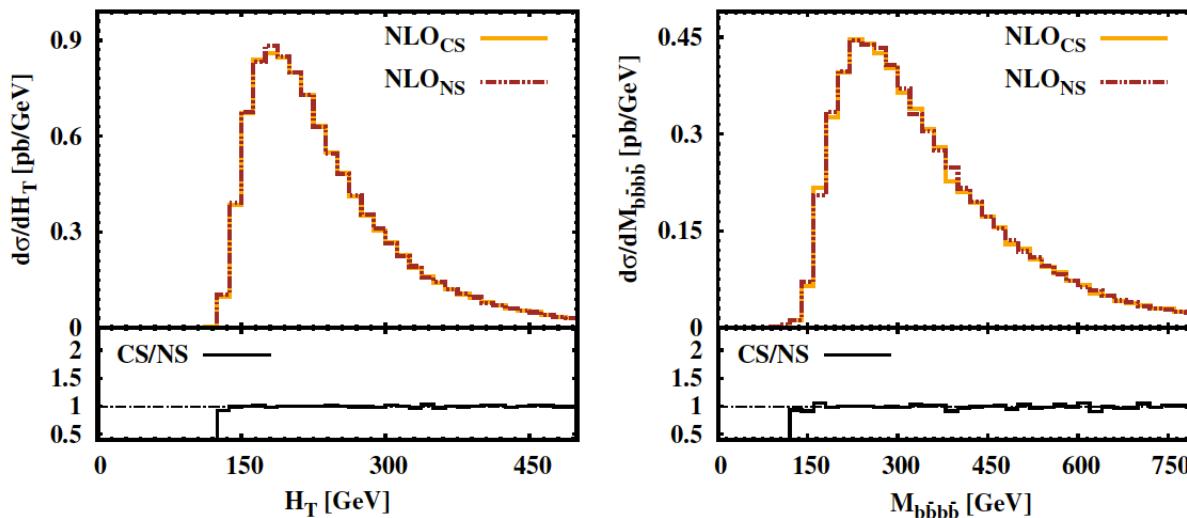


- Splitting functions have equal singular limits, but different finite parts
- Different number of mappings from $(m+1)$ to m -parton kinematics
- Different dipole phase space factorization and kinematics

CS vs. NS [5FS]

- Comparison between two schemes for the inclusive and differential cross sections

$pp \rightarrow b\bar{b}b\bar{b} + X$	$\sigma_{\text{NLO}}^{\text{CS } (\alpha_{\max}=1)} [\text{pb}]$	$\sigma_{\text{NLO}}^{\text{CS } (\alpha_{\max}=0.01)} [\text{pb}]$	$\sigma_{\text{NLO}}^{\text{NS}} [\text{pb}]$
CT10	123.6 ± 0.4	124.9 ± 0.9	124.8 ± 0.3
MSTW2008NLO	136.7 ± 0.3	136.1 ± 0.5	137.6 ± 0.5



AGREEMENT BETWEEN TWO SCHEMES
VALIDATION OF THE IMPLEMENTATION OF THE NS SCHEME

Comparison CS vs. NS

PROCESS	NR. OF DIPOLES	NR. OF SUBTRACTIONS	NR. OF FEYNMAN DIAGRAMS
	CATANI-SEYMOUR	NAGY-SOPER	
$gg \rightarrow t\bar{t}b\bar{b}g$	55	11	341
$gg \rightarrow t\bar{t}t\bar{t}g$	30	6	682
$gg \rightarrow b\bar{b}b\bar{b}g$	90	18	682
$gg \rightarrow t\bar{t}ggg$	75	15	1240

Number of CS and NS subtraction terms and Feynman diagrams

The CPU time needed to evaluate the subtracted real emission for one phase space point

Intel 3.40 GHz & Intel Fortran

Bevilacqua, Czakon, Kubocz, Worek (2013)

Comparison CS vs. NS

PROCESS	$\varepsilon_{\text{SR}}^{\text{CS}(\alpha_{\max}=0.01)} \text{ [pb]}$	$\varepsilon_{\text{SR}}^{\text{CS}(\alpha_{\max}=1)} \text{ [pb]}$	$\varepsilon_{\text{SR}}^{\text{NS}} \text{ [pb]}$
$gg \rightarrow t\bar{t}b\bar{b}g$	$4.405 \cdot 10^{-5}$	$4.108 \cdot 10^{-5}$	$5.424 \cdot 10^{-5}$
$gg \rightarrow t\bar{t}t\bar{t}g$	$1.356 \cdot 10^{-7}$	$2.298 \cdot 10^{-8}$	$2.377 \cdot 10^{-8}$
$gg \rightarrow b\bar{b}b\bar{b}g$	$1.271 \cdot 10^{-3}$	$1.494 \cdot 10^{-3}$	$2.027 \cdot 10^{-3}$
$gg \rightarrow t\bar{t}ggg$	$7.560 \cdot 10^{-3}$	$2.290 \cdot 10^{-3}$	$6.507 \cdot 10^{-3}$

