

# Low-energy searches for New Physics



**Advances in Computational Particle Physics  
Durbach, September 16, 2014**

Andrzej Czarnecki  University of Alberta

# Outline

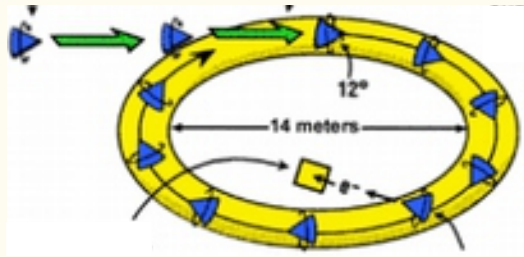
Three promising directions in low-energy searches:

- \* anomalous magnetic moments (muon vs electron)
- \* electric dipole moments (electron)
- \* lepton flavor violation (muon  $\rightarrow$  electron)

# Anomalous magnetic dipole moments

# The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy persists,



$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 287(80) \times 10^{-11}$$

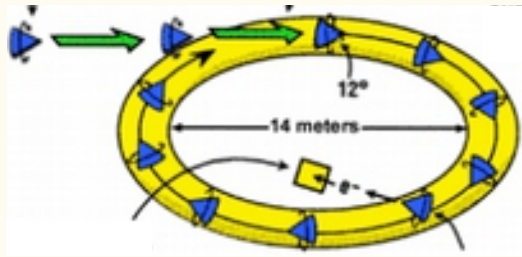
PRD 86, 095009 (2012)

Much work done in the SFB/TR9, mainly the crucial hadronic contribution:

- \* Kurz, Liu, Marquard, Steinhauser
- \* Baikov, Chetyrkin, Kühn, Sturm, Maier perturbative, up to six loops!
- \* Burger, Feng, Hotzel, Jansen, Petschlies, Renner: lattice Ken Wilson Lattice Award
- \* Kühn, Czyż, Rodrigo, Melnikov, et al: radiative return method

# The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy persists,



$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 287(80) \times 10^{-11}$$

PRD 86, 095009 (2012)

This is rather large when compared with other bounds on New Physics:

Muon MDM

$$d_{\mu} \sim \frac{e}{2m_{\mu}} a_{\mu}^{\text{NP}} \sim 3 \cdot 10^{-22} e \cdot \text{cm}$$

Muon-electron transition moment

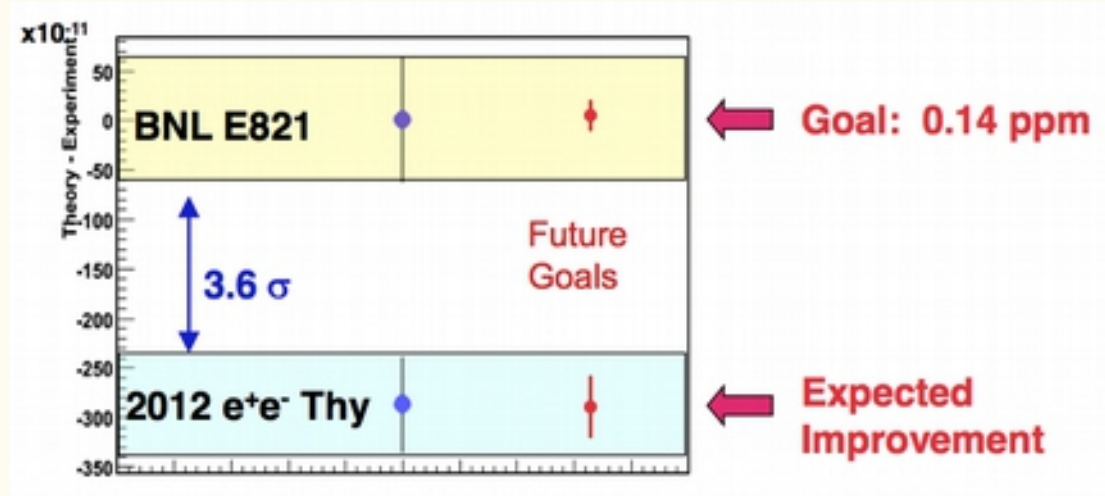
$$|d_{\mu \rightarrow e}| < 4 \cdot 10^{-27} e \cdot \text{cm} \quad \text{MEG 2013}$$

Electron EDM

$$|d_e| < 8.7 \cdot 10^{-29} e \cdot \text{cm} \quad \text{ACME 2013}$$

# How can $g_\mu - 2$ be checked?

New experiment at Fermilab



New experimental concept at J-PARC

Can we use  $g_e - 2$ ?

# New approach to $g_\mu-2$ at J-PARC

Slower muons 300 MeV (instead of the "magic" 3.1 GeV)

Ultracold muons; no electric focusing!

Smaller ring  $r = 33$  cm (instead of 7 m)

$$r \text{ [in meters]} \simeq \frac{\gamma}{3B \text{ [in Tesla]}}$$

Strong, very precisely controlled magnetic field.

~ 10 times more muons than at Fermilab (compensates shorter lifetime).

	Brookhaven	Fermilab	J-PARC
Muon momentum	3.09 GeV/c		0.3 GeV/c
gamma	29.3		3
Storage field	B=1.45 T		3.0 T
Focusing field	Electric quad		None
# of detected $\mu^+$ decays	5.0E9	1.8E11	1.5E12
# of detected $\mu^-$ decays	3.6E9	-	-
Precision (stat)	0.46 ppm	0.1 ppm	0.1 ppm

$$\simeq \sqrt{\frac{2\pi}{\alpha}}$$

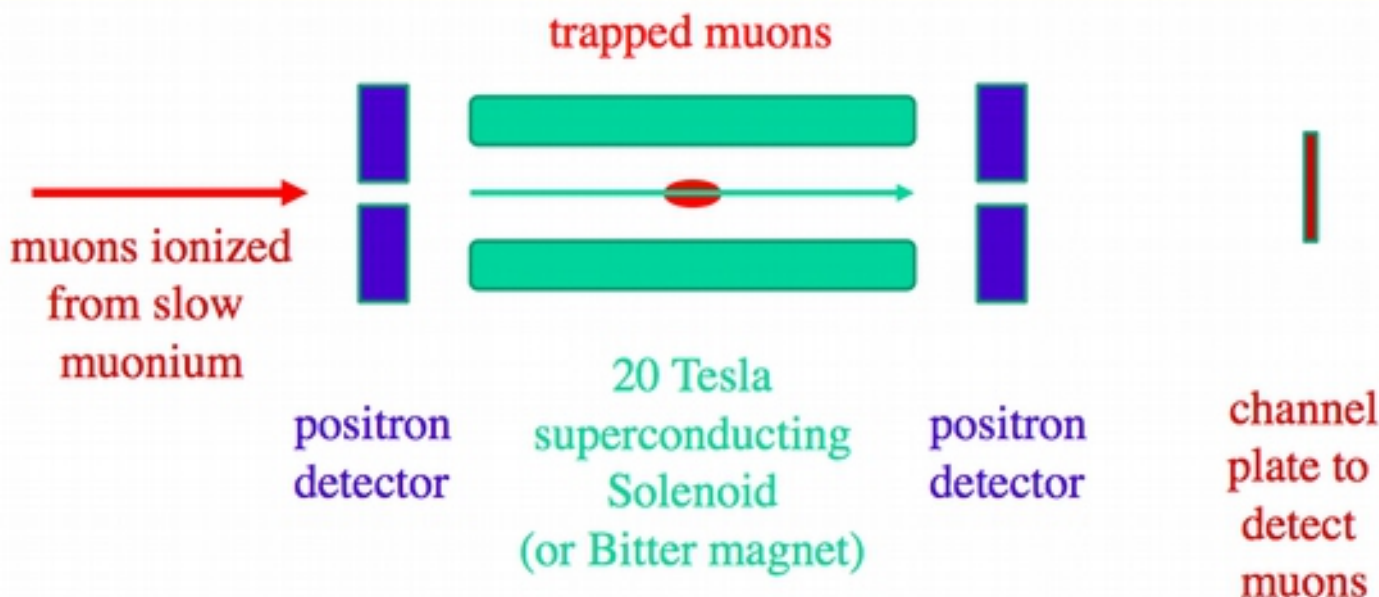


# (Futuristic?) approach to $g_\mu - 2$

USM2013

Gabrielse

## Measure Muon Magnetic Moment in a Trap



Advantages: Use muons to measure the magnetic field in situ  
Less expensive  
Different systematics

