

Hadron Structure from lattice QCD

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*Computational Particle Physics
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Outline

1 Introduction

2 Recent achievements

- Simulations with physical values of the quark masses
- Masses of Hyperons and Charmed baryons
- Isospin effects

3 Nucleon Structure

- Axial charge g_A
- Scalar and tensor charges
- Momentum fraction and spin
- Electromagnetic form factors

4 Conclusions

Quantum Chromodynamics (QCD)

QCD-Gauge theory of the strong interaction

Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A_\mu^a$$



Harald Fritzsch



Murray Gell-Mann



Heinrich Leutwyler

Phys.Lett. B47 (1973) 365

This “simple” Lagrangian produces the amazingly rich structure of strongly interacting matter in the universe.

Numerical simulation of QCD provides essential input for a wide class of complex strong interaction phenomena

→ In this talk: Hadron structure of interest to both the phenomenological and experimental communities.

Acknowledgments & Statistics

European Twisted Mass Collaboration (ETMC)



Cyprus (Univ. of Cyprus, Cyprus Inst.),
France (Orsay, Grenoble), **Germany**
(Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), **Italy** (Rome I, II, III, Trento),
Netherlands (Groningen), **Poland** (Poznan),
Spain (Valencia), **Switzerland** (Bern), **UK**
(Liverpool)

Collaborators:

A. Abdel-Rehim, K. Cichy, M. Constantinou,
V. Drach, E. Garcia Ramos, K. Hadjiyianakou,
K.Jansen Ch. Kallidonis, G. Koutsou, K. Ottnad, M. Petschlies, F. Steffens, A. Strelchenko, A. Vaquero, C. Wiese

B3 statistics - Lattice part only

- 8 postdocs
- Awards: Cyprus Senior Researcher RPF award ([K. Jansen](#))
- numerous computertime awards at [NIC](#), [PRACE](#), [HLRN](#), [SuperMUC](#), [Cy-Tera](#), [JUROPA](#)
- 4 Ph.D. theses
- 20 publications in refereed journals
- 31 Proceedings contributions at international conferences

Systematic uncertainties

- Finite lattice spacing a - take the continuum limit $a \rightarrow 0$
- Finite volume L - take infinite volume limit $L \rightarrow \infty$
- Identification of hadron state of interest

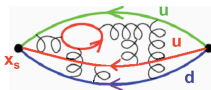
Systematic uncertainties

- Finite lattice spacing a - take the continuum limit $a \rightarrow 0$
- Finite volume L - take infinite volume limit $L \rightarrow \infty$
- Identification of hadron state of interest

Creation operator for zero momentum: $J_p^\dagger(t_s) = \sum_{\vec{x}_s} J_p^\dagger(\vec{x}_s, t_s)$

Proton propagator:

$$\begin{aligned} \langle J_p(t_s) J_p^\dagger(0) \rangle &= \sum_n \langle 0 | J_p e^{-H_{QCD} t_s} | n \rangle \langle n | J_p^\dagger | 0 \rangle \\ &= \sum_n |\langle 0 | J_p | n \rangle|^2 e^{-E_n t_s} \xrightarrow{t_s \rightarrow \infty} |\langle 0 | J_p | p \rangle|^2 e^{-m_p t_s} \end{aligned}$$



Noise to signal increases with t_s :
 $\sim e^{(m_p - \frac{3}{2} m_\pi) t_s}$

Systematic uncertainties

- Finite lattice spacing a - take the continuum limit $a \rightarrow 0$
- Finite volume L - take infinite volume limit $L \rightarrow \infty$
- Identification of hadron state of interest
- Simulation at physical quark masses - now feasible
- Computation of valence quark loops - now feasible

Recent achievements

Fermion action

Observables: $\langle \mathcal{O} \rangle = \frac{1}{Z} \sum_{\{U_\mu\}} \mathcal{O}(D^{-1}, U_\mu)$

Use $\mathcal{O}(a)$ -improved fermion action: $\langle \mathcal{O} \rangle_{\text{cont}} = \langle \mathcal{O} \rangle_{\text{latt}} + \mathcal{O}(a^2)$

Use the Wilson twisted mass formulation at maximal twist, R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007

- Automatic $\mathcal{O}(a)$ improvement
- No operator improvement needed, renormalization simplified \rightarrow important for hadron structure

Fermion action

Simulations by the European Twisted mass Collaboration: [see Talk by K. Jansen](#)

- $N_f = 2$, 4 lattice spacings, different volumes, Ph. Boucaud *et al.*, Comput.Phys.Commun. 179 (2008) 695; Phys.Lett. B650 (2007) 304
- $N_f = 2 + 1 + 1$, 3 lattice spacings, different volumes, R. Baron *et al.*, JHEP 1008 (2010) 097
- $N_f = 2$ at the physical point, one lattice spacing and volume, A. Abdel-Rehim *et al.*, arXiv:1311.4522
→ [preliminary results at physical point](#), M. Constantinou (ETMC), Plenary, Lattice 2014; C. Alexandrou *et al.*, PoS LATTICE2013 (2013) 292

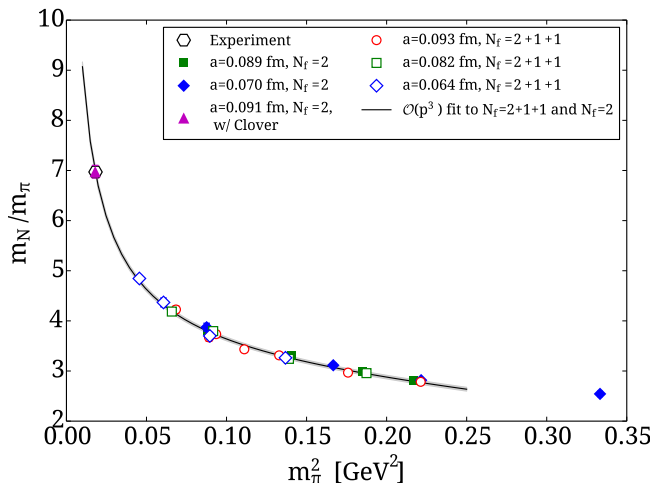


COURTESY: FORSCHUNGSZENTRUM JÜLICH

5.0 Pflop/s, second biggest in Europe, 8th in the world - TOP 500 June 2014

Simulations with physical quark masses

The nucleon



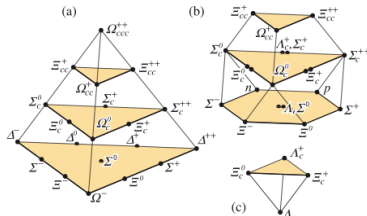
$L \sim 3$ fm and $a \sim 0.1$ fm; Lowest order heavy baryon chiral perturbation theory with experimental value excluded

Hyperons and Charmed baryons

SU(4) representations:

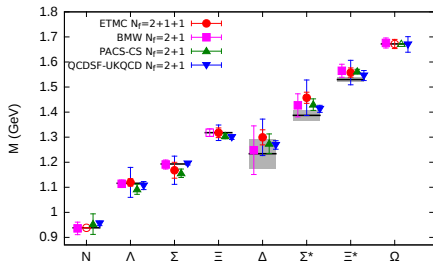
$$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \bar{4}$$

$$\square \otimes \square \otimes \square = \square\square\square \oplus \square\square \oplus \square \oplus \square$$