Bevilacqua, Czakon, Kubocz, Worek (2013)

Absolute error for subtracted real emission cross sections for dominant partonic subprocesses contributing at $\mathcal{O}(\alpha_s^5)$

BOTH SCHEMES, WITH THEIR DIFFERENT MOMENTUM MAPPINGS AND SUBTRACTION TERMS, HAVE SIMILAR PERFORMANCE

Comparison CS vs. NS

FULL COLOR SUMMATION

PROCESS	$\sigma_{\text{RE}}^{\text{CS}(\alpha_{\max}=0.01)} \text{ [pb]}$	$\sigma_{\text{RE}}^{\text{CS}(\alpha_{\max}=1)} \text{ [pb]}$	$\sigma_{\text{RE}}^{\text{NS}} \text{ [pb]}$
$gg \rightarrow t\bar{t}b\bar{b}g$	$(28.43 \pm 0.13) \cdot 10^{-3}$	$(28.39 \pm 0.04) \cdot 10^{-3}$	$(28.59 \pm 0.06) \cdot 10^{-3}$
$gg \rightarrow t\bar{t}t\bar{t}g$	$(17.03 \pm 0.08) \cdot 10^{-5}$	$(16.98 \pm 0.02) \cdot 10^{-5}$	$(17.01 \pm 0.03) \cdot 10^{-5}$
$gg \rightarrow b\bar{b}b\bar{b}g$	$(65.71 \pm 0.30) \cdot 10^{-2}$	$(66.24 \pm 0.16) \cdot 10^{-2}$	$(66.06 \pm 0.22) \cdot 10^{-2}$
$gg \rightarrow t\bar{t}ggg$	$(87.91 \pm 0.17) \cdot 10^{-1}$	$(87.96 \pm 0.07) \cdot 10^{-1}$	$(88.16 \pm 0.08) \cdot 10^{-1}$

RANDOM COLOR SAMPLING

PROCESS	$\sigma_{\text{RE, COL}}^{\text{CS}(\alpha_{\max}=0.01)} \text{ [pb]}$	$\sigma_{\text{RE, COL}}^{\text{CS}(\alpha_{\max}=1)} \text{ [pb]}$	$\sigma_{\text{RE, COL}}^{\text{NS}} \text{ [pb]}$
$gg \rightarrow t\bar{t}b\bar{b}g$	$(28.91 \pm 0.32) \cdot 10^{-3}$	$(28.35 \pm 0.14) \cdot 10^{-3}$	$(28.77 \pm 0.14) \cdot 10^{-3}$
$gg \rightarrow t\bar{t}t\bar{t}g$	$(16.99 \pm 0.10) \cdot 10^{-5}$	$(17.00 \pm 0.03) \cdot 10^{-5}$	$(17.01 \pm 0.04) \cdot 10^{-5}$
$gg \rightarrow b\bar{b}b\bar{b}g$	$(67.01 \pm 0.64) \cdot 10^{-2}$	$(65.71 \pm 0.50) \cdot 10^{-2}$	$(67.00 \pm 0.66) \cdot 10^{-2}$
$gg \rightarrow t\bar{t}ggg$	$(88.05 \pm 0.45) \cdot 10^{-1}$	$(88.04 \pm 0.37) \cdot 10^{-1}$	$(87.76 \pm 0.31) \cdot 10^{-1}$

Bevilacqua, Czakon, Kubocz, Worek (2013)

Real emission cross sections for dominant partonic subprocesses contributing to the subtracted real emissions at $\mathcal{O}(\alpha_s^5)$

Comparison CS vs. NS

RANDOM HELICITY SAMPLING

PROCESS	$\sigma_{\text{RE}}^{\text{NS}} [\text{pb}]$	$\varepsilon_{\text{RE}}^{\text{NS}} [\%]$
$gg \rightarrow t\bar{t}b\bar{b}g$	$(28.59 \pm 0.06) \cdot 10^{-3}$	0.22
$gg \rightarrow t\bar{t}t\bar{t}g$	$(17.01 \pm 0.03) \cdot 10^{-5}$	0.19
$gg \rightarrow b\bar{b}b\bar{b}g$	$(66.06 \pm 0.22) \cdot 10^{-2}$	0.33
$gg \rightarrow t\bar{t}ggg$	$(88.16 \pm 0.08) \cdot 10^{-1}$	0.09

Real emission cross sections for dominant partonic subprocesses contributing to the subtracted real emissions at $\mathcal{O}(\alpha_s^5)$

