Hard scattering cross sections in QCD

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Highest energies at colliders until 201x

Energy frontier

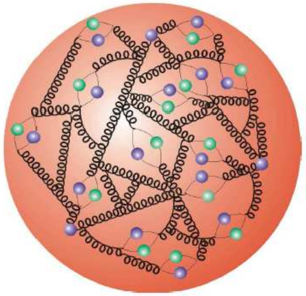
- Search for Higgs boson, new massive particles at highest energies

$$\mathrm{E} = \mathrm{m\,c}^2$$

Hadron colliders

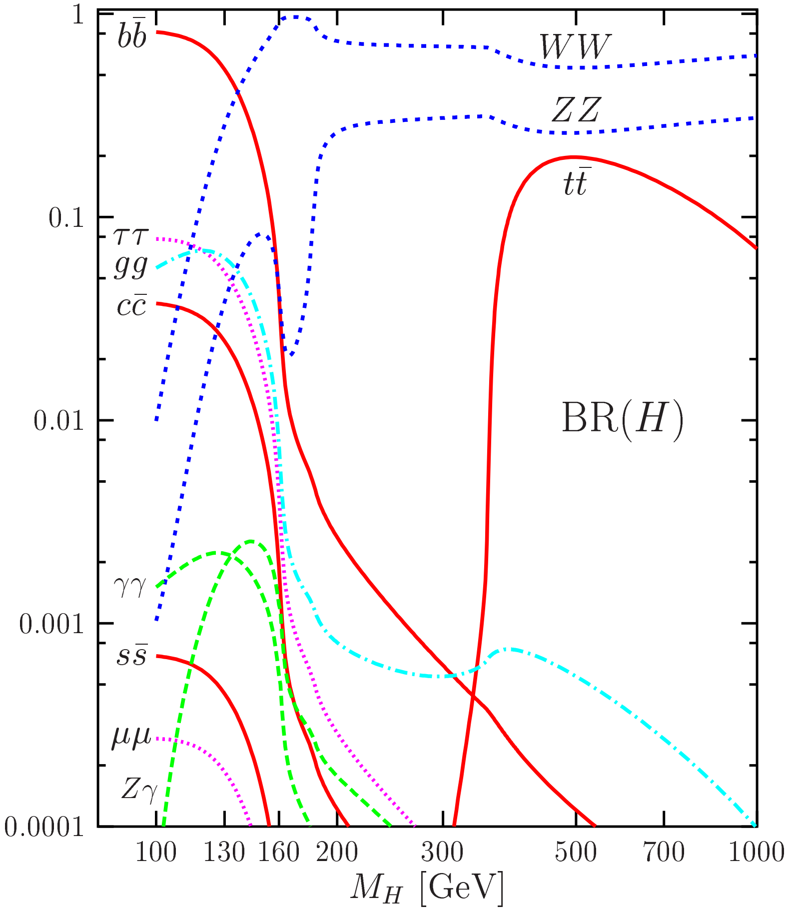
- Proton–(anti)-proton collisions reach TeV-scale
  - Tevatron $\sqrt{S} = 1.96$TeV (until 2009), LHC with $\sqrt{S} = 14$TeV

- Proton: composite multi-particle bound state
  - collider: "wide-band beams" of quarks and gluons
- Protons are heavy
  - no significant synchrotron radiation $\sim \left(\frac{E}{m}\right)^4 / r$



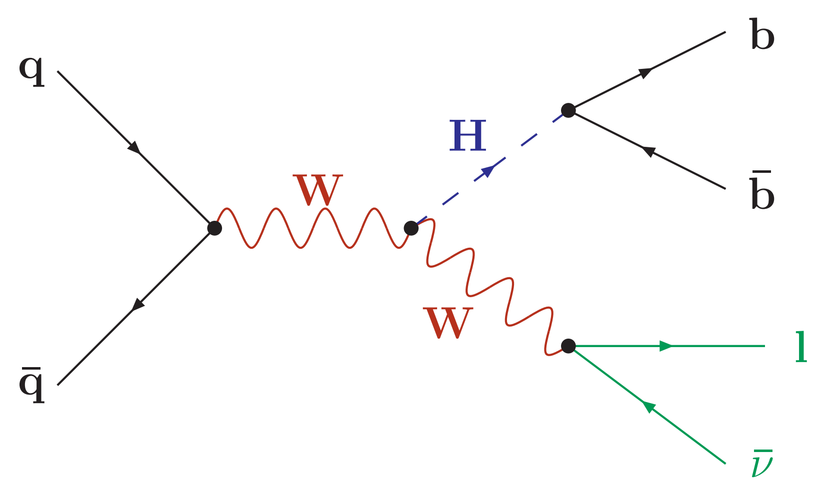
Large Hadron Collider

Higgs production at LHC



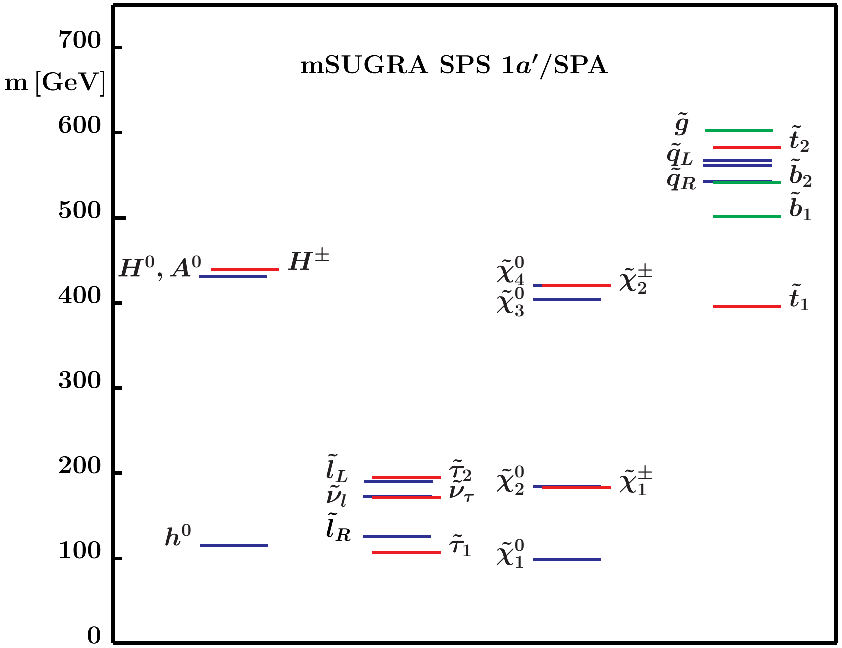
- Branching ratios for decay of Standard Model Higgs

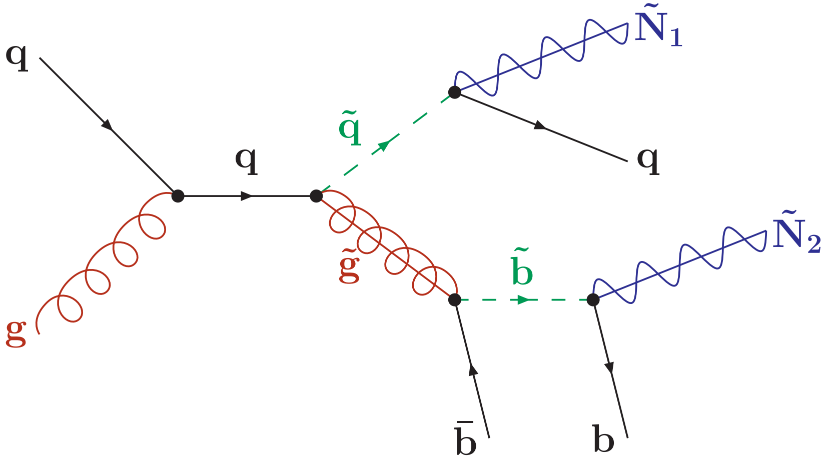
- High-multiplicity final states
  - typical SM process is accompanied by radiation of multiple jets
- Example: Higgs-strahlung
  - channel $q\bar{q} \to W(Z)H$ (third largest rate at LHC)
  - dominant decay $H \to b\bar{b}$



Supersymmetry

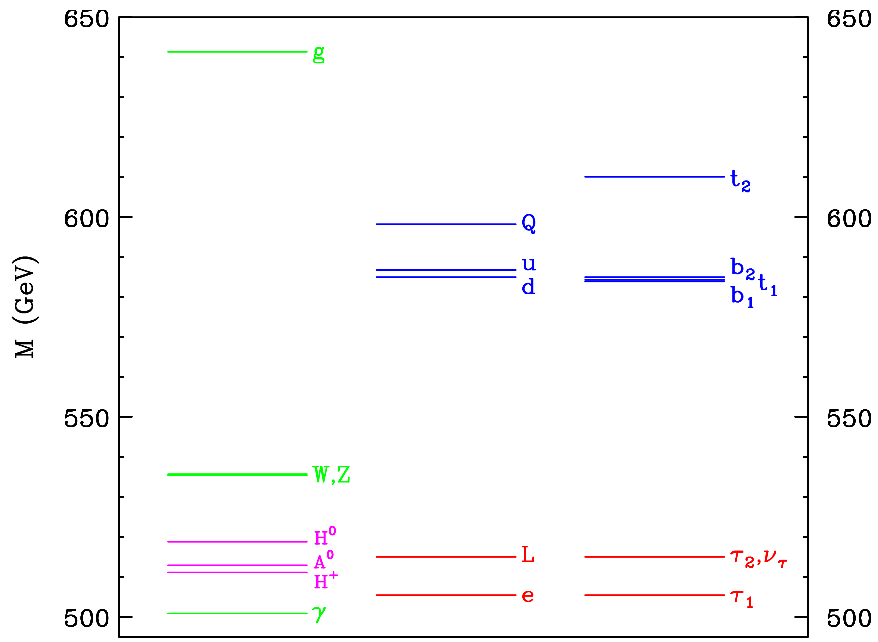
- Pair-production of supersymmetric particles (R-parity)
  - lightest supersymmetric particle (LSP) must be absolutely stable
- MSSM spectrum
  - typical signature: multiple jets, leptons and missing energy
- Example: neutralino production $\tilde{N}^0_{1,2}$
  - electric and color-neutral (dark matter candidate)

