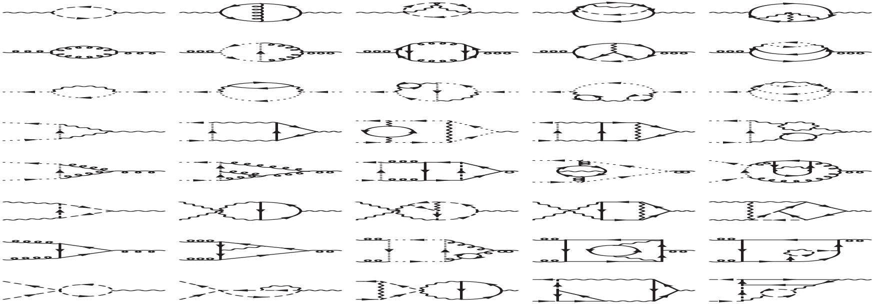
Project C5: Higgs production and renormalization group functions to high loop orders

Principle investigators: Matthias Steinhauser and Luminita Mihaila

Matthias Steinhauser | TTP Karlsruhe

SFB ABSCHLUß KOLLOQUIUM, SEPTEMBER 2014, DURBACH



Project C5

- since Oktober 2004: part of the CRC/TR 9
- since January 2011: Luminita Mihaila joins as project leader
- Contributions from
  - Postdocs: Eiras, Grozin, Kiyo, Pak, Reißer, Rogal, Seidel
  - PhD studtents: Bekavac, Grigo, Hermann, Höschele, Hoff, Kant, Kunz, Kurz, Martens, Piclum, Salomon, Zerf
- Multi-loop calculation with heavy fermions in the SM and MSSM
  - β functions in SM and MSSM
  - decoupling in SM, from MSSM to SM, from SU(5) GUT to MSSM
  - GUT scenarios
  - matching coefficients (heavy-light cur., chromomagn. moment, ...)
  - Higgs boson mass in MSSM to 3 loops
  - Higgs boson production in SM and MSSM
  - $\bar{B} \to X_s\gamma$ and $B_s \to \mu^+\mu^-$ to NNLO

Outline

- Renormalization group functions and decoupling
  - SM beta functions
  - decoupling in QCD
  - matching MSSM $\rightarrow$ SM
- Higgs boson production
  - SM: NNLO
  - SM: some N^{3}LO steps
  - MSSM: NNLO

SM beta functions

QCD

KIT Karlsruhe Institute of Technology

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s(\mu)}{\pi} = \beta(\alpha_s)$$
$$= -\left(\frac{\alpha_s}{\pi}\right)^2 \left[\beta_0 + \frac{\alpha_s}{\pi}\beta_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \beta_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \beta_3 + \ldots\right]$$

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} T n_f$$

[Gross,Wilczek'73; Politzer'73]

QCD

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s(\mu)}{\pi} = \beta(\alpha_s)$$
$$= -\left(\frac{\alpha_s}{\pi}\right)^2 \left[\beta_0 + \frac{\alpha_s}{\pi}\beta_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \beta_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \beta_3 + \ldots\right]$$

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} T n_f$$

[Gross,Wilczek'73; Politzer'73]

β_1 [Jones'74; Caswell'74; Egorian,Tarasov'78]

β_2 [Tarasov,Vladimirov,Zharkov'80; Larin,Vermaseren'93]

β_3 [van Ritbergen,Vermaseren,Larin'97; Czakon'04]

SM: couplings

Gauge:

$$\alpha_1 = \frac{5}{3}\frac{\alpha}{\cos^2\theta_W}$$
$$\alpha_2 = \frac{\alpha}{\sin^2\theta_W}$$
$$\alpha_3 = \alpha_s$$

[SU(5)-like normalization]

Yukawa:

$$\alpha_{t,b,\tau} = \frac{\alpha m_{t,b,\tau}^2}{2\sin^2\theta_W M_W^2}$$

Higgs:

$$\alpha_7 = \frac{\lambda}{4\pi} \qquad [\mathcal{L} = -\lambda(\Phi^\dagger\Phi)^2 + \ldots]$$

SM: β functions

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_i}{\pi} = \beta_i(\alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_b, \alpha_\tau, \lambda)$$

- β_i from renormalization constants Z_{α_i}
- Example: calculation of $\beta_1, \beta_2, \beta_3$ requires:
  - $Z_{\alpha_1}, Z_{\alpha_2}, Z_{\alpha_3}$ to 3 loops
  - $\beta_t, \beta_b, \beta_\tau$ to 1 loop

    $\alpha_{t,b,\tau}$ dependence of $Z_{\alpha_1}, Z_{\alpha_2}, Z_{\alpha_3}$ starts at 2 loops
  - β_λ to tree level

    λ dependence of $Z_{\alpha_1}, Z_{\alpha_2}, Z_{\alpha_3}$ starts at 3 loops

SM: 1- and 2-loop results

Gauge:

1 loop: [Gross,Wilczek'73; Politzer'73]

2 loops: [Jones'81; Fischler,Hill'82; Machacek,Vaughn'83; Jack,Osborn'84]

Yukawa:

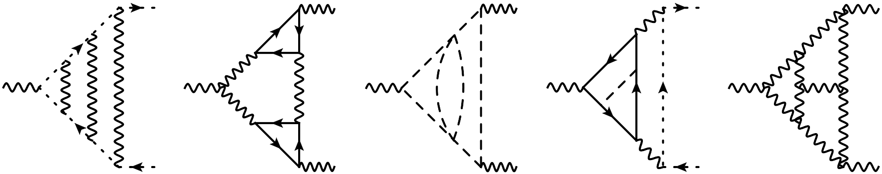
2 loops: [Fischler,Oliensis'82; Machacek,Vaughn'83; Jack,Osborn'84]

Higgs:

2 loops: [Machacek,Vaughn'84; Jack,Osborn'84; Ford,JackJones'92; Luo,Xiao'02]