Challenge: 2.2 microsecond muon lifetime

Thanks to A. Mills for helpful conversations



# Magnetic moment of the electron

$$a_e = \frac{g_e - 2}{2}$$

Measured with relative error  $25 \cdot 10^{-11}$

Phys. Rev. Lett. 100, 120801 (2008)

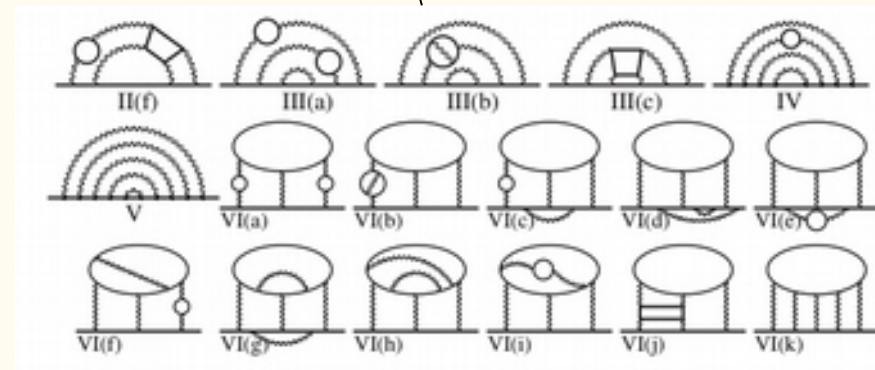
Provides the fine structure constant with the same precision,

$$\alpha^{-1}(a_e) = 137.035\,999\,1736(331)(86)$$

Phys. Rev. Lett. 109, 111807 (2012)

Experimental error dominates (for now)

Numerical errors from 4- and 5-loop diagrams



Kinoshita

# How to use $g_e-2$ to check $g_\mu-2$ ?

If the muon anomaly is due to New Physics, the expected effect for the electron is likely smaller by

$$\frac{m_e^2}{m_\mu^2} \sim \frac{1}{43000}$$

$$\Delta a_\mu \sim 287 \cdot 10^{-11} \rightarrow \Delta a_e \sim 7 \cdot 10^{-14}$$

This means relative uncertainty

$$\frac{\Delta a_e}{a_e} \sim 7 \cdot 10^{-11}$$

and requires a factor of 4 improvement of the latest measurement.

In addition, an independent determination of the fine structure constant is needed, with matching precision.

# How to use $g_e-2$ to check $g_\mu-2$ ?

Nature 442, 516 (2006)  
PRA 89, 052118 (2014)

The second best determination of alpha:  
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

$$14 \cdot 10^{-11}$$

$$\alpha^2 = \frac{R_\infty}{2} \cdot \frac{u}{m_e} \cdot \frac{M_X}{u} \cdot \frac{h}{M_X}$$

$$7 \cdot 10^{-12}$$

(but is it  
for sure?)

$$8 \cdot 10^{-11}$$

**NEW** Nature 2014  
Sturm et al

$$12 \cdot 10^{-11}$$

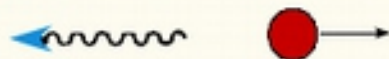
for Rb  
(better for He)

$$124 \cdot 10^{-11}$$

improvement  
needed by  
factor ~10



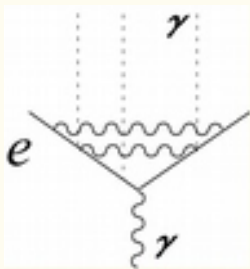
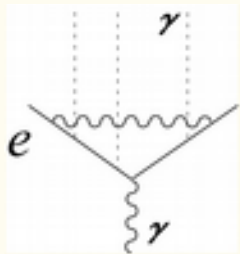
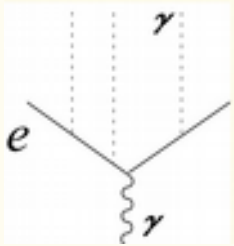
gives  $h/m$



$$\alpha(\text{Rb}) = 1/137.035\,999\,049(90) \quad [66 \cdot 10^{-11}]$$

PRL 106, 080801 (2011)

# Bound-electron g-2: theory needed for $u/m_e$



$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots$$

$$+ \frac{\alpha}{\pi} \left[ 1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4 (a_{41} \ln Z\alpha + a_{40}) + \dots \right]$$

$$+ \left( \frac{\alpha}{\pi} \right)^2 \left[ -0.65.. \left( 1 + \frac{(Z\alpha)^2}{6} \right) + (Z\alpha)^4 (b_{41} \ln Z\alpha + b_{40}) + \dots \right]$$

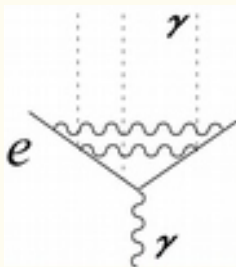
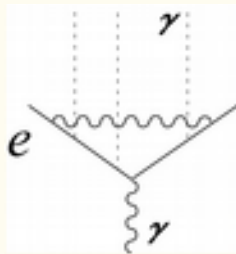
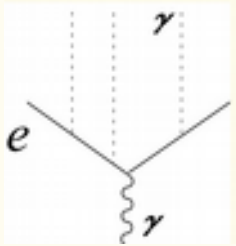
two-loop corrections

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

Pachucki,  
AC  
Jentschura,  
Yerokhin

# Bound-electron g-2: theory needed for $u/m_e$



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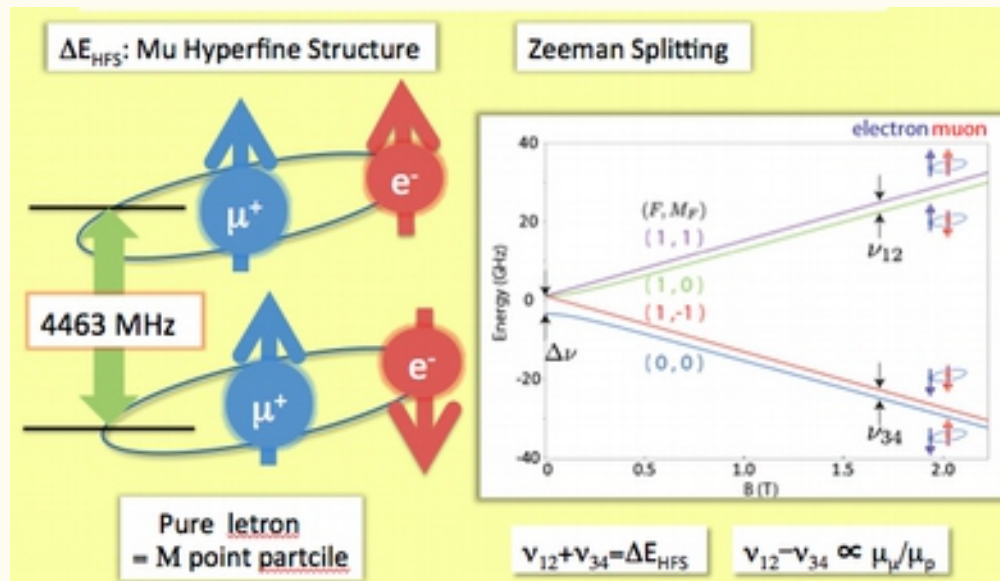
$$+ \left( \frac{\alpha}{\pi} \right)^2 \left[ -0.65.. \left( 1 + \frac{(Z\alpha)^2}{6} \right) + (Z\alpha)^4 (b_{41} \ln Z\alpha + b_{40}) + \dots \right]$$

With Matt Dowling and Jan Piclum,  
we are now computing  $a_{50}$  and  $b_{50}$ ,  
using many tools developed in SFB/TR9:  
q2e, exp, MATAD, FIRE, FIESTA  
and of course FORM

