Next-to-next-to-leading order QCD techniques and predictions for the LHC

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Results within the SFB project B1

- Top-quark physics
  - ✓ Threshold expansions of hadronic cross sections
  - ✓ Results based on soft-gluon resummation at NNLL
  - ✓ Two-loop virtual corrections
  - ✓ Exact results for total cross sections at NNLO
  - ✓ Public software Top++
  - ✓ Complete off-shell effects in hadronic production with NLO accuracy
  - ✓ Production in association with two jets with NLO accuracy
  - ✓ Applications to PDF studies
  - ✓ Applications to BSM studies

- Subtraction scheme at NNLO
  - ✓ Development of STRIPPER (sector-improved residue subtraction)
  - ✓ Formulation in ‘t Hooft-Veltman regularization

- General interest
  - ✓ Soft-currents at one-loop with massive partons
  - ✓ Soft function for color octet production at threshold to NNLO

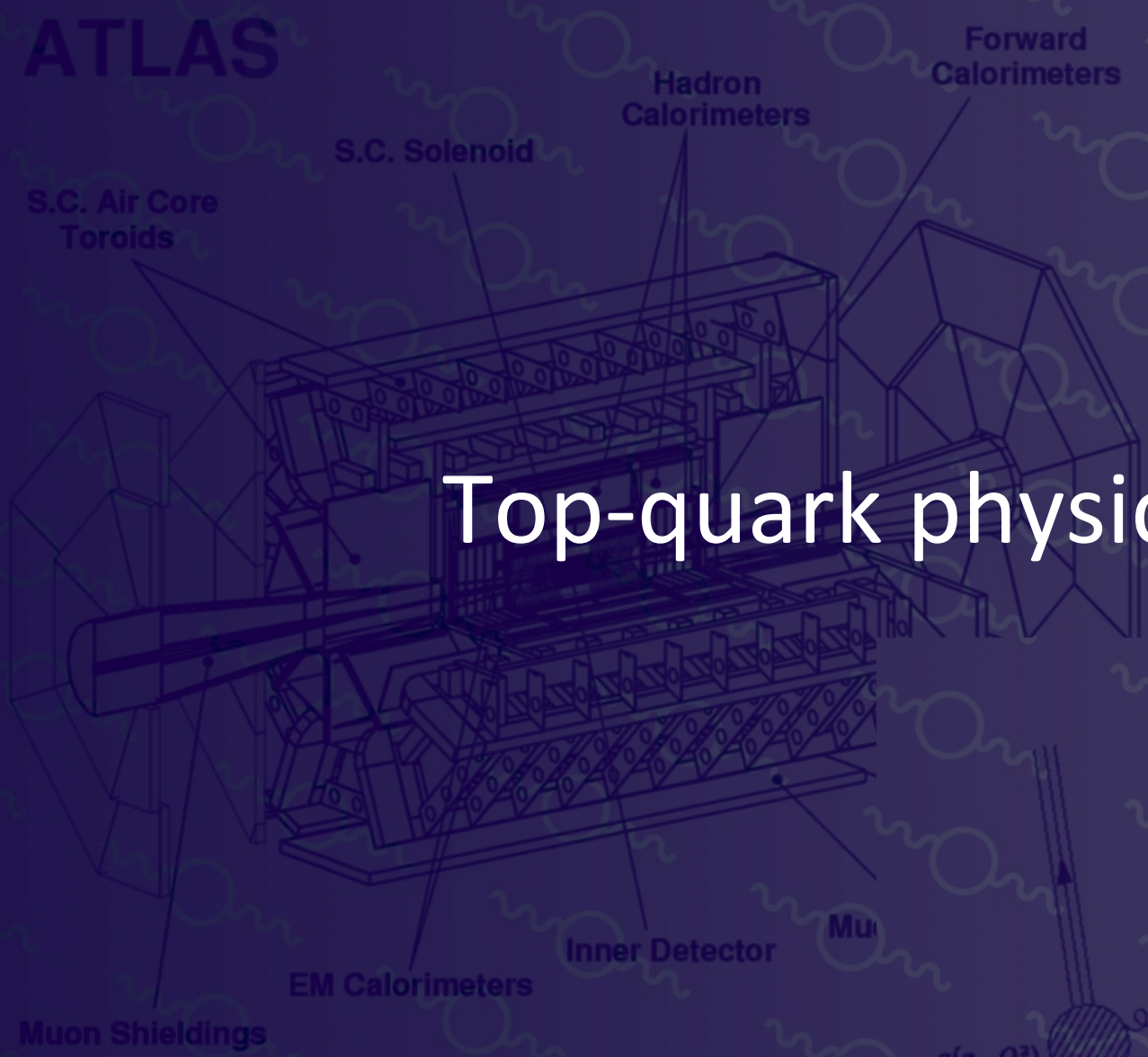
Collaborators

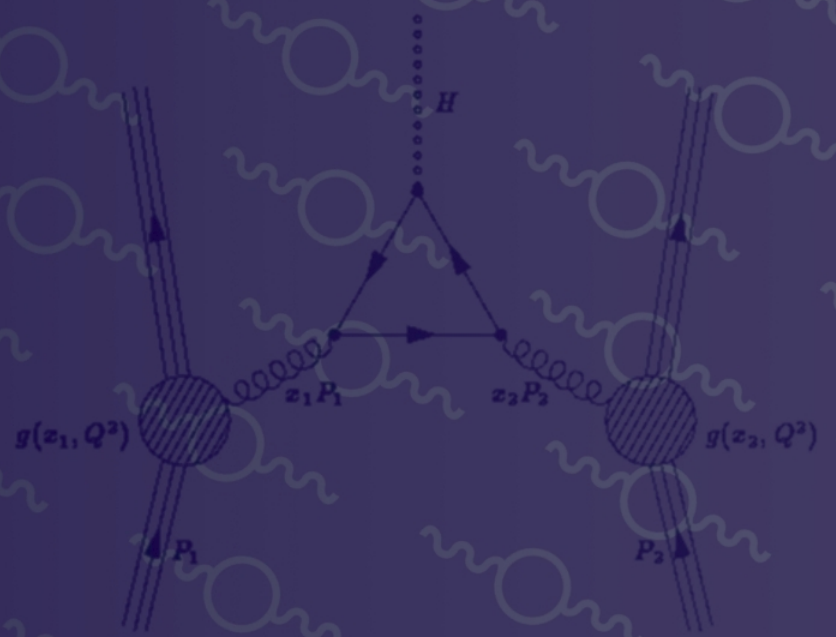
- PhD Students within the SFB
  1. Peter Bärnreuther
  2. Paul Fiedler
  3. David Heymes

- Postdocs within the SFB
  1. Giuseppe Bevilacqua

- Main external collaborators
  1. Alexander Mitov
  2. Costas Papadopoulos
  3. Malgorzata Worek

Top-quark physics

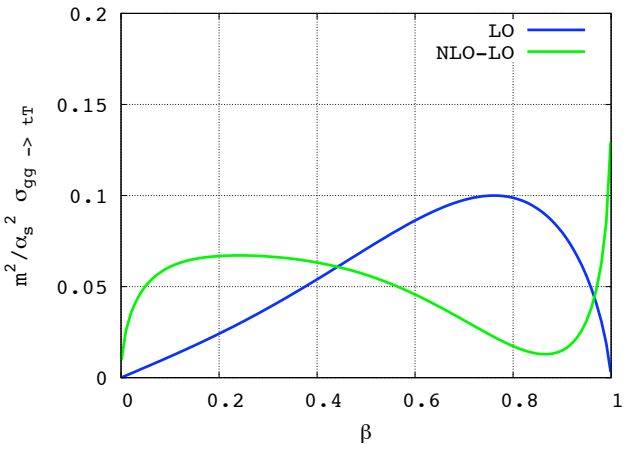




Dominant effects

soft gluon emission

large effects



Coulomb attraction/repulsion

small effects

t-channel gluon exchange

negligible effects

All effects can be resummed !!!