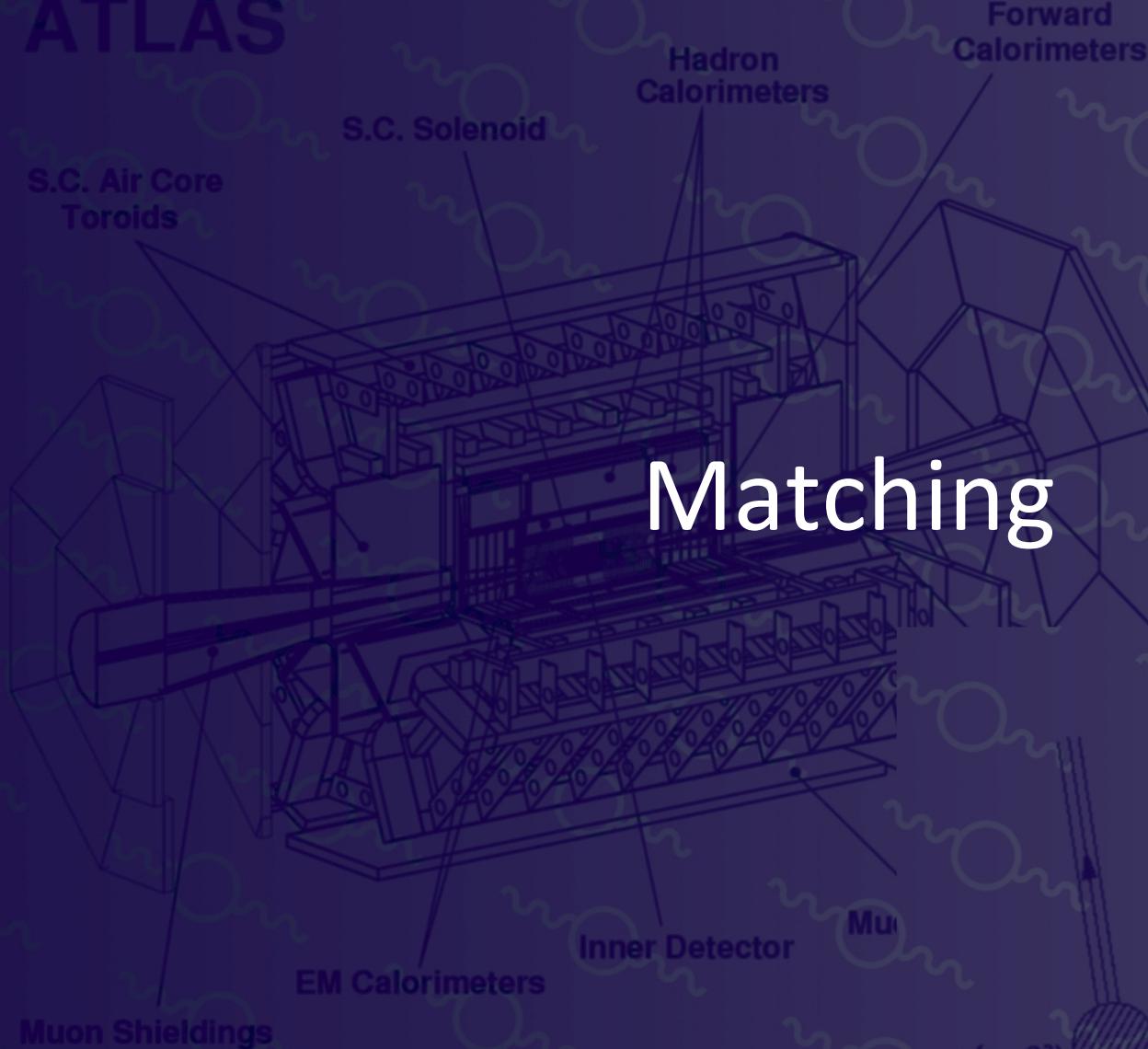
RANDOM POLARIZATION SAMPLING

PROCESS	$\sigma_{\text{RE, POL}}^{\text{NS}} [\text{pb}]$	$\varepsilon_{\text{RE, POL}}^{\text{NS}} [\%]$
$gg \rightarrow t\bar{t}b\bar{b}g$	$(28.50 \pm 0.06) \cdot 10^{-3}$	0.21
$gg \rightarrow t\bar{t}t\bar{t}g$	$(17.01 \pm 0.03) \cdot 10^{-5}$	0.19
$gg \rightarrow b\bar{b}b\bar{b}g$	$(66.23 \pm 0.20) \cdot 10^{-2}$	0.30
$gg \rightarrow t\bar{t}ggg$	$(88.16 \pm 0.07) \cdot 10^{-1}$	0.08