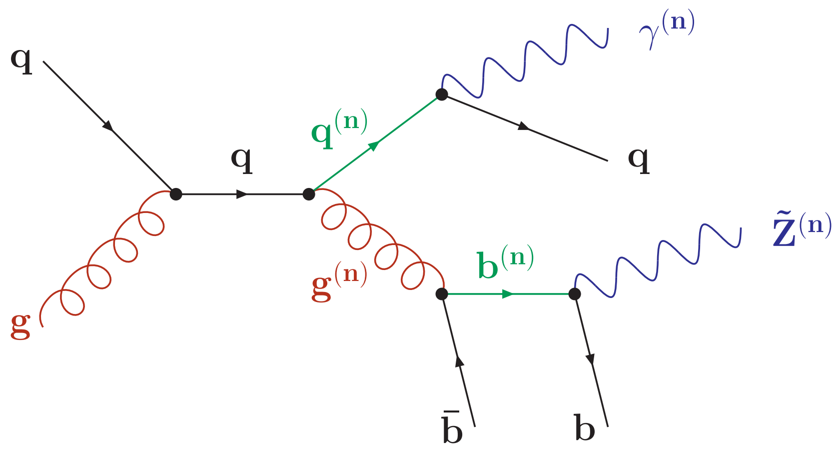




Large extra dimensions

- Spectrum of first Kaluza-Klein excitations
  - effective mass $\simeq$ (compactification radius)$^{-1}$, $m^{(n)} \simeq 1/R$
- Pair-production of excited KK-modes in interactions
  - phenomenology: missing energy in subsequent chain decays





BSM theory landscape (Murayama)

Theoretical predictions for the LHC

Challenge

- Solve master equation

$$\text{new physics} = \text{data} - \text{Standard Model}$$

- New physics searches require understanding of SM background
- LHC explores the energy frontier
  - theory has to match or exceed accuracy of LHC data

Theoretical predictions for the LHC

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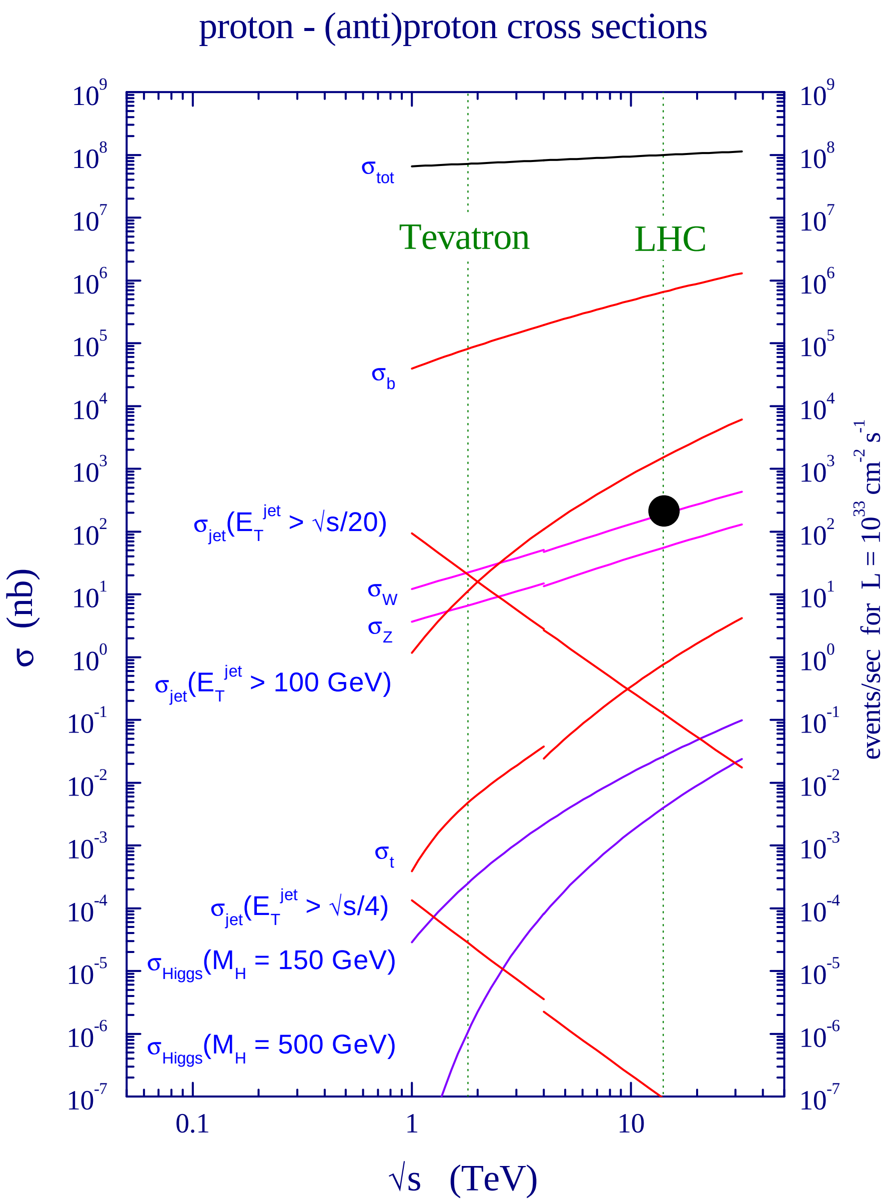
Tools

- LHC is a QCD machine
  - perturbative QCD is essential and established part of toolkit (we no longer “test” QCD)
- Electroweak corrections important for precision predictions

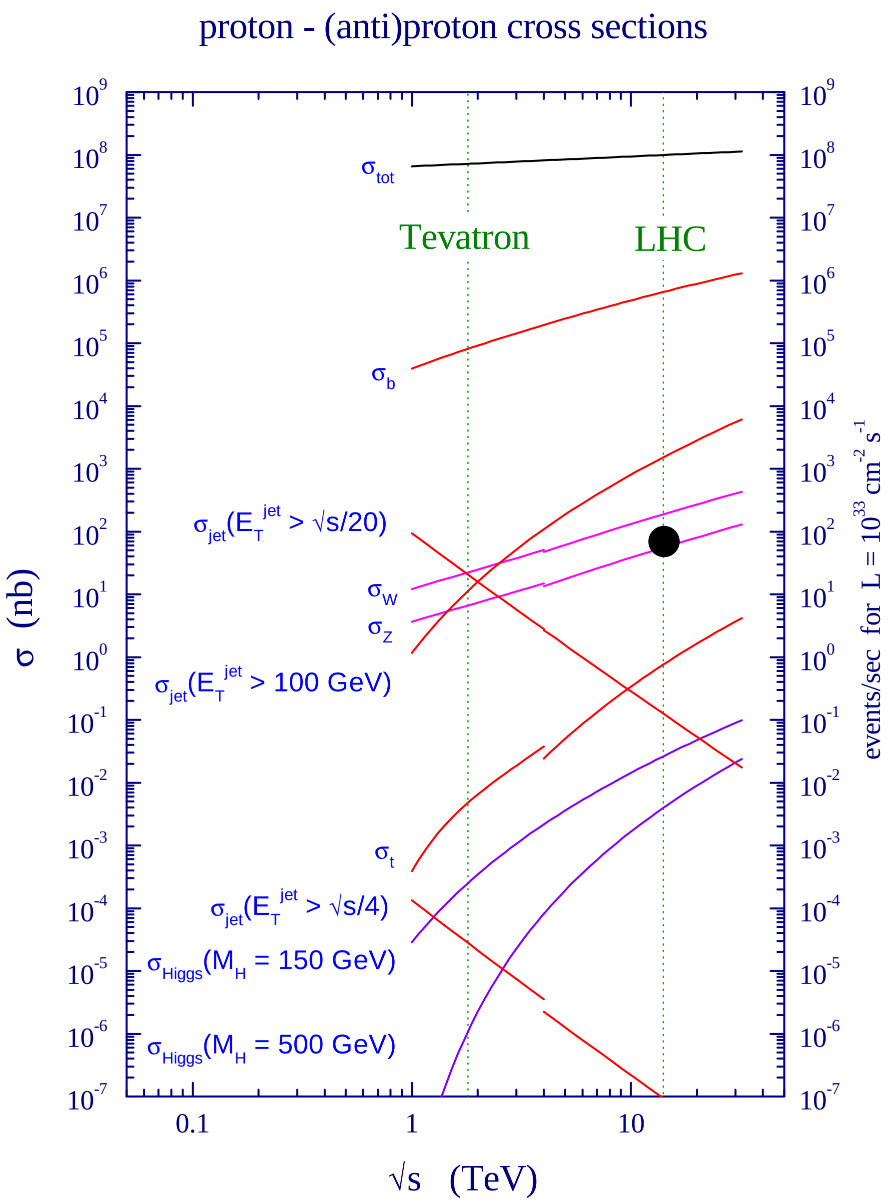
Proton-proton scattering

- Large rates expected for many Standard Model processes
- $\sigma_W \sim 150$ nb
  - B$R(W \to e + \mu) \sim 20\%$
    10 fb^{-1} gives 300M leptonic events
    rate(10^{33} cm^{-2} s^{-1}) $\sim$ 30 Hz
    rate(10^{34} cm^{-2} s^{-1}) $\sim$ 300 Hz