Predictions for hadron colliders

Bärnreuther, MC, Fiedler, Mitov `12 `13

NNLO + NNLL

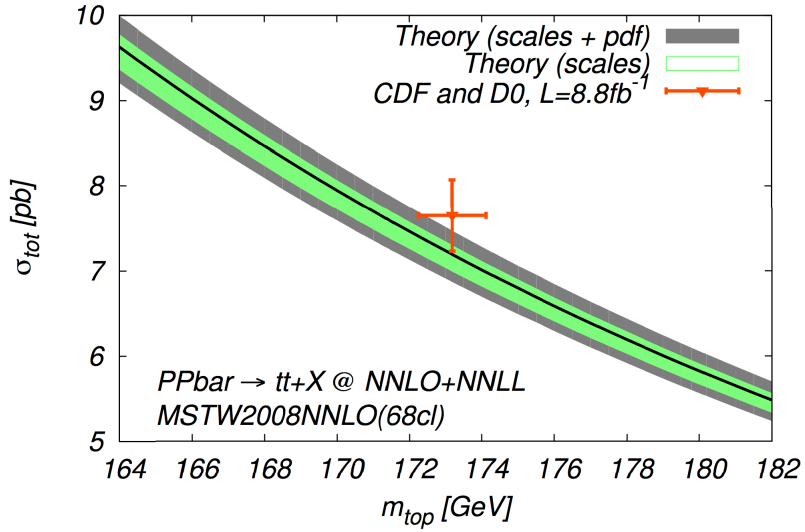
Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$^{+0.110(1.5\%)}_{-0.200(2.8\%)}$	$^{+0.169(2.4\%)}_{-0.122(1.7\%)}$
LHC 7 TeV	172.0	$^{+4.4(2.6\%)}_{-5.8(3.4\%)}$	$^{+4.7(2.7\%)}_{-4.8(2.8\%)}$
LHC 8 TeV	245.8	$^{+6.2(2.5\%)}_{-8.4(3.4\%)}$	$^{+6.2(2.5\%)}_{-6.4(2.6\%)}$
LHC 14 TeV	953.6	$^{+22.7(2.4\%)}_{-33.9(3.6\%)}$	$^{+16.2(1.7\%)}_{-17.8(1.9\%)}$

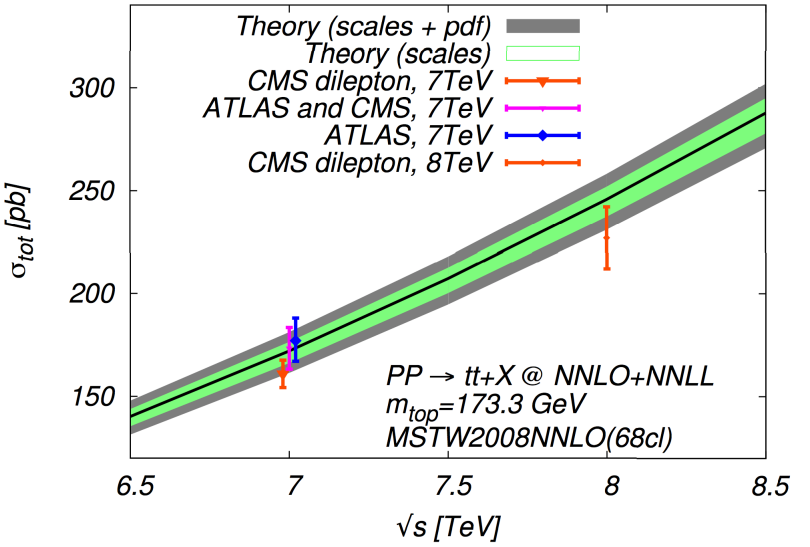
NNLO

Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$^{+0.259(3.7\%)}_{-0.374(5.3\%)}$	$^{+0.169(2.4\%)}_{-0.121(1.7\%)}$
LHC 7 TeV	167.0	$^{+6.7(4.0\%)}_{-10.7(6.4\%)}$	$^{+4.6(2.8\%)}_{-4.7(2.8\%)}$
LHC 8 TeV	239.1	$^{+9.2(3.9\%)}_{-14.8(6.2\%)}$	$^{+6.1(2.5\%)}_{-6.2(2.6\%)}$
LHC 14 TeV	933.0	$^{+31.8(3.4\%)}_{-51.0(5.5\%)}$	$^{+16.1(1.7\%)}_{-17.6(1.9\%)}$

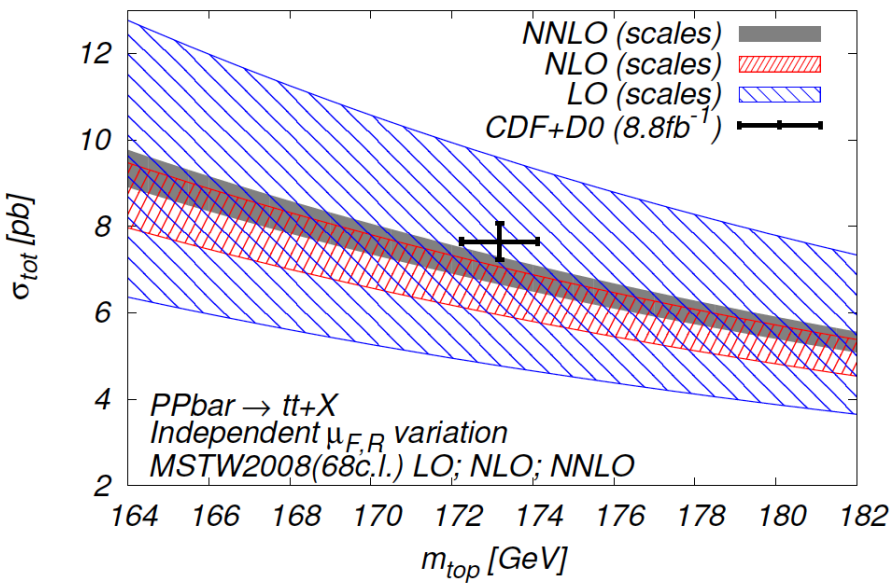
All numerical results obtained with top++