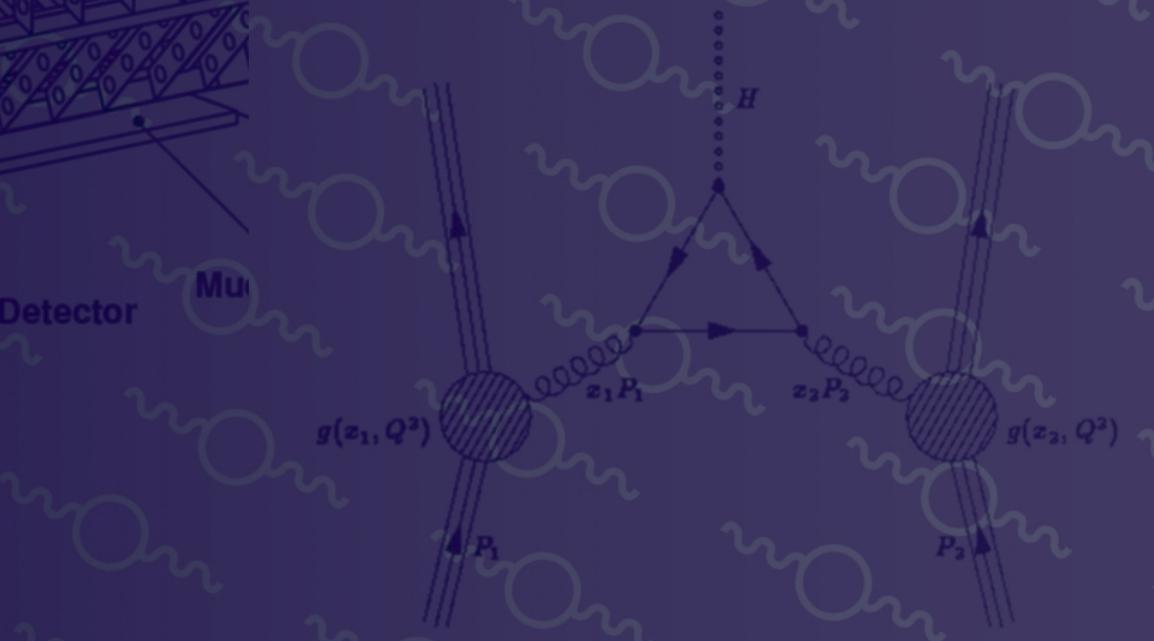
Results are shown for random helicity & polarization sampling

BOTH APPROACHES ARE SIMILAR IN EFFICIENCY

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Matching



MC, Hartanto, Kraus, Worek, in preparation

- NLO density matrix

$$|\rho\rangle \equiv \underbrace{|\rho_m^{(0)}\rangle}_{\text{Born, } \mathcal{O}(1)} + \underbrace{|\rho_m^{(1)}\rangle}_{\text{Virtual, } \mathcal{O}(\alpha_s)} + \underbrace{|\rho_{m+1}^{(0)}\rangle}_{\text{Real, } \mathcal{O}(\alpha_s)} + \mathcal{O}(\alpha_s^2)$$

- Expanded evolution operator

$$\begin{aligned} U(t_F, t_0) &= N(t_F, t_0) + \int_{t_0}^{t_F} d\tau U(t_F, \tau) [H_I(\tau) - V_S(\tau)] N(\tau, t_0) \\ &\approx \left[1 - \int_{t_0}^{t_F} d\tau V_E(\tau) \right] + \int_{t_0}^{t_F} d\tau [H_I(\tau) - V_S(\tau)] + \mathcal{O}(\alpha_s^2) \\ &= 1 + \int_{t_0}^{t_F} d\tau [H_I(\tau) - V(\tau)] + \mathcal{O}(\alpha_s^2), \end{aligned}$$

- NLO density matrix from the parton shower

$$|\rho(t_F)\rangle = U(t_F, t_0)|\rho\rangle \approx |\rho\rangle + \int_{t_0}^{t_F} d\tau [H_I(\tau) - V(\tau)] |\rho_m^{(0)}\rangle + \mathcal{O}(\alpha_s^2)$$

MC, Hartanto, Kraus, Worek, in preparation

- Starting density matrix without double counting

$$|\bar{\rho}\rangle \equiv |\rho\rangle - \int_{t_0}^{t_F} d\tau [H_I(\tau) - V(\tau)] |\rho_m^{(0)}\rangle + \mathcal{O}(\alpha_s^2).$$

- Modified cross section contributions with showering

$$\begin{aligned} \bar{\sigma}[F] = & \frac{1}{m!} \int [d\Phi_m] (F|U(t_F, t_0)|\Phi_m) \left[(\Phi_m|\rho_m^{(0)}) + (\Phi_m|\rho_m^{(1)}) + \int_{t_0}^{t_F} d\tau (\Phi_m|V(\tau)|\rho_m^{(0)}) \right] \\ & + \frac{1}{(m+1)!} \int [d\Phi_{m+1}] (F|U(t_F, t_0)|\Phi_{m+1}) \left[(\Phi_{m+1}|\rho_{m+1}^{(0)}) - \int_{t_0}^{t_F} d\tau (\Phi_{m+1}|H_I(\tau)|\rho_m^{(0)}) \right] \end{aligned}$$