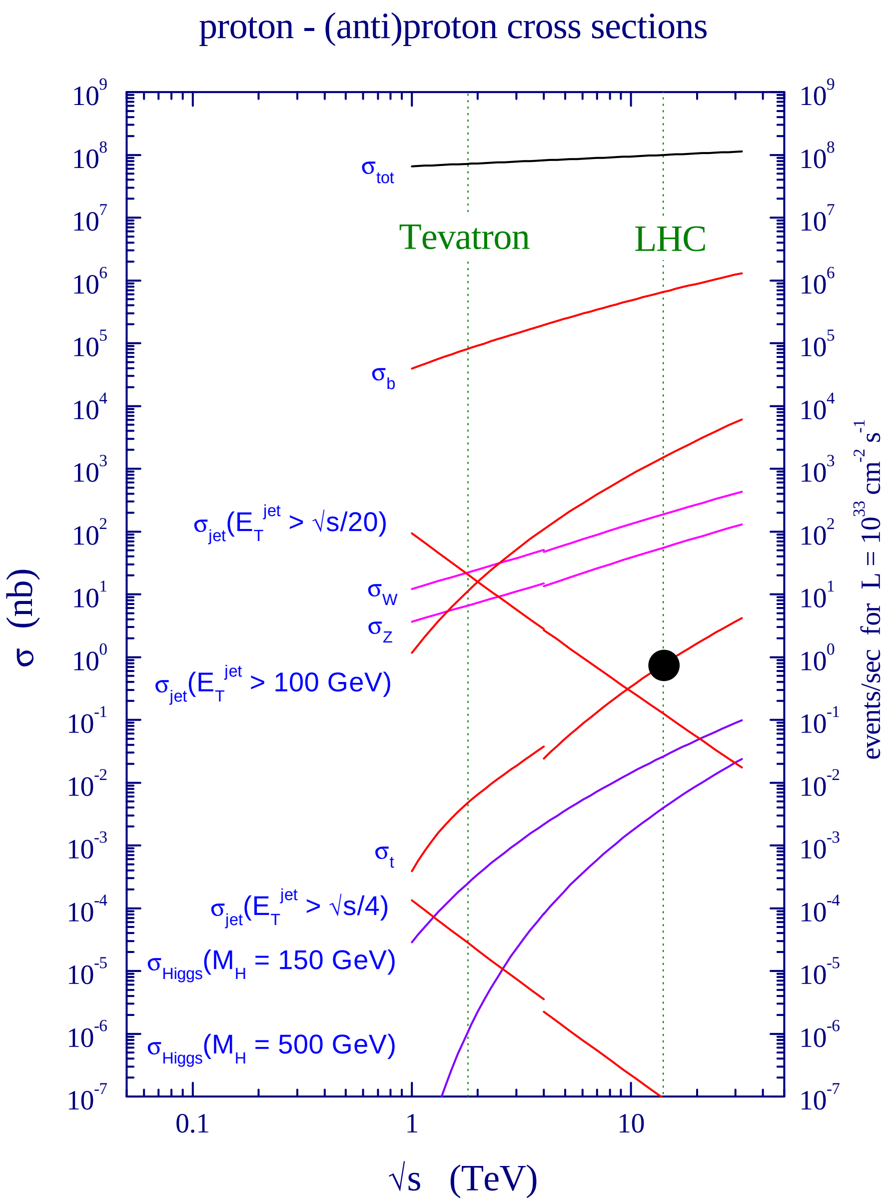
Proton-proton scattering

- Large rates expected for many Standard Model processes
- $\sigma_Z \sim 50$ nb
  - B$R(W \to ee + \mu\mu) \sim 6.6\%$
    10 fb^{-1} gives 33M leptonic events
    rate(10^{33} cm^{-2} s^{-1}) $\sim$ 3.5 Hz
    rate(10^{34} cm^{-2} s^{-1}) $\sim$ 35 Hz



Proton-proton scattering

- Large rates expected for many Standard Model processes
- $\sigma_{t\bar{t}} \sim 800$ pb
  - B$R(W \to e + \mu) \sim 30\%$
    10 fb^{-1} gives 2.4M leptonic events
    rate(10^{33} cm^{-2} s^{-1}) $\sim$ 0.2 Hz
    rate(10^{34} cm^{-2} s^{-1}) $\sim$ 2 Hz



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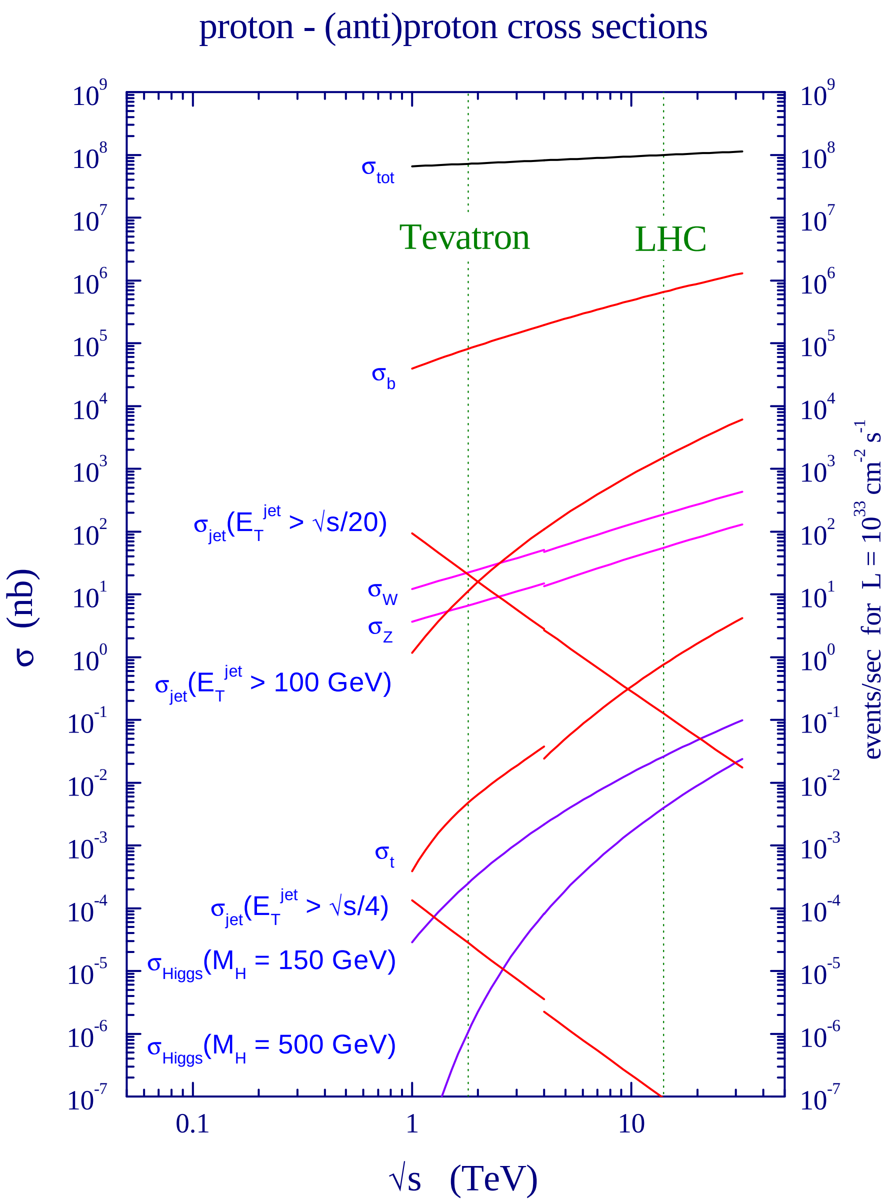
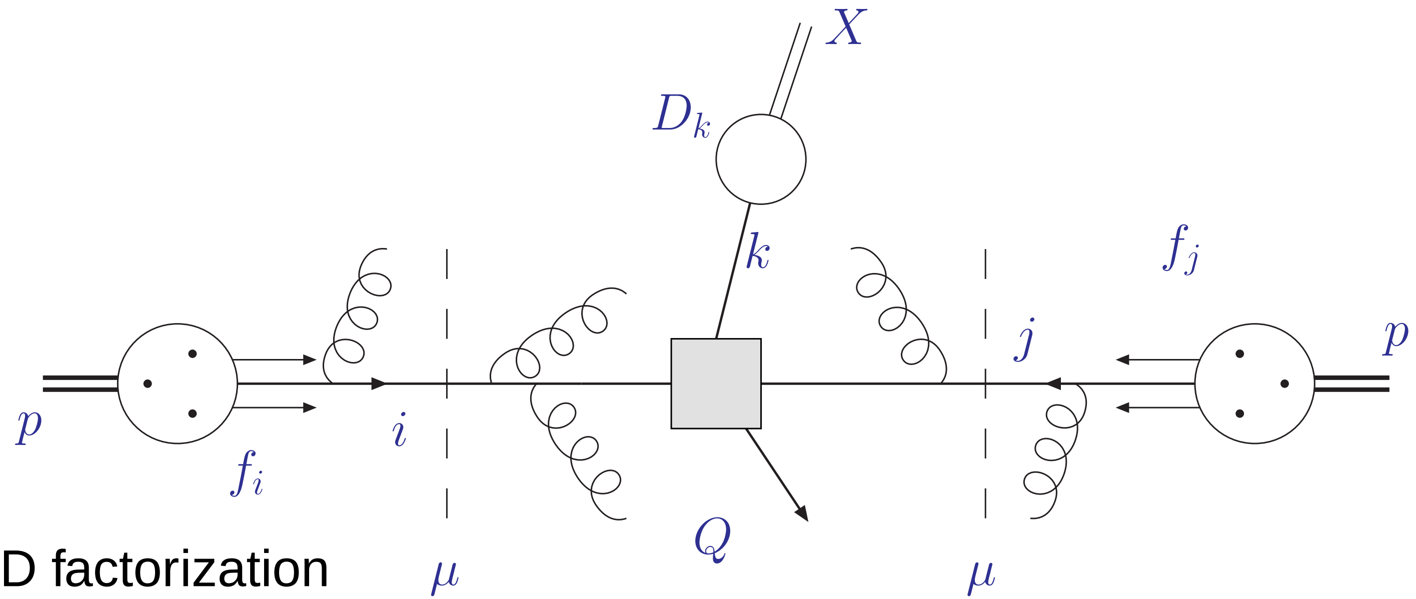
- New physics signals
  - cross section predictions
    $\sigma_{\text{new physics}} \sim \mathcal{O}(1 - 10)$ pb
  - superpartners in MSSM (neutralinos, charginos, squarks, gluinos, ...), KK modes
  - searches often assume 100 fb^{-1}