Steinhauser, Smirnov  
Vermaseren

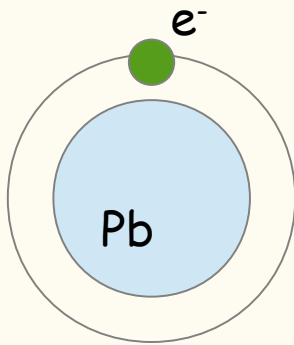
talk by Takahiro Ueda

# Similar binding corrections needed for the muonium: input to g-2

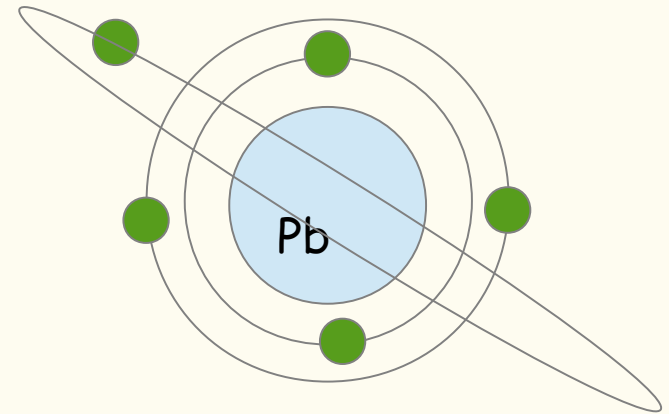


# Another source of alpha: highly-charged ions

$$g \simeq 2 - \frac{2(Z\alpha)^2}{3} \longrightarrow \frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2} \sqrt{(\delta g_{\text{exp}})^2 + (\delta g_{\text{th}})^2} \quad \text{large } Z \text{ favorable}$$



Hydrogen-like lead



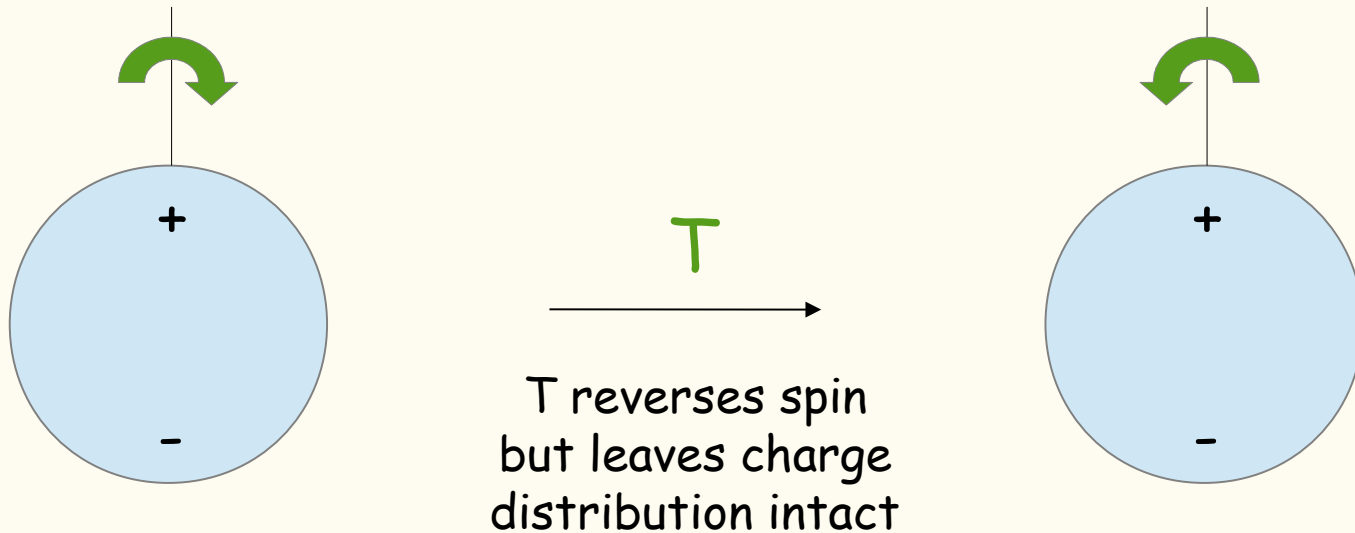
Boron-like lead

There is a combination of  $g$ -factors in both ions where the sensitivity to the nuclear structure largely cancels, but the sensitivity to alpha remains.

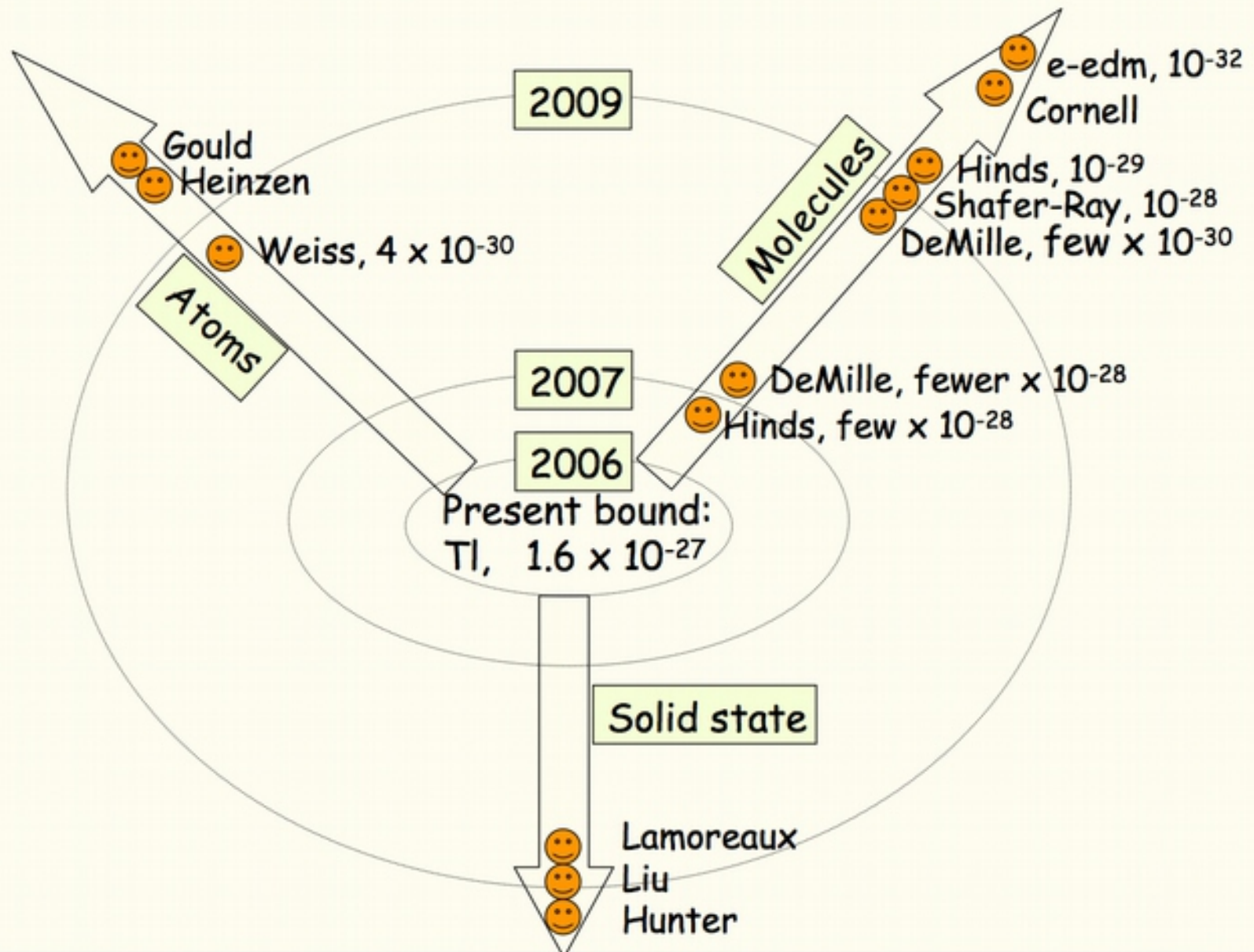
Much interesting theoretical work remains to be done!