take to infinity



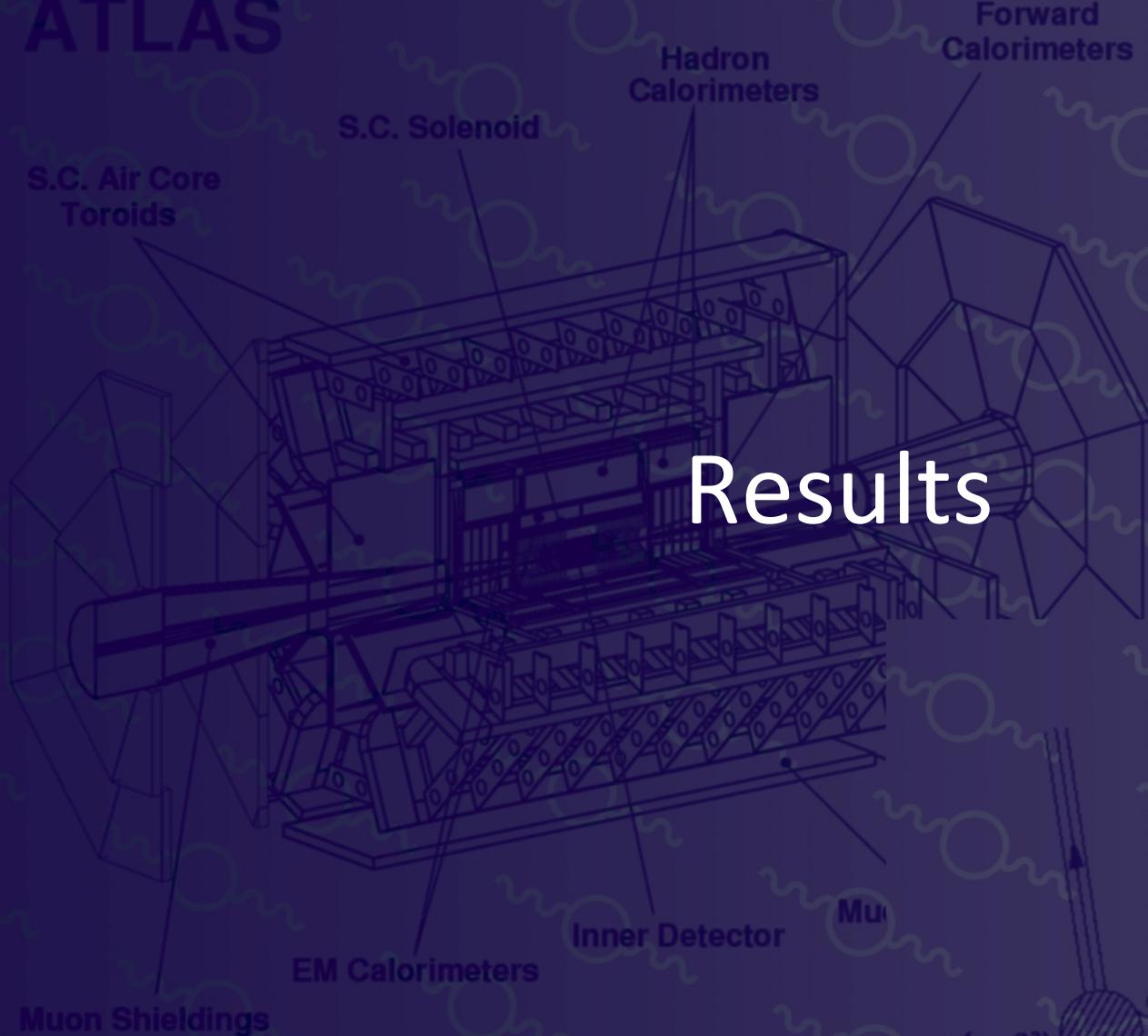
- Matched sample

$$|\bar{\rho}\rangle = |\rho_m^{(0)}\rangle + \left[|\rho_m^{(1)}\rangle + [\mathbf{I}(t_0) + \mathbf{K}(t_0) + \mathbf{P}]|\rho_m^{(0)}\rangle \right] + \left[|\rho_{m+1}^{(0)}\rangle - \sum_l \mathbf{S}_l \Theta(t_l - t_0) |\rho_m^{(0)}\rangle \right]$$