First goal: reproduce the low-lying masses

Also $N_f = 2 + 1 + 1$ results: C.A., V. Drach, K. Jansen, Ch. Kallidonis, G. Koutsou, arXiv:1406.4310



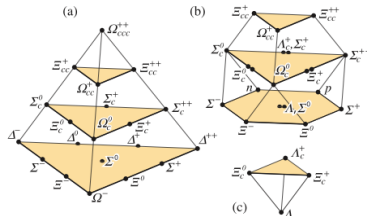
- continuum limit taken
- finite volume effects checked
- systematic error due to the chiral extrapolation estimated - biggest uncertainty

Hyperons and Charmed baryons

SU(4) representations:

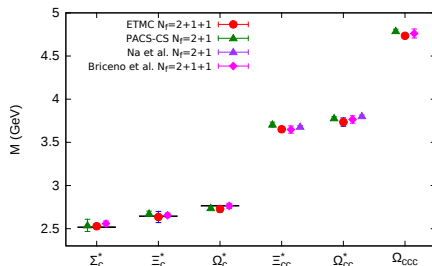
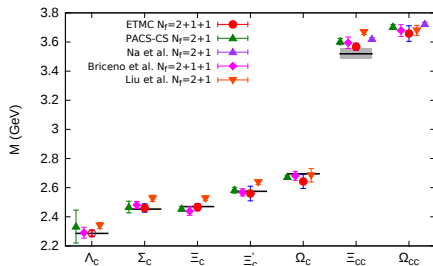
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$$\square \otimes \square \otimes \square = \square\square\square \oplus \square\square \oplus \square \oplus \square$$



First goal: reproduce the low-lying masses **and make predictions**

Also $N_f = 2 + 1 + 1$ results: C.A., V. Drach, K. Jansen, Ch. Kallidonis, G. Koutsou, arXiv:1406.4310



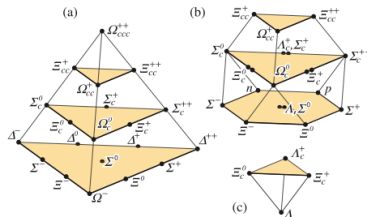
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Hyperons and Charmed baryons

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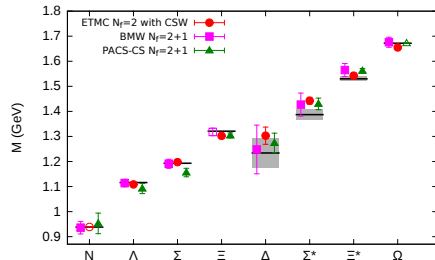
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Results by ETM Collaboration using $N_f = 2$ simulations with **physical pion mass** for one lattice volume and lattice spacing $a = 0.093$ fm

Preliminary results

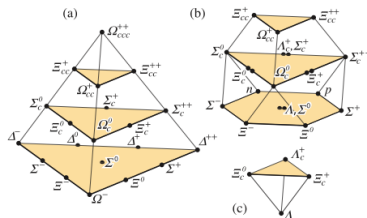


Hyperons and Charmed baryons

SU(4) representations:

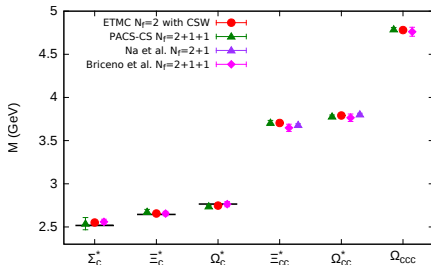
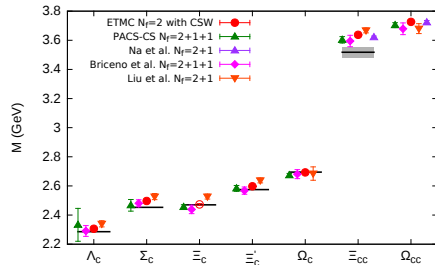
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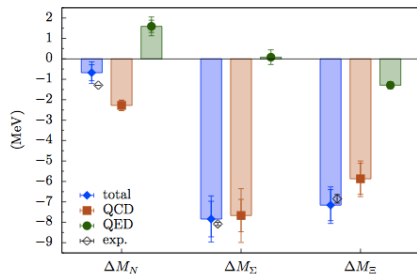
Results by ETM Collaboration using $N_f = 2$ simulations with **physical pion mass** for one lattice volume and lattice spacing $a = 0.093$ fm

Preliminary results



Isospin and electromagnetic mass splitting

RBC and BMW collaborations: Treat isospin and electromagnetic effects to LO



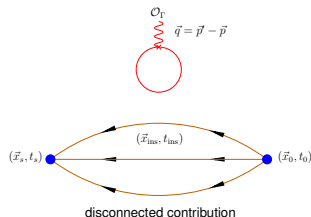
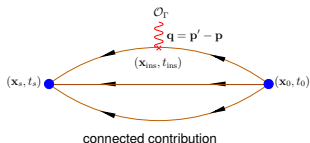
Baryon spectrum with mass splitting from BMW, Sz. Borsanyi *et al.*, Phys. Rev. Lett. 111 (2013) 252001

- Nucleon mass: isospin and electromagnetic effects with opposite signs
- Physical splitting reproduced

Nucleon structure

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

$$R(t_s, t_{\text{ins}}, t_0) \xrightarrow[(t_s - t_{\text{ins}})\Delta \gg 1]{(t_{\text{ins}} - t_0)\Delta \gg 1} \mathcal{M} [1 + \dots e^{-\Delta(\mathbf{p})(t_{\text{ins}} - t_0)} + \dots e^{-\Delta(\mathbf{p}')(t_s - t_{\text{ins}})}]$$



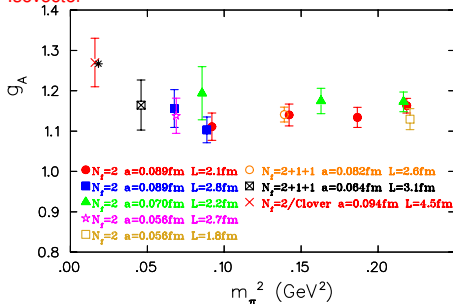
- \mathcal{M} the desired matrix element; t_s, t_{ins}, t_0 the sink, insertion and source time-slices; $\Delta(\mathbf{p})$ the energy gap with the first excited state
- Identification of hadron state of interest - dependent on \mathcal{O}_Γ i.e. different for g_A , σ -terms, EM form factors
- Connect lattice results to measurements:
 $\mathcal{O}_{\overline{\text{MS}}}(\mu) = Z(\mu, a) \mathcal{O}_{\text{latt}}(a) \implies \text{evaluate } Z(\mu, a) \text{ non-perturbatively}$

Axial charge g_A

The good news:

Axial-vector FFs: $A_\mu^3 = \bar{\psi} \gamma_\mu \gamma_5 \frac{\tau^3}{2} \psi(x) \Rightarrow \frac{1}{2} \bar{u}_N(\vec{p}') \left[\gamma_\mu \gamma_5 G_A(q^2) + \frac{q^\mu \gamma_5}{2m} G_P(q^2) \right] u_N(\vec{p})|_{q^2=0}$
 \rightarrow yields $G_A(0) \equiv g_A$: i) well known experimentally & ii) no quark loop contributions