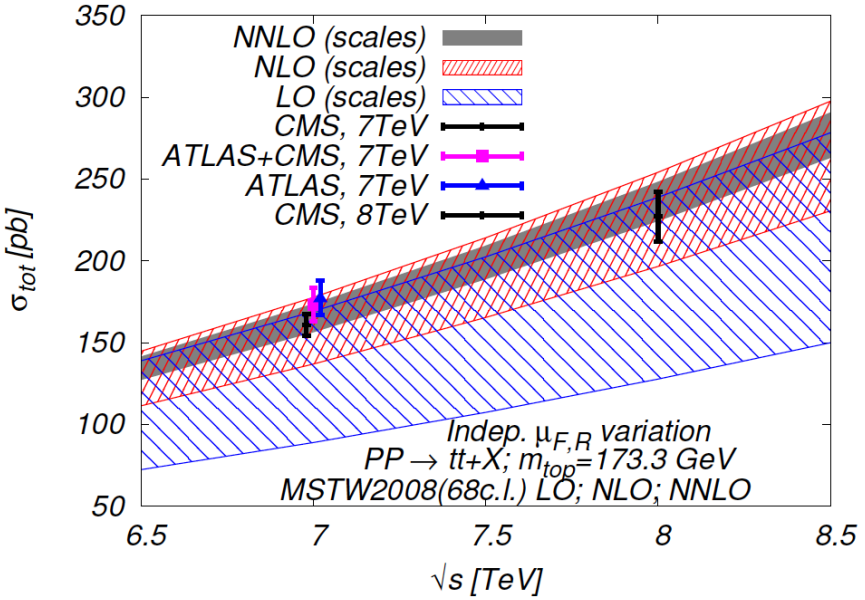
MC, Mitov `11

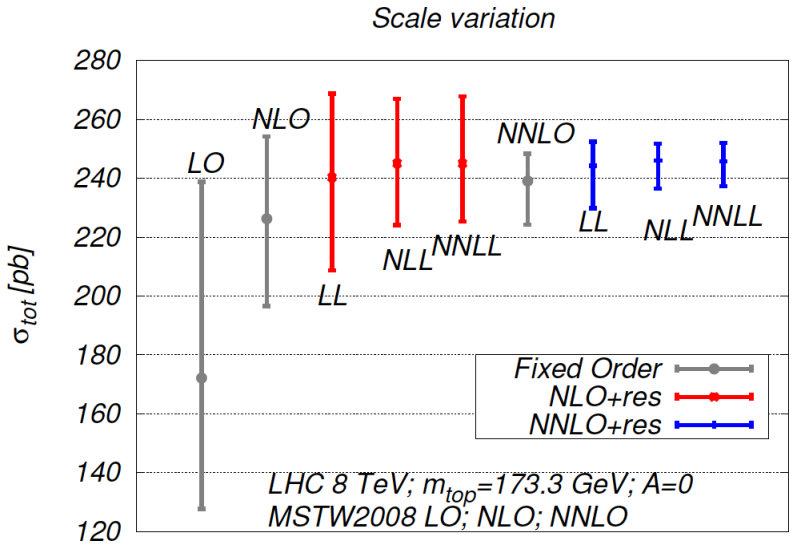




Perturbative convergence







Concurrent uncertainties:

Scales	~ 3%
pdf (at 68%cl)	~ 2-3%
α_s (parametric)	~ 1.5%
m_{top} (parametric)	~ 3%

Soft gluon resummation makes a difference:

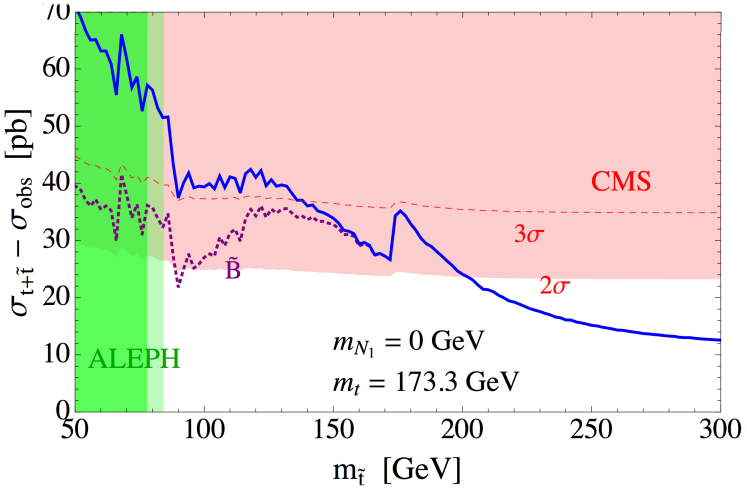
5% -> 3%

Status experiment

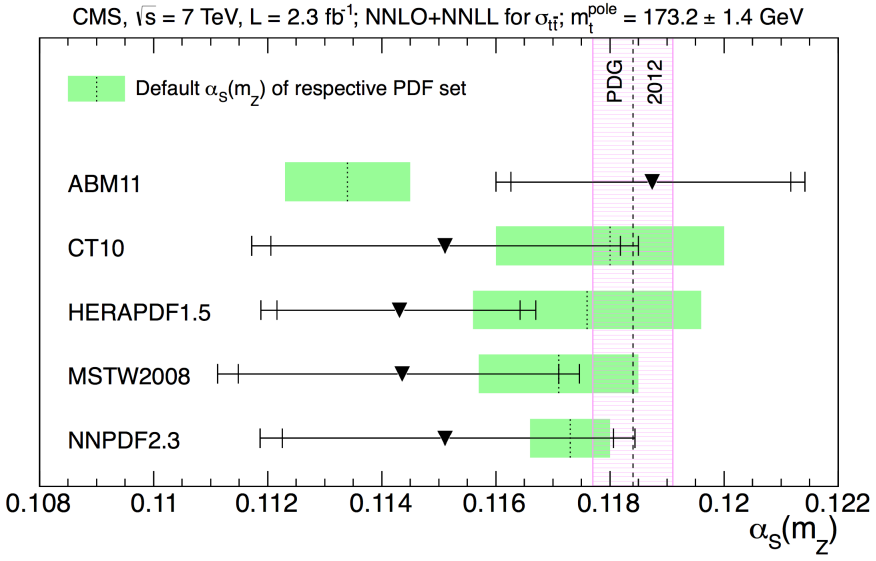
Summary of combinations of total cross section measurements

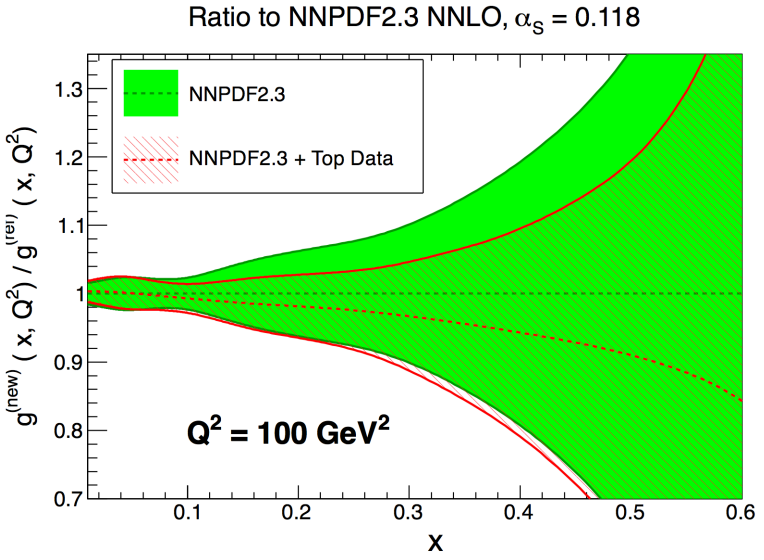
- **Combining measurements from CDF and DØ gives a Tevatron cross section at 1.96 TeV c-o-m of $7.60 \pm 0.41(5.4\%)$ pb , to be compared with the theoretical calculation (NNLO+NNLL) $7.24^{+0.23}_{-0.27}(3.4\%)$pb (Czakon et. al).**
- **Combining measurements from ATLAS and CMS gives a LHC cross section at 7 TeV c-o-m of $173 \pm 10(5.8\%)$ pb, to be compared to the theoretical calculation (NNLO+NNLL) of $172.0^{+6.4}_{-7.5}(4.1\%)$ pb.**
- **The most precise measurements at 8 TeV are from the ATLAS and CMS dilepton channel: $238 \pm 11(4.6\%)$ pb and $227 \pm 15(6.6\%)$ pb.**
  The NNLO+NNLL SM prediction is $245.8^{+8.8}_{-10.6}(4.0\%)$ pb.