Perturbative QCD at colliders

- Hard hadron-hadron scattering
  - constituent partons from each incoming hadron interact at short



- QCD factorization
  - separate sensitivity to dynamics from different scales

$$\sigma_{pp\to X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to k}\left(\alpha_s(\mu^2), Q^2, \mu^2\right) \otimes D_{k\to X}(\mu^2)$$

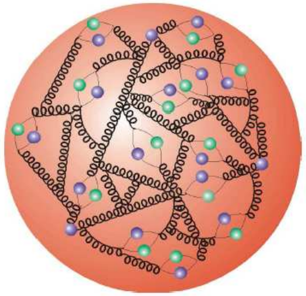
  - factorization scale μ, subprocess cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X

Hard scattering cross section

- Standard approach to uncertainties in theoretical predictions
  - variation of factorization scale μ: $\frac{d}{d \ln \mu^2} \sigma_{pp \to X} = \mathcal{O}(\alpha_s^{l+1})$

$$\sigma_{pp \to X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to k} \left(\alpha_s(\mu^2), Q^2, \mu^2 \right) \otimes D_{k \to X}(\mu^2)$$

- Parton cross section $\hat{\sigma}_{ij \to k}$ calculable pertubatively in powers of α_s
  - constituent partons from incoming protons interact at short distances of order $\mathcal{O}(1/Q)$
- Parton luminosity $f_i \otimes f_j$
  - convolution of parton distribution functions
  - quarks/gluons carry fraction x of proton momentum

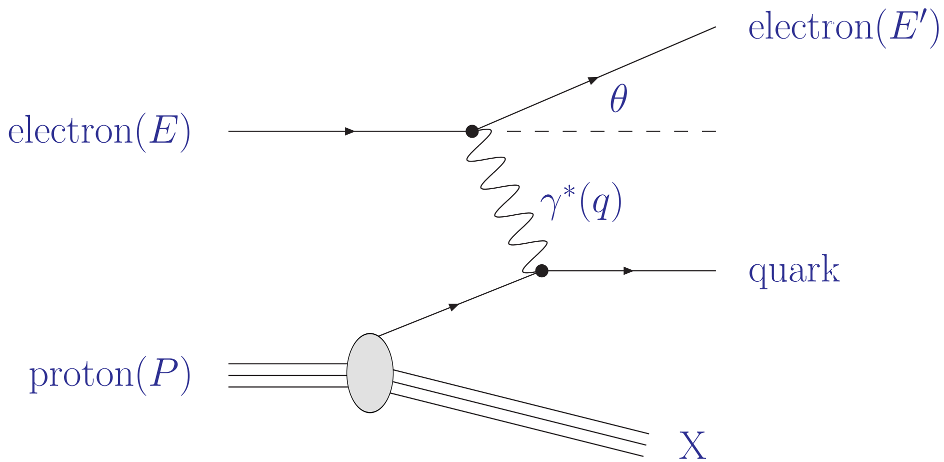


- Final state X: hadrons, mesons, jets, ...
  - fragmentation function $D_{k \to X}(\mu^2)$ or jet algorithm
  - interface with showering algorithms (Monte Carlo)

Accuracy of perturbative predictions for σ_{had}

- LO (leading order)
  - Automated tree level calculations in Standard Model, MSSM, . . . (Madgraph, Sherpa, Alpgen, CompHEP, . . .)
  - LO + parton shower
  - String inspired techniques
- NLO (next-to-leading order)
  - Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, . . .)
  - NLO + parton shower (MC@NLO, VINCIA)
- NNLO (next-to-next-to-leading order)
  - selected results known (mostly inclusive kinematics)
- N^{3}LO (next-to-next-to-next-to-leading order)
  - very few . . .

Inelastic electron-proton scattering



- Virtuality of photon: resolution

$$Q^2 \equiv -q^2 = 4EE' \sin^2(\theta/2)$$

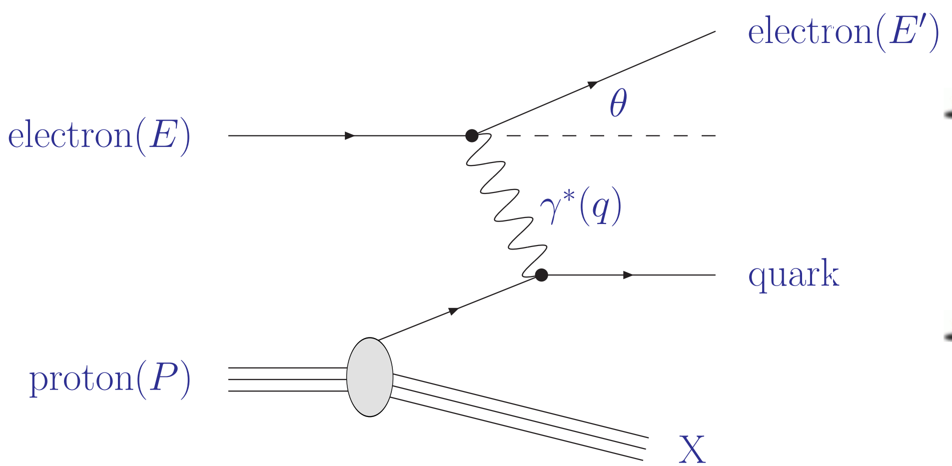
- Bjorken variable: inelasticity

$$x = \frac{Q^2}{2P \cdot q} < 1$$

- Cross section (X inclusive): proton structure function F_i^p

$$(E - E') \frac{d\sigma}{d\Omega \, dE'} \stackrel{\text{lab}}{=} \underbrace{\frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}}_{\text{Mott-scattering (point-like)}} \left\{ F_2^p(x, Q^2) + \tan^2 \frac{\theta}{2} \, F_1^p(x, Q^2) \right\}$$

Inelastic electron-proton scattering



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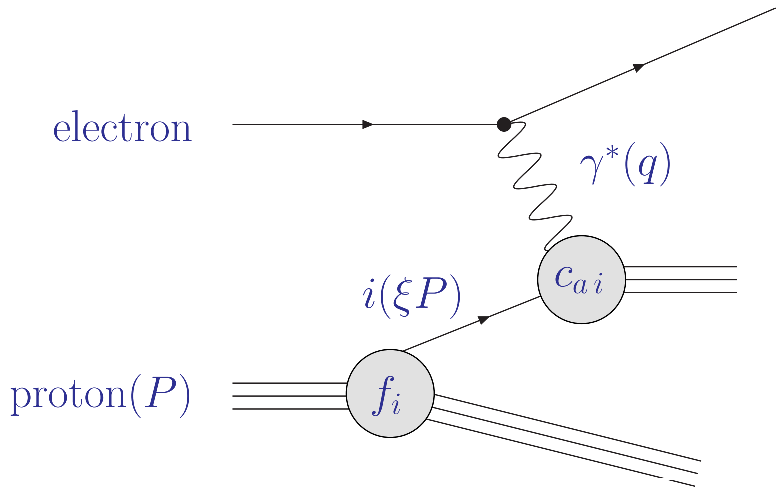
- Deep-inelastic scattering (Bjorken limit: $Q^2 \to \infty$ and x fixed)
  Parton modell (quasi-free point-like constituents, incoherence)

$$F_2(x,Q^2) \simeq F_2(x) = \sum_i e_i^2\, x f_i(x)$$

  - $x f_i(x)$ distribution for momentum fraction x of parton i

QCD corrections in deep-inelastic scattering



- Structure function F_2 (up to terms $\mathcal{O}(1/Q^2)$)
  - Renormalization/factorization scale $\mu = \mathcal{O}(Q)$

$$x^{-1} F_2^p(x, Q^2) \;=\; \sum_i \int_x^1 \frac{d\xi}{\xi}\, c_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^p(\xi, \mu^2)$$

