Unstable particles with effective field theory

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Outline

- Motivation and theoretical framework
- Line-shape

  MB, Chapovsky, Signer, Zanderighi, hep-ph/03120331, hep-ph/0401002.
- W pair production near threshold
- Invariant mass cuts

  MB, Kauer, Signer, Zanderighi, hep-ph/0411108, MB, Falgari, Schwinn, Signer, Zanderighi, 0707.0773 [hep-ph], Actis, MB, Falgari, Schwinn, 0807.102 [hep-ph].
- Non-resonant effects in $t\bar{t}$ production near threshold

  MB, Jantzen, Ruiz-Femenia 1004.2188 [hep-ph]; Jantzen, Ruiz-Femenia 1307.4337 [hep-ph]
- Summary and further results

Motivation

- "Fundamental question in QFT" – Perturbation expansions do not work for the production of resonances ("unstable particles") even for weak coupling, because

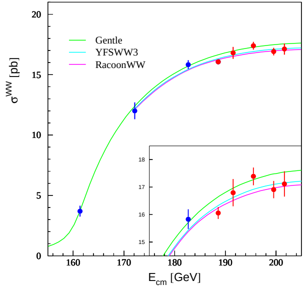
$$M^2/(s-M^2) \sim M^2/(M\Gamma) \sim 1/g^2$$

Systematic expansion?

- "Precision physics" – with electroweak gauge bosons W, Z, the top quark.
Decay rapidly ($\tau < 10^{-25} s$) such that

$$\frac{\text{width}}{\text{mass}} \equiv \frac{\Gamma}{M} \sim \mathcal{O}(\alpha_{\mathrm{EW}}) \ll 1$$

but non-negligible.
(Higgs width is very small)



What's the problem?

- Singularity of propagator indicates sensitivity to two very different scales: short-distance production ($1/\sqrt{s}$, $1/M$) and the lifetime $1/\Gamma \gg 1/M$ (unless the contour can be deformed away from the singularity).

- "Dyson resummation" of self-energy insertions

$$\frac{1}{p^2 - M^2} \to \frac{1}{p^2 - M^2 - \Pi(p^2)}$$

  regularizes the singularity, since $\Pi(M^2) \approx \delta M^2 - iM\Gamma$, but upsets the perturbative expansion. Gauge-dependence of $\Pi(s)$ and the propagator of a gauge boson resonance.
  Need a systematic approximation in g^2 and Γ/M to the scattering amplitude/cross section.

- Note: unstable particles have no asymptotic states and their lines are never cut in Cutkosky's rules [Veltman, 1963]. Theory is unitary in the Hilbert space of asymptotic states. "On-shell" production of unstable particles corresponds to the leading-order approximation

$$\frac{M\Gamma}{(p^2 - M^2)^2 + M^2\Gamma^2} \stackrel{\Gamma \to 0}{\to} \pi\delta(p^2 - M^2)$$

Methods/approaches

- "Complex mass scheme" [Denner, Dittmaier, Roth, Wackeroth, 1999 ... Denner, Lang, 2014]
  - Standard perturbative calculation with complex mass counterterms,

    $$M^2_{\text{bare}} = \mu^2 + \delta\mu^2, \qquad \mu^2 = M^2 - iM\Gamma,$$

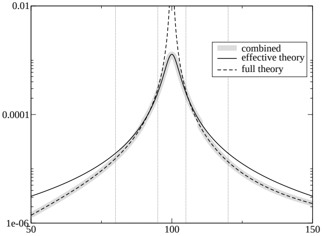
    so $p^2 - \mu^2$ is never zero.
  - With M_Z, M_W and G_F as inputs for the renormalized electroweak parameters $\rightarrow \sin\theta_W$ and coupling constants become complex (essential for Ward identities to hold).
  - Straightforward for standard NLO calculations, including fully differential quantities.