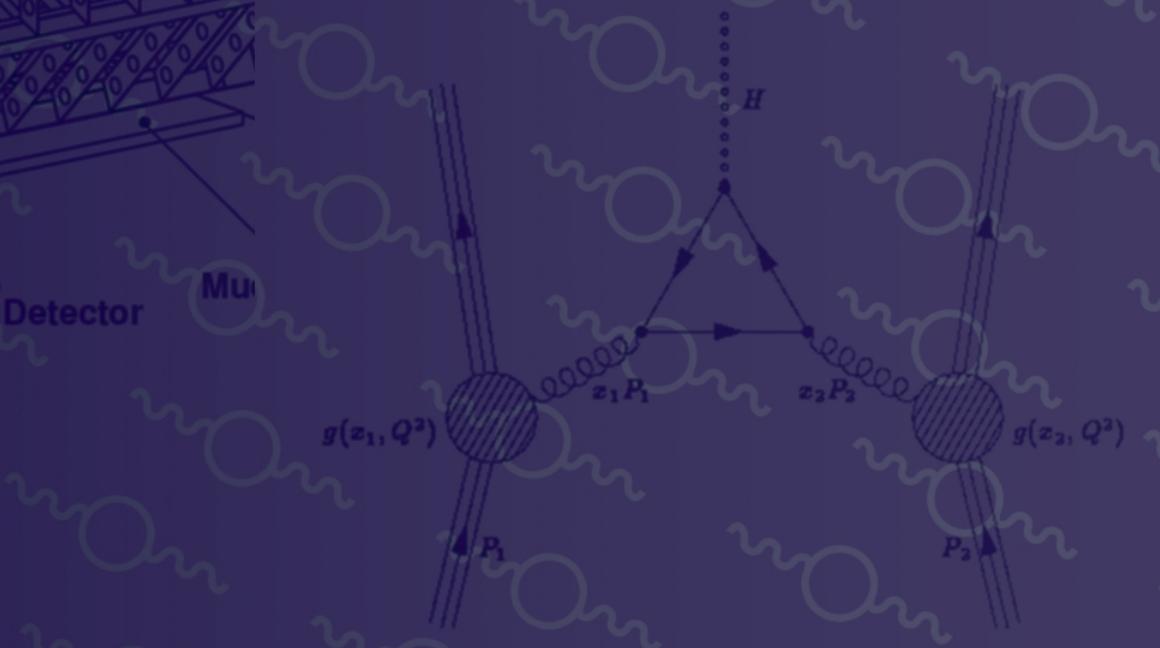
MC, Hartanto, Kraus, Worek, in preparation

1. What to do with Born cross sections, which require cuts?
 2. What to do with PDFs in the backward evolution?
 3. How to match massless partons onto massive in the shower?
- Implementation in HELAC (completed)
 1. Use reweighting for all m-parton samples
 2. Use unweighting for m+1-parton samples
 3. Current matching has the LC+ spin-averaged accuracy of the parton shower implementation in DEDUCTOR

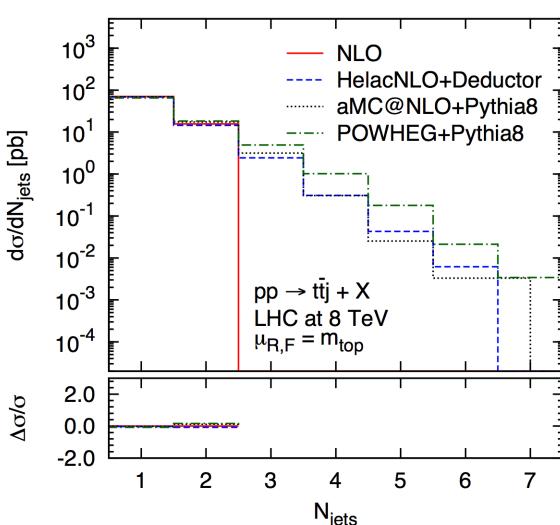
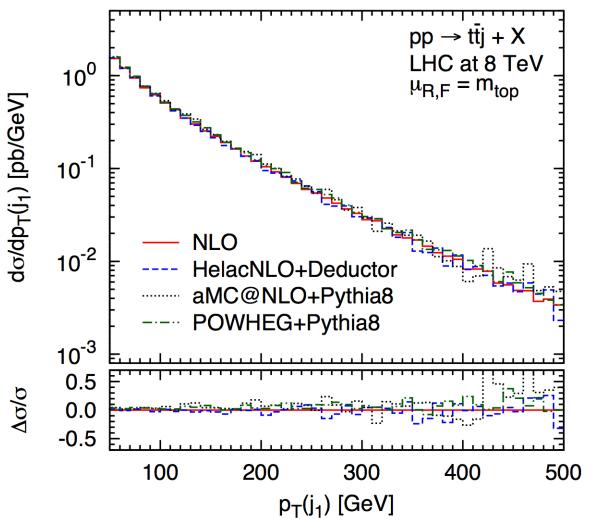
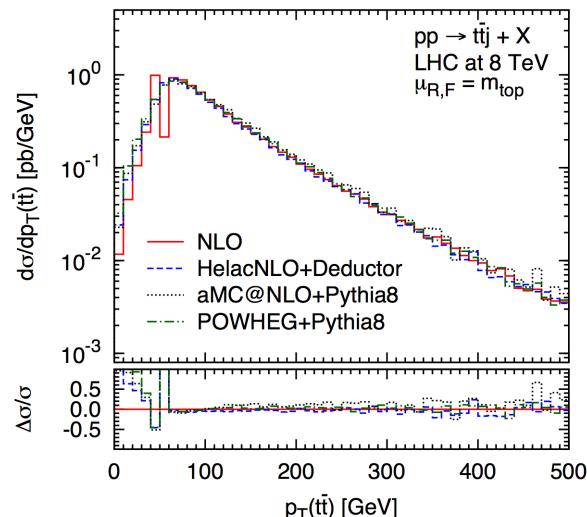
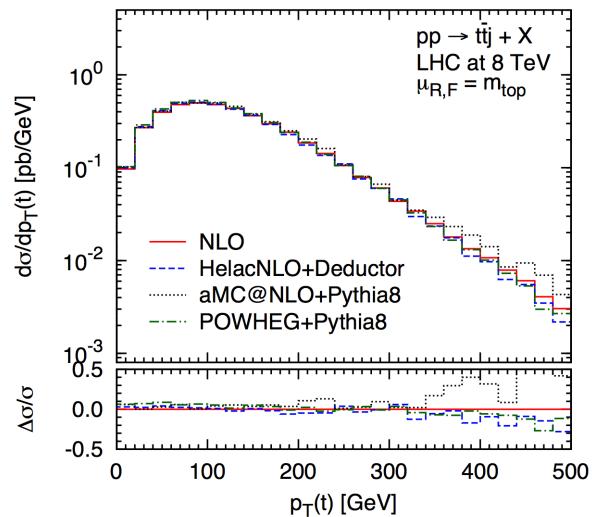
ATLAS