- Coefficient functions c_a

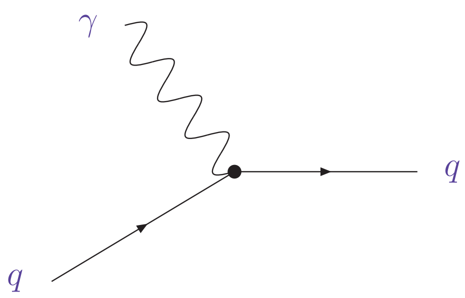
$$c_a \;=\; \underbrace{\alpha_s^{n_a}\Big[c_a^{(0)} + \alpha_s\, c_a^{(1)}}_{} \;+\; \alpha_s^2\, c_a^{(2)} \;+\; \ldots \Big]$$

NLO: standard approximation (large uncertainties)

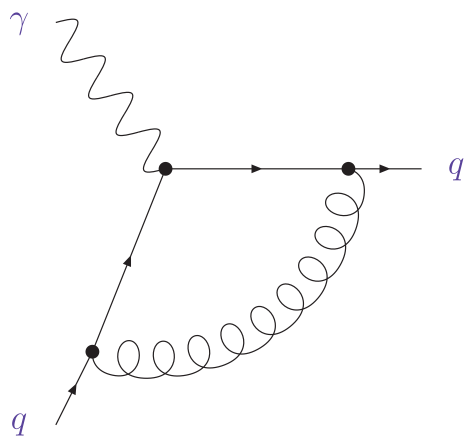
Radiative corrections in a nutshell

- Leading order
  - partonic structure function

    $\hat{F}^{(0)}_{2,q} \;=\; e_q^2 \delta(1-x)$

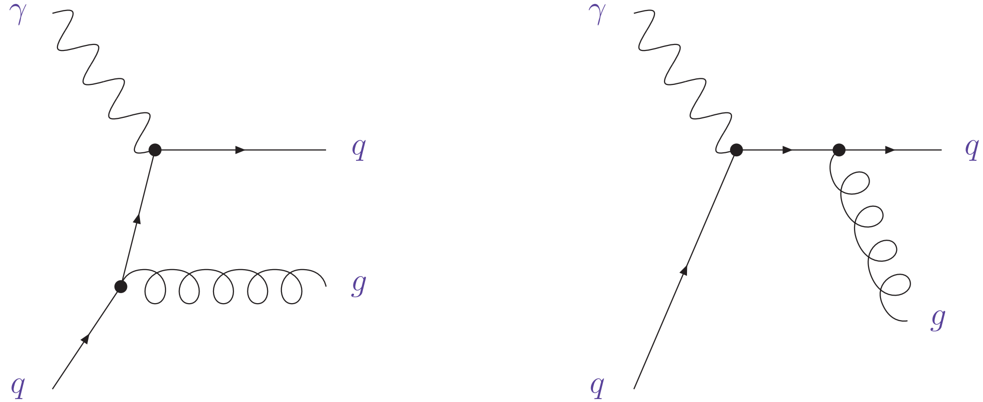


- Next-to-leading order
  - virtual correction
    (infrared divergent; proportional to Born)
  - dimensional regularization $D = 4 - 2\epsilon$



$$\hat{F}^{(1),v}_{2,q} \;=\; e_q^2\, C_F\, \frac{\alpha_s}{4\pi}\, \delta(1-x) \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \zeta_2 + \mathcal{O}(\epsilon)\right)$$

- Next-to-leading order



  - add real and virtual corrections $\hat{F}_{2,q}^{(1)} = \hat{F}_{2,q}^{(1),r} + \hat{F}_{2,q}^{(1),v}$
  - collinear divergence remains splitting functions $P_{qq}^{(0)}$

$$\hat{F}_{2,q}^{(1)} = e_q^2 C_F \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left\{ \frac{1}{\epsilon} \left(\frac{4}{1-x} - 2 - 2x + 3\delta(1-x) \right) + 4\frac{\ln(1-x)}{1-x} - 3\frac{1}{1-x} - (9 + 4\zeta_2)\delta(1-x) - 2(1+x)(\ln(1-x) - \ln(x)) - 4\frac{1}{1-x}\ln(x) + 6 + 4x + \mathcal{O}(\epsilon) \right\}$$

- Structure of NLO correction
  - absorb collinear divergence $P_{qq}^{(0)}$ in renormalized parton distributions

$$\hat{F}_{2,q}^{(1),bare} = e_q^2 \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left\{ \frac{1}{\epsilon} P_{qq}^{(0)}(x) + c_{2,q}^{(1)}(x) + \mathcal{O}(\epsilon) \right\}$$

$$q^{ren}(\mu_F^2) = q^{bare} - e_q^2 \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} P_{qq}^{(0)}(x) \left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}$$

  - partonic (physical) structure function at factorization scale μ_F

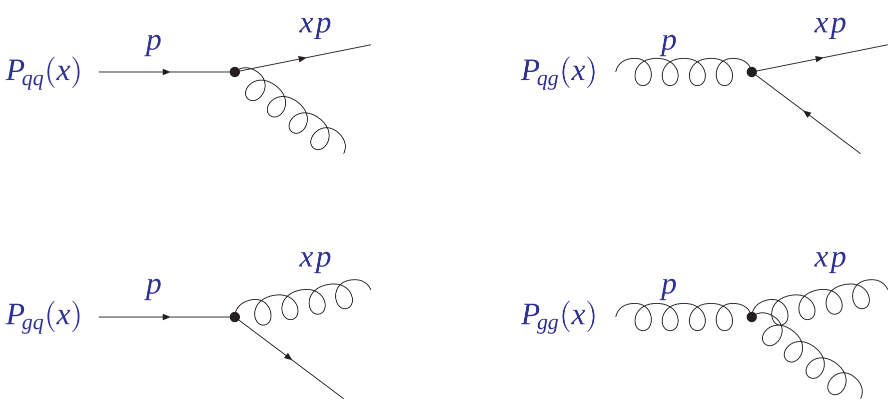
$$\hat{F}_{2,q} = e_q^2 \left(\delta(1-x) + \frac{\alpha_s}{4\pi} \left\{ c_{2,q}^{(1)}(x) - \ln\left(\frac{Q^2}{\mu_F^2}\right) P_{qq}^{(0)}(x) \right\} \right)$$

Parton distributions in proton

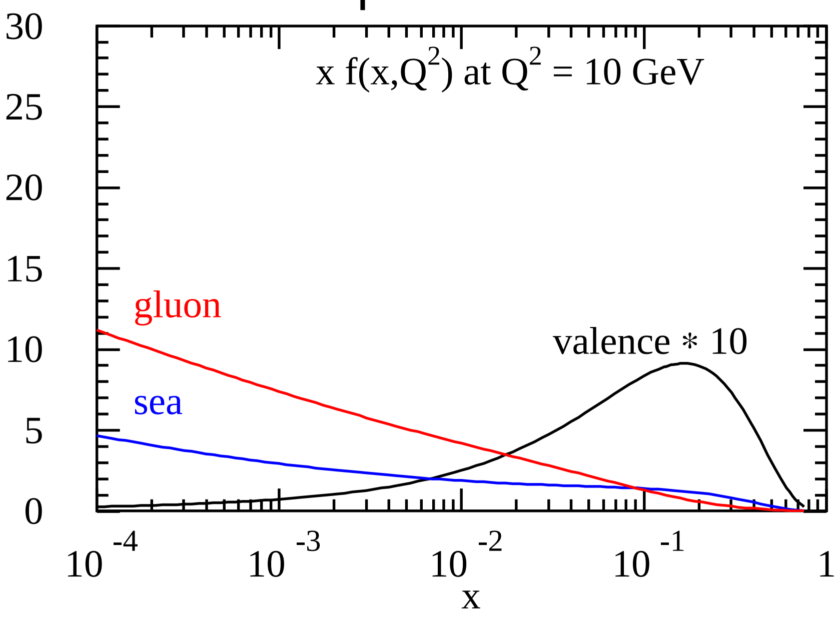
- Evolution equations for parton distributions f_i
  - splitting functions $P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots$
    (calculable in perturbative QCD)

$$\frac{d}{d \ln \mu^2} f_i(x, \mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

parton splitting in leading order



universality: predictions from fits to reference processes



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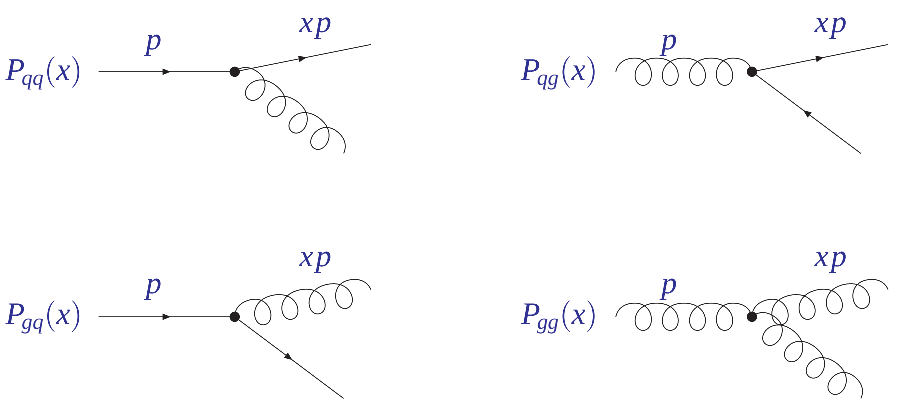