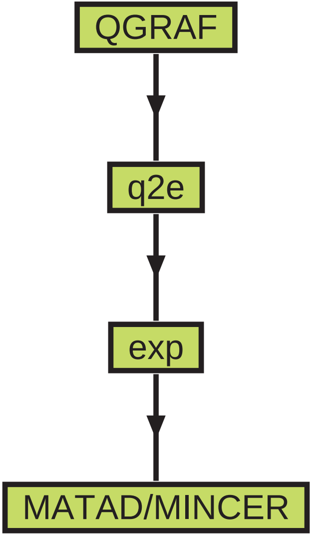
Calculation (gauge couplings)

- $\overline{\mathrm{MS}}$ scheme
- unbroken phase of SM
- $$Z_{\alpha_i} = \frac{Z_V}{\prod_k \sqrt{Z_{k,WF}}} = \frac{Z_{1,\alpha_i c\bar{c}}}{Z_{2c}\sqrt{Z_{3,\alpha_i}}} = \frac{Z_{1,\alpha_i\alpha_i\alpha_i}}{\left(\sqrt{Z_{3,\alpha_i}}\right)^3}$$
- $\approx 1\,000\,000$ Feynman diagrams



- 1 non-zero external momentum ⇨ MINCER [Larin,Tkachov,Vermaseren'91]
- γ_5

Automation



QGRAF [Nogueira’91]

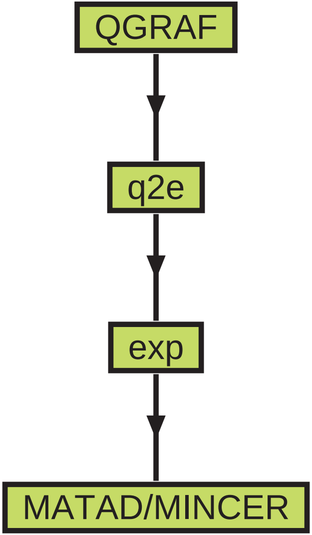
q2e/exp [Harlander,Seidelsticker,Steinhauser’97;Seidelsticker’97]

MINCER [Larin,Tkachov,Vermaseren’91]

MATAD [Steinhauser’00]

FORM/TFORM/ParFORM [Vermaseren,Ueda,. . .]

Automation



FeynArts

QGRAF [Nogueira'91]

q2e/exp [Harlander,Seidelsticker,Steinhauser'97;Seidelsticker'97]

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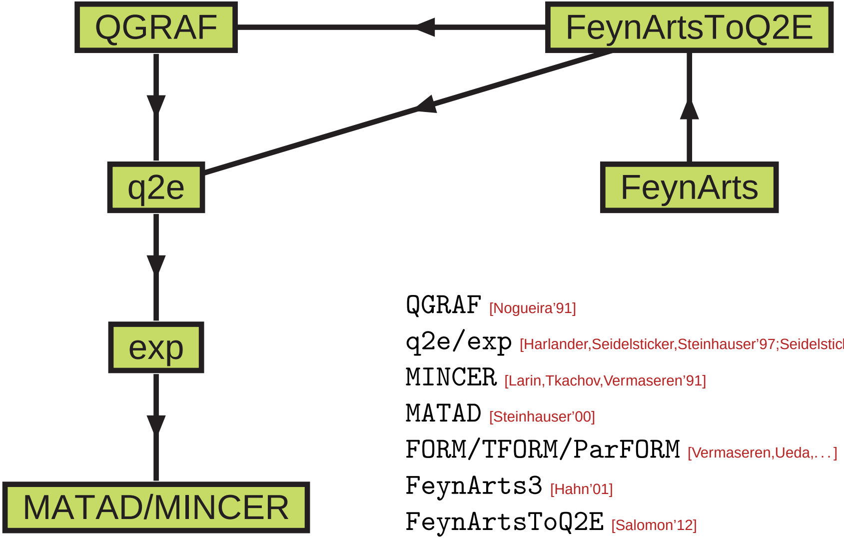
MATAD [Steinhauser'00]

FORM/TFORM/ParFORM [Vermaseren,Ueda,. . .]

FeynArts3 [Hahn'01]

Automation



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MATAD [Steinhauser'00]

FORM/TFORM/ParFORM [Vermaseren,Ueda,. . .]

FeynArts3 [Hahn'01]

FeynArtsToQ2E [Salomon'12]

Checks

- 2-loop results for $\beta_{\rm Yukawa}$ [Fischler,Oliensis'82; Machacek,Vaughn'83; Jack,Osborn'84]
- 3-loop QCD β function [Tarasov,Vladimirov,Zharkov'80; Larin,Vermaseren'93]
- 3-loop $\mathcal{O}(\alpha_3^3\alpha_t)$ to β_3 [Steinhauser'98]
- 3-loop results for $SU(3)$, $SU(2)$, $U(1)$ (only one gauge coupling)

  [Pickering,Gracey,Jones'01]
- $Z_{\alpha_i} = \frac{Z_{1,\alpha_i c\bar{c}}}{Z_{2c}\sqrt{Z_{3,\alpha_i}}} = \frac{Z_{1,\alpha_i\alpha_i\alpha_i}}{\left(\sqrt{Z_{3,\alpha_i}}\right)^3}$
- calculation for arbitrary $\xi_{\rm QCD}, \xi_W, \xi_B$

  ➪ β functions are ξ independent
- *BBB* vertex ➪ zero to 3 loops ($=$ sum of 300 000 diagrams)
- IR safe: introduce mass $m \neq 0$; asymptotic expansion for $q^2 \gg m^2$

  ➪ NO $\ln(m^2/\mu^2)$ terms!

  (Note: up to 35 sub-diagrams/diagram!)