- "Effective theory approach" [this project]
  - Starts from $\Gamma \ll M$ and idea of scale separation. Strict expansion and power counting.
  - Especially useful for threshold, combination with resummation, beyond NLO for sufficiently inclusive quanitites.
  - Field theory realisation and systematic extension of the "(Double) pole approximation" [Stuart, 1991; Aeppli, van Oldenborgh, Wyler, 1994]

Matching kinematic regions

- Consider line-shape $A + B \to \text{resonance} \to X$

$$\delta \equiv \frac{s - M^2}{M^2}$$



- Off resonance, $\delta \sim 1$, conventional perturbation theory applies

$$\sigma = g^4 f_1(\delta) + g^6 f_2(\delta) + \ldots$$

- Near resonance, $\delta \ll 1$, expand in δ and reorganize

$$\sigma \sim \sum_n \left(\frac{g^2}{\delta}\right)^n \times \{1\ (\text{LO}); g^2, \delta\ (\text{NLO}), \ldots\} = h_1(g^2/\delta) + g^2 h_2(g^2/\delta) + \ldots$$

- The two approximations can be matched in an intermediate region, where δ and g^2/δ are small.
  In the following concentrate on the resonance region (threshold for pair production).

Unstable particle EFT (I)

Step 1: Integrate out hard fluctuations $k \sim M$
The EFT contains

- No top, Z, W, Higgs.
- Resonant field ϕ_v ($p = Mv + k$, as in HQET) with soft ($k \sim \Gamma$) fluctuations. Non-relativistic fields for pair production near threshold.
- Soft $k \sim \Gamma$ massless fields (photons, gluons, light fermions)
- Hard-collinear ($k_+ \sim M, k_\perp \sim \sqrt{M\Gamma}, k_- \sim \Gamma$) massless fields (photons, gluons, light fermions)
- Effective interactions
- The production of the W bosons is short-distance and must be incorporated into the EFT by local operators (more precisely, local modulo collinear Wilson lines).

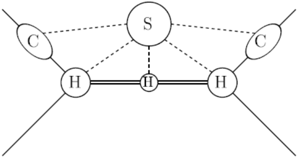
Step 2: Integrate out hard-collinear fluctuations, leaving

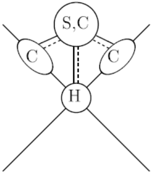
- resonant and soft fields as above
- external-collinear fields ψ_{n_-} ($p = Mn_-/2 + k$) and χ_{n_+} ($p = Mn_+/2 + k$)

i.e. only soft fluctuations around classical scattering trajectory.

Unstable particle EFT (II)

For $\nu e \to W \to X$ line-shape





General formula for the forward-scattering amplitude including non-resonant production

$$i\,\mathcal{A} = \sum_{k,l} \int d^4x\, \langle \nu e | T(i\mathcal{O}_p^{(k)}(0) i\mathcal{O}_p^{(l)}(x)) | \nu e \rangle + \sum_k \langle \nu e | i\mathcal{O}_{\rm nr}^{(k)}(0) | \nu e \rangle.$$

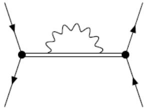
- Matrix elements are evaluated in the EFT: HPET (NREFT) + SCET
- The local "non-resonant" operator includes off-shell W or "background" processes.

Unstable particle EFT (III)

$$\begin{aligned}\mathcal{L}_{\rm eff} \;=\;& 2\hat{M}\phi_v^\dagger\left(iv\cdot D_s-\frac{\Delta}{2}\right)\phi_v+2\hat{M}\phi_v^\dagger\left(\frac{(iD_{s\top})^2}{2\hat{M}}+\frac{\Delta^2}{8\hat{M}}\right)\phi_v\\ &-\frac{1}{4}F_{s\mu\nu}F_s^{\mu\nu}+\bar{\psi}_s i\not{D}_s\psi_s+\bar{\chi}_s i\not{\partial}\chi_s+\bar{\psi}_{n_-}in_-D_s\frac{\not{n}_+}{2}\psi_{n_-}\\ &+C\left[y\,\phi_v\bar{\psi}_{n_-}\chi_{n_+}+\text{h.c.}\right]+\frac{yy^*D}{4\hat{M}^2}(\bar{\psi}_{n_-}\chi_{n_+})(\bar{\chi}_{n_+}\psi_{n_-})+\ldots\end{aligned}$$