# Electron electric dipole moment



# Electron EDM: what used to be expected (2006)



# Recent great progress

2011: YbF @ Imperial

$$|d_e| < 10.5 \times 10^{-28} e \text{ cm}$$

LETTER

doi:10.1038/nature10104

## Improved measurement of the shape of the electron

J. J. Hudson<sup>1</sup>, D. M. Kara<sup>1</sup>, I. J. Smallman<sup>1</sup>, B. E. Sauer<sup>1</sup>, M. R. Tarbutt<sup>1</sup> & E. A. Hinds<sup>1</sup>

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2014: ThO by ACME

$$|d_e| < 8.7 \times 10^{-29} e \cdot \text{cm}$$



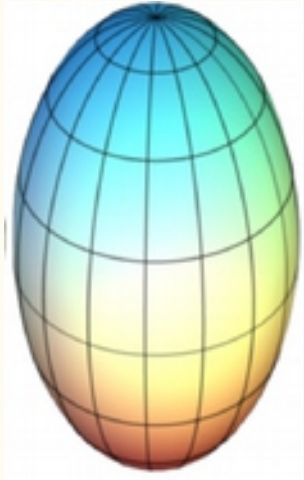
## Order of Magnitude Smaller Limit on the Electric Dipole Moment of the Electron

The ACME Collaboration,\* J. Baron,<sup>1</sup> W. C. Campbell,<sup>2</sup> D. DeMille,<sup>3†</sup> J. M. Doyle,<sup>1†</sup> G. Gabrielse,<sup>1‡</sup> Y. V. Gurevich,<sup>1‡</sup> P. W. Hess,<sup>1</sup> N. R. Hutzler,<sup>1</sup> E. Kirilov,<sup>3§</sup> I. Kozyryev,<sup>3||</sup> B. R. O'Leary,<sup>3</sup> C. D. Panda,<sup>1</sup> M. F. Parsons,<sup>1</sup> E. S. Petrik,<sup>1</sup> B. Spaun,<sup>1</sup> A. C. Vutha,<sup>4</sup> A. D. West<sup>3</sup>

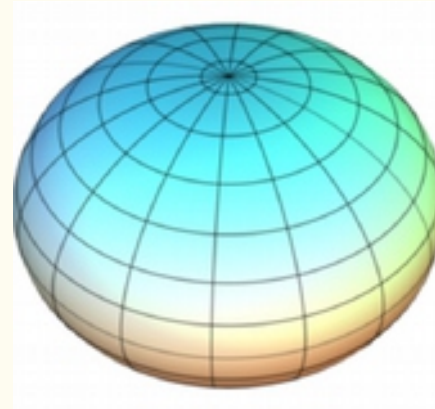
# What will be on the cover when $d_e$ discovered?

Analogous discussion about the shape of the earth (18<sup>th</sup> century):  
which theory of gravity is correct?

Descartes:



Newton:



# When will we get there? Plans for $d_e$ :

	Near future	Ultimate goal	
Molecular ion	$< ACME$	$< 10^{-29}$	Ye Cornell
Atoms in an optical lattice	$< ACME$	$\text{few} \cdot 10^{-30}$	Weiss
YbF	$10^{-29}$	$\sim 10^{-30}$	Sauer
Future ACME	$10^{-29}$	$< 10^{-30}$	DeMille Gabrielse

# Remarks on $d_e$

Molecules: great new player; but enhancement factors very tricky

One way to check: hyperfine structure (YbF)

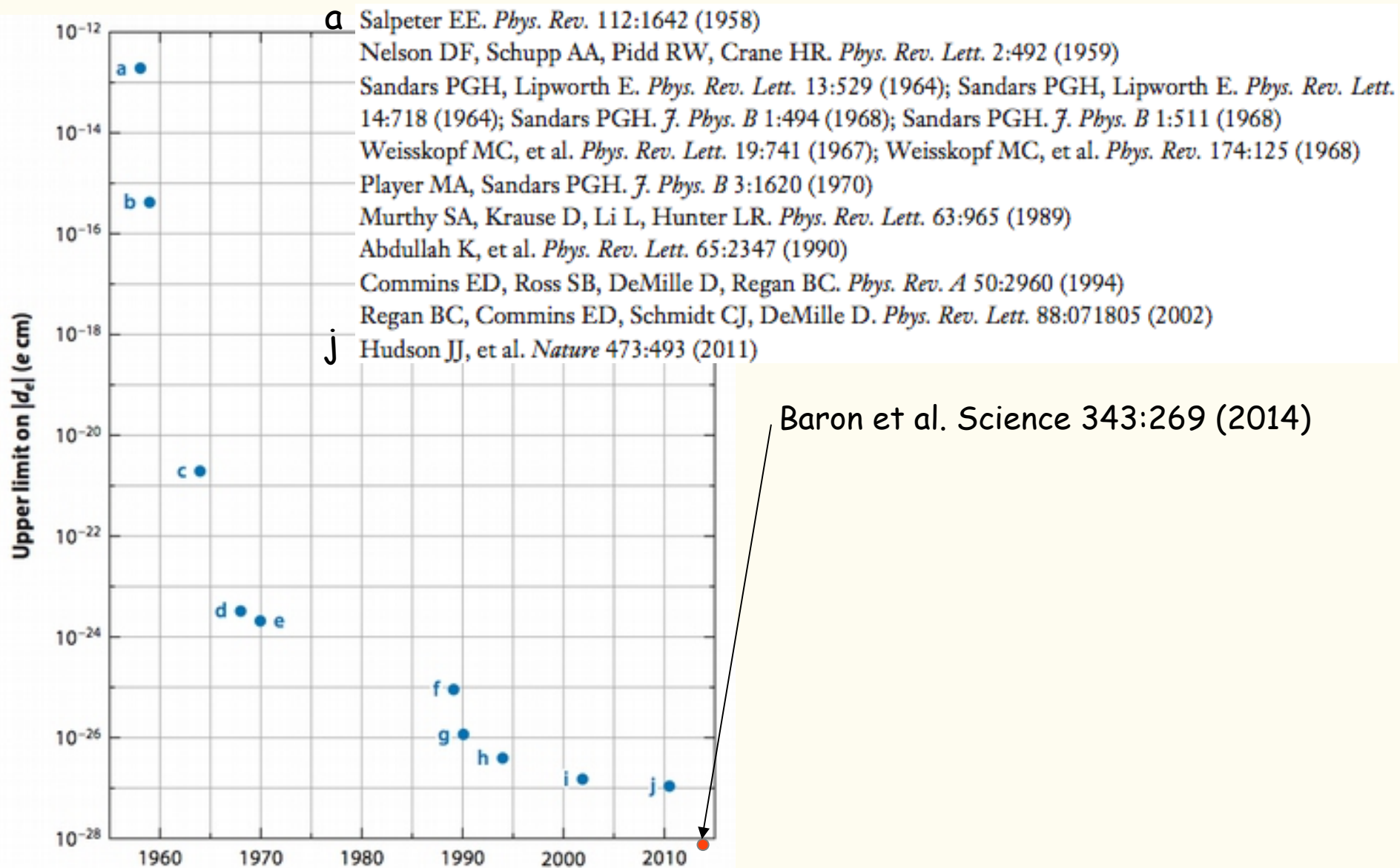
Molecules are a very new field; many improvements possible  
(sources!)

An even newer field: molecular ions.

Challenge for theorists: "enhancement factors": this is a new frontier for quantitative studies in molecules.