Results: β_2 as an example

$$\beta_2 = \left(\frac{\alpha_2}{\pi}\right)^2 \Bigg\{ -\frac{43}{24} + \frac{n_G}{3} + \frac{\alpha_1}{\pi}\left(\frac{3}{160} + \frac{n_G}{80}\right) + \frac{\alpha_2}{\pi}\left(-\frac{259}{96} + \frac{49 n_G}{48}\right) + \frac{\alpha_3}{\pi}\frac{n_G}{4} + \left(\frac{\alpha_1}{\pi}\right)^2\left(\frac{163}{102400} - \frac{7 n_G}{960} - \frac{11 n_G^2}{2880}\right)$$
$$+ \frac{\alpha_1}{\pi}\frac{\alpha_2}{\pi}\left(\frac{561}{10240} + \frac{13 n_G}{1280}\right) - \frac{\alpha_1}{\pi}\frac{\alpha_3}{\pi}\frac{n_G}{960} + \left(\frac{\alpha_2}{\pi}\right)^2\left(-\frac{667111}{110592} + \frac{1603 n_G}{432} - \frac{415 n_G^2}{1728}\right) + \frac{\alpha_2}{\pi}\frac{\alpha_3}{\pi}\frac{13 n_G}{64}$$
$$+ \left(\frac{\alpha_3}{\pi}\right)^2\left(\frac{125 n_G}{192} - \frac{11 n_G^2}{144}\right) + n_t \frac{\alpha_t}{\pi}\left[-\frac{3}{32} - \frac{\alpha_1}{\pi}\frac{593}{10240} - \frac{\alpha_2}{\pi}\frac{729}{2048} - \frac{\alpha_3}{\pi}\frac{7}{64} + \frac{\alpha_t}{\pi}\left(\frac{57}{1024} + \frac{45 n_t}{512}\right)\right]$$
$$+ \frac{\lambda}{4\pi^2}\left(\frac{\alpha_1}{\pi}\frac{3}{640} + \frac{\alpha_2}{\pi}\frac{3}{128} - \frac{\lambda}{4\pi^2}\frac{3}{64}\right)\Bigg\}$$

[α_b and α_τ set to zero]

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[α_b and α_τ set to zero]

n_G: # of generations

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[α_b and α_τ set to zero]

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2 loops: $\propto \alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_b, \alpha_\tau$

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[α_b and α_τ set to zero]

n_G: # of generations

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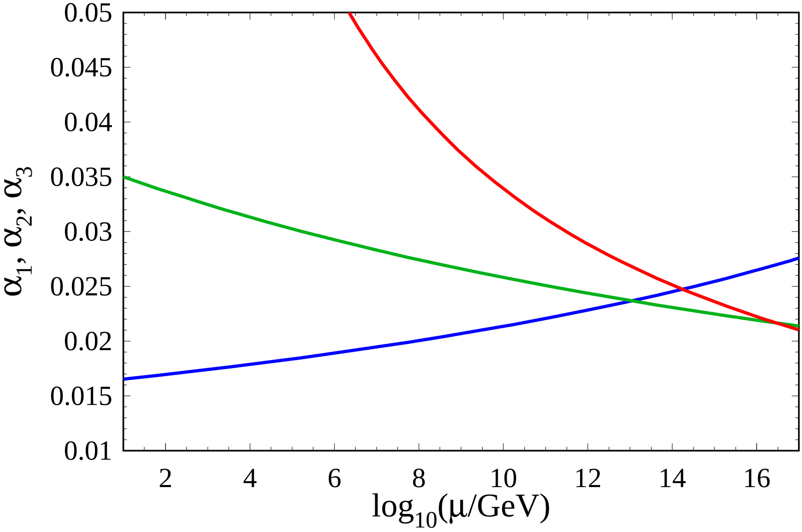
1 loop: $-\frac{43}{24} + \frac{n_G}{3}$

2 loops: $\propto \alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_b, \alpha_\tau$

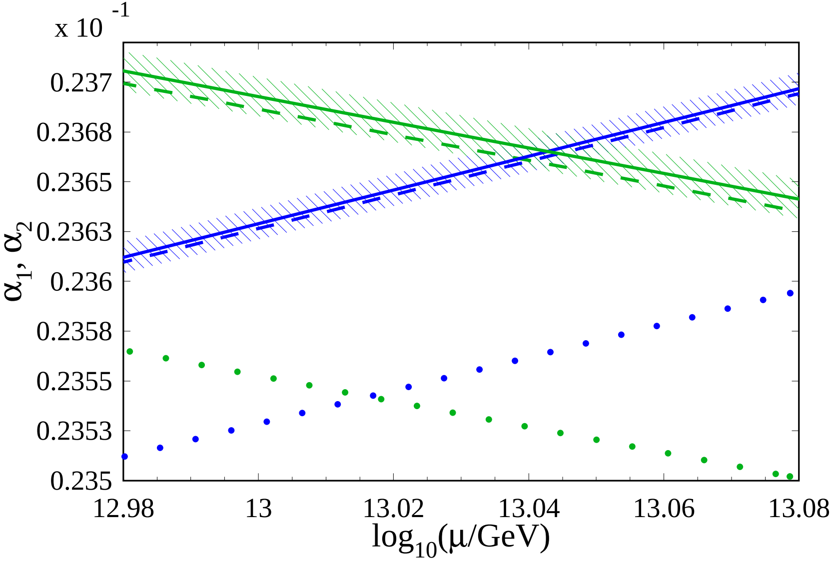
3 loops: $\alpha_i\alpha_j(\ldots) + \lambda\alpha_i(\ldots) + \lambda^2(\ldots)$

Numerics



0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
$\alpha_1, \alpha_2, \alpha_3$
2
4
6
8
10
12
14
16
$\log_{10}(\mu/\text{GeV})$

Numerics



x 10 -1
0.237
0.2368
0.2365
0.2363
0.236
0.2358
0.2355
0.2353
0.235
α_1, α_2
12.98
13
13.02
13.04
13.06
13.08
$\log_{10}(\mu/\mathrm{GeV})$

3-loop results for β_i

Gauge: [Mihaila,Salomon,Steinhauser'12; Bednyakov,Pikelner,Velizhanin'13]

Yukawa: [Bednyakov,Pikelner,Velizhanin'13]

β_t: [Chetyrkin,Zoller'13]

Higgs: [Chetyrkin,Zoller'13; Bednyakov,Pikelner,Velizhanin'13]

Decoupling

Formalism

- $\overline{\rm MS}$ scheme ➪ Appelquist-Carrazone decoupling theorem not valid ➪ has to be done “by hand”
- field and parameters in full and effective theory are related by so-called “decoupling constants”
- example: $\alpha_s^{(5)} = \zeta_{\alpha_s} \alpha_s^{(6)}$
- *n*-loop decoupling constant ζ_i computed from *n*-loop vacuum integral