- At NLO need
  - Δ to order g^4 (two-loop on-shell, hard self-energy)
    In the pole scheme $\Delta=-i\Gamma$ exactly with Γ the on-shell width
  - $C=1+\ldots$ to one-loop
  - D at tree-level, $D=1$
- The unstable particle propagator is $$\frac{i}{2\hat{M}(v\cdot k-\Delta^{(1)}/2)}$$
- After deriving $\mathcal{L}_{\rm eff}$ to the required accuracy by matching calculations, calculate the scattering amplitude in the effective theory – both is done in conventional PT

Sample diagram



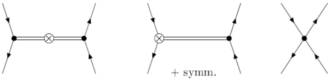
Separate hard and soft contributions to the 1-loop self-energy $\Pi(s) = \Pi_h(s) + \Pi_s(s)$, then expand

$$\Pi_h(s) = \hat{M}^2 \sum_l \delta^l \, \Pi^{(1,l)}$$

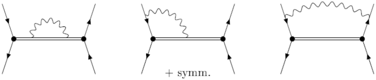
- The different terms are distributed as follows:
  - $\Pi^{(1,0)}$ (gauge-invariant) $\rightarrow \Delta^{(1)}$ (LO)
  - $\Pi^{(1,1)}$ (gauge-dependent) $\rightarrow C^{(1)}$ (NLO)
  - $\Pi^{(1,2)}$ (gauge-dependent) $\rightarrow D^{(1)}$ (NNLO)
  - Π_s is reproduced by the effective theory self-energy

  And so on in higher order in δ and α

- The matching procedure guarantees that the coefficients of the effective Lagrangian are automatically gauge-invariant (because so is the Lagrangian), and that no double-counting occurs.



$$i\,\mathcal{T}_h^{(1)} = i\,\mathcal{T}^{(0)} \times \left[2\,C^{(1)} - \frac{[\Delta^{(1)}]^2}{8\mathcal{D}\hat{M}} + \frac{\Delta^{(2)}}{2\mathcal{D}} - \frac{\mathcal{D}}{2\hat{M}}\right]$$

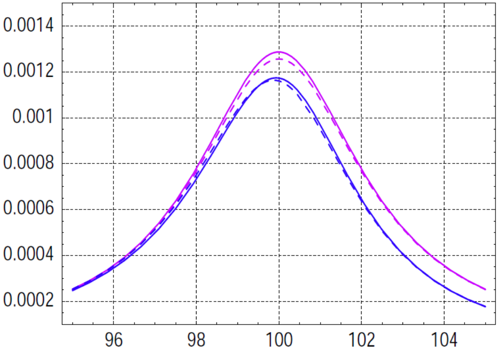


$$i\,\mathcal{T}_s^{(1)} = i\,\mathcal{T}^{(0)} \times a_g \left[4\ln^2\left(\frac{-2\mathcal{D}}{\mu}\right) - 4\ln\left(\frac{-2\mathcal{D}}{\mu}\right) + \frac{5\pi^2}{6}\right]$$

$$\mathcal{D} \equiv \sqrt{s} - \hat{M} - \frac{\Delta^{(1)}}{2}$$

- Leading-order line-shape $\mathcal{T}^{(0)}$ has exact Breit-Wigner form
- $1/\epsilon$ poles cancel when adding hard and soft contributions up to initial state collinear divergence (standard)
- Simple (single-scale) calculations
- NLO line-shape $\mathcal{T}^{(0)}$ no longer Breit-Wigner-shaped. Fitting to Breit-Wigner leads to errors in mass determinations of $\mathcal{O}(100\,\mathrm{MeV})$.

