

Leptonic Light-by-Light Scattering Contributions to g-2

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DESY

in collaboration with

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Outline

- 1 Introduction
- 2 Leptonic contributions
- 3 Hadronic contributions
- 4 Conclusions

Lepton anomalous magnetic moment

Best experimentally measured and theoretically predicted quantity

- electron

$$a_e|_{\text{exp}} = 0.001\ 159\ 652\ 180\ 73(28)$$

$$a_e|_{\text{theo}} = 0.001\ 159\ 652\ 181\ 78(6)(4)(3)(77)$$

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- muon

$$a_\mu|_{\text{exp}} = 0.001\ 165\ 920\ 80(54)(33)[63]$$

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$$a_\mu|_{\text{exp}} - a_\mu|_{\text{theo}} = 240 \times 10^{-11} \quad \text{2.9}\sigma \text{ diff.}$$

Pure Leptonic Contributions

- analytical results

- one loop: $a_\mu^{(1)} = \frac{1}{2}$

[Schwinger 1948]

- two loop

[Petermann; Sommerfeld 1957]

- three loop

[Laporta, Remiddi 1996]

- four loop: see this talk

- numerical results

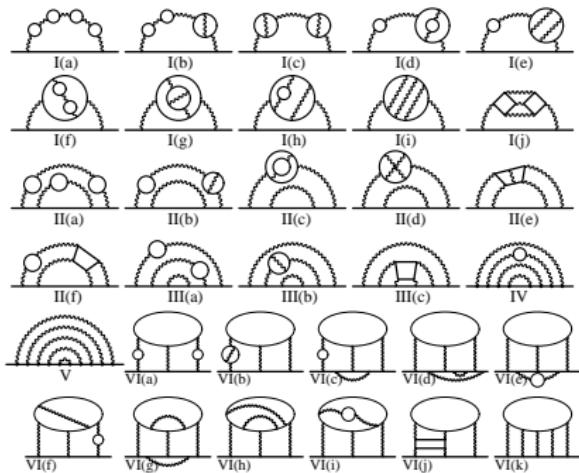
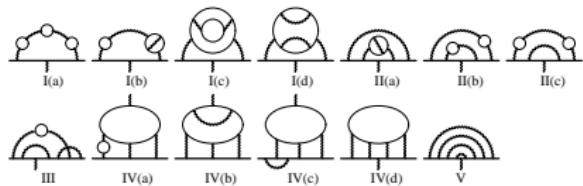
- four loop

[Kinoshita et al]

- five loop

[Aoyama, Hayakawa, Kinoshita, Nio 2012]

QED contributions @ 4 and 5 loops



$$a_\mu^{4\ell} = 130.8796(63) \left(\frac{\alpha}{\pi}\right)^4$$

$$a_\mu^{5\ell} = 753.29(1.04) \left(\frac{\alpha}{\pi}\right)^5$$

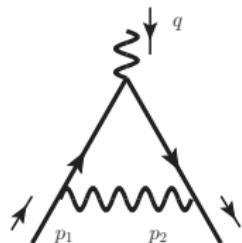
$$\begin{aligned} a_\mu(QED, \alpha(Rb)) &= 116584718951(9)(19)(7)(77) \times 10^{-14} \\ a_\mu(QED, \alpha(a_e)) &= 116584718845(9)(19)(7)(30) \times 10^{-14} \end{aligned}$$

Contributions from different orders

order	with $\alpha^{-1}(Rb)[\times 10^{-11}]$	with $\alpha^{-1}(a_e)[\times 10^{-11}]$
2	116 140 973.318 (77)	116 140 973.212 (30)
4	413 217.6291 (90)	413 217.6284 (89)
6	30 141.902 48 (41)	30 141.902 39 (40)
8	381.008 (19)	381.008 (19)
10	5.0938 (70)	5.0938 (70)
a_μ (QED)	116 584 718.951 (80)	116 584 718.845 (37)

Note: four-loop (eighth order contribution) is larger than the difference between SM prediction and measured value.