# History of the electron EDM



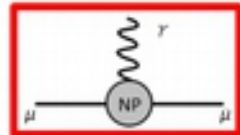
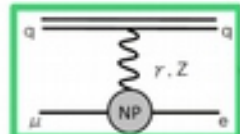
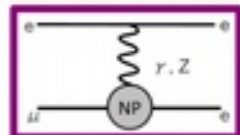
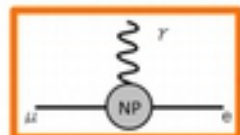
from Commins: Electron spin and its history

# Lepton flavor violation

# Efforts in LFV searches

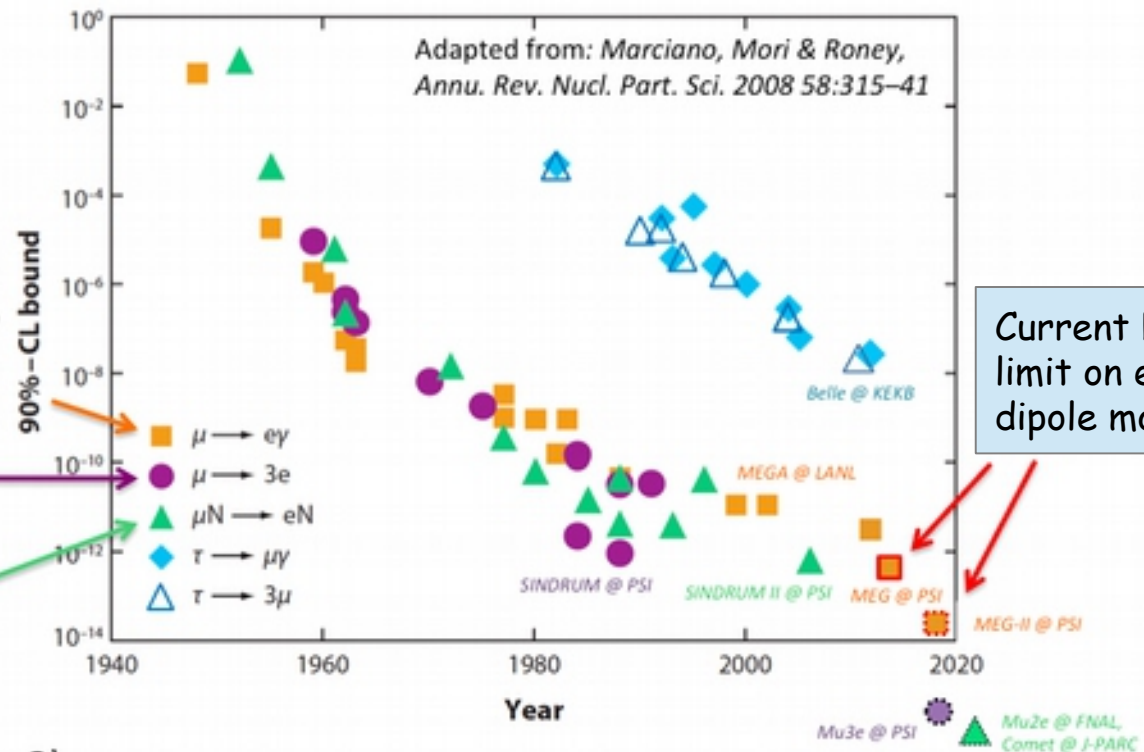
Historical overview  
of selected CLFV  
searches:

New physics:



branching ratio  
90% - CL bound

$(g-2)_\mu$



Current best  
limit on exotic  
dipole moments

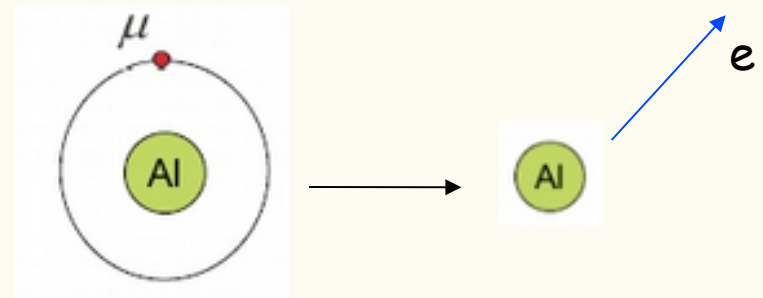
from Gordon Lim

# Conversion: probes also non-dipole interactions

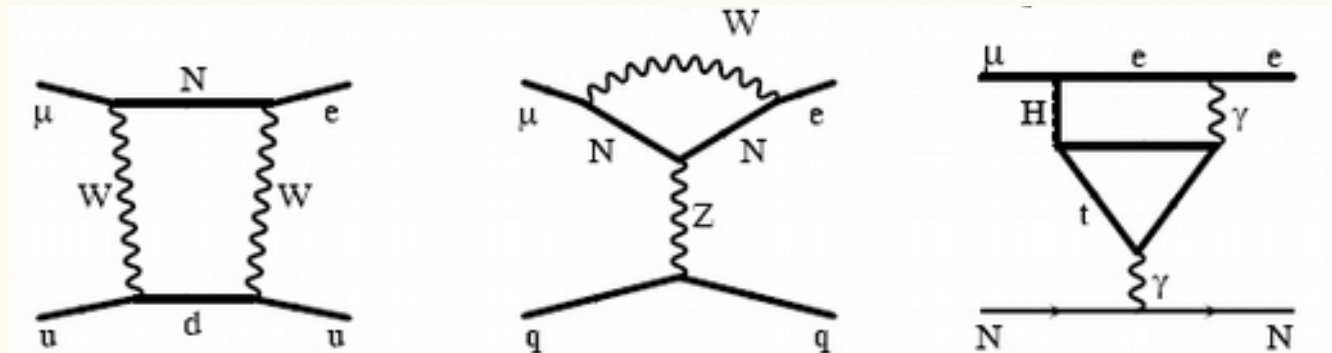
So far, we have only talked about dipole interactions.  
There are also vectors and scalars.

They are not (directly) probed by processes with external photons,  
by gauge invariance requirements.

New process: muon-electron conversion  
(as well as  $\mu \rightarrow eee$ )

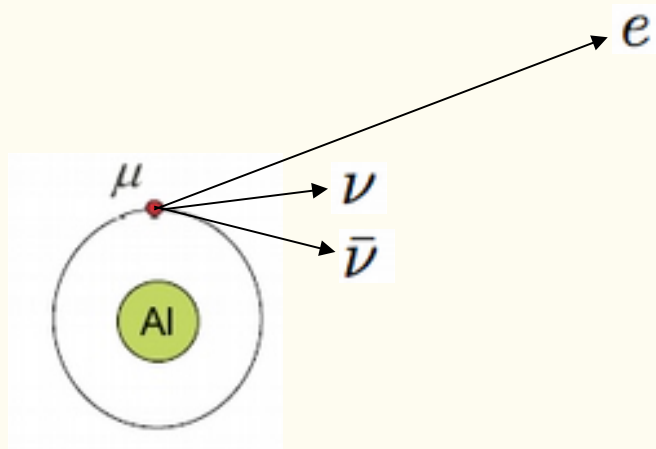


Variety of mechanisms:



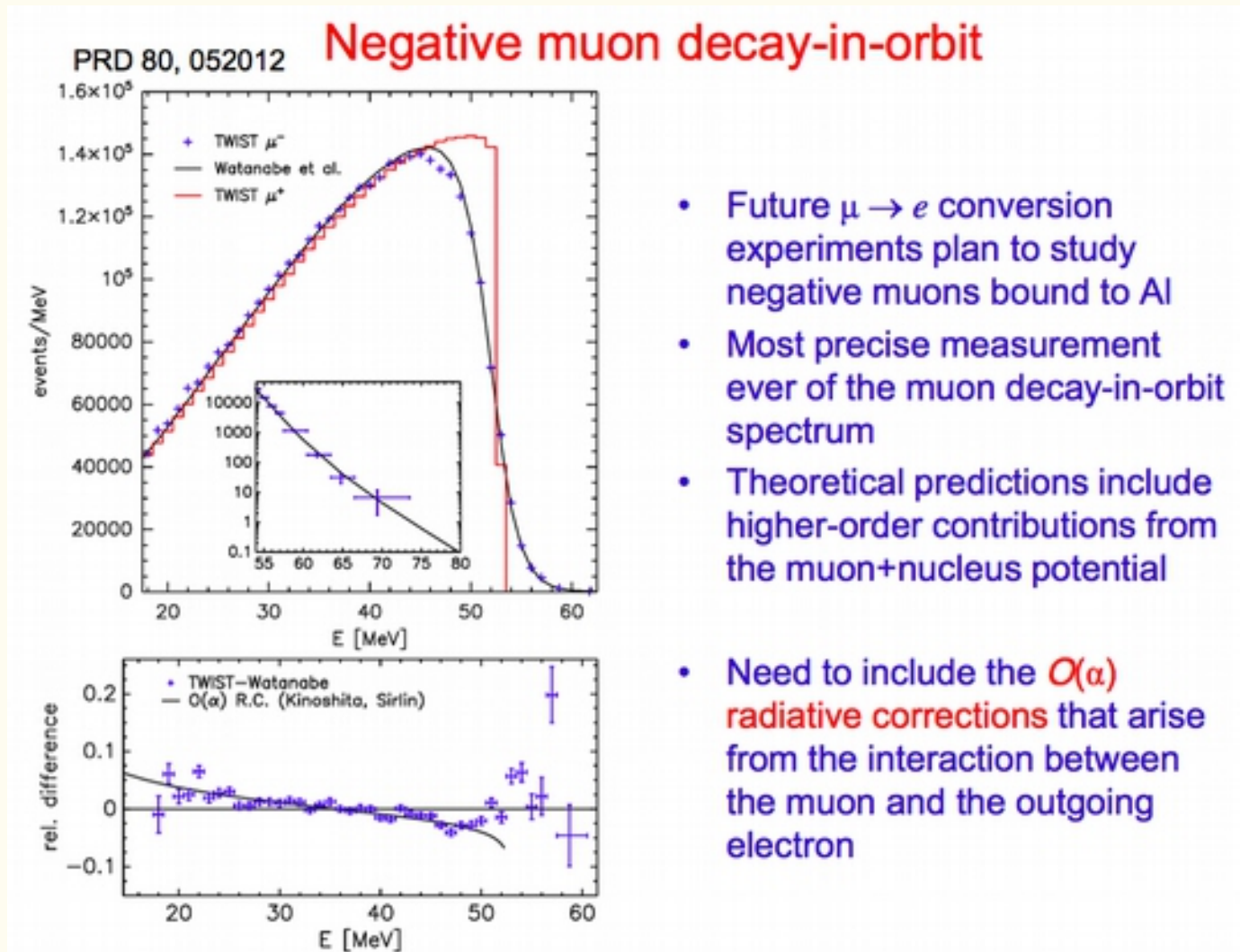
# Background for the conversion search

Normal decay of the muon bound in the atom can produce high-energy electron,

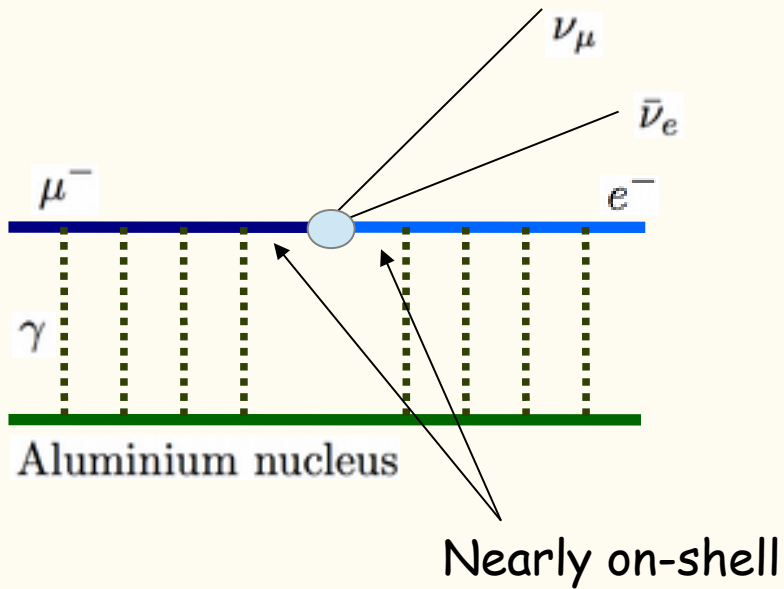


Spectrum has to be well understood.