Applications

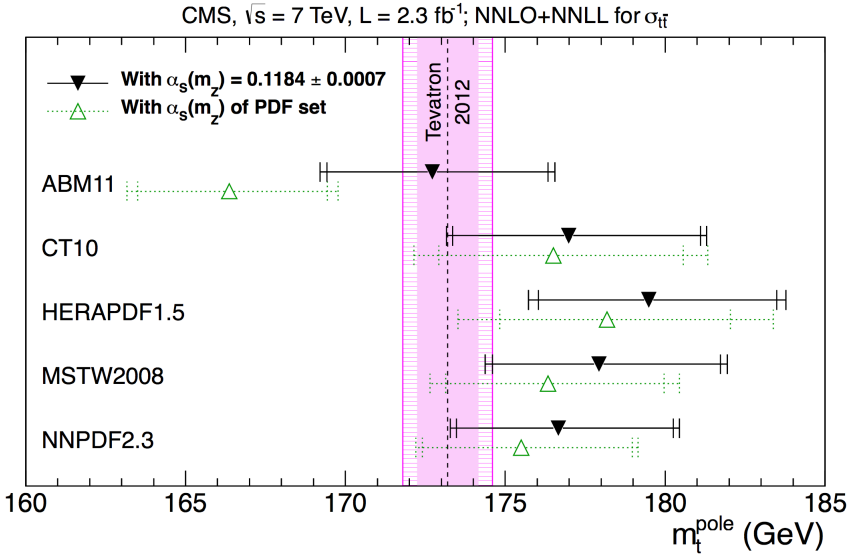


MC, Mitov, Papucci, Ruderman, Weiler, '14





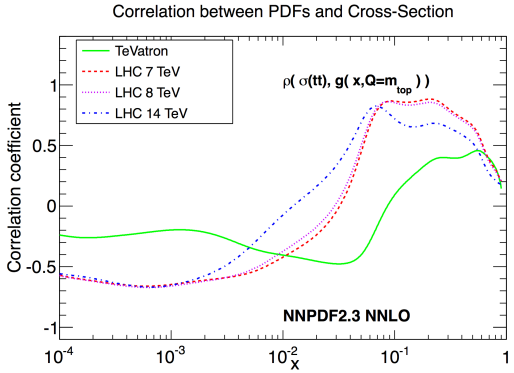
MC, Mangano, Mitov, Rojo `13

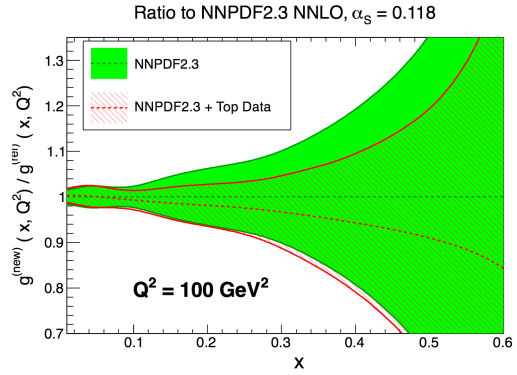


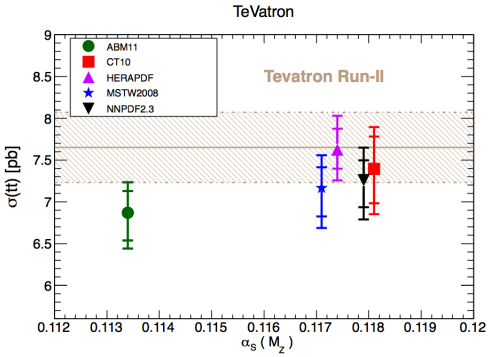
arXiv:1307.1907 (CMS-TOP-12-022)