Definition + Calculation

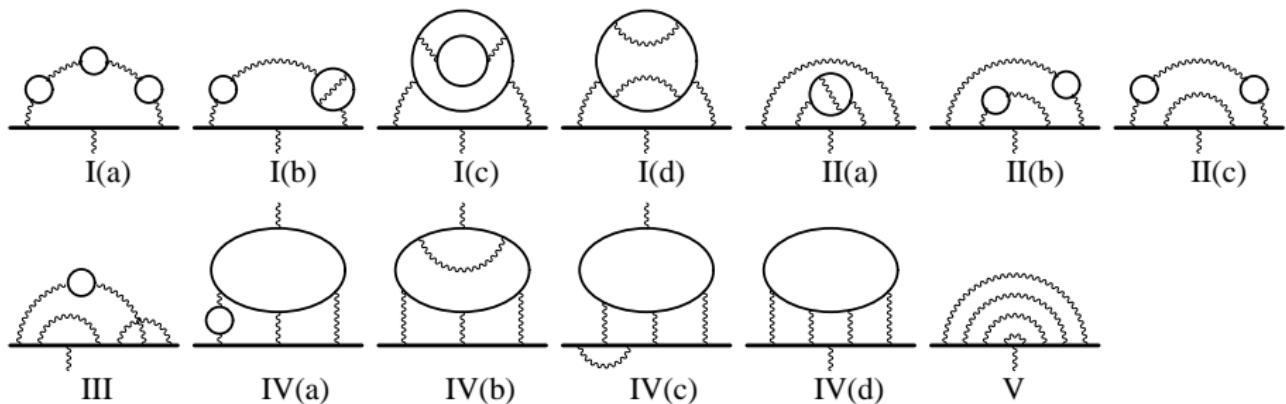


$$= (-ie)\bar{u}(p_2) \left\{ \gamma^\mu F_E(q^2) + i \frac{\sigma^{\mu\nu} q^\nu}{2m} F_M(q^2) \right\} u(p_1)$$

$$a_\mu = F_M(0)$$

Contributions from diagrams with electrons and/or tau leptons can be calculated using asymptotic expansions \Rightarrow reduction to single-scale problem.

Diagram classes at four loops



[Aoyama et al '2012]

Tau contributions

- perform asymptotic expansion in m_μ^2/m_τ^2 using q2e/exp
[Harlander, Seidensticker, Steinhauser]
- ⇒ big simplification: only massive tadpoles remain @ four loops
- reduce to master integrals using Crusher and FIRE [PM, Seidel; Smirnov]
- all needed master integrals known analytically
- very good convergence since $m_\mu^2/m_\tau^2 \ll 1$

Results for Tau contribution

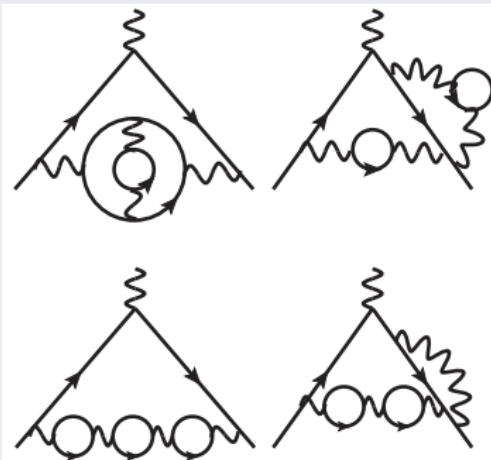
group	$10^2 \cdot A_{2,\mu}^{(8)}(M_\mu/M_\tau)$	
	[Kurz et al]	[Aoyama et al]
I(a)	0.00324281(2)	0.0032(0)
I(b) + I(c) + II(b) + II(c)	-0.6292808(6)	-0.6293(1)
I(d)	0.0367796(4)	0.0368(0)
III	4.5208986(6)	4.504(14)
II(a) + IV(d)	-2.316756(5)	-2.3197(37)
IV(a)	3.851967(3)	3.8513(11)
IV(b)	0.612661(5)	0.6106(31)
IV(c)	-1.83010(1)	-1.823(11)

fast convergence

$$A_{2,\mu}^{(8)}(M_\mu/M_\tau) = 0.0421670 + 0.0003257 + 0.00000015 = 0.0424941(2)(53)$$

Contributions involving electrons

n_e^3 and n_e^2 part



- leading term in expansion in m_e/m_μ calculable using massless electrons
- logarithmic terms can be recovered by going from the $\overline{\text{MS}}$ scheme to the on-shell scheme.