[Chetyrkin,Kniehl,Steinhauser’98]

- matching done at (unphysical) scale $\mu_{\rm dec}$

QCD:

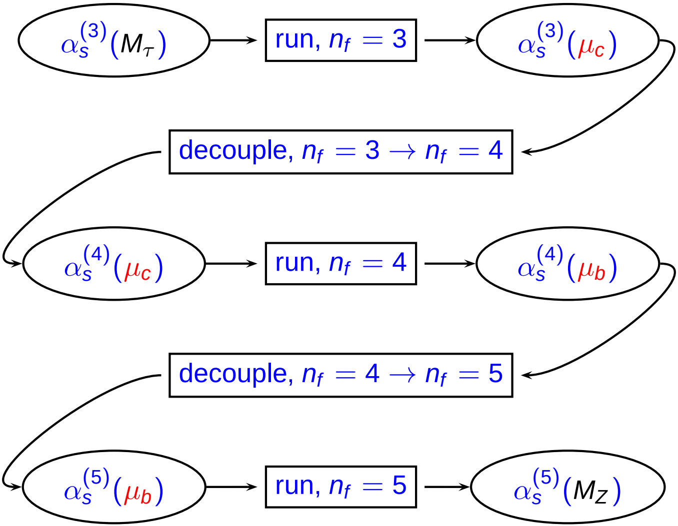
2 loops [Bernreuther,Wetzel’82; Larin,van Ritbergen,Vermaseren’95]

3 loops [Chetyrkin,Kniehl,Steinhauser’98]

4 loops [Schröder,Steinhauser’06; Chetyrkin,Kühn,Sturm’06]

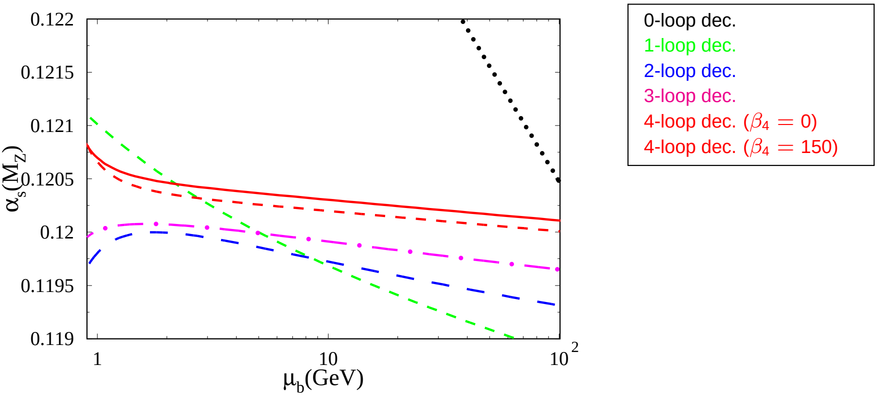
3 loops simultaneous charm and bottom [Grozin,Höschele,Hoff,Steinhauser’11]

QCD: $\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$



$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$

$\mu_{c,\mathrm{dec}} = 3$ GeV, $\quad \mu_{b,\mathrm{dec}} \equiv \mu_b$



$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$

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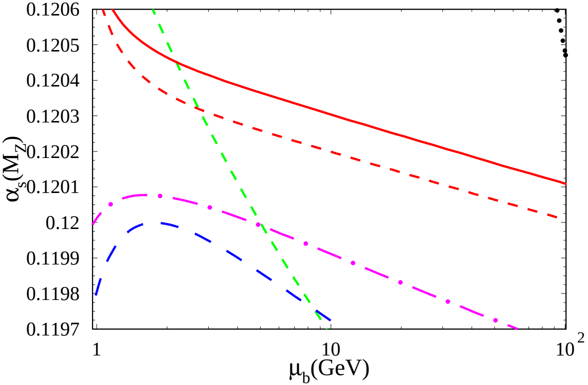
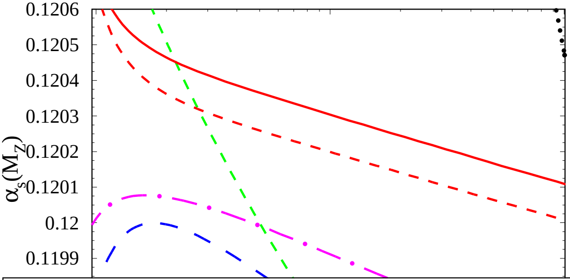


0-loop dec.
1-loop dec.
2-loop dec.
3-loop dec.
4-loop dec. ($\beta_4 = 0$)
4-loop dec. ($\beta_4 = 150$)

$\delta\alpha_s(\text{trunc. p.t.}) \approx 0.0002$
$\delta\alpha_s(\mu_{b,\mathrm{dec}}) \approx 0.0002$

$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$

$\mu_{c,\mathrm{dec}} = 3$ GeV, $\mu_{b,\mathrm{dec}} \equiv \mu_b$



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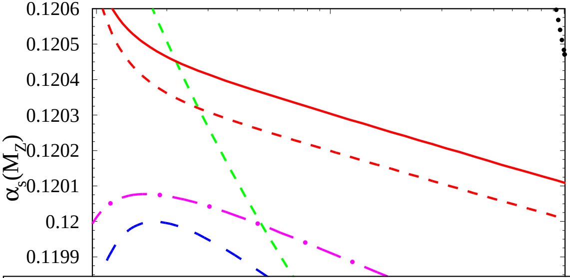
$\delta\alpha_s(\text{trunc. p.t.}) \approx 0.0002$
$\delta\alpha_s(\mu_{b,\mathrm{dec}}) \approx 0.0002$

$\alpha_s^{(3)}(M_\tau) = 0.332 \pm 0.005_{\mathrm{exp}} \pm 0.015_{\mathrm{th}}$ [Baikov,Chetyrkin,Kühn'08]
$\Rightarrow$
$\alpha_s^{(5)}(M_Z) = 0.1202 \pm 0.0006_{\mathrm{exp}} \pm 0.0018_{\mathrm{th}} \pm 0.0003_{\mathrm{evol}}$