Isovector



ETM Collaboration

- g_A at the physical point mass indicates agreement with the physical value \rightarrow important to reduce error
- many results from other collaborations, e.g.
 - $N_f = 2 + 1$ Clover, LHPC, J. R. Green *et al.*, arXiv:1209.1687
 - $N_f = 2$ Clover, QCDSF, R. Hosley *et al.*, arXiv:1302.2233
 - $N_f = 2$ Clover, CLS, S. Capitani *et al.* arXiv:1205.0180
 - $N_f = 2 + 1$ Clover, CSSM, B. J. Owen *et al.*, arXiv:1212.4668
 - $N_f = 2 + 1 + 1$ Mixed action (HISQ/Clover), PNDME, T. Bhattacharya *et al.*, arXiv:1306.5435

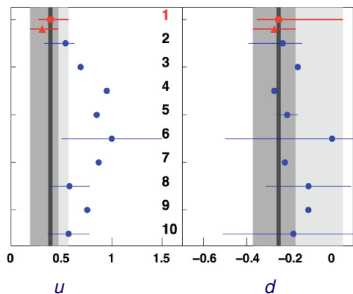
Nucleon charges: g_A , g_s , g_T

- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x) \frac{\tau^a}{2} \psi(x)$
- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x) \gamma^\mu \gamma_5 \frac{\tau^a}{2} \psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x) \sigma^{\mu\nu} \frac{\tau^a}{2} \psi(x)$

$\Rightarrow \langle N(\vec{p}') | \mathcal{O}_\Gamma N(\vec{p}) \rangle|_{q^2=0}$ yields g_s , g_A , g_T

(i) isovector combination has no disconnect contributions; (ii) g_A well known experimentally, g_T to be measured at JLab

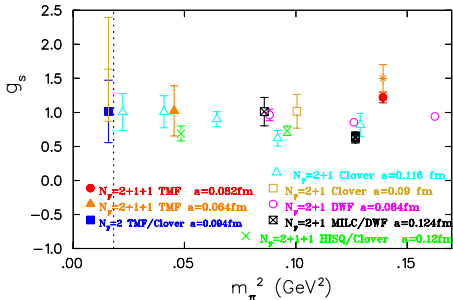
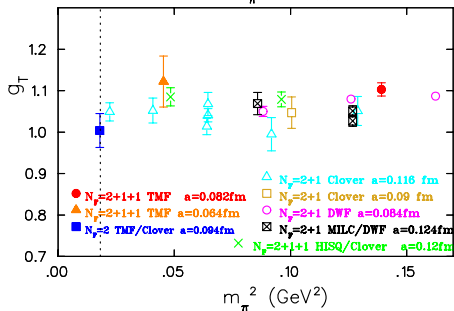
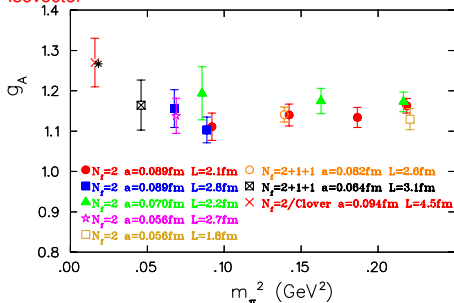
Planned experiment at JLab, SIDIS on ^3He /Proton at 11 GeV:



Experimental values: $g_T^u = 0.39^{+0.18}_{-0.12}$ and $g_T^d = -0.25^{+0.3}_{-0.1}$

Nucleon charges: g_A , g_S , g_T

Isovector

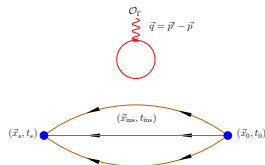


- g_A at the physical point mass indicates agreement with the physical value → **important to reduce error** - many results from other collaborations
- Experimental value of $g_T \sim 0.54^{+0.30}_{-0.13}$ from global analysis of HERMES, COMPASS and Belle e^+e^- data, *M. Anselmino et al. (2013)*
- Large excited state contributions to g_S : increasing the sink-source time separation to $\sim 1.5\text{ fm}$ is crucial

Disconnected quark loop contributions

Notoriously difficult

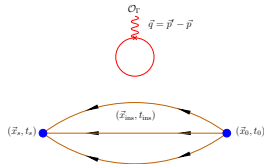
- $L(x_{\text{ins}}) = \text{Tr} [\Gamma G(x_{\text{ins}}; x_{\text{ins}})] \rightarrow$ need quark propagators from all \vec{x}_{ins} or L^3 more expensive as compared to the calculation of hadron masses
- Large gauge noise \rightarrow large statistics
- Use special techniques that utilize stochastic noise on all spatial lattice sites $\rightarrow N_r$ more expensive than hadron masses with $N_r \ll L^3$
- Reduce noise by increasing statistics
 \Rightarrow take advantage of graphics cards (GPUs) \rightarrow need to develop special multi-GPU codes



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A Fermi card



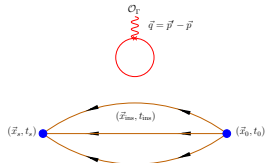
Cluster of 8 nodes of Fermi GPUs

C. A., M. Constantinou, S. Dinter, V. Drach, K. Hadjiyiannakou, K. Jansen, G. Koutsou, A. Strelchenko, A. Vaquero arXiv:1211.0126
C.A., K. Hadjiyiannakou, G. Koutsou, A. O'Cais, A. Strelchenko, arXiv:1108.2473

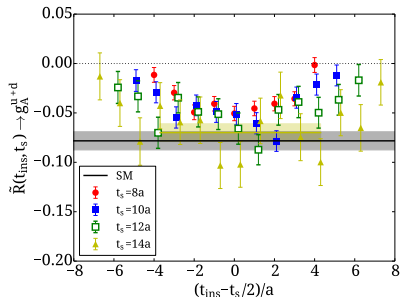
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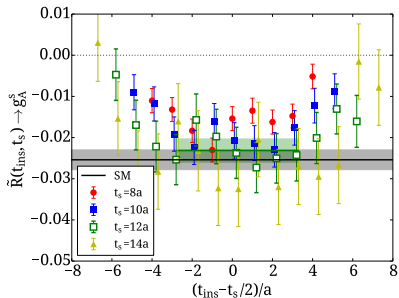


$N_r = 2 + 1 + 1$ twisted mass, $a = 0.082$ fm, $m_\pi = 373$ MeV, $\sim 150,000$ statistics (on 4700 confs)



Disconnected isoscalar, agrees with [Bali et al.](#)

(QCDSF), Phys.Rev.Lett. 108 (2012) 222001

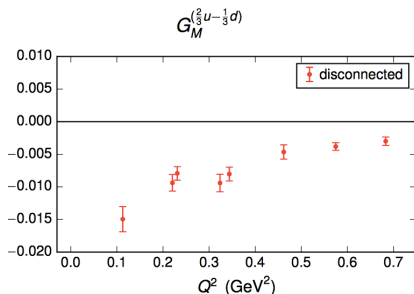
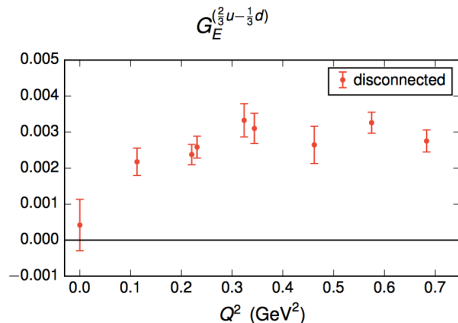
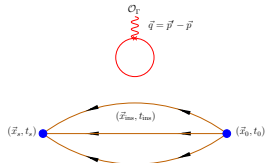