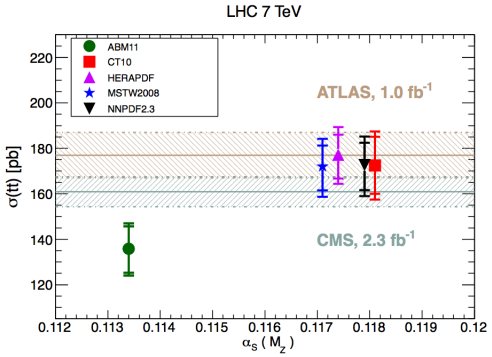
Application to PDF determination

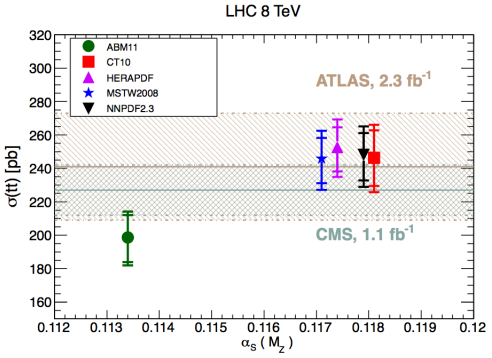
MC, Mangano, Mitov, Rojo `13

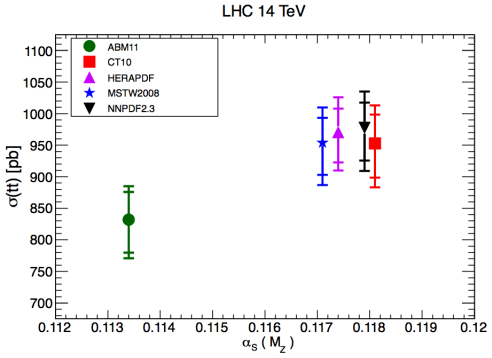


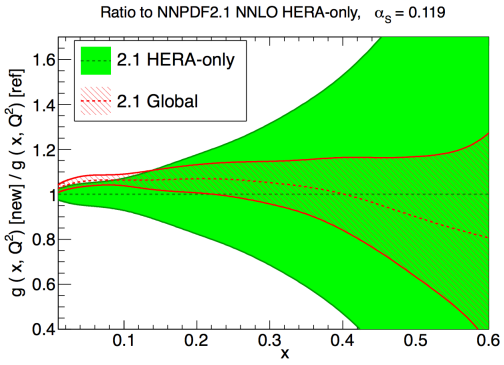


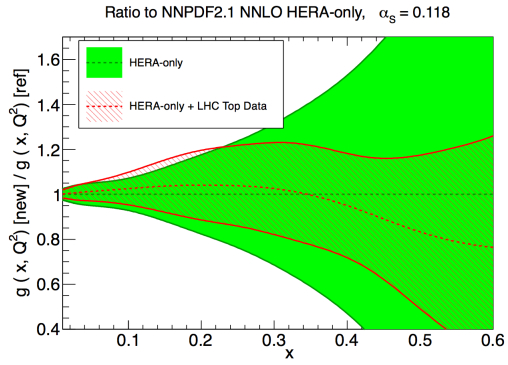




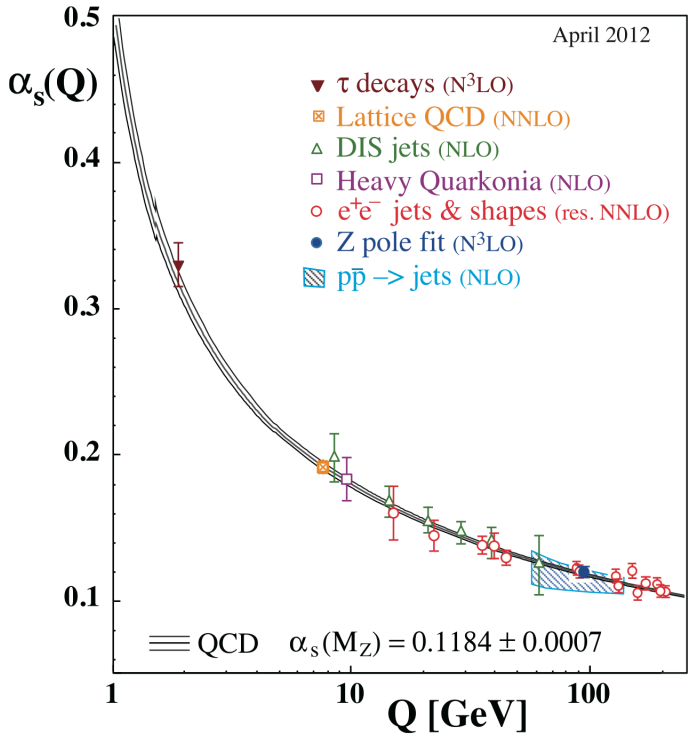


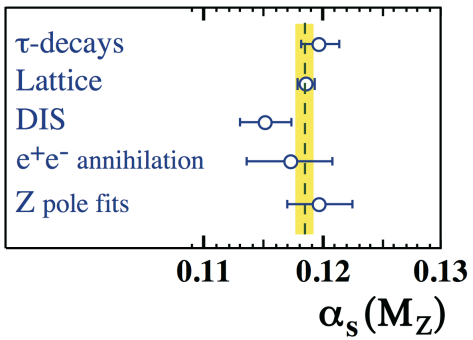






Application to α_s determination

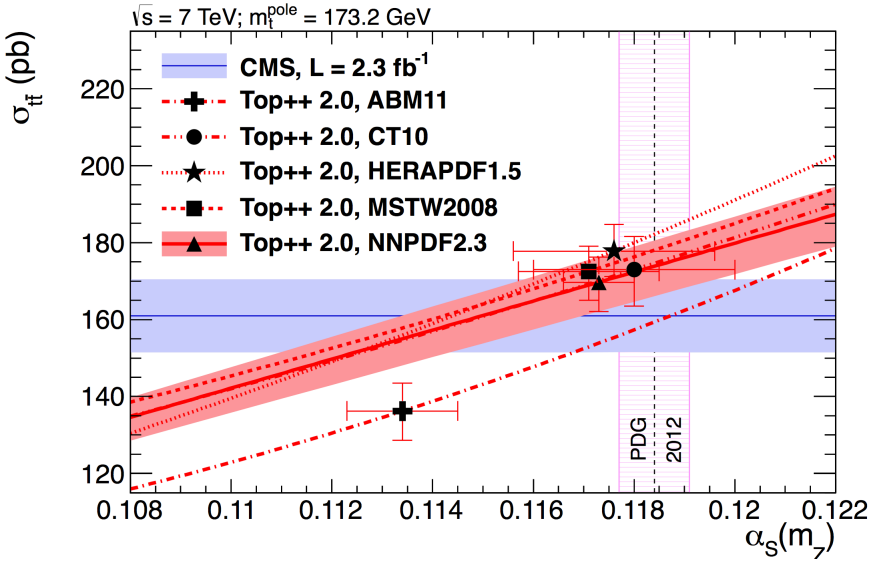


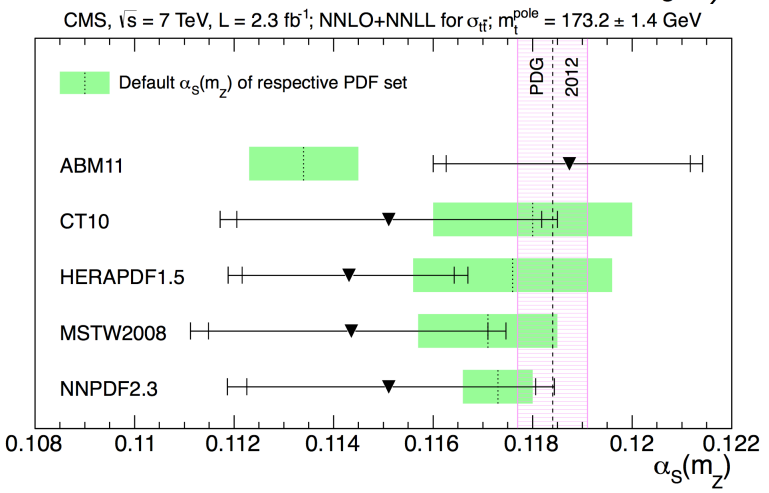


First determination of the strong coupling at NNLO from a hadron collider

Competitive to other collider determinations

arXiv:1307.1907 (CMS-TOP-12-022)



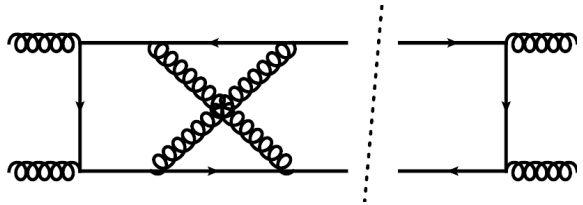


With NNPDF: $\alpha_S(m_Z) = 0.1151^{+0.0033}_{-0.0032}$

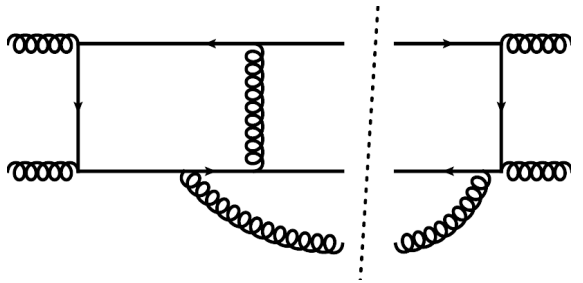
NNLO methods

There are 3 principal contributions:

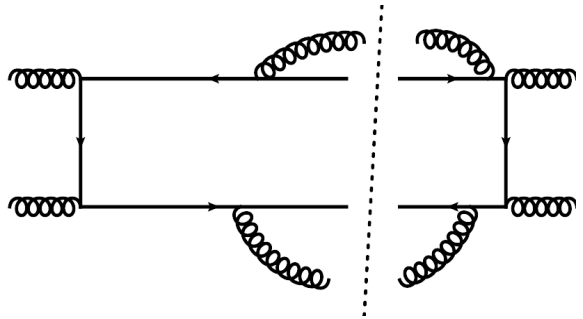
- 2-loop virtual corrections (V-V)
  MC `07 (quark annihilation)
  Bärnreuther, MC, Fiedler, in preparation (gluon fusion)



- 1-loop virtual with one extra parton (R-V)
  code by Stefan Dittmaier
  new subtraction terms: Bierenbaum, Czakon, Mitov `11



- 2 extra emitted partons at tree level (R-R)
  MC `10 `11 invention of a new subtraction scheme called STRIPPER



And 2 secondary contributions:

- Collinear subtraction for the initial state — Known, in principle. Done numerically. (the only non-differential contribution)
- One-loop squared amplitudes — Körner, Merebashvili, Rogal `07 (quark annihilation) done from scratch for gluon fusion

Additionally: divergences of two-loop amplitudes in quark annihilation: Ferroglia, Neubert, Pecjak, Yang '09