$\alpha_s^{(3)}(M_\tau) = 0.3156 \pm 0.0030_{\mathrm{exp}} \pm 0.0051_{\mathrm{th}}$ [Beneke,Jamin'08]
$\Rightarrow$
$\alpha_s^{(5)}(M_Z) = 0.11795 \pm 0.00038_{\mathrm{exp}} \pm 0.00063_{\mathrm{th}} \pm 0.00020_{\mathrm{evol}}$

$\alpha_s^{(3)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$

$\mu_{c,\mathrm{dec}} = 3$ GeV, $\quad \mu_{b,\mathrm{dec}} \equiv \mu_b$



0-loop dec.
1-loop dec.
2-loop dec.
3-loop dec.
4-loop dec. ($\beta_4 = 0$)
4-loop dec. ($\beta_4 = 150$)

$\delta\alpha_s(\text{trunc. p.t.}) \approx 0.0002$
$\delta\alpha_s(\mu_{b,\mathrm{dec}}) \approx 0.0002$

$\alpha_s^{(3)}(M_\tau) = 0.332 \pm 0.005_{\mathrm{exp}} \pm 0.015_{\mathrm{th}}$ [Baikov,Chetyrkin,Kühn'08]
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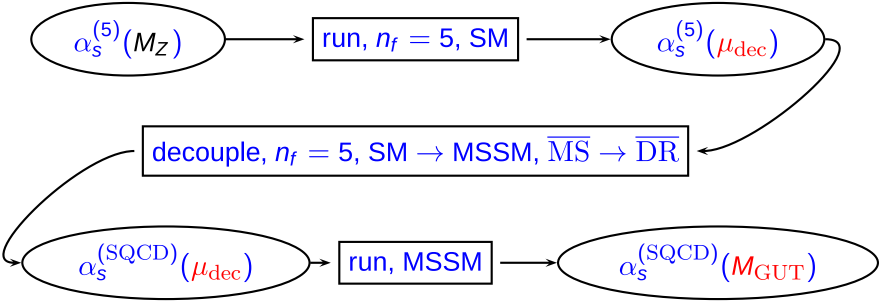
$\Rightarrow$
$\alpha_s^{(5)}(M_Z) = 0.11795 \pm 0.00038_{\mathrm{exp}} \pm 0.00063_{\mathrm{th}} \pm 0.00020_{\mathrm{evol}}$

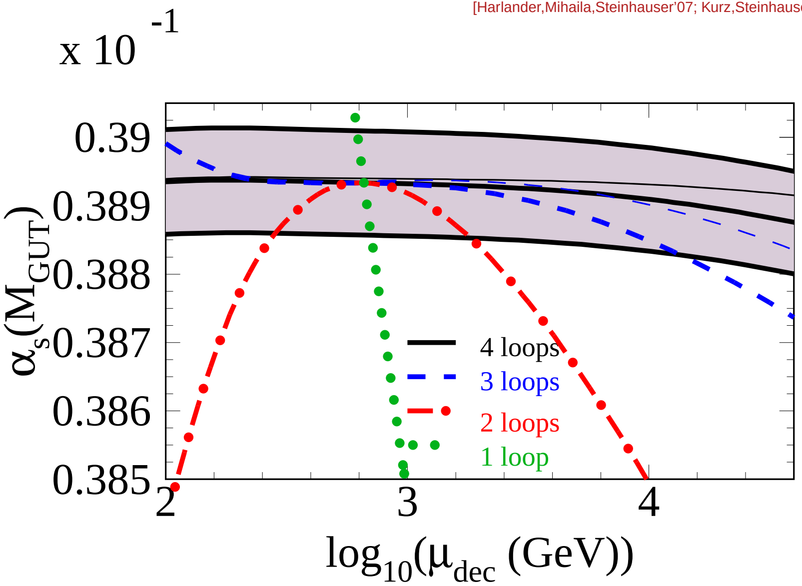
RunDec [Chetyrkin,Kühn,Steinhauser'00], CRunDec [Schmidt,Steinhauser'12]

QCD: $\alpha_s^{(5)}(M_Z) \Rightarrow \alpha_s^{(\mathrm{SQCD})}(M_{\mathrm{GUT}})$



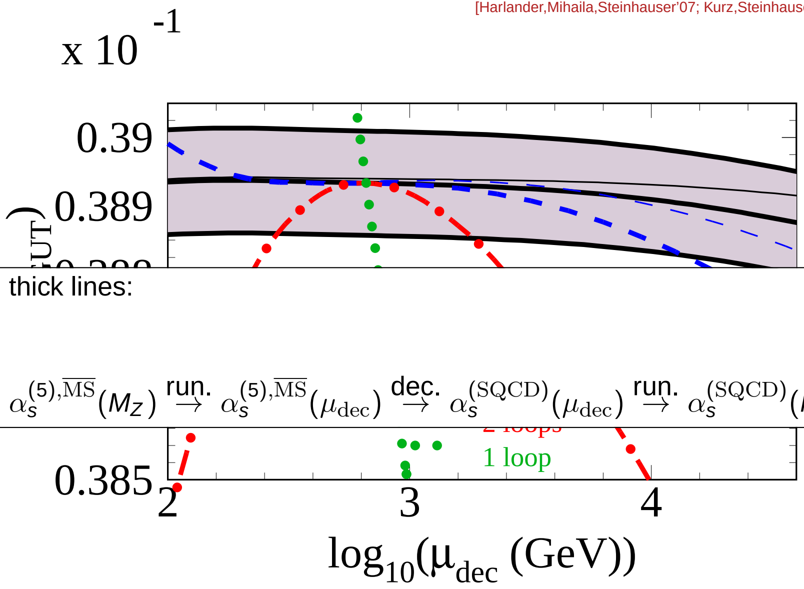
SUSY-QCD: $\alpha_s^{(5)}(M_Z) \Rightarrow \alpha_s(M_{\rm GUT})$

[Harlander,Mihaila,Steinhauser'07; Kurz,Steinhauser,Zerf'12]



SUSY-QCD: $\alpha_s^{(5)}(M_Z) \Rightarrow \alpha_s(M_{\rm GUT})$

[Harlander,Mihaila,Steinhauser'07; Kurz,Steinhauser,Zerf'12]