Strange quark loop

Disconnected quark loop contributions

Notoriously difficult

- $L(x_{\text{ins}}) = \text{Tr} [\Gamma G(x_{\text{ins}}; x_{\text{ins}})] \rightarrow$ need quark propagators from all \vec{x}_{ins} or L^3 more expensive as compared to the calculation of hadron masses
- Large gauge noise \rightarrow large statistics
- Use special techniques that utilize stochastic noise on all spatial lattice sites $\rightarrow N_f$ more expensive than hadron masses with $N_f \ll L^3$
- Reduce noise by increasing statistics
 \Rightarrow take advantage of graphics cards (GPUs) \rightarrow need to develop special multi-GPU codes



100,000 Statistics using hierarchical probing, $N_f = 2 + 1$ clover (one level of stout smearing), $V = 32^3 \times 96$, $a \sim 0.114 \text{ fm}$, $m_\pi \sim 320 \text{ MeV}$, St. Meinel *et al.*, Lattice 2014, N. York, June, 2014

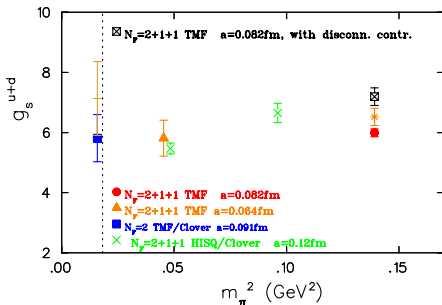
Isoscalar nucleon charges: g_A , g_s , g_T

- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x) \frac{\tau^a}{2} \psi(x)$
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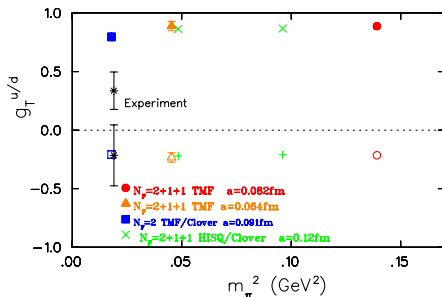
- $N_f = 2 + 1 + 1$ twisted mass, $a = 0.082$ fm, $m_\pi = 373$ MeV
- Disconnected part, $\sim 150\,000$ statistics using GPUs

Results shown in \overline{MS} at 4 GeV^2

Analysis at the physical point still preliminary



Large source-sink separation and inclusion of disconnected is required



Experimental values from global analysis of HERMES, COMPASS and Belle e^+e^- data, M. Anselmino *et al.* (2013)

Nucleon momentum fraction and spin

What is the distribution of the nucleon momentum among the nucleon constituents?

→ needs knowledge of the parton distribution functions (PDFs)

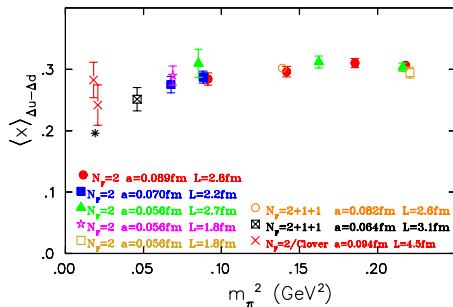
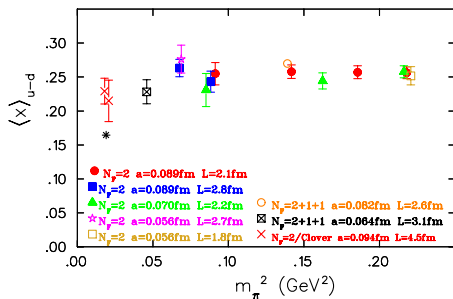
One measures moments of parton distributions, in DIS:

- Unpolarized moments: $\langle x^n \rangle_q = \int_0^1 dx x^n [q(x) - (-1)^n \bar{q}(x)]$, $q(x) = q(x)_\downarrow + q(x)_\uparrow$
- Helicity moments: $\langle x^n \rangle_{\Delta q} = \int_0^1 dx x^n [\Delta q(x) + (-1)^n \Delta \bar{q}(x)]$, $\Delta q(x) = q(x)_\downarrow - q(x)_\uparrow$
- Transversity moments: $\langle x^n \rangle_{\delta q} = \int_0^1 dx x^n [\delta q(x) - (-1)^n \delta \bar{q}(x)]$, $\delta q(x) = q(x)_\perp + q(x)_\top$

Nucleon momentum fraction and spin

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Consider $n = 1$; results obtained in the \overline{MS} scheme at $\mu = 2$ GeV.



- $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u-\Delta d}$ approach physical value for bigger source-sink separations \rightarrow need an equivalent high statistics study as at $m_\pi = 373$ MeV
- Can provide a prediction for $\langle x \rangle_{\delta u-\delta d}$
- These are computed within B3 for both Lattice and Phenomenology

Experimental values:

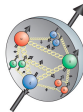
$\bullet \langle x \rangle_{u-d}$ from S. Alekhin *et al.* arXiv:1202.2281

$\bullet \langle x \rangle_{\Delta u-\Delta d}$ from Blumlein *et al.* arXiv:1005.3113

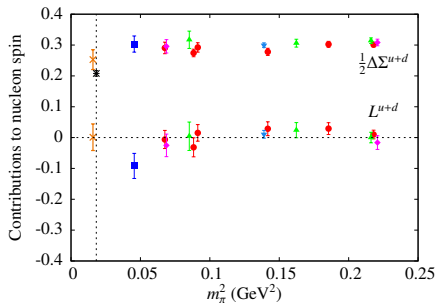
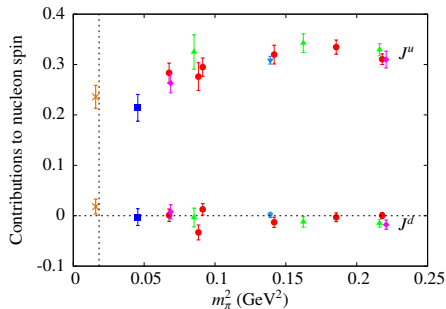
Where is the nucleon spin?

$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2} \Delta\Sigma^q + L^q \right)}_{J^q} + J^G$$

$$J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \text{ and } \Delta\Sigma^q = g_A^q$$



Connected contributions



⇒ Total spin for u-quarks $J^u \lesssim 0.25$ and for d-quark $J^d \sim 0$

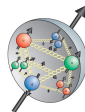
- $L^{u+d} \sim 0$ at physical point
- $\Delta\Sigma^{u+d}$ in agreement with experimental value at physical point
- The total spin $J^{u+d} \sim 0.25 \Rightarrow$ **Where is the other half?**

However, more statistics and checks of systematics are needed for final results at the physical point

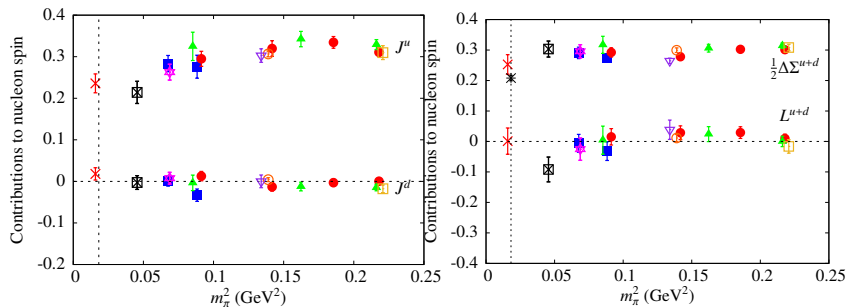
Where is the nucleon spin?