Results: $n_e^3, n_e^2 n_\mu, n_e^2$

$$\begin{aligned} a_\mu^{(43)} &= \frac{1}{54} L_{\mu e}^3 - \frac{25}{108} L_{\mu e}^2 + \left(\frac{317}{324} + \frac{\pi^2}{27} \right) L_{\mu e} - \frac{2\zeta_3}{9} - \frac{25\pi^2}{162} - \frac{8609}{5832} \\ &\approx 7.19666, \end{aligned}$$

[Lee et al; Laporta; Aguilar, Greynat, De Rafael]

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[Lee et al; Laporta; Aguilar, Greynat, De Rafael]

$$a_\mu^{(42)} = a_\mu^{(42)a} + \textcolor{red}{n_h} a_\mu^{(42)b}$$

$$a_\mu^{(42)a} = L_{\mu e}^2 \left[\pi^2 \left(\frac{5}{36} - \frac{\log 2}{6} \right) + \frac{\zeta_3}{4} - \frac{13}{24} \right] + \dots \approx -3.624\,27,$$

[Lee et al]

$$a_\mu^{(42)a} \Big|_{\text{num}} = -3.642\,04(1\,12),$$

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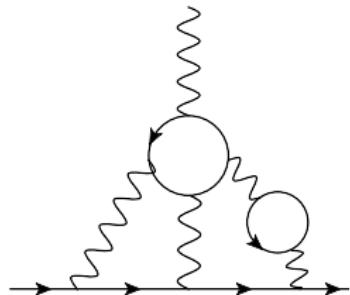
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$$\begin{aligned} a_\mu^{(42)b} &= \left(\frac{119}{108} - \frac{\pi^2}{9} \right) L_{\mu e}^2 + \left(\frac{\pi^2}{27} - \frac{61}{162} \right) L_{\mu e} - \frac{4\pi^4}{45} + \frac{13\pi^2}{27} + \frac{7627}{1944} \\ &\approx 0.494\,05 \end{aligned}$$

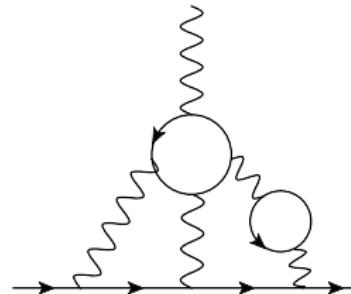
Leptonic light-by-light contributions

- Preliminary analysis for Kinoshita class IV(a)
dominant contribution @ four loops
- Calculation involves two parts
 - two electron loops
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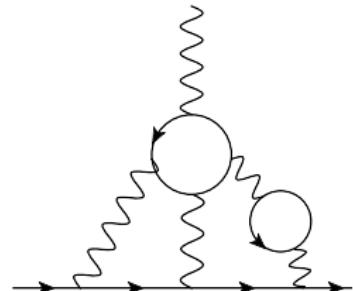
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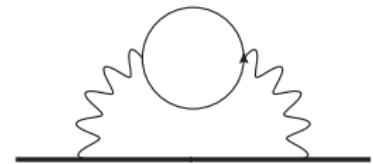


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 - specialized trial and error approach
- reduction to master integrals using Crusher and FIRE
- master integrals either calculated analytically or using FIESTA



Asymptotic Expansion



Consider the typical on-shell integral ($q^2 = M^2$)

$$\int dk^d dl^d \frac{1}{(k^2 + 2k \cdot q) k^4 ((k - l)^2 - m^2)(l^2 - m^2)}$$

we get the regions

- $k^2, l^2 \approx M^2$: expand in $m^2 \rightarrow$ massless on-shell propagator
- $k^2 \approx M^2, l^2 \approx m^2$: one-loop on-shell \times massive tadpole
- $k^2, l^2 \approx m^2$: expand in $k^2 \rightarrow$ new class of diagrams

$$\int dk^d dl^d \frac{1}{(k \cdot q) k^4 ((k - l)^2 - m^2)(l^2 - m^2)}$$

Preliminary results for class IV(a)

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- poles sum up to 0 (n_e^2 -part)
 - $1/\epsilon^2$

$$(-0.8206 \pm 0.0174) + (0.8223 \pm 0.0002)$$

$1/\epsilon$	$- 14.55$	$+ 3.29 \ell_e$	$- 3.29 \ell_\mu$
	$+ 6.30 \pm 0.12$	$+ (-3.28 \pm 0.07) \ell_e$	
	$+ 8.28 \pm 0.001$		$+ (3.29 \pm 0.001) \ell_\mu$

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- logs $\ln m_e/\mu$ and $\ln m_\mu/\mu$ combine to $\ln m_e/m_\mu$

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- logs $\ln m_e/\mu$ and $\ln m_\mu/\mu$ combine to $\ln m_e/m_\mu$
- final result **PRELIMINARY**

$$122 \pm 2 \quad \Leftrightarrow \quad 123.78551 \pm 0.00044 \text{ Aoyama et al}$$

Hadronic vacuum-polarization insertions

Can be obtained by integrating experimentally measured

$$R(s) = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma_{pt}}$$

over a kernel function

$$a_\mu^{(1)} = \frac{1}{3} \left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^\infty ds \frac{R(s)}{s} K^{(1)}(s)$$

with

$$K^{(1)}(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{M_\mu^2}}$$

For higher-order corrections need higher-order kernels.