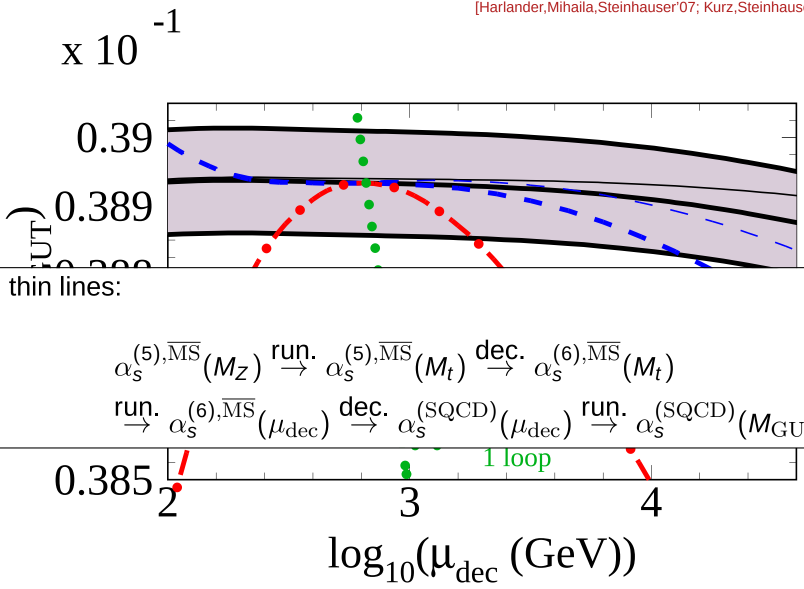


thick lines:

$$\alpha_s^{(5),\overline{\rm MS}}(M_Z) \overset{\text{run.}}{\rightarrow} \alpha_s^{(5),\overline{\rm MS}}(\mu_{\rm dec}) \overset{\text{dec.}}{\rightarrow} \alpha_s^{(\rm SQCD)}(\mu_{\rm dec}) \overset{\text{run.}}{\rightarrow} \alpha_s^{(\rm SQCD)}(M_{\rm GUT})$$

SUSY-QCD: $\alpha_s^{(5)}(M_Z) \Rightarrow \alpha_s(M_{\mathrm{GUT}})$

[Harlander,Mihaila,Steinhauser'07; Kurz,Steinhauser,Zerf'12]



thin lines:

$$\alpha_s^{(5),\overline{\mathrm{MS}}}(M_Z) \overset{\text{run.}}{\rightarrow} \alpha_s^{(5),\overline{\mathrm{MS}}}(M_t) \overset{\text{dec.}}{\rightarrow} \alpha_s^{(6),\overline{\mathrm{MS}}}(M_t)$$

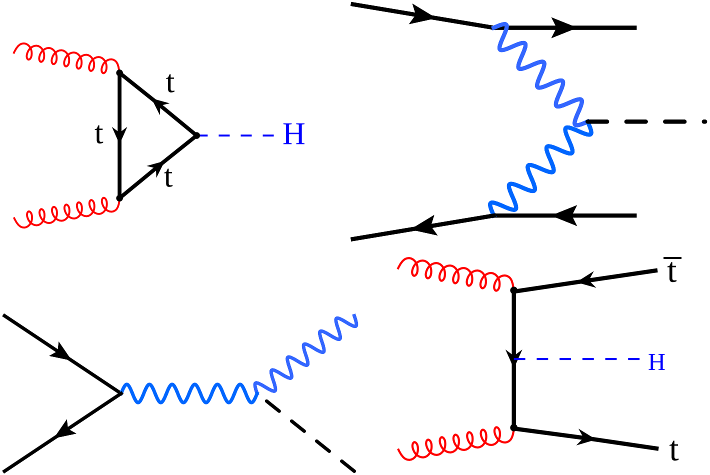
$$\overset{\text{run.}}{\rightarrow} \alpha_s^{(6),\overline{\mathrm{MS}}}(\mu_{\mathrm{dec}}) \overset{\text{dec.}}{\rightarrow} \alpha_s^{(\mathrm{SQCD})}(\mu_{\mathrm{dec}}) \overset{\text{run.}}{\rightarrow} \alpha_s^{(\mathrm{SQCD})}(M_{\mathrm{GUT}})$$

LET: Low-Energy-Theorem

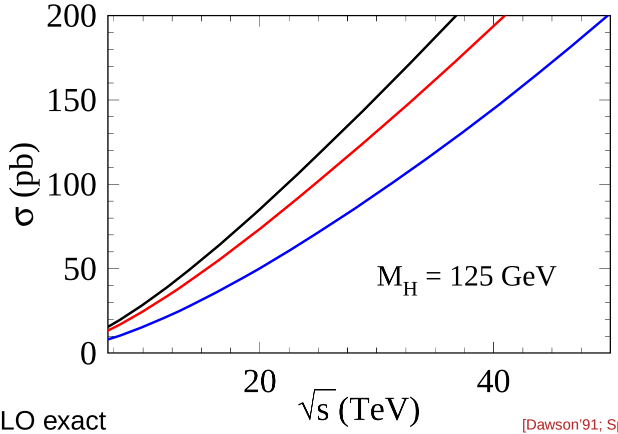
- effective Higgs-gluon coupling: $\mathcal{L}^{\rm eff} = \frac{H}{v} C_1 \frac{1}{4} \left(G^a_{\mu\nu}\right)^2$
- [Chetyrkin,Kniehl,Steinhauser'98]: $C_1 = \frac{\partial \ln \zeta_{\alpha_s}}{\partial \ln m_t}$
- simple-minded explanation: $\frac{\partial}{\partial m_t} \frac{1}{m_t - \not{p}} \sim \frac{1}{m_t - \not{p}} \frac{1}{m_t - \not{p}}$
- see also [Kilian'95; Kniehl,Spira'95]
- applications: [Schröder,Steinhauser'06; Chetyrkin,Kühn,Sturm'06; Degrassi,Slavich'08; Mihaila,Reisser'12; Kurz,Steinhauser,Zerf'12; . . .]