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For one ensemble at $m_\pi = 373$ MeV we have the disconnected contribution \rightarrow we can check the effect on the observables, $\mathcal{O}(150,000)$ statistics

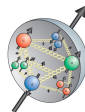


- Disconnected quark loop contributions non-zero for $\Delta \Sigma^{u,d,s}$
- $L^d \sim -L^u$
- The total spin $J^{u+d} \sim 0.25 \Rightarrow$ **Where is the other half?**
- Contributions from J^G ? \rightarrow on-going efforts to compute them, K.-F. Liu *et al.* (χ QCD), arXiv:1203.6388; C.A. *et al.*, arXiv:1311.3174

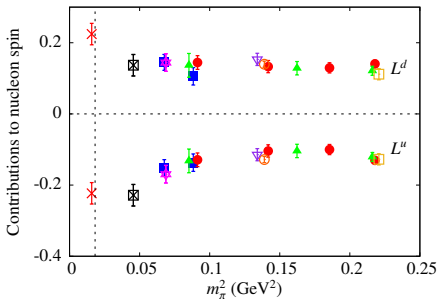
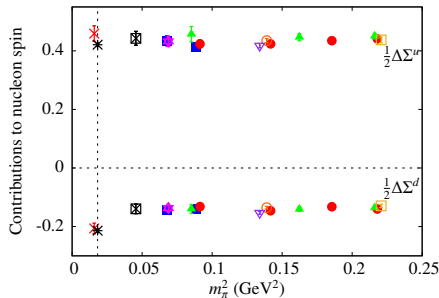
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Direct computation of PDFs

Ongoing work: C.A., K. Cichy, E. Garcia Ramos, K.Jansen, K. Hadjiyiannakou, F. Steffens, C. Wiese
Quasi distributions, X. Ji, arXiv:1305.1539:

$$\tilde{q}(x, \mu^2, p_z) = \frac{1}{4\pi} \int dz e^{-izxp_z} \langle N(p_z) | \psi(z) \gamma_z \mathcal{P}(z, 0) \psi(0) | N(p_z) \rangle + \mathcal{O} \left(\frac{\Lambda^2}{p_z^2}, \frac{m_N^2}{p_z^2} \right)$$

Needs to be matched to the distribution extracted from experiment

$$\tilde{q}(x, \mu^2, p_z) = \int \frac{dy}{y} Z \left(\frac{x}{y}, \frac{\mu}{p_z} \right) q(y, \mu) + \mathcal{O} \left(\frac{\Lambda^2}{p_z^2}, \frac{m_N^2}{p_z^2} \right)$$

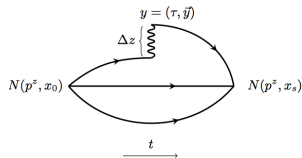
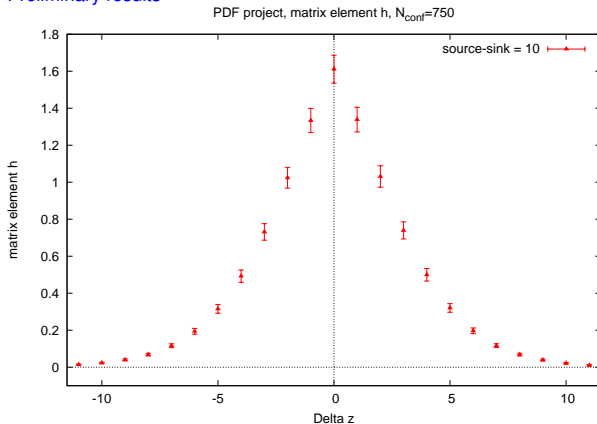
Real question is how large should the momentum be?

Direct computation of PDFs

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Preliminary results

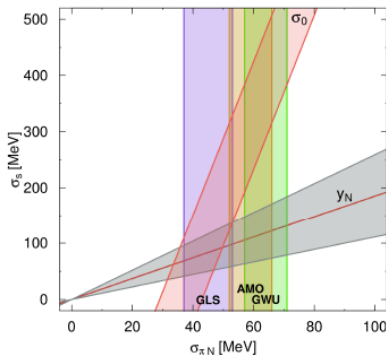
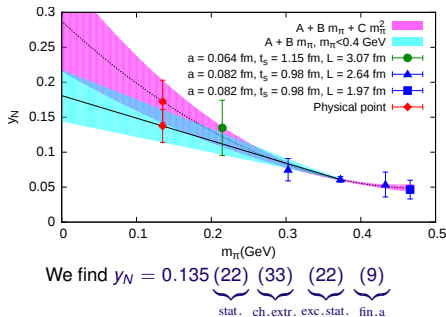


The quark content of the nucleon

- $\sigma_I \equiv m_l \langle N | \bar{u}u + \bar{d}d | N \rangle$: measures the explicit breaking of chiral symmetry
Extracted from analysis of low-energy pion-proton scattering data
Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on σ_I , J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD it can be obtained via the Feynman-Hellman theorem: $\sigma_I = m_l \frac{\partial m_N}{\partial m_l}$
- Similarly $\sigma_s \equiv m_s \langle N | \bar{s}s | N \rangle = m_s \frac{\partial m_N}{\partial m_s}$
- The strange quark content of the nucleon: $y_N = \frac{2 \langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle} = 1 - \frac{\sigma_0}{\sigma_I}$, where σ_0 is the flavor non-singlet
- A number of groups have used the spectral method to extract the σ -terms, R. Young, Lattice 2012
But they can be also calculated directly

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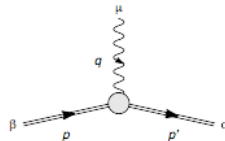


Using $\sigma_s = \frac{1}{2} \frac{m_s}{m_l} y_N \sigma_I$ we find σ_s to be less ~ 200 MeV

C.A., M. Constantinou, s. Dinter, V. Drach, K. Hadjiyiannakou, K. Jansen, G.Koutsou, A. Vagueiro, arXiv:1309.7768

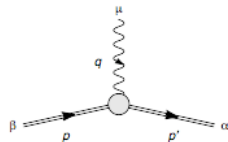
Electromagnetic form factors

$$\langle N(p', s') | j^\mu(0) | N(p, s) \rangle = \bar{u}_N(p', s') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u_N(p, s)$$