Two-loop amplitudes

Illustration: gluon fusion case

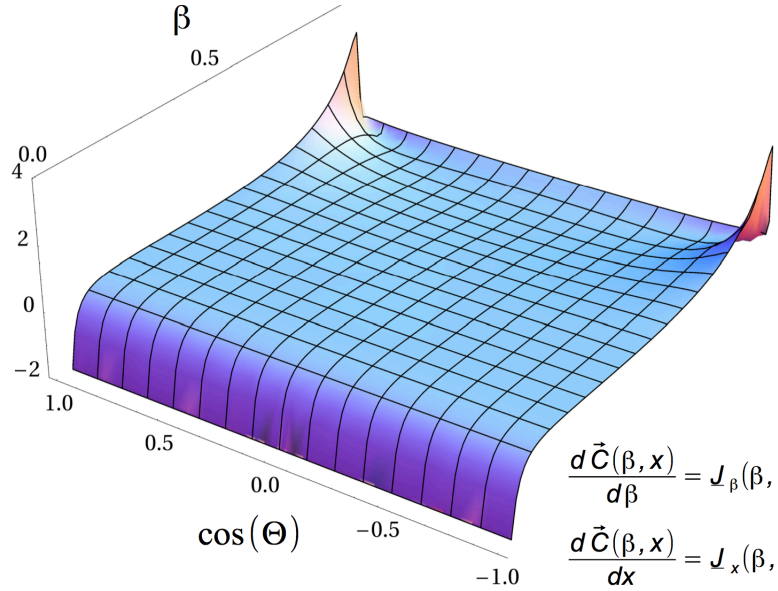
- 422 master integrals
- Solution by numerical differential equations & semi-analytic expansions
- Boundaries with Mellin-Barnes techniques
- About 1h per phase space point

$$M_i(\beta, x, \epsilon) = \sum_{k=Min}^{Max} C_{ik}(\beta, x)\, \epsilon^k$$

$M_i(\beta, x, \epsilon)$ Master integral

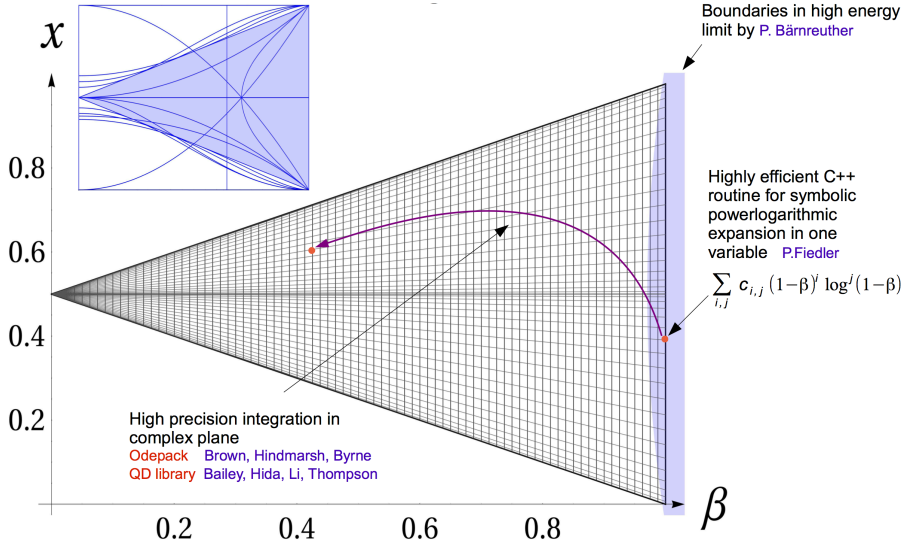
$C_{ik}(\beta, x)$ Coefficient of Laurent series

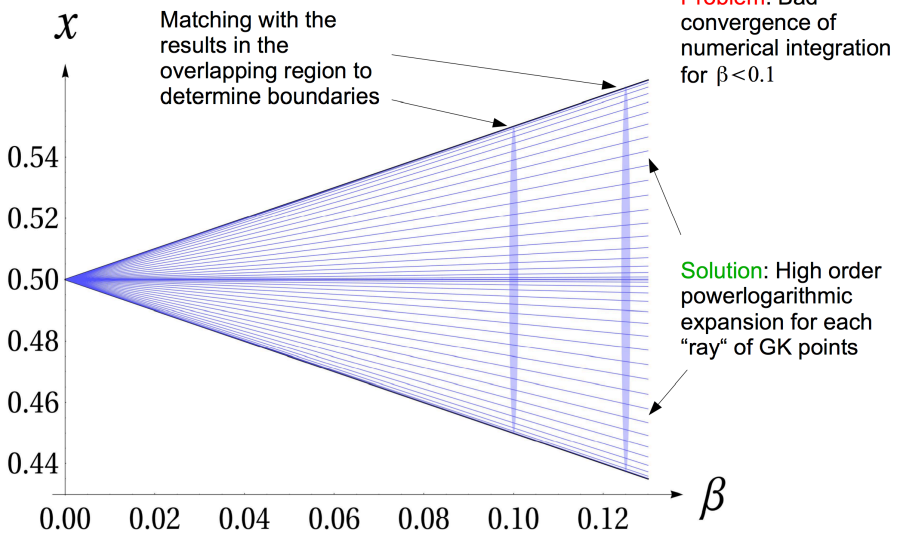
$$x \equiv \frac{m^2 - \hat{t}}{\hat{s}} = \frac{1}{2}(1 - \beta \cos(\Theta))$$



$$\frac{d\vec{C}(\beta, x)}{d\beta} = J_\beta(\beta, x)\, \vec{C}(\beta, x)$$

$$\frac{d\vec{C}(\beta, x)}{dx} = J_x(\beta, x)\, \vec{C}(\beta, x)$$





Finite remainder

Bärnreuther, MC, Fiedler '13

Matching coefficients @ NNLO

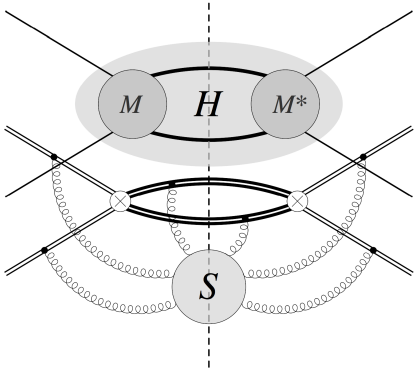
Beneke, MC, Falgari, Mitov, Schwinn `09

$$\sigma_{q\bar{q}}^{(2)} = \frac{3.60774}{\beta^2} + \frac{1}{\beta}\Big(- 140.368 \ln^2\beta + 32.106 \ln\beta + 3.95105\Big)$$
$$+910.222 \ln^4\beta - 1315.53 \ln^3\beta + 592.292 \ln^2\beta + 528.557 \ln\beta + C_{qq}^{(2)}$$

$$\sigma_{gg}^{(2)} = \frac{68.5471}{\beta^2} + \frac{1}{\beta}\Big(496.3 \ln^2\beta + 321.137 \ln\beta - 8.62261\Big)$$
$$+4608 \ln^4\beta - 1894.91 \ln^3\beta - 912.349 \ln^2\beta + 2456.74 \ln\beta + C_{gg}^{(2)}$$

- Matching coefficients can be obtained from factorization
  MC, Fiedler `13

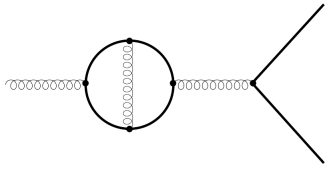
- Requires virtual corrections and soft function
  Bärnreuther, MC, Fiedler `13