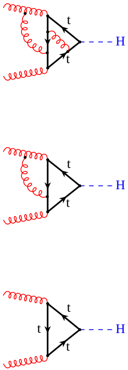
Higgs boson production

Higgs production in SM



Gluon fusion to NNLO





NLO exact

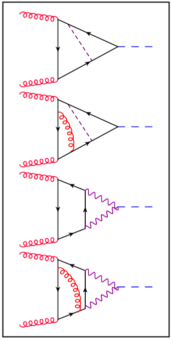
NNLO $m_t \to \infty$

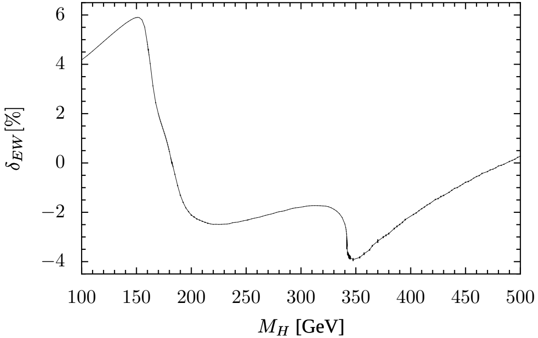
[Dawson'91; Spira,Djouadi,Graudenz,Zerwas'91'93]

[Harlander,Kilgore'02; Anastasiou,Melnikov'02; Ravindran,Smith,v.Neerven'03]

Gluon fusion: EW corrections

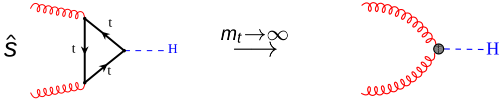
[Actis,Passarino,Sturm,Uccirati'08]





- EW $\mathcal{O}(G_F m_t^2)$: $\approx 0.5\%$ [Djouadi,Gambino'94]
- EW/QCD $\mathcal{O}(\alpha_s G_F m_t^2)$: $\approx 0.5\%$ [Steinhauser'98]
- EW $\mathcal{O}(\alpha)$: $\approx 5\%$ [Aglietti et al.'04; Degrassi,Maltoni'04; Actis,Passarino,Sturm,Uccirati'08]
- EW/QCD $\mathcal{O}(n_f \alpha \alpha_s)$: $\approx 1\%$ ➪ "complete factorization" [Anastasiou,Boughezal,Petriello'09]
- EW to real rad.: $\approx 1\%$ [. . . Keung,Petriello'09; Brein'10; Anastasiou, et al.'11]

Beyond “heavy-top approximation”

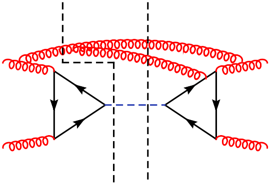


What about $\boxed{\frac{\hat{s}}{m_t^2}}$ and $\boxed{\frac{M_H^2}{m_t^2}}$ terms?

➪ Do calculation in full theory in the limit of large m_t

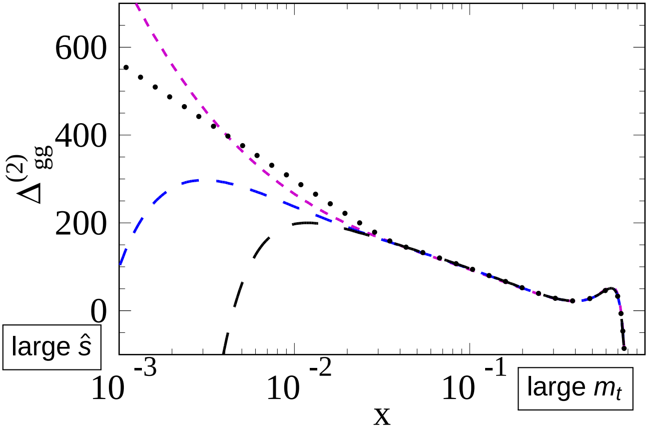
➪ Check effective-theory result.
How big are the $1/m_t^2$ corrections?

[Harlander,Ozeren’09; Pak,Rogal,Steinhauser’09’11; Harlander,Mantler,Marzani,Ozeren’10]



NNLO partonic cross section

[Harlander,Ozeren'09; Pak,Rogal,Steinhauser'09'11; Harlander,Mantler,Marzani,Ozeren'10]

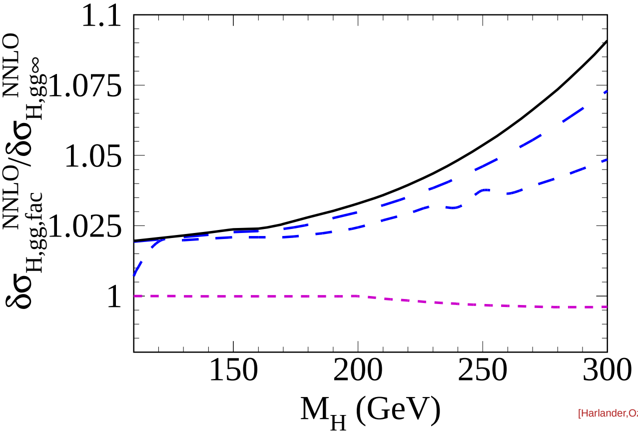


$M_H = 120$ GeV

$x = \frac{M_H^2}{\hat{s}}$

Dotted: match to $\hat{s} \to \infty$ result [Marzani,Ball,Del Duca,Forte,Vicini'08]

NNLO hadronic results



[Harlander,Ozeren'09; Harlander,Mantler,Marzani,Ozeren'10]

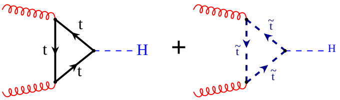
[Pak,Rogal,Steinhauser'09'11]

$M_H = 125$ GeV: NNLO corrections change by $\approx 2\%$

➪ $\sigma_{\rm tot}$ changes by $< 0.5\%$

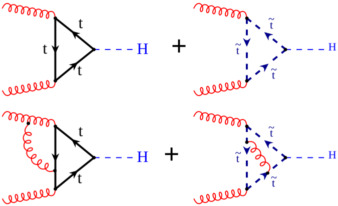
➪ effective theory works well!