Electromagnetic form factors

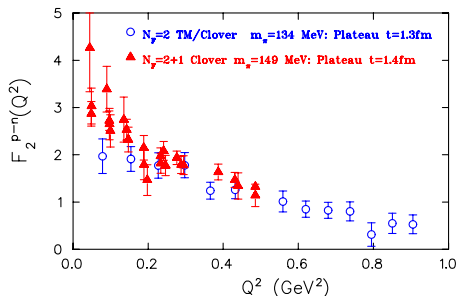
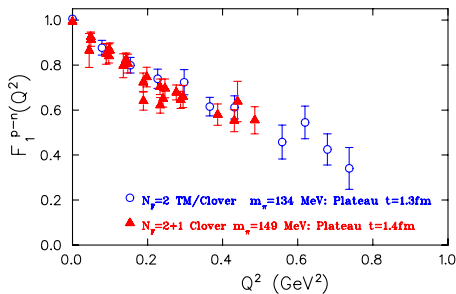
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The good news

Two studies at near physical pion mass:

- ETMC: $N_f = 2$ twisted mass with clover, $a = 0.091$ fm, $m_\pi = 134$ MeV, 1020 statistics
- MIT: $N_f = 2 + 1$ clover produced by the BMW collaboration, $a = 0.116$ MeV, $m_\pi = 149$ MeV, ~ 7750 statistics, J.M. Green *et al.* 1404.4029

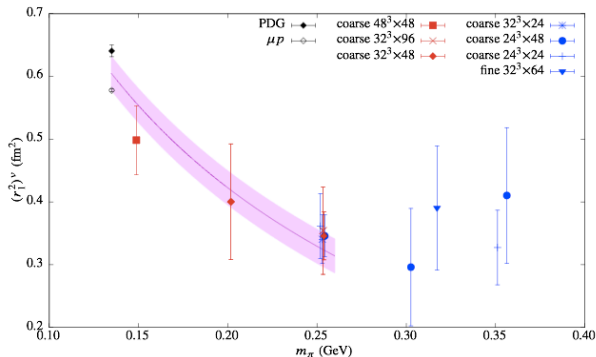


Agreement even before taking the continuum limit

Dirac and Pauli radii

Dipole fits: $\frac{G_0}{(1+Q^2/M^2)^2} \Rightarrow \langle r_i^2 \rangle = -\frac{6}{F_i} \frac{dF_i}{dQ^2} \Big|_{Q^2=0} = \frac{12}{M_i^2}$

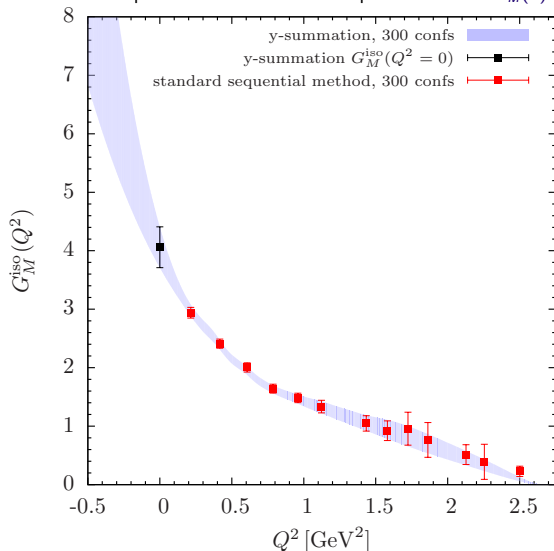
Need better accuracy at the physical point



Using results from summation method, J. M. Green *et al.*, 1404.4029

Momentum dependence of form factors

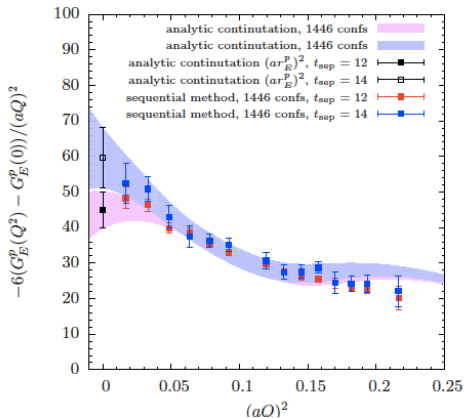
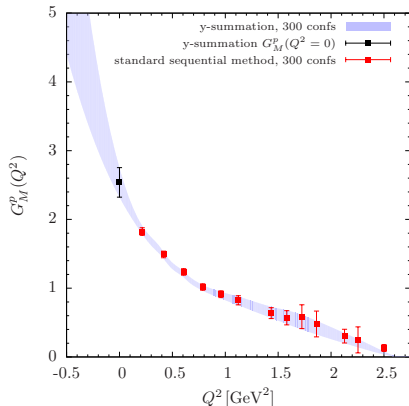
Avoid model dependence-fits: As a first step we calculated $G_M(0)$ (equivalently $F_2(0)$) at $m_\pi = 373$ MeV



Work in progress, C.A., G. Koutsou, K. Ottnad, M. Petschlies

Momentum dependence of form factors

Avoid model dependence-fits: As a first step we calculated $G_M(0)$ (equivalently $F_2(0)$) at $m_\pi = 373$ MeV
Disconnected contributions small



Work in progress, C.A., G. Koutsou, K. Ottnad, M. Petschlies

Conclusions

Simulations at the physical point → that's where we always wanted to be! An excellent outcome of SFB TR9

Future Perspectives

- Confirm g_A , $\langle x \rangle_{u-d}$, etc, at the physical point
- Provide predictions for g_S , g_T , tensor moment, sigma-terms, etc.
- Provide accurate results on proton radius
- Develop methods for resonances
- Develop methods for calculating the neutron electric dipole moment and other challenging observables

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Many challenges ahead ...

But as simulations at the physical pion mass and more computer resources are becoming available we expect many physical results on key hadron observables that will impact both experiments and phenomenology



Backup slides

Hadron mass

First goal: reproduce the low-lying masses

As in the meson sector we need:

Use Euclidean correlation functions

$$\begin{aligned}
 G(\vec{q}, t_s) &= \sum_{\vec{x}_s} e^{-i\vec{x}_s \cdot \vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0) \rangle \\
 &= \sum_{n=0, \dots, \infty} A_n e^{-E_n(\vec{q}) t_s} \xrightarrow{t_s \rightarrow \infty} A_0 e^{-E_0(\vec{q}) t_s}
 \end{aligned}$$

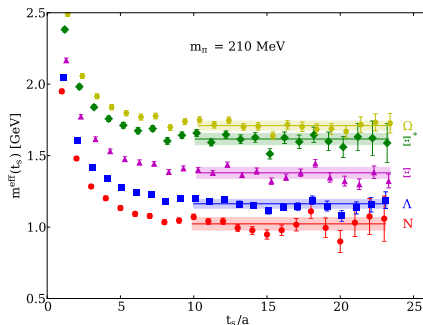


- Noise to signal increases with t_s
 - ▶ use techniques to improve ground state dominance in correlators
 - ▶ enough statistics so that the signal extends to large enough t_s at which any remaining contamination from higher states is negligible

- Large Euclidean time evolution gives ground state for given quantum numbers \Rightarrow enables determination of low-lying hadron properties

Special techniques to extract excited states

- $aE_{\text{eff}}(\vec{q}, t_s) = \ln [G(\vec{q}, t_s)/G(\vec{q}, t_s + a)]$
 $= aE_0(\vec{q}) + \text{excited states}$
 $\rightarrow aE_0(\vec{q}) \xrightarrow{\vec{q}=0} am$