# Bound muon decay has been measured: TWIST/TRIUMF

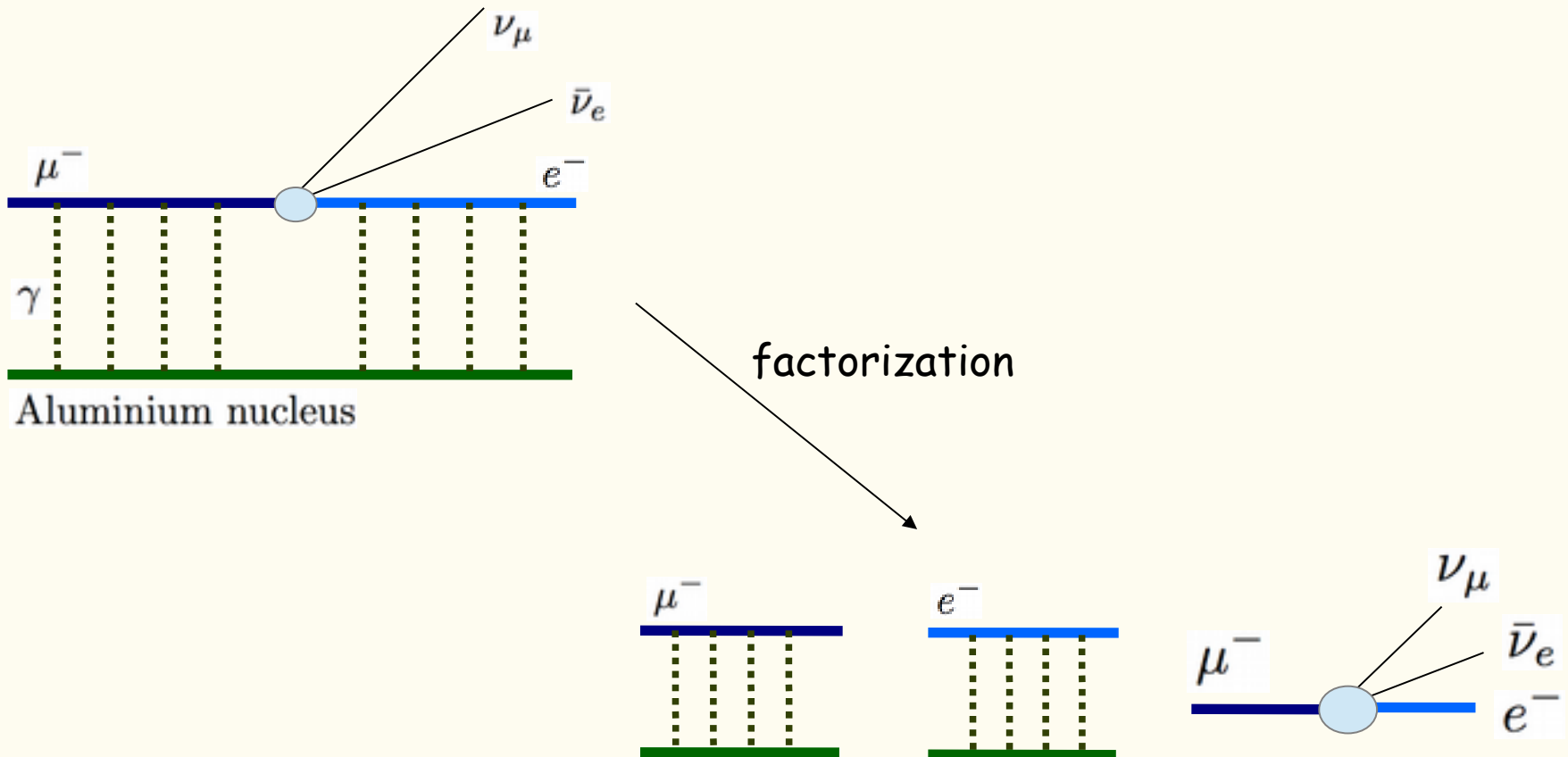


# Decay of a muon bound in aluminium

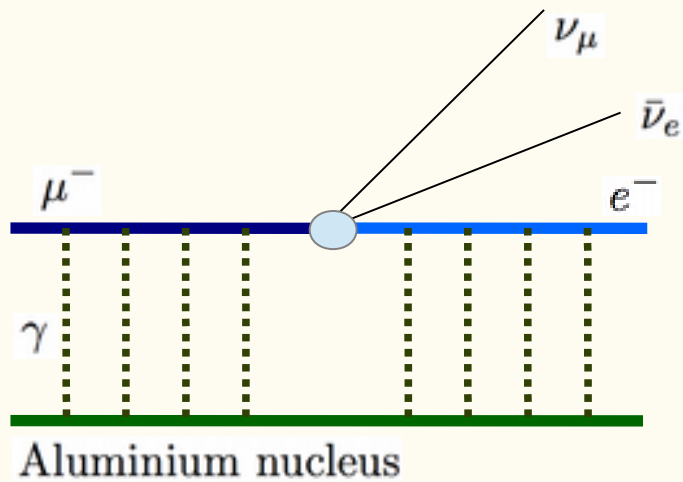




# Decay of a muon bound in aluminium

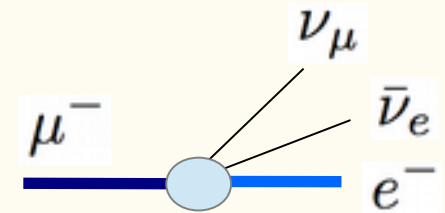
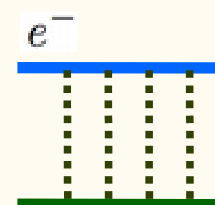
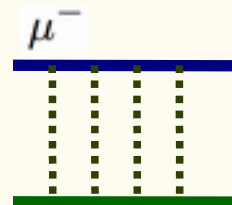


# Decay of a muon bound in aluminium



factorization

Free muon decay rate,  
with all corrections!

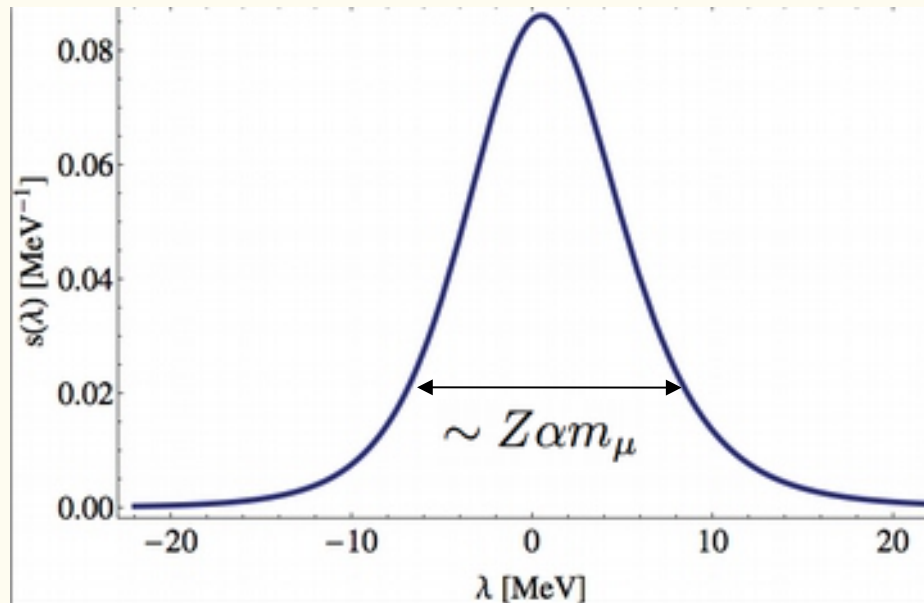


$$\frac{d\Gamma}{dE_e} = \int d\lambda s(\lambda) \frac{d\Gamma_{\text{free}}}{dz} \frac{dz}{dE_e} \Big|_{z \rightarrow z(\lambda)}$$

$$z(\lambda) = \frac{2(E_e + \lambda) + (Z\alpha)^2 m_\mu}{m_\mu + \lambda}$$

# Shape function

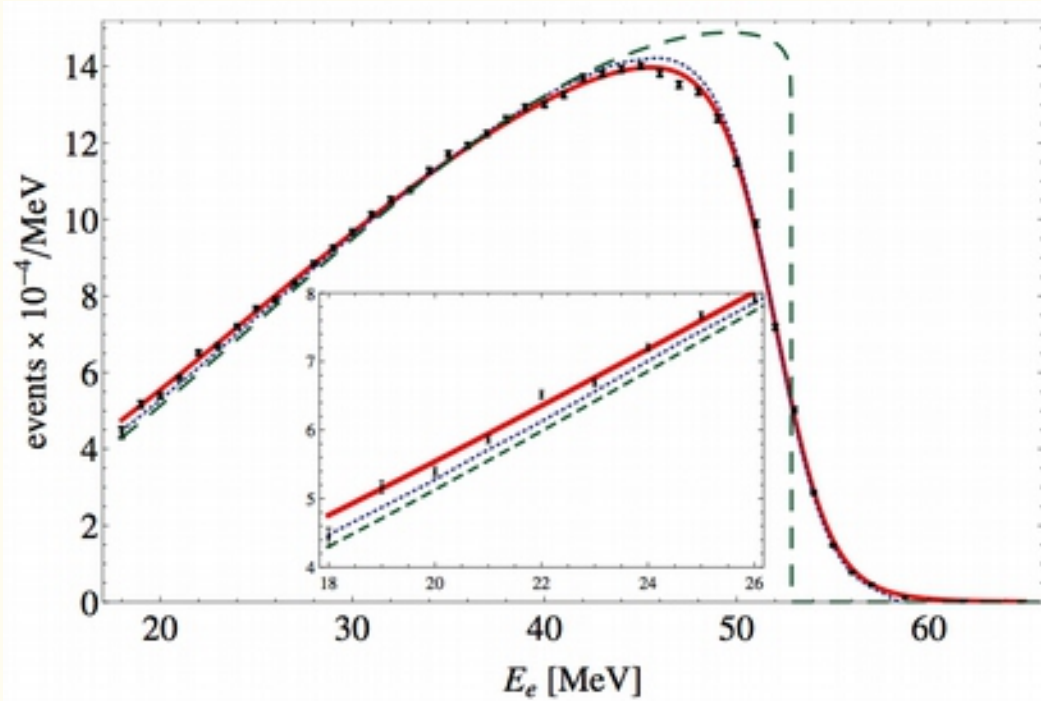
$$s(\lambda) = \int \frac{d^3k}{(2\pi)^3} \psi_g^* (\vec{k}) \delta(\lambda + \vec{n} \cdot \vec{k}) \psi_g (\vec{k})$$



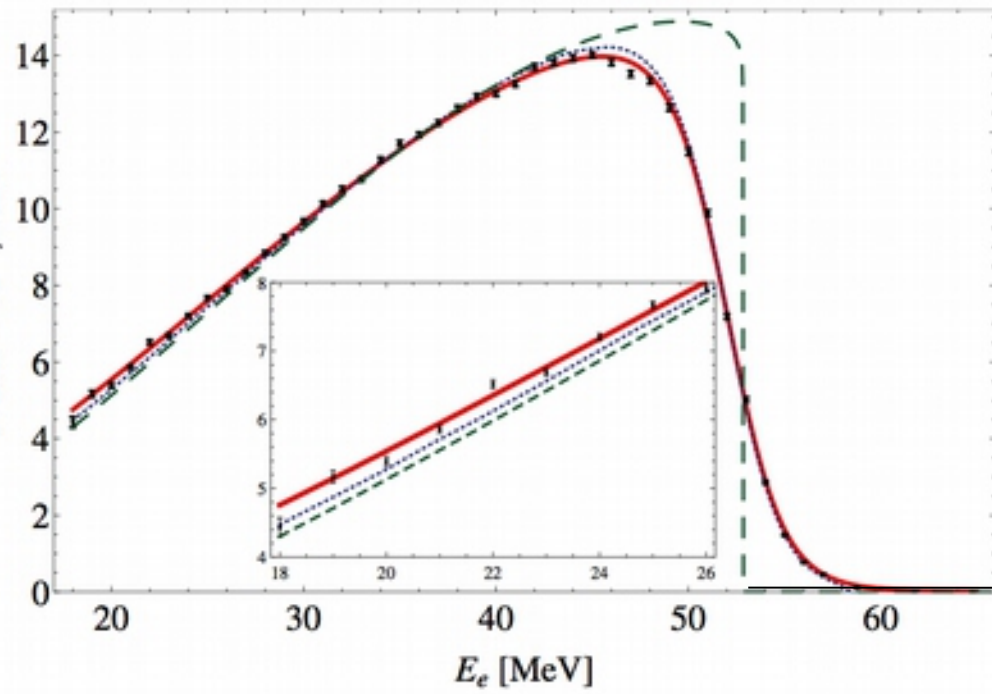
Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.