NLO line shape



Scales, parameters, power counting – WW and $t\bar{t}$ threshold

- WW pair production near threshold is dominated by electroweak interactions (in leading orders), top pair production by the strong interaction.
-

	$\Phi = Z, W, \ldots$	WW	$t\bar{t}$
α_{ew}	δ	δ	δ
α_{em}	δ	δ	δ
α_s	$(\sqrt{\delta})$	$(\sqrt{\delta})$	$\sqrt{\delta}$
Γ/M	δ	δ	δ
$v^2 \equiv (\sqrt{s} - [(2)M + i\Gamma])/M$	δ	δ	δ
g^2/v (Coulomb)	–	$\sqrt{\delta}$	1

- Both require non-relativistic + unstable particle EFT, but for top the former is more essential, while for W unstable particle effects are more important, and the Coulomb interaction does not have to be summed (screened by width).
  Expansion runs in $\sqrt{\delta}$ for pair production: LO, $\mathrm{N}^{1/2}$LO, NLO, ... (WW)

Inclusive $e^- e^+ \to 4f$

- Consider

$$e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d}\, X$$

near threshold. Dominated by nearly on-shell $W^- W^+$. Large sensitivity to M_W.
ILC with GIGAZ option: $\delta M_W \approx 6\,\mathrm{MeV}$ experimentally [Wilson, 2001]. Or TLEP.
Rule of thumb: $\delta\sigma \approx 1\% \Leftrightarrow \delta M_W \approx 15\,\mathrm{MeV}$.

- Calculate totally inclusive final state, except for flavour quantum numbers.
Extract cross section from the forward-scattering amplitude

$$\hat{\sigma} = \frac{1}{s}\,\mathrm{Im}\,\mathcal{A}(e^- e^+ \to e^- e^+)_{|\mu^- \bar{\nu}_\mu u \bar{d}}$$

- Perform a “QCD-style” calculation of the short-distance cross section with massless electrons in the $\overline{\mathrm{MS}}$ scheme, then

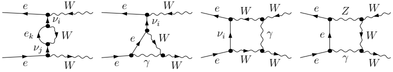
$$\sigma(s) = \int_0^1 dx_1 dx_2\, f_{e/e}(x_1)\, f_{e/e}(x_2)\, \hat{\sigma}(x_1 x_2 s).$$

$\overline{\mathrm{MS}}$ electron distribution function depends on m_e, but not on $\sqrt{s}$, M, Γ.

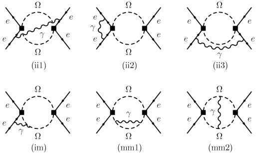
Radiative corrections

- Two-loop $\Delta^{(2)} = M_W(\Pi^{(2,0)} + \Pi^{(1,1)}\Pi^{(1,0)}) = -i\Gamma_W^{(1)}$, i.e. one-loop EW correction to on-shell W decay in the pole mass renormalization scheme.
- One-loop EW correction to the LO production operator

$$\mathcal{O}_p^{(1)} \;=\; \frac{\pi\alpha_{ew}}{\hat{M}_W^2}\left[C_{p,LR}^{(1)}\left(\bar{e}_L\gamma^{[i}n^{j]}e_L\right) + C_{p,RL}^{(1)}\left(\bar{e}_R\gamma^{[i}n^{j]}e_R\right)\right]\left(\Omega_-^{\dagger i}\Omega_+^{\dagger j}\right)$$



- Up to two insertions of the Coulomb potential interaction.
- Soft and collinear photon corrections to the EFT forward-scattering amplitude.



$$\Delta\mathcal{A}_{LR,\,\mathrm{soft}}^{(1),\,\mathrm{fin}} \;=\; \mathcal{A}_{LR}^{(0)}\,\frac{2\alpha}{\pi}\left[L^2 - 4L + 8 + \frac{13}{24}\pi^2\right]$$

$$L = \ln\left[-\,\frac{8(E + i\Gamma_W^{(0)})}{\mu}\right]$$

- Resummation of large collinear logarithms $\ln(M_w/m_e)$ from initial-state radiation.

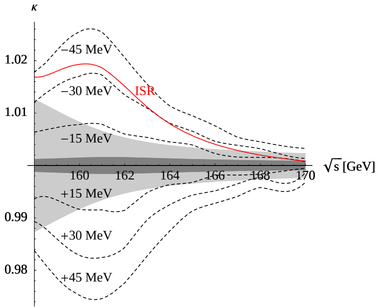
NLO Result

Comparison with of Born, EFT, full four fermion [Denner, Dittmaier, Roth, Wieders, 2005] and DPA NLO calculations, ISR resummed. Same input parameters.