Results

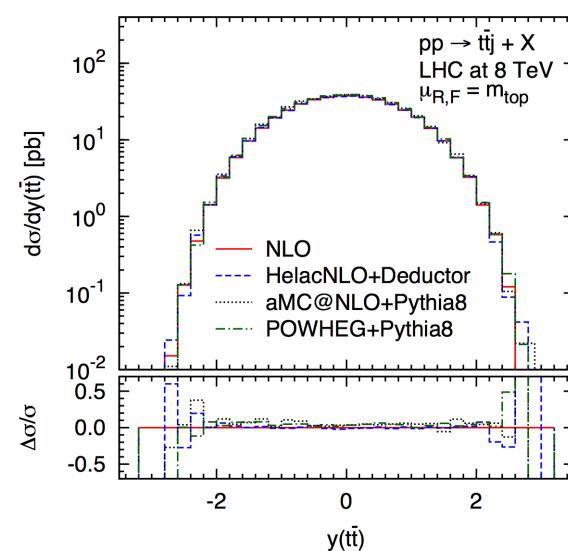
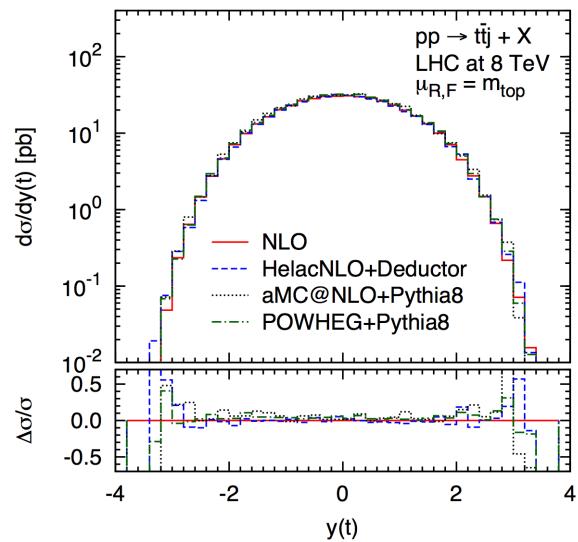
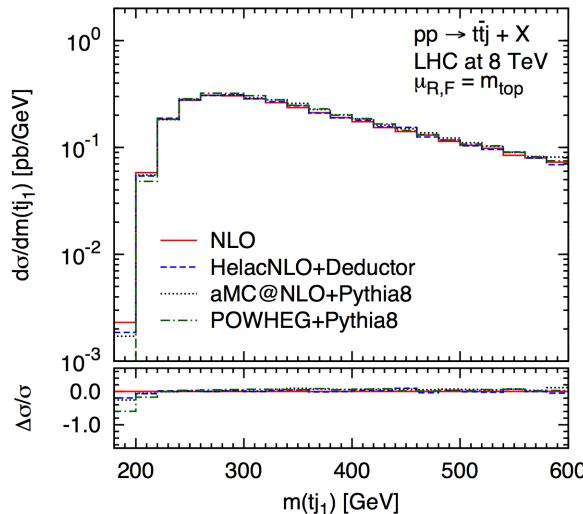
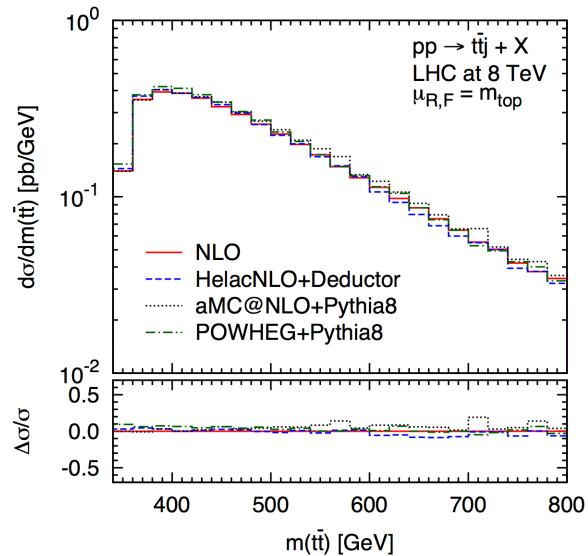