Hadronic vacuum polarization insertions

- Analysis limited by measurement of $R(s)$, results to about half of the total error of a_μ
- LO analysis

$$a_\mu^{LO}(\text{had.v.p.}) = 6949.1(37.2)_{\text{exp}}(21.0)_{\text{rad}} \times 10^{-11}$$

[Hagiwara et al]

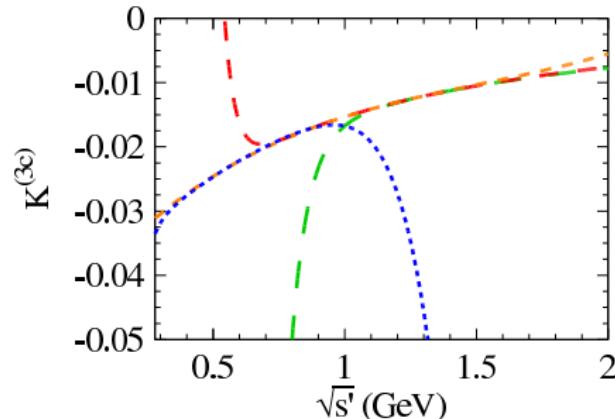
- NLO analysis

$$a_\mu^{\text{NLO}}(\text{had.v.p.}) = -98.4(0.6)_{\text{exp}}(0.4)_{\text{rad}} \times 10^{-11}$$

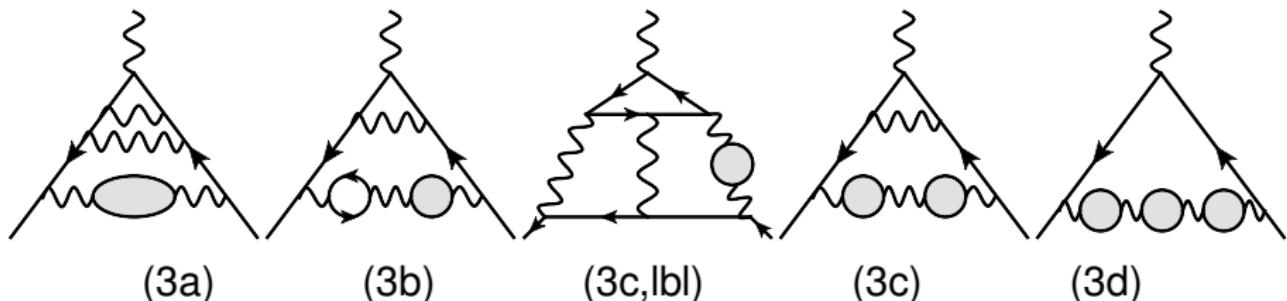
[Krause; Hagiwara et al]

Calculation of the three-loop kernels

- Hadronic vacuum polarization takes the form of a massive photon ($M^2 = s$)
- perform asymptotic expansion in m_μ^2/s . $m_\mu^2 < m_\pi^2 \leq s$
- for diagrams containing electrons expand in addition in $m_e \ll m_\mu$
- for double insertions expand in $s \ll s'$, $s \gg s'$ and $s \approx s'$



Hadronic vacuum polarization insertion @ NNLO



$$a_\mu^{(3a)} = 8.0 \times 10^{-11}$$

$$a_\mu^{(3b)} = -4.1 \times 10^{-11}$$

$$a_\mu^{(3b,\text{lbl})} = 9.1 \times 10^{-11}$$

$$a_\mu^{(3c)} = -0.6 \times 10^{-11}$$

$$a_\mu^{(3d)} = 0.005 \times 10^{-11}$$

first appearance of light-by-light
diagrams leads to an enhanced
NNLO contribution.

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[Krause; Hagiwara et al]

- NNLO analysis

$$a_\mu^{\text{NNLO}}(\text{had.v.p.}) = 12.4 \pm 0.1 \times 10^{-11}$$

slight reduction of discrepancy by 0.2σ

[Kurz,Liu,PM,Steinhauser]

Conclusions

- Calculated τ -lepton contribution to g-2 @ four loops.
- Calculated NNLO hadronic contributions to g-2
⇒ larger than expected impact
- Preliminary result for n_f^2 light-by-light contribution to g-2 @ four loops.
Dominant contribution @ four loops
⇒ agreement with results by Aoyama et al.