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} X)$(fb)			
$\sqrt{s}$ [GeV]	Born (SM)	EFT	full ee4f	DPA
161	107.06(4)	117.38(4)	118.12(8)	115.48(7)
170	381.0(2)	399.9(2)	401.8(2)	402.1(2)

Sensitivity to M_W and theoretical uncertainty
Variation of cross section wrt to standard input

- Large uncertainty from current implementation of ISR ($\delta M_W \approx 30\,\text{MeV}$)
- Uncertainties from $\text{N}^{3/2}$LO radiative effects are estimated 10 MeV from hard corrections and 4 MeV from Coulomb times hard + soft
- Target accuracy (6 MeV) can be reached by NLL ISR implementation and inclusion of $\text{N}^{3/2}$LO – use existing full NLO $4f$ calc. plus dominant NNLO terms from EFT approach.



Dominant NNLO (I)

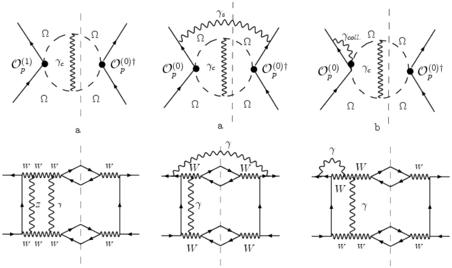
Dominant NNLO = N$^{3/2}$LO in EFT counting

N$^{3/2}$LO terms already included in full NLO 4f calculation

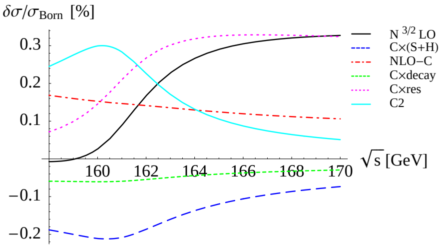
- NLO correction to non-resonant four-electron operator (non-resonant Born terms were N$^{1/2}$LO).
- Interference of 1-loop Coulomb exchange with tree-level higher-dimensional production operators.

N$^{3/2}$LO terms from true NNLO diagrams (2-loop virtual and 1-loop virtual $\times$ real) contain at least one Coulomb photon:

- Mixed hard/Coulomb corrections
- Interference of Coulomb exchange with soft and collinear radiative corrections
- Correction to the Coulomb potential itself.



Beyond NLO (II)



In total a small correction:

$[\delta M_W]_{\text{BeyondNLO}} \approx (3-5)$ MeV

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} X)$(fb)				
$\sqrt{s}$ [GeV]	Born	Born (ISR)	NLO	$\hat{\sigma}^{(3/2)}$	$\sigma_{\mathrm{ISR}}^{(3/2)}$
158	61.67(2)	45.64(2) [−26.0%]	49.19(2) [−20.2%]	−0.001 [−0.00%]	0.000 [+0.00%]
161	154.19(6)	108.60(4) [−29.6%]	117.81(5) [−23.6%]	0.147 [+0.10%]	0.087 [+0.06%]
164	303.0(1)	219.7(1) [−27.5%]	234.9(1) [−22.5%]	0.811 [+0.27%]	0.544 [+0.18%]
167	408.8(2)	310.2(1) [−24.1%]	328.2(1) [−19.7%]	1.287 [+0.31%]	0.936 [+0.23%]
170	481.7(2)	378.4(2) [−21.4%]	398.0(2) [−17.4%]	1.577 [+0.33%]	1.207 [+0.25%]

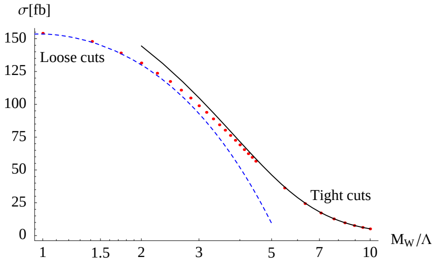
Cuts are not straightforward in the EFT approach: may introduce new scales regions.
Example: Invariant mass cuts $|M^2_{u\bar{d}} - M^2_W|, |M^2_{\mu\bar{\nu}_\mu} - M^2_W| < \Lambda^2$

- Loose cut: $\Lambda \sim M_W$

  No modification of potential loops (momenta always within the cut by power counting).
  Cut affects the matching coefficient of the four-electron operator (non-resonant terms).