Mannel, Neubert, Bigi, Shifman, Uraltsev, Vainshtein

# Result: spectrum of the bound muon decay



# What about the upper half of the spectrum?



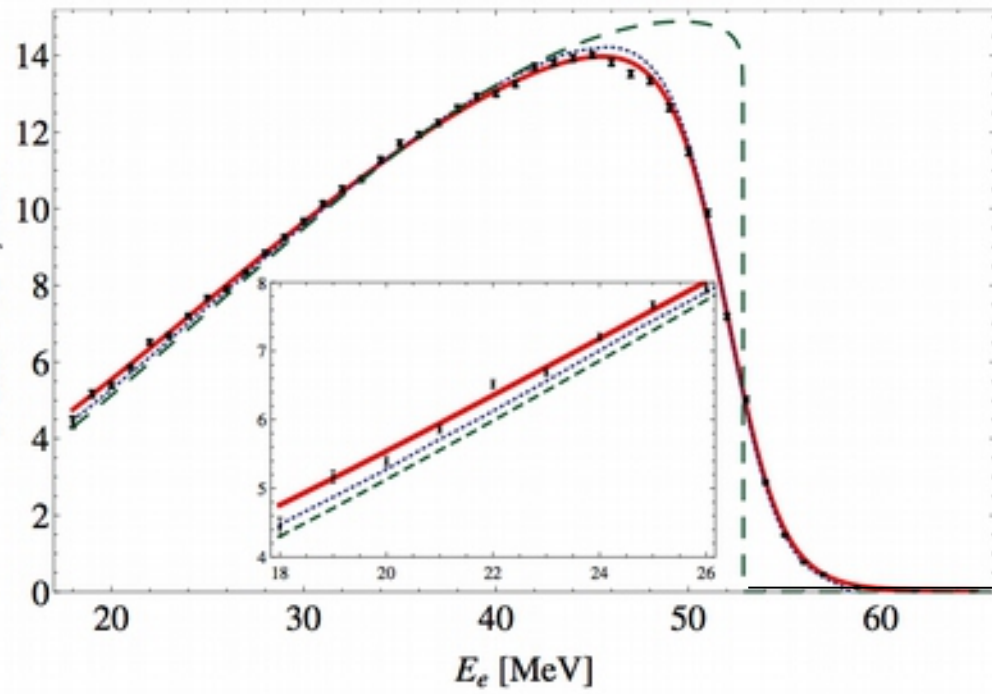
$$\frac{m_\mu}{2} \lesssim E_e \lesssim m_\mu$$

$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{\max} - E)^5$$

105 MeV

It seems to be VERY suppressed; so who cares?

# What about the upper half of the spectrum?

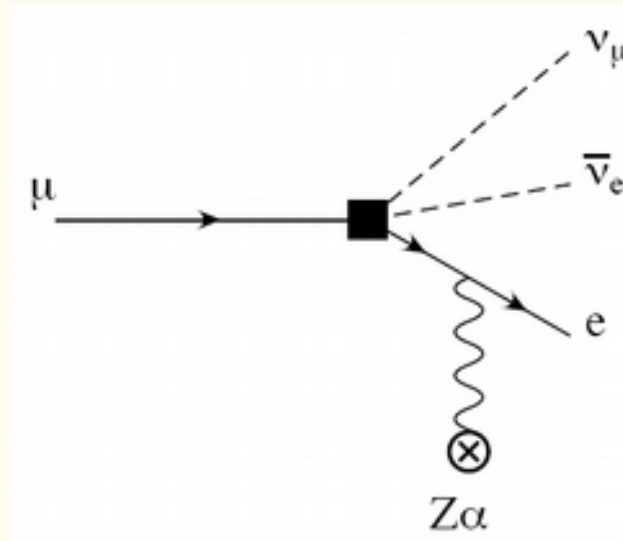


$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{\max} - E)^5$$

105 MeV

It is the main background for the expected conversion signal

# Next step: radiative corrections to the electron spectrum



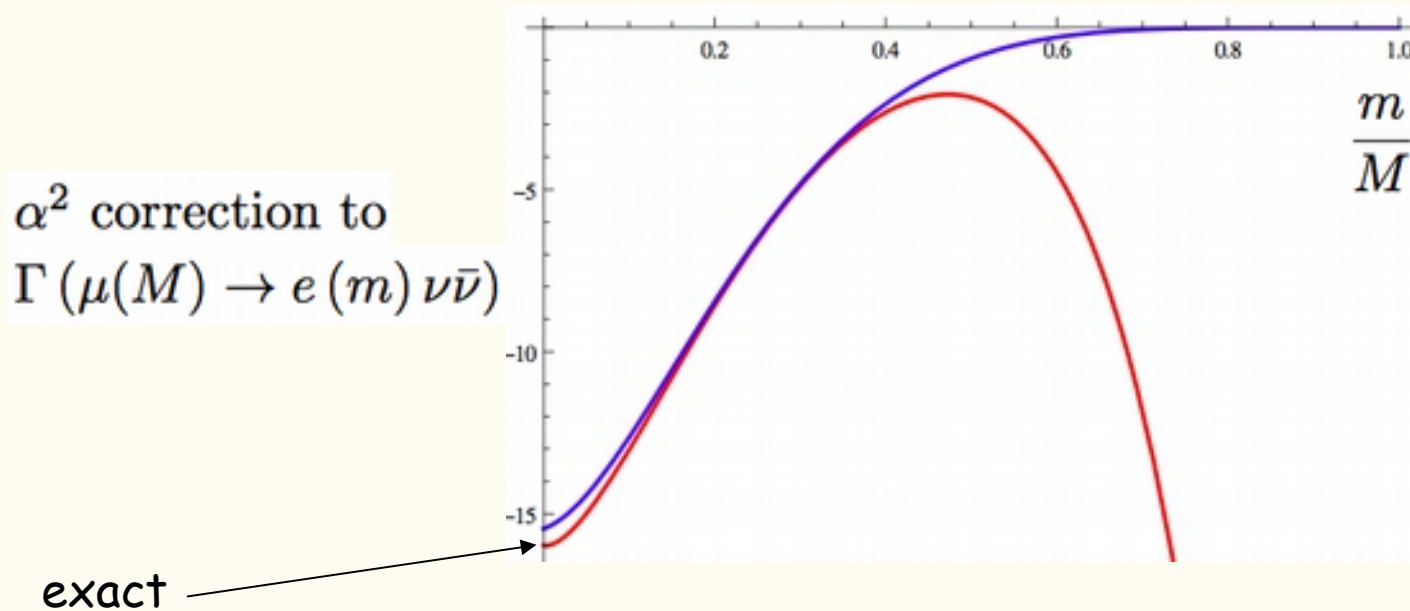
Competing effects:

- vacuum polarization in the hard photon; and
- self-energy and real radiation

Ultimate goal: smooth matching of all energy regions, from the bound electron at low energy to the end-point.



# Free muon lifetime



Note: the blue curve is designed for  $m \sim M$ ,  
but is good even for  $m \ll M$ .

So we want to exploit the expansion around  $m = M$   
to get  $\alpha^3$

## Conclusions

Great opportunities for precise theoretical studies in low-energy physics:

- bound electron  $g$ -factor: complements a vigorous experimental program in Mainz, Heidelberg and GSI Darmstadt
- muon decay: background for  $\mu$ -e conversion searches; theoretically interesting
- many atomic/molecular structure studies needed, eg for electric dipole searches