MC, Hartanto, Kraus, Worek, in preparation



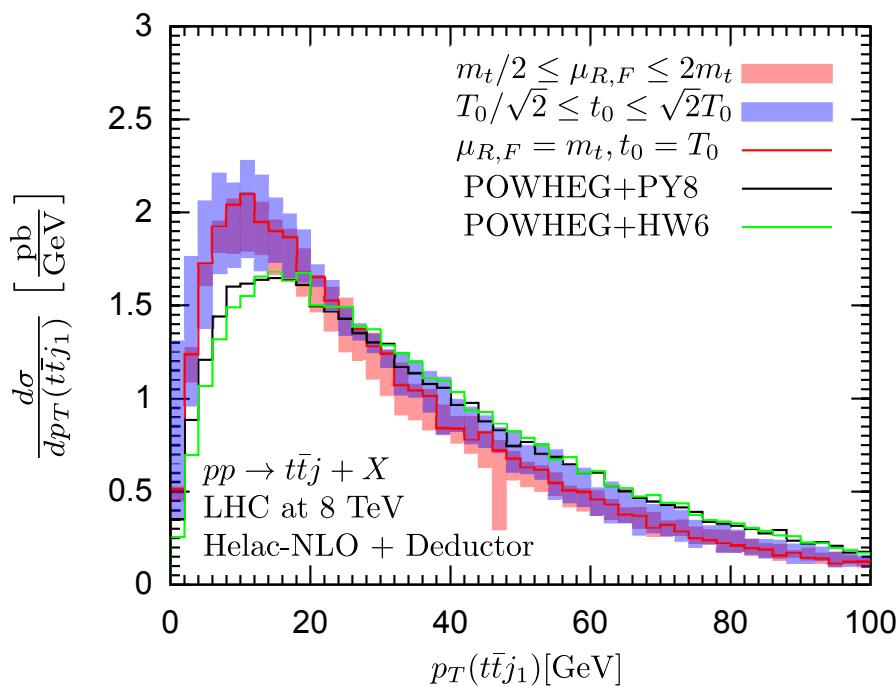
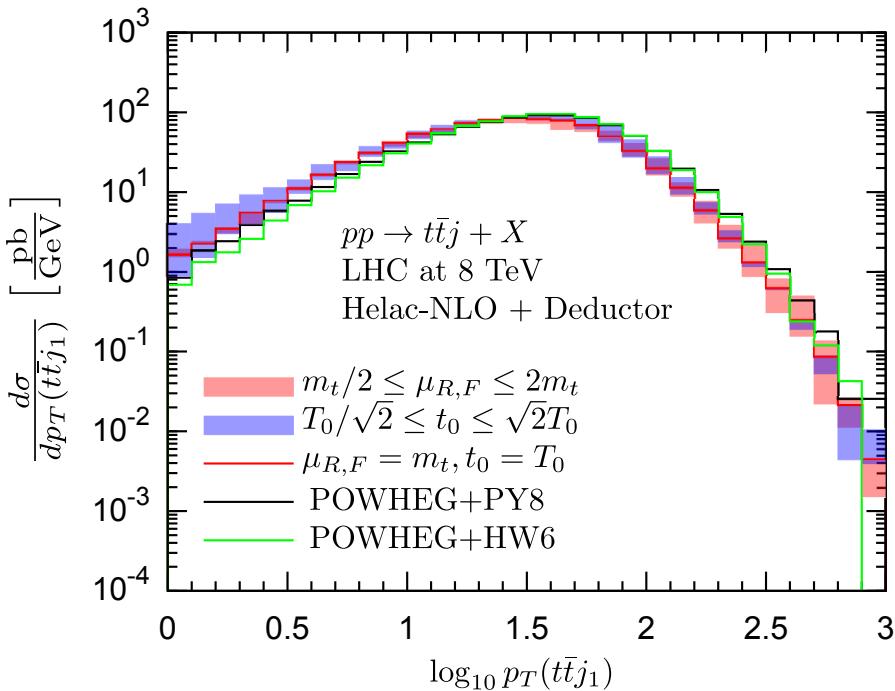
Matching in $t\bar{t}$ + jet

MC, Hartanto, Kraus, Worek, in preparation



Starting time dependence

MC, Hartanto, Kraus, Worek, in preparation



- Generation cut dependence

p_T^{gen}	$\sigma_{\text{NLO+PS}} [\text{pb}]$
5 GeV	86.49 ± 0.48
10 GeV	85.94 ± 0.38
15 GeV	86.04 ± 0.32
30 GeV	85.99 ± 0.23

Conclusions and outlook

- Implemented new subtraction scheme in HELAC-DIPOLES
- Implemented matching to DEDUCTOR at NLO
- Performed first tests on non-trivial process
- Lots to understand
- Almost ready for phenomenology