- Tight cut: $\Lambda \sim \sqrt{M_W \Gamma_W}$

  Four-electron operator (non-resonant terms) does not contribute at all.
  Cut affects loop calculations in the effective theory.

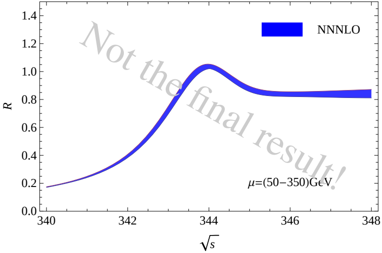


Red dots: Born cross section ($\sqrt{s} = 161$ GeV, WHIZARD)

Top-pair production near threshold

From Project C3:

[MB, Kiyo, Schuller; MB ... Steinhauser, in progress]



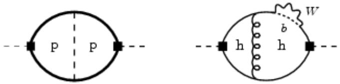
$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}, \Gamma_t = 1.33\,\mathrm{GeV}$

- Width relevant at LO
- 3rd order QCD *defined* by QCD correlation function with $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$.
- Not the full story. Uncancelled ultraviolet divergences (from NNLO).
- Accuracy of 3rd order QCD makes consideration of $e^+e^- \to W^+W^-b\bar{b}$ mandatory.

Finite-width divergences and non-resonant effects

- Finite-width divergences (overall log divergence, already at NNLO):

$$[\delta G(E)]_{\text{overall}} \propto \frac{\alpha_s}{\epsilon} \cdot E$$



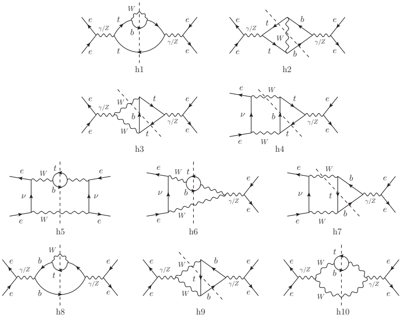
Since $E = \sqrt{s} - 2m_t + i\Gamma$, the divergence survives in the imaginary part:

$$\text{Im}\,[\delta G(E)]_{\text{overall}} \propto m_t \times \frac{\alpha_s \alpha_{ew}}{\epsilon}$$

- Electroweak effect. Must consider $e^+e^- \to W^+W^- b\bar{b}$.

$$\sigma_{e^+e^- \to W^+W^- b\bar{b}} = \underbrace{\sigma_{e^+e^- \to [t\bar{t}]_{\text{res}}}(\mu_w)}_{\text{pure (PNR)QCD}} + \sigma_{e^+e^- \to W^+W^- b\bar{b}_{\text{nonres}}}(\mu_w)$$

Non-resonant starts at NLO (overall linear divergence) [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]. Finite-width scale dep must cancel. Need consistent dim reg calculation.



Equivalent to the dimensionally regulated $e^+e^- \to bW^+\bar{t}$ process with $\Gamma_t = 0$, expanded in the hard region around $s = 4m_t^2$.

$$\int_{\Delta^2}^{m_t^2} dp_t^2\,(m_t^2 - p_t^2)^{\frac{d-3}{2}} H_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

$$p_t^2 \equiv (p_b + p_{W+})^2$$

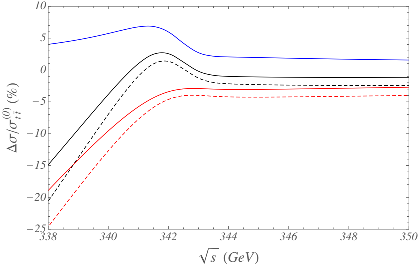
$$H_1\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right) \quad \stackrel{p_t^2 \to m_t^2}{\to} \quad \text{const} \times \frac{1}{(m_t^2 - p_t^2)^2}$$

Linearly IR divergent. Finite in dim reg.