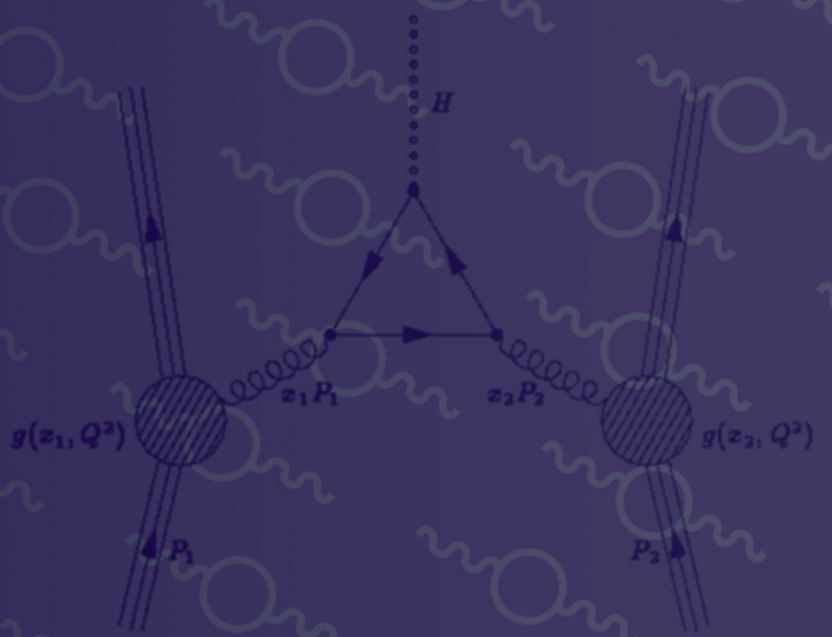
$$C_{gg}^{(2)} = 503.664 - 29.9249\, n_l + 0.142857\, {n_l}^2 = 357.611$$

$$C_{q\bar{q}}^{(2)} = 1104.08 - 42.9666\, n_l - 4.28168\, {n_l}^2 = 782.208$$

$$\sigma_{q\bar{q}}^{(2)} = \frac{3.60774}{\beta^2} + \frac{1}{\beta}\Big(- 140.368 \ln^2\beta + 32.106 \ln\beta + 3.95105\Big)$$
$$+910.222 \ln^4\beta - 1315.53 \ln^3\beta + 592.292 \ln^2\beta + 515.397 \ln\beta + C_{q\bar{q}}^{(2)}$$



Sector-improved residue subtraction scheme
STRIPPER



Subtraction scheme

General subtraction scheme invented as consequence of difficulties in treating double-real radiation for top quark pair production MC '10

Real radiation for tops in most channels calculated as first application in MC '11

Subsequently applied by others to several non-trivial problems:

- Z -> e^+e^- (as a warmup) Boughezal, Melnikov, Petriello '11
- top quark decay Brucherseifer, Caola, Melnikov '13
- b -> $X_u$ev Brucherseifer, Caola, Melnikov '13
- H + jet Boughezal, Caola, Melnikov, Petriello, Schulze '13
- Muon decay spin asymmetry Caola, Czarnecki, Liang, Melnikov, Szafron '14
- Single top-quark production Brucherseifer, Caola, Melnikov '14

	analytic	FS colour	IS colour	local
antenna subtraction	✓	✓	✓	✗
STRIPPER	✗	✓	✓	✓
q_T subtraction	✓	✗	✓	✓
reverse unitarity	✓	✗	✓	-
Trócsányi et al	✗	✓	✗	✓

Currie, LoopFest `14

Main ideas

1. parameterization of the massless system with energies and angles modified to allow for a description of all collinear singular configurations with only two variables

2. level 1 decomposition into sectors allowing for only one type of collinear singularities

3. level 2 decomposition into sectors defining the order of singular limits

4. Subtraction at the endpoint derived from known soft and collinear limits of QCD amplitudes

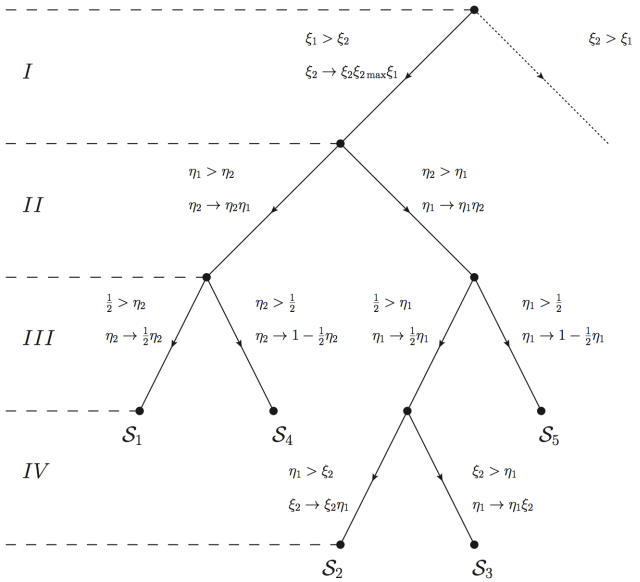
5. No analytic integration of the subtraction terms

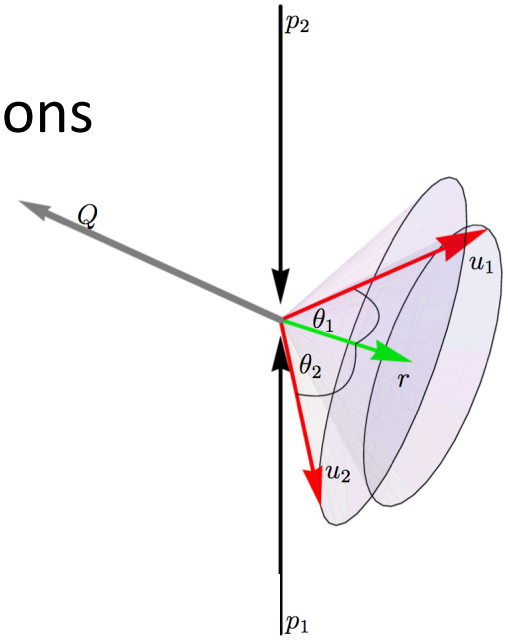
4D formulation

MC, Heymes `14

- Complete set of phase space parameterizations

$$r^\mu = r^0\,\hat{r}^\mu = r^0\begin{pmatrix}1\\ \hat{\boldsymbol{r}}\end{pmatrix}\,, \quad u_1^\mu = u_1^0\,\hat{u}_1^\mu = u_1^0\begin{pmatrix}1\\ \hat{\boldsymbol{u}}_1\end{pmatrix}\,, \quad u_2^\mu = u_2^0\,\hat{u}_2^\mu = u_2^0\begin{pmatrix}1\\ \hat{\boldsymbol{u}}_2\end{pmatrix}$$





$$\hat{\boldsymbol{r}} = \hat{\boldsymbol{n}}^{(3-2\epsilon)}(\alpha_1, \alpha_2, \dots)\,,$$

$$\hat{\boldsymbol{u}}_1 = \boldsymbol{R}_1^{(3-2\epsilon)}(\alpha_1, \alpha_2, \dots)\hat{\boldsymbol{n}}^{(3-2\epsilon)}(\theta_1, \phi_1, \rho_1, \rho_2, \dots)\,,$$

$$\hat{\boldsymbol{u}}_2 = \boldsymbol{R}_1^{(3-2\epsilon)}(\alpha_1, \alpha_2, \dots)\boldsymbol{R}_2^{(3-2\epsilon)}(\phi_1, \rho_1, \rho_2, \dots)\hat{\boldsymbol{n}}^{(3-2\epsilon)}(\theta_2, \phi_2, \sigma_1, \sigma_2, \dots)$$

$$\int \mathrm{d}\boldsymbol{\Phi}_{\text{unresolved}}\,\theta(u_1^0 - u_2^0) =$$