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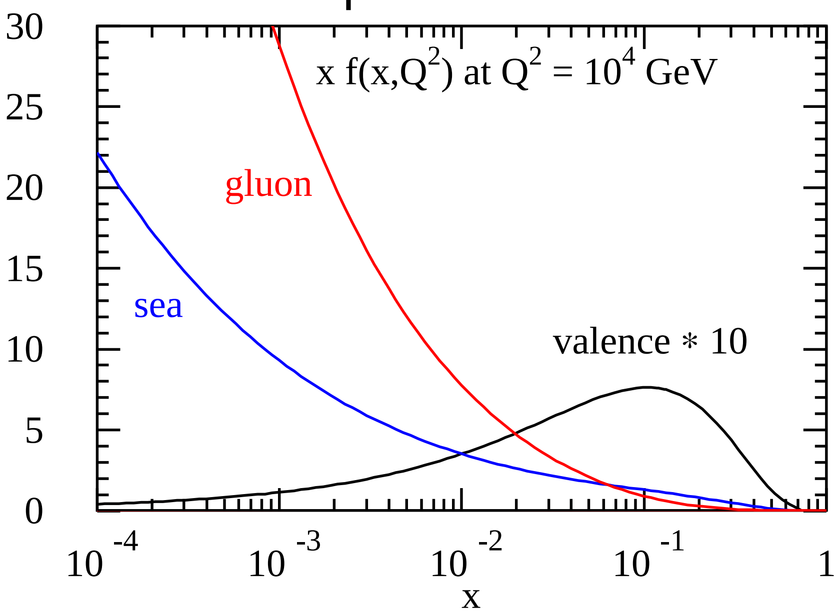
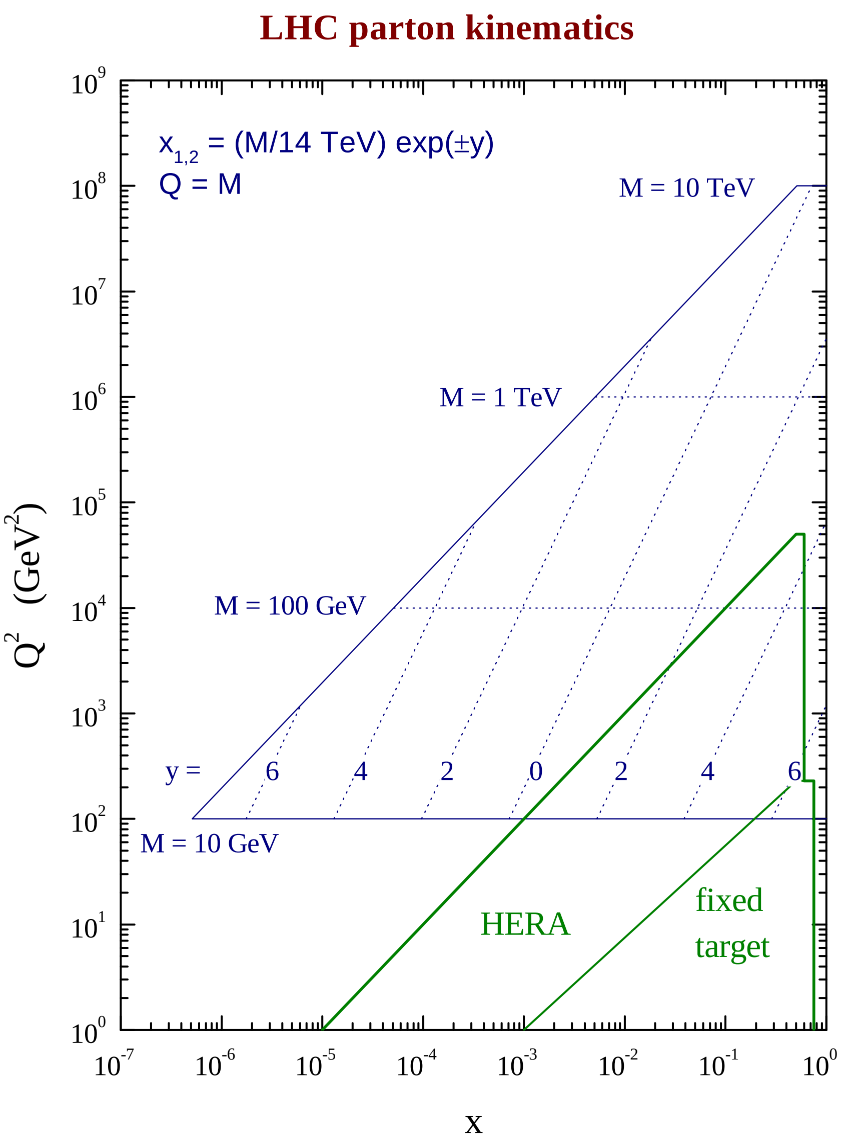


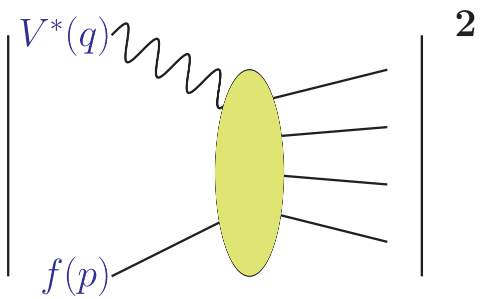
Parton luminosity at LHC

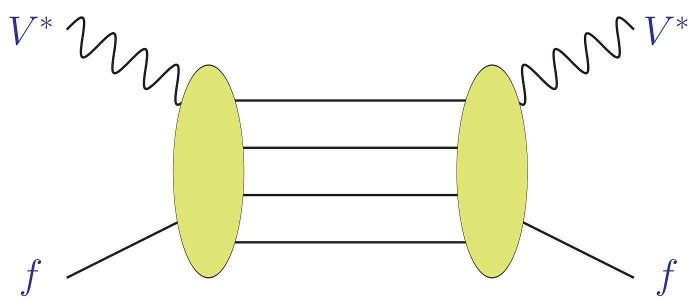
- Precision HERA data on F_2 covers most of the LHC x-range
- Scale evolution of PDFs in Q over two to three orders
- Sensitivity at LHC
  - 100 GeV physics: small-x, sea partons
  - TeV scales: large-x
  - rapidity distributions probe extreme x-values
- Stable evolution in QCD
  - splitting functions to NNLO
    S.M. Vermaseren, Vogt '04



Our calculation in deep-inelastic scattering

- **"Loop technology"**: optical theorem
  total cross section $\longleftrightarrow$ imaginary part of Compton amplitude





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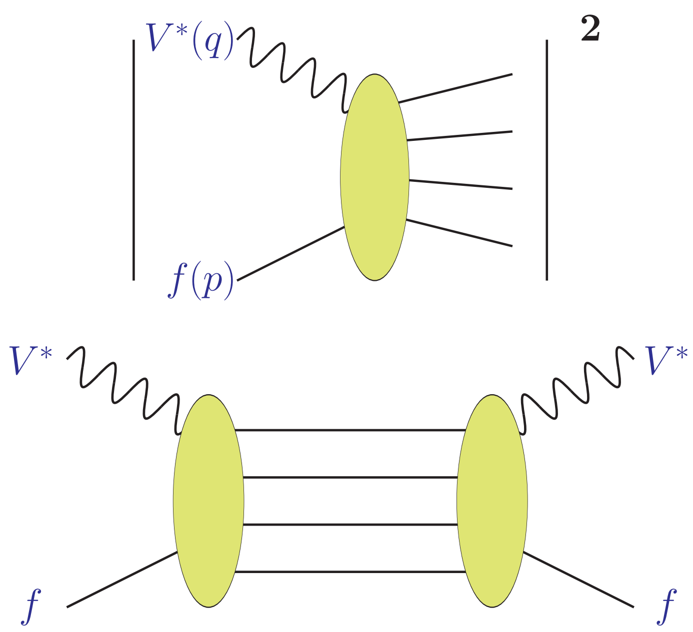


	tree	1-loop	2-loop	3-loop
qγ	1	3	25	359
gγ		2	17	345
hγ			2	56
qW	1	3	32	589
qϕ		1	23	696
gϕ	1	8	218	6378
hϕ		1	33	1184
sum	3	18	350	9607

- more than 10 FTE years and a few CPU years
  - computer algebra updates: $\rightarrow$ 3.1 $\rightarrow$ 3.2 $\rightarrow \ldots$
  - $> 10^5$ tabulated symbolic integrals (> 3GB)

Splitting functions for a quarter of a century

$$
\begin{aligned}
P^{(0)}_{\rm ns}(x) &= C_F\,(2p_{\rm qq}(x)+3\delta(1-x)) \\
P^{(0)}_{\rm ps}(x) &= 0 \\
P^{(0)}_{\rm qg}(x) &= 2n_f\,p_{\rm qg}(x) \\
P^{(0)}_{\rm gq}(x) &= 2C_F\,p_{\rm gq}(x) \\
P^{(0)}_{\rm gg}(x) &= C_A\Big(4p_{\rm gg}(x)+\frac{11}{3}\delta(1-x)\Big)-\frac{2}{3}n_f\,\delta(1-x)
\end{aligned}
$$