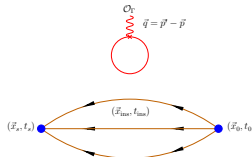
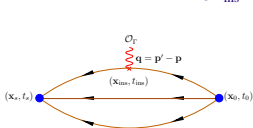


$N_f = 2 + 1 + 1$ TM fermions

Challenges: II. Nucleon structure

Evaluation of three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_S, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_S, t_s) \mathcal{O}^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



- Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

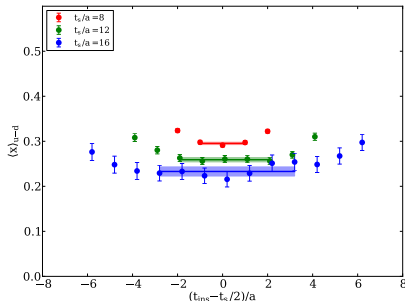
$$R(t_s, t_{\text{ins}}, t_0) \xrightarrow[(t_s - t_{\text{ins}})\Delta \gg 1]{(t_{\text{ins}} - t_0)\Delta \gg 1} \mathcal{M} [1 + \langle 0 | J_N | N \rangle \langle N | \mathcal{O}_T | N' \rangle \langle N' | J_h^+ | 0 \rangle e^{-\Delta(\mathbf{p})(t_{\text{ins}} - t_0)} + \langle 0 | J_N | N' \rangle \langle N' | \mathcal{O}_T | N \rangle \langle N | J_h^+ | 0 \rangle e^{-\Delta(\mathbf{p}')(t_s - t_{\text{ins}})} + \dots]$$

- \mathcal{M} the desired matrix element;
- t_s, t_{ins}, t_0 the sink, insertion and source time-slices;
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

- Connect lattice results to measurements:

$$\mathcal{O}_{\overline{\text{MS}}}(\mu) = Z(\mu, a) \mathcal{O}_{\text{latt}}(a)$$

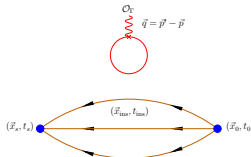
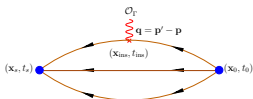
\Rightarrow evaluate $Z(\mu, a)$ non-perturbatively



Challenges: II. Nucleon structure

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$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



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$$R(t_s, t_{\text{ins}}, t_0) \xrightarrow[(t_s - t_{\text{ins}})\Delta \gg 1]{(t_{\text{ins}} - t_0)\Delta \gg 1} \mathcal{M} [1 + \langle 0 | J_N | N \rangle \langle N | \mathcal{O}_\Gamma | N' \rangle \langle N' | J_h^+ | 0 \rangle e^{-\Delta(\mathbf{p})(t_{\text{ins}} - t_0)} + \langle 0 | J_N | N' \rangle \langle N' | \mathcal{O}_\Gamma | N \rangle \langle N | J_h^+ | 0 \rangle e^{-\Delta(\mathbf{p}')(t_s - t_{\text{ins}})} + \dots]$$

- \mathcal{M} the desired matrix element; t_s, t_{ins}, t_0 the sink, insertion and source time-slices; $\Delta(\mathbf{p})$ the energy gap with the first excited state
- Summing over t_{ins} :

$$\sum_{t_{\text{ins}}=t_0}^{t_s} R(t_s, t_{\text{ins}}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

So the excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{\text{ins}}$ and/or $t_{\text{ins}} - t_0$. However, one needs to fit the slope rather than to a constant.

- Fit $R(t_s, t_{\text{ins}}, t_0)$ including the first excited state

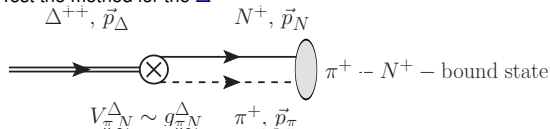
All methods should yield the same result if the ground state is identified

Connect lattice results to measurements: $\mathcal{O}_{\overline{\text{MS}}}(\mu) = Z(\mu, a) \mathcal{O}_{\text{latt}}(a) \implies$ evaluate $Z(\mu, a)$ non-perturbatively

Challenges: IV. Decay width of baryons

Use the transition amplitude method, C. McNeile, C. Michael, P. Pennanen, PRD 65 094505 (2002)

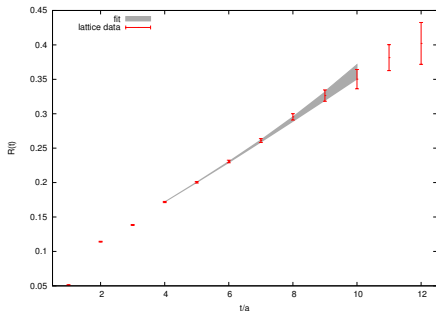
Test the method for the Δ



\Rightarrow compute transition amplitude $x = \langle \Delta | N\pi \rangle$ from the correlator $G^{\Delta \rightarrow N\pi}$

- Need $E_\Delta \sim E_\pi + E_N$
- Applicable for $xt \ll 1$

Hybrid calculation at $m_\pi \sim 360$ MeV with $L = 3.6$ fm



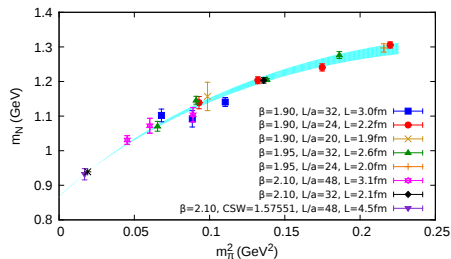
$$R(t) = \frac{G^{\Delta \rightarrow \pi N}(t, \vec{Q}, \vec{q})}{\sqrt{G^\Delta(t, \vec{q}) G^{N\pi}(t, \vec{Q}, \vec{q})}} \sim A + B \frac{\sinh(\Delta t/2)}{\sin(\Delta/2)} \sim A + Bt$$

$$g_{\Delta\pi N} = 27.0(0.6)(1.5) \rightarrow \Gamma_\Delta = 99(12) \text{ MeV}$$

C. A., J. W. Negele, M. Petschlies, A. Tsapalis, PRD 88 031501 (2013)

Setting the scale

- For baryon observables use nucleon mass at physical limit
- Extrapolate using lowest one-loop result: $m_N = m_N^0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2} m_\pi^3$
- Estimate systematic error from next order in HB χ PT that includes explicit Δ -degrees of freedom



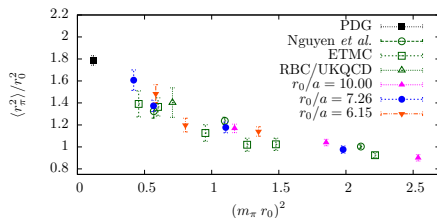
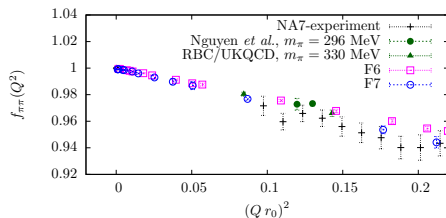
- For $N_f = 2 + 1 + 1$ three different lattice spacings smaller than 0.1 fm \rightarrow allows continuum extrapolation
- For $N_f = 2$ with clover term $a = 0.0937(2)(2)$ fm and pion mass 130 MeV.