$$\left(\frac{\mu_R^2 e^{\gamma_\mathrm{E}}}{4\pi}\right)^{2\epsilon} \int_{\mathcal{S}_1^{2-2\epsilon}} \mathrm{d}\boldsymbol{\Omega}(\theta_1, \phi_1, \rho_1, \dots) \int_{\mathcal{S}_1^{2-2\epsilon}} \mathrm{d}\boldsymbol{\Omega}(\theta_2, \phi_2, \sigma_1, \sigma_2, \dots) \int_0^{u_{\max}^0} \frac{\mathrm{d}u_1^0\,(u_1^0)^{1-2\epsilon}}{2(2\pi)^{3-2\epsilon}} \int_0^{u_{2\max}^0} \frac{\mathrm{d}u_2^0\,(u_2^0)^{1-2\epsilon}}{2(2\pi)^{3-2\epsilon}}\,\theta(u_1^0 - u_2^0) =$$

$$\frac{E_{\max}^4}{(2\pi)^6}\left(\frac{\pi\mu_R^2 e^{\gamma_\mathrm{E}}}{8E_{\max}^2}\right)^{2\epsilon} \int_{\mathcal{S}_1^{1-2\epsilon}} \mathrm{d}\boldsymbol{\Omega}(\phi_1, \rho_1, \dots) \int_{\mathcal{S}_1^{-2\epsilon}} \mathrm{d}\boldsymbol{\Omega}(\sigma_1, \sigma_2, \dots) \int_0^1 \mathrm{d}\zeta \left(\zeta(1-\zeta)\right)^{-\frac{1}{2}-\epsilon} \iiiint_0^1 \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2 \sum_{i=1}^{5} \mu_{\mathcal{S}_i}\,,$$

4D formulation

MC, Heymes `14

- Azimuthal averaging

- Separation of finite contributions

LO	$\hat{\sigma}^{B}$
NLO	$\hat{\sigma}^{\mathrm{R}}_{\mathrm{F}}\,,\quad \hat{\sigma}^{\mathrm{V}}_{\mathrm{F}}\,,\quad \hat{\sigma}_{\mathrm{U}} = \hat{\sigma}^{\mathrm{R}}_{\mathrm{U}} + \hat{\sigma}^{\mathrm{V}}_{\mathrm{U}} + \hat{\sigma}^{\mathrm{C}}$
NNLO	$\hat{\sigma}^{\mathrm{RR}}_{\mathrm{F}}\,,\quad \hat{\sigma}^{\mathrm{RV}}_{\mathrm{F}}\,,\quad \hat{\sigma}^{\mathrm{VV}}_{\mathrm{F}}\,,\quad \hat{\sigma}_{\mathrm{FR}} = \hat{\sigma}^{\mathrm{RV}}_{\mathrm{FR}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{FR}} + \hat{\sigma}^{\mathrm{C2}}_{\mathrm{FR}}\,,$
	$\hat{\sigma}_{\mathrm{SU}} = \hat{\sigma}^{\mathrm{RR}}_{\mathrm{SU}} + \hat{\sigma}^{\mathrm{RV}}_{\mathrm{SU}} + \hat{\sigma}^{\mathrm{C1}}_{\mathrm{SU}}\,,\quad \hat{\sigma}_{\mathrm{DU}} = \hat{\sigma}^{\mathrm{RR}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{RV}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{C1}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{C2}}_{\mathrm{DU}}$

- `t Hooft-Veltman regularization

$$\mathrm{F}_n \longrightarrow \mathrm{F}_n \left(\frac{\mu_R^2 e^{\gamma_\mathrm{E}}}{4\pi}\right)^{-(n-1)\epsilon} \left[\prod_{i=1}^{n-1} (2\pi)^{-2\epsilon} \delta^{(-2\epsilon)}(q_i)\right]$$

$$\int \mathrm{d}\Phi_n\Big(p_1 + p_2 \to \sum_{i=1}^{n} q_i\Big) \longrightarrow \int \prod_{i=1}^{n} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2q_i^0} (2\pi)^4 \delta^{(4)}\Big(\sum_{i=1}^{n} q_i - p_1 - p_2\Big)$$

Example: gg -> tT + ng

- SU – contribution

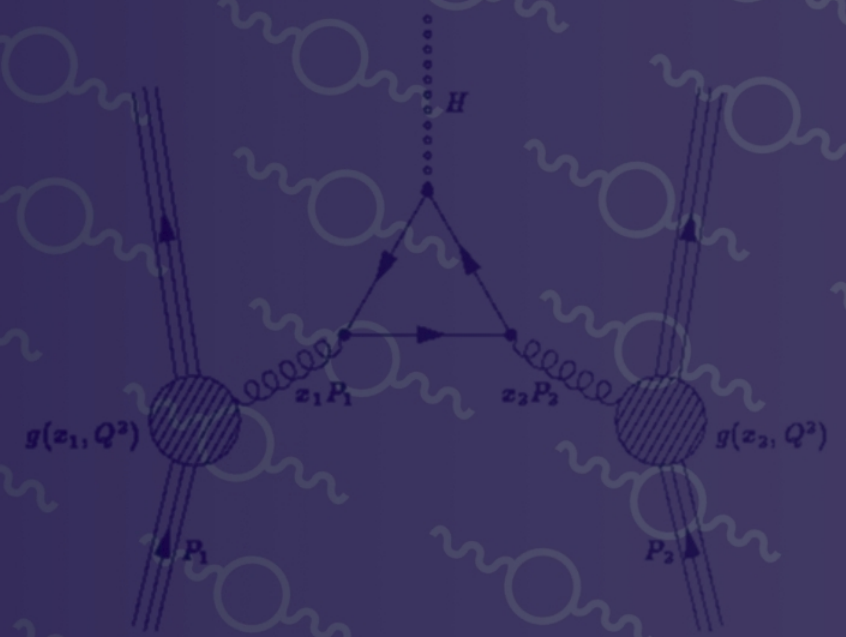
$$\sigma = \frac{\alpha_s^4}{m_t^2}\hat{\sigma} \qquad \beta = \sqrt{1 - \frac{4m_t^2}{s}} = 0.5$$

	CDR	't Hooft Veltman	Agreement (CDR - HV)
$1/\varepsilon^2$	$(-8.1 \pm 7.5) \cdot 10^{-6}$	$(1.1 \pm 0.8) \cdot 10^{-5}$	
$1/\varepsilon$	$(-5.7 \pm 5.9) \cdot 10^{-5}$	$(4.2 \pm 4.1) \cdot 10^{-5}$	
Finite Term	(0.2580 ± 0.0003)	(0.2584 ± 0.0002)	(-0.0004 ± 0.0004)

- DU -contribution

	CDR	't Hooft Veltman	Agreement (CDR -HV)
$1/\varepsilon^4$	$(-1.6 \pm 0.9) \cdot 10^{-6}$	$(-8.6 \pm 8.9) \cdot 10^{-7}$	
$1/\varepsilon^3$	$(-5.2 \pm 6.1) \cdot 10^{-6}$	$(3.2 \pm 5.2) \cdot 10^{-6}$	
$1/\varepsilon^2$	$(1.3 \pm 2.4) \cdot 10^{-5}$	$(-1.0 \pm 1.7) \cdot 10^{-5}$	
$1/\varepsilon$	$(7.4 \pm 9.3) \cdot 10^{-5}$	$(1.9 \pm 6.2) \cdot 10^{-5}$	
Finite Term	(-0.03041 ± 0.00035)	(-0.03045 ± 0.00042)	(0.00004 ± 0.00042)

Conclusions and outlook



Conclusions and outlook

- Provided state of the art results for top-quark cross sections
- Developed a new subtraction scheme with non-trivial applications
- Differential description of top-pair production nearly finished
- Future work on
  1. Combined NNLO production and decay for top-pairs
  2. Publicly available implementation of STRIPPER