1973

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross | H. David Politzer | Frank Wilczek

$$
\begin{aligned}
P^{(1)+}_{\rm ns}(x) &= 4C_AC_F\Big(p_{\rm qq}(x)\Big[\frac{67}{18}-\zeta_2+\frac{11}{6}{\rm H}_0+{\rm H}_{0,0}\Big]+p_{\rm qq}(-x)\Big[\zeta_2+2{\rm H}_{-1,0}-{\rm H}_{0,0}\Big] \\
&\quad+\frac{14}{3}(1-x)+\delta(1-x)\Big[\frac{17}{24}+\frac{11}{3}\zeta_2-3\zeta_3\Big]\Big)-4C_Fn_f\Big(p_{\rm qq}(x)\Big[\frac{5}{9}+\frac{1}{3}{\rm H}_0\Big]+\frac{2}{3}(1-x) \\
&\quad+\delta(1-x)\Big[\frac{1}{12}+\frac{2}{3}\zeta_2\Big]\Big)+4{C_F}^2\Big(2p_{\rm qq}(x)\Big[{\rm H}_{1,0}-\frac{3}{4}{\rm H}_0+{\rm H}_2\Big]-2p_{\rm qq}(-x)\Big[\zeta_2+2{\rm H}_{-1,0} \\
&\quad-{\rm H}_{0,0}\Big]-(1-x)\Big[1-\frac{3}{2}{\rm H}_0\Big]-{\rm H}_0-(1+x){\rm H}_{0,0}+\delta(1-x)\Big[\frac{3}{8}-3\zeta_2+6\zeta_3\Big]\Big) \\
P^{(1)-}_{\rm ns}(x) &= P^{(1)+}_{\rm ns}(x)+16C_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[\zeta_2+2{\rm H}_{-1,0}-{\rm H}_{0,0}\Big]-2(1-x) \\
&\quad-(1+x){\rm H}_0\Big)
\end{aligned}
$$

$$
P^{(1)}_{\rm ps}(x) = 4C_Fn_f\Big(\frac{20}{9}\frac{1}{x}-2+6x-4{\rm H}_0+x^2\Big[\frac{8}{3}{\rm H}_0-\frac{56}{9}\Big]+(1+x)\Big[5{\rm H}_0-2{\rm H}_{0,0}\Big]\Big)
$$

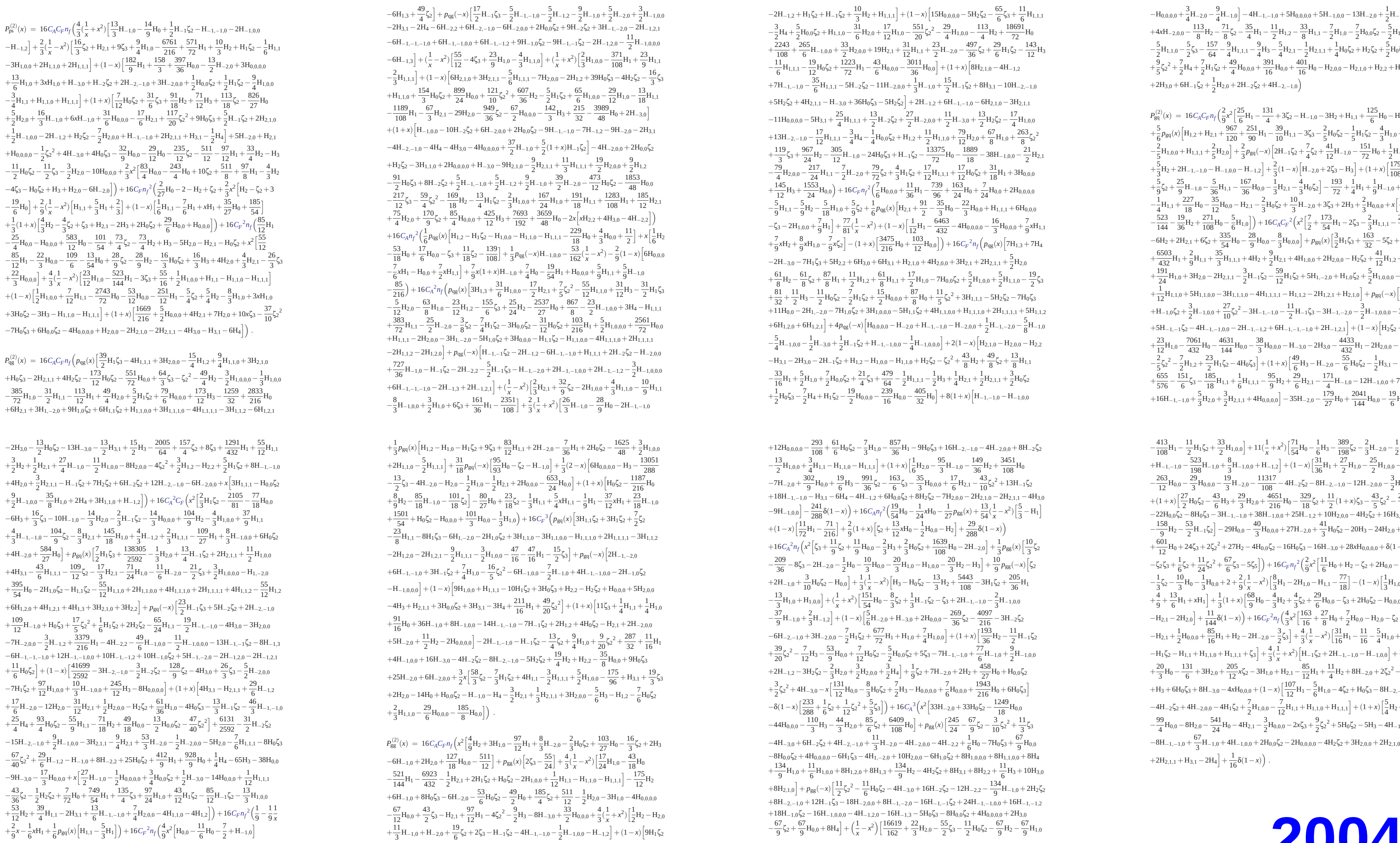
$$
\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4C_An_f\Big(\frac{20}{9}\frac{1}{x}-2+25x-2p_{\rm qg}(-x){\rm H}_{-1,0}-2p_{\rm qg}(x){\rm H}_{1,1}+x^2\Big[\frac{44}{3}{\rm H}_0-\frac{218}{9}\Big] \\
&+4(1-x)\Big[{\rm H}_{0,0}-2{\rm H}_0+x{\rm H}_1\Big]-4\zeta_2x-6{\rm H}_{0,0}+9{\rm H}_0\Big)+4C_Fn_f\Big(2p_{\rm qg}(x)\Big[{\rm H}_{1,0}+{\rm H}_{1,1}+{\rm H}_2 \\
&-\zeta_2\Big]+4x^2\Big[{\rm H}_0+{\rm H}_{0,0}+\frac{5}{2}\Big]+2(1-x)\Big[{\rm H}_0+{\rm H}_{0,0}-2x{\rm H}_1+\frac{29}{4}\Big]-\frac{15}{2}-{\rm H}_{0,0}-\frac{1}{2}{\rm H}_0\Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4C_AC_F\Big(\frac{1}{x}+2p_{\rm gq}(x)\Big[{\rm H}_{1,0}+{\rm H}_{1,1}+{\rm H}_2-\frac{11}{6}{\rm H}_1\Big]-x^2\Big[\frac{8}{3}{\rm H}_0-\frac{44}{9}\Big]+4\zeta_2-2 \\
&-7{\rm H}_0+2{\rm H}_{0,0}-2{\rm H}_1x+(1+x)\Big[2{\rm H}_{0,0}-5{\rm H}_0+\frac{37}{9}\Big]-2p_{\rm gq}(-x){\rm H}_{-1,0}\Big)-4C_Fn_f\Big(\frac{2}{3}x \\
&-p_{\rm gq}(x)\Big[\frac{2}{3}{\rm H}_1-\frac{10}{9}\Big]\Big)+4{C_F}^2\Big(p_{\rm gq}(x)\Big[3{\rm H}_1-2{\rm H}_{1,1}\Big]+(1+x)\Big[{\rm H}_{0,0}-\frac{7}{2}+\frac{7}{2}{\rm H}_0\Big]-3{\rm H}_{0,0} \\
&+1-\frac{3}{2}{\rm H}_0+2{\rm H}_1x\Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4C_An_f\Big(1-x-\frac{10}{9}p_{\rm gg}(x)-\frac{13}{9}\Big(\frac{1}{x}-x^2\Big)-\frac{2}{3}(1+x){\rm H}_0-\frac{2}{3}\delta(1-x)\Big)+4{C_A}^2\Big(27 \\
&+(1+x)\Big[\frac{11}{3}{\rm H}_0+8{\rm H}_{0,0}-\frac{27}{2}\Big]+2p_{\rm gg}(-x)\Big[{\rm H}_{0,0}-2{\rm H}_{-1,0}-\zeta_2\Big]-\frac{67}{9}\Big(\frac{1}{x}-x^2\Big)-12{\rm H}_0 \\
&-\frac{44}{3}x^2{\rm H}_0+2p_{\rm gg}(x)\Big[\frac{67}{18}-\zeta_2+{\rm H}_{0,0}+2{\rm H}_{1,0}+2{\rm H}_2\Big]+\delta(1-x)\Big[\frac{8}{3}+3\zeta_3\Big]\Big)+4C_Fn_f\Big(2{\rm H}_0 \\
&+\frac{2}{3}\frac{1}{x}+\frac{10}{3}x^2-12+(1+x)\Big[4-5{\rm H}_0-2{\rm H}_{0,0}\Big]-\frac{1}{2}\delta(1-x)\Big)\,.
\end{aligned}
$$