Gluon fusion to NNLO SUSY-QCD



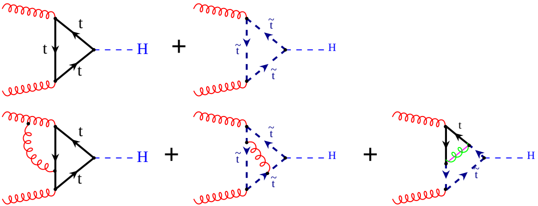
LO

Gluon fusion to NNLO SUSY-QCD



LO

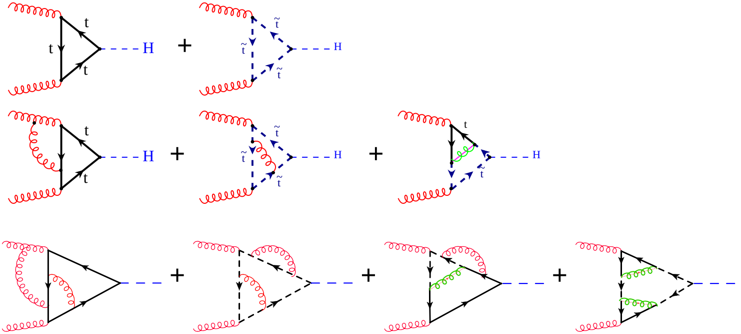
Gluon fusion to NNLO SUSY-QCD



LO

NLO

Gluon fusion to NNLO SUSY-QCD



LO

NLO

NNLO

+ 57936 more Feynman diagrams

Gluon fusion to NNLO SUSY-QCD

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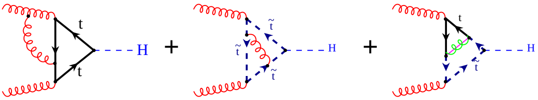
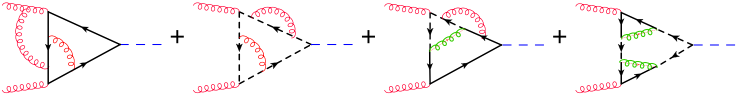


NLO

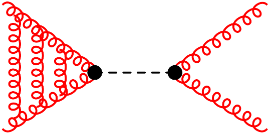


NNLO

- NLO SUSY-QCD, $M_H \ll M_{SUSY}$, $(t, \tilde{t})$ sector [Harlander,Steinhauser'03'04; Degrassi,Slavich'08]
- NLO Squark loops [Aglietti,Bonciani,Degrassi,Vivini'07; Mühlleitner,Spira'08]
- NLO SUSY-QCD, $(b, \tilde{b})$ sector [Anastasiou et al.'07; Degrassi,Slavich'11; Harlander,Hofmann,Mantler'11]
- NLO SUSY-QCD “full theory” [Anastasiou,Bucherer,Daleo'07; Mühlleitner,Rzehak,Spira]
- `SusHi` [Harlander,Liebler,Mantler'13]
- NNLO SUSY-QCD, $M_H \ll M_{SUSY}$, $(t, \tilde{t})$ sector [Pak,Steinhauser,Zerf'10'12]

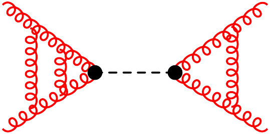
Towards N^{3}LO

Estimated uncertainty from higher orders: $\approx$ 10% ➪ N^{3}LO needed

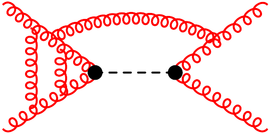


[Baikov,Chetyrkin,Smirnov,Smirnov, Steinhauser'09], [Gehrmann,Glover,Huber,IkizlerliStuderus'10]

complete

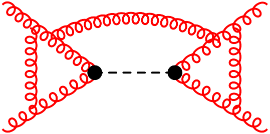


complete



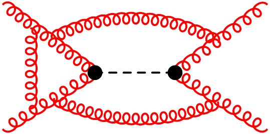
[Duhr,Gehrmann'13], [Li,Zhu'13]

soft limit



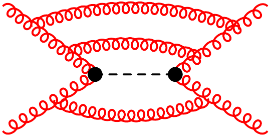
[Anastasiou,Duhr,Dulat,Herzog, Mistlberger'13], [Kilgore'13]

complete



[Anastasiou,Duhr,Dulat,Furlan,Gehrmann, Herzog,Mistlberger'14], [Li,von Manteuffel,Schabinger,Zhu'14]

soft limit



[Anastasiou,Duhr,Dulat,Mistlberger'13]

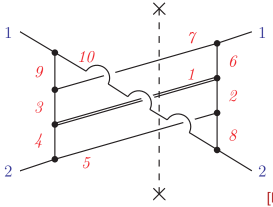
soft limit

Leading term for $\sigma_{\rm tot}$ in soft expansion: [Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Mistlberger'14]

"N^{3}LO approx.": [Ball,Bonvini,Forte,Marzani,Ridolfi'13'14], [de Florian,Mazzitelli,Moch,Vogt'14]

Towards N^{3}LO @ SFB/TR 9

- $\mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 \frac{1}{4} \left(G^a_{\mu\nu} \right)^2$ available to N^{3}LO (i.e. C_1 to 4 loops)
  [Chetyrkin,Kniehl,Steinhauser'98; Schröder,Steinhauser'06; Chetyrkin,Kühn,Sturm'06]
- N^{3}LO virtual corrections [Baikov,Chetyrkin,Smirnov,Smirnov,Steinhauser'09]
- NNLO cross section to $\mathcal{O}(\epsilon)$ [Pak,Rogal,Steinhauser'12; Höschele,Hoff,Pak,Steinhauser,Ueda'12]
- Convolutions of LO, NLO and NNLO partonic cross section with 1-, 2- and 3-loop splitting functions [Höschele,Hoff,Pak,Steinhauser,Ueda'12'13]
  public computer code: `MT.m`
- Techniques for MIs: choose "canonical basis"



[Höschele,Hoff,Ueda'14] (uses ideas of [Henn'13])

Conclusions

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- β functions in SM to 3 loops
- decoupling for α_s (SM, MSSM)
- H production in SM (NNLO, N^3LO) and MSSM (NNLO)