Can impose invariant mass cuts on top decay products. $\Delta^2 = M_W^2$ for inclusive cross section. EFT works differently for loose and wide cuts [Actis, MB, Falgari, Schwinn, 2008]

Non-resonant corrections at NLO

[MB, Jantzen, Ruiz-Femenia, 2010]

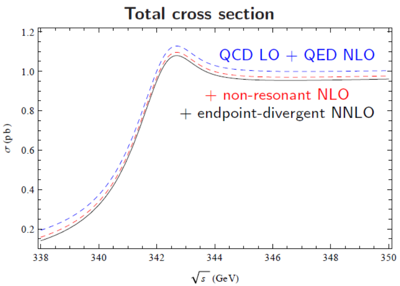


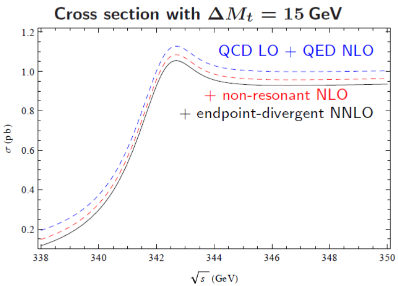
QED and non-resonant corrections relative to the $t\bar{t}$ LO cross section in percent: $\sigma^{(1)}_{\mathrm{QED}}/\sigma^{(0)}_{t\bar{t}}$ (upper solid blue), $\sigma^{(1)}_{\text{non-res}}/\sigma^{(0)}_{t\bar{t}}$ for the total cross section (lower solid red) and $\Delta M_t = 15$ GeV (lower dashed red). The relative size of the sum of the QED and non-resonant corrections is represented by the middle (black) lines, for $\Delta M_{t,\max}$ (solid) and $\Delta M_t = 15$ GeV (dashed). $m_{t,\text{pole}} = 172$ GeV.

Large correction below threshold.
Much larger than QCD scale-dependence at 3rd order ($\pm(2-3)\%$)

$e^+e^- \to W^+W^-b\bar{b}$ near $s = 4m_t^2$

NLO + NNLO singular terms [Jantzen, Ruiz-Femenia, 2013; see also Hoang, Reisser, Ruiz-Femenia, 2010]
(Singular refers to expansion in Λ/m_t where Λ is an invariant mass cut such that $m_t\Gamma_t \ll \Lambda^2 \ll m_t^2$.)





NNLO non-resonant still -2% at threshold and larger below.
Accurate description of region below peak is required for precise determination of m_t.

Summary and further results

I Developed a new approach to treating unstable particles consistently.

- Systematic, power counting, gauge-invariant, minimal, non-diagrammatic.
- Especially useful for inclusive observables, resummations (non-relativistic and logs of Γ/M), beyond NLO.
- Less (so far) for distributions. Presence of further scales in different regions of phase-space makes expansions complicated. General feature of EFT/SCET computations.

II Further results

- Single-top production [Falgari, Mellor, Signer, 1007.0893 [hep-ph]; Falgari, Gianuzzi, Mellor, Signer, 1102.5267 [hep-ph]]
- Finite-width effects on threshold corrections to squark and gluino production [Falgari, Schwinn, Wever, 1211.3408 [hep-ph]]
- General formalism for distributions at hadron colliders at NLO and application to $t\bar{t}$ [P. Falgari, A.S. Papanastasiou, A. Signer, 1303.5299 [hep-ph]]

II Further results (continued)

- Two-loop, $\mathcal{O}(\alpha_s \alpha)$ corrections to Drell-Yan production in the resonance region [Dittmaier, Huss, Schwinn, 1403.3216 [hep-ph]]
- Cascade decays of supersymmetric particles.
  Mass determination from kinematic edges in two-jet mass distribution at $M_{\text{had}}^2 = (M_{\tilde{g}}^2 - M_{\tilde{q}}^2)(1 - M_{\chi}^2/M_{\tilde{q}}^2)$. [MB, Jenniches, Mück, Ubiali, in progress]