- σ -term from m_N using $\mathcal{O}(p^3)$ and $m_\pi \lesssim 300$ MeV: $\sigma_{\pi N} = 58(8)(7)$ MeV
- Using the nucleon mass we find $r_0 \sim 0.495(5)$ fm in the continuum limit

Pion form factor

Several Collaborations e.g. ETMC, $N_f = 2$, R. Frezzotti, V. Lubicz and S. Simula, PRD 79, 074506 (2009); B.B. Brandt, A. Juttner, H. Wittig (CLS) arXiv1306.2916, using three lattice spacings smaller than 0.1 fm and pion masses ~ 250 MeV and ~ 600 MeV

- Examine volume and cut-off effects \Rightarrow estimate continuum and infinite volume values
- Twisted boundary conditions to probe small $Q^2 = -q^2$
- All-to-all propagators and 'one-end trick' to obtain accurate results
- Chiral extrapolation using NNLO $\rightarrow \langle r^2 \rangle$ and $F_\pi(Q^2) = (1 + \langle r^2 \rangle Q^2/6)^{-1}$

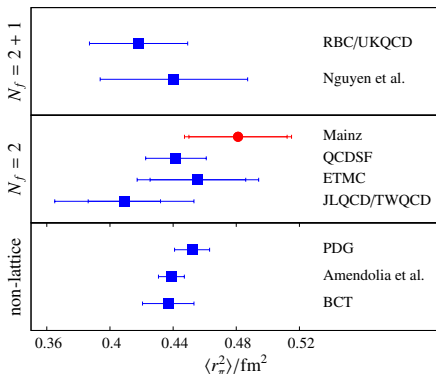


B.B. Brandt, A. Juttner, H. Wittig (CLS) arXiv1306.2916

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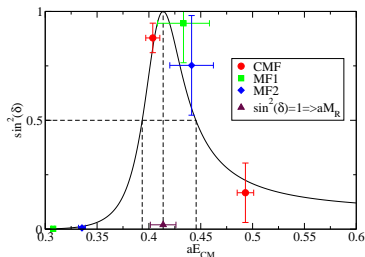
B.B. Brandt, A. Juttner, H. Wittig (CLS) arXiv1306.2916

ρ -meson width

- Consider $\pi^+\pi^-$ in the $l = 1$ -channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $\tan\delta_{11}(k) = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{E(m_R^2 - E^2)}$, $k = \sqrt{E^2/4 - m_\pi^2} \rightarrow$ determine m_R and

$$g_{\rho\pi\pi} \text{ and then extract } \Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k_R^3}{m_R^2}, \quad k_R = \sqrt{m_R^2/4 - m_\pi^2}$$

$$m_\pi = 309 \text{ MeV}, L = 2.8 \text{ fm}$$



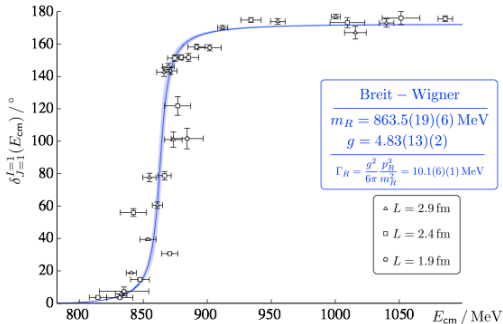
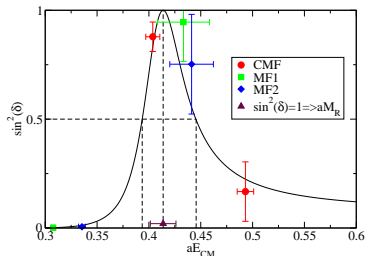
$N_F = 2$ twisted mass fermions, Xu Feng, K. Jansen and
D. Renner, Phys. Rev. D83 (2011) 094505

ρ -meson width

- Consider $\pi^+\pi^-$ in the $I = 1$ -channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $\tan\delta_{11}(k) = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{E(m_R^2 - E^2)}$, $k = \sqrt{E^2/4 - m_\pi^2} \rightarrow$ determine m_R and

$$g_{\rho\pi\pi} \text{ and then extract } \Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k_R^3}{m_R^2}, \quad k_R = \sqrt{m_R^2/4 - m_\pi^2}$$

$m_\pi = 309$ MeV, $L = 2.8$ fm

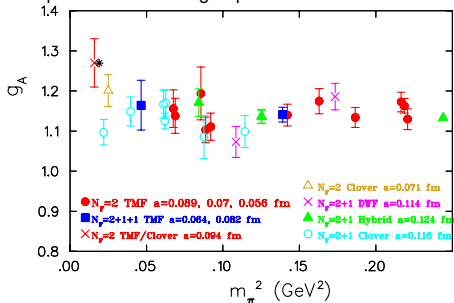


$N_F = 2$ twisted mass fermions, Xu Feng, K. Jansen and
D. Renner, Phys. Rev. D83 (2011) 094505

Impressive results using $N_f = 2 + 1$ clover fermions
and 3 asymmetric lattices, J. J. Dudek, R. G. Edwards and C.E.
Thomas, Phys. Rev. D 87 (2013) 034505

Volume dependence axial charge g_A

Comparison with other groups



Results obtained using the plateau method with sink-source time separation $\sim (1.0 - 1.2)$ fm

- Results at near physical pion mass are now becoming available \rightarrow need dedicated study at physical point with high statistics and larger volumes

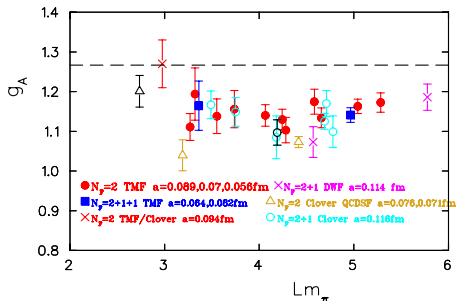
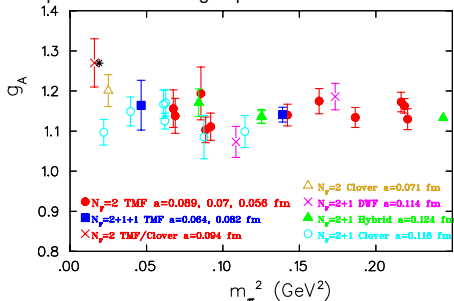
- A number of collaborations are engaging in systematic studies, e.g.

- $N_f = 2 + 1$ Clover, J. R. Green *et al.*, arXiv:1209.1687
- $N_f = 2$ Clover, R. Hosley *et al.*, arXiv:1302.2233
- $N_f = 2$ Clover, S. Capitani *et al.*, arXiv:1205.0180
- $N_f = 2 + 1$ Clover, B. J. Owen *et al.*, arXiv:1212.4668
- $N_f = 2 + 1 + 1$ Mixed action (HISQ/Clover), T. Bhattacharya *et al.*, arXiv:1306.5435
- Also several talks in Lattice 2013 e.g. S. Ohta, M. Lin, RBC-UKQCD

C.A., M. Constantinou, S. Dinter, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, arXiv:1303.5979

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• Volume effects may not be the full story if we compare the result by QCDSF ($Lm_\pi \sim 2.7$) and LHPC ($Lm_\pi \sim 4.2$) at near physical pion mass