1980

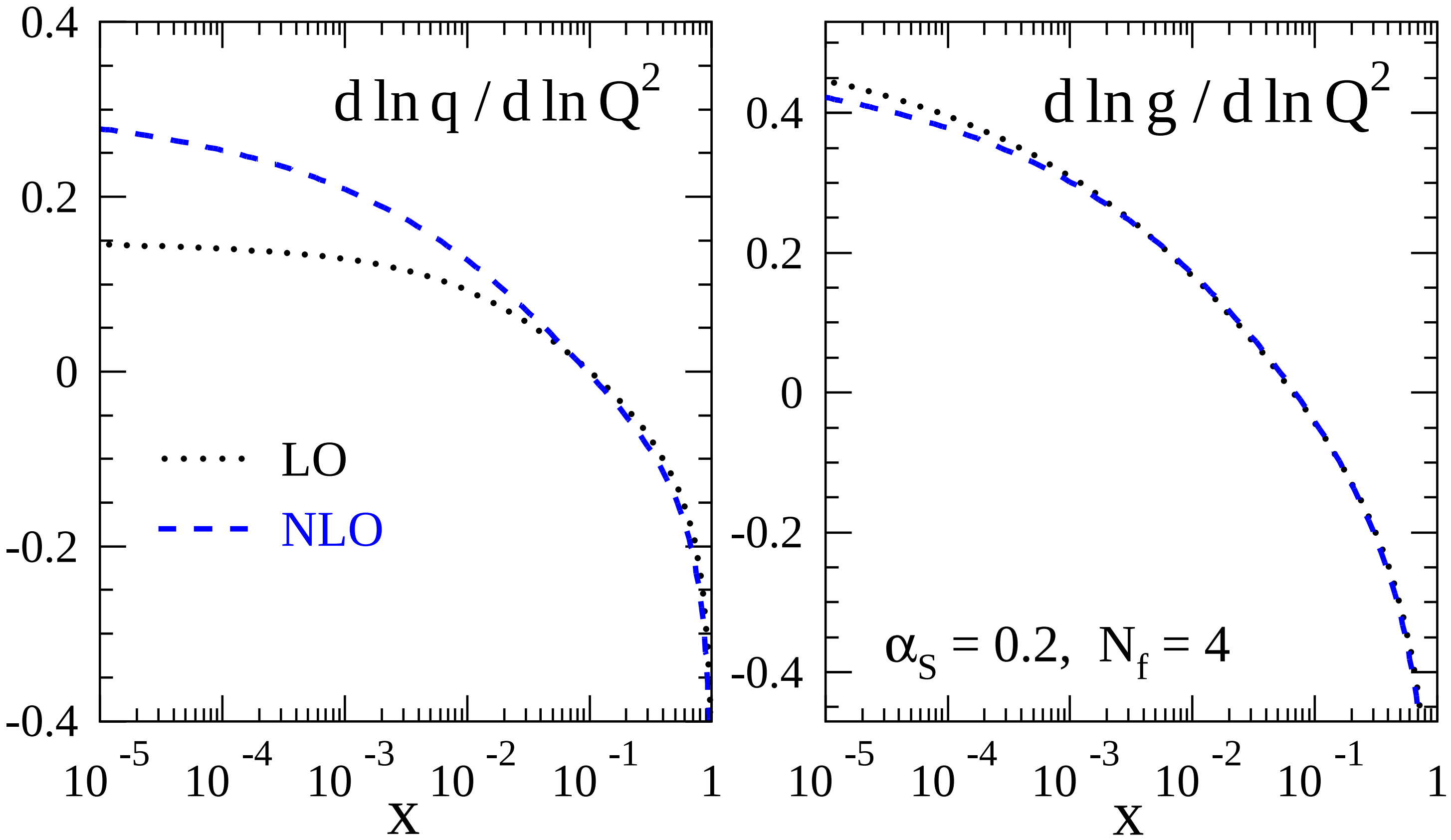
NNLO splitting functions S.M. Vermaseren, Vogt



2004

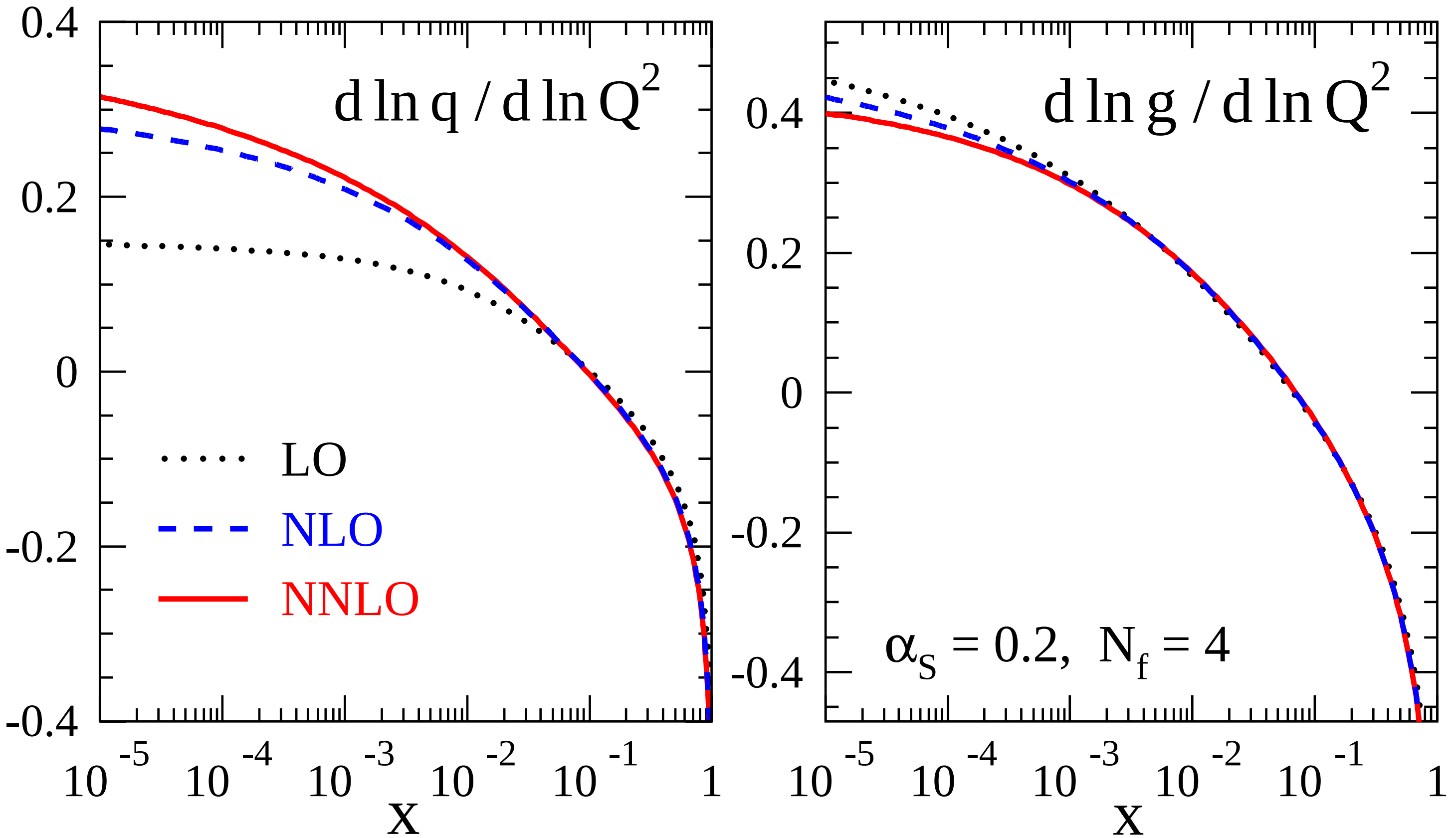
Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2



Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2



- Expansion very stable except for very small momenta $x \lesssim 10^{-4}$
  S.M. Vermaseren, Vogt '04

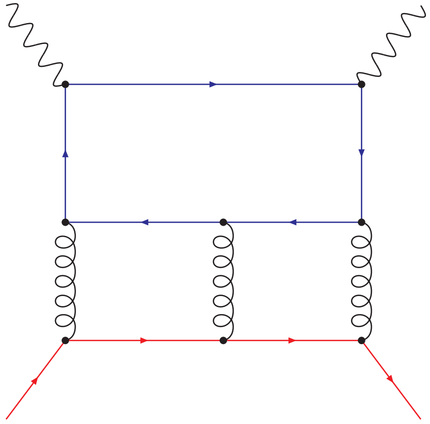
Strange asymmetry

- Probability of a splitting $q \rightarrow q'$ different from that of $q \rightarrow \bar{q}'$ at higher orders (starting at three loops)
  - dynamical generation of asymmetric sea $q - \bar{q}$
    Catani, De Florian, Rodrigo, Vogelsang '04
- Non-singlet distributions $q^{\pm}$ and q^{v}
  - splitting function combinations $P^{\pm}_{\mathrm{ns}}$ and $P^{\mathrm{v}}_{\mathrm{ns}} \;=\; P^{-}_{\mathrm{ns}} + P^{\mathrm{s}}_{\mathrm{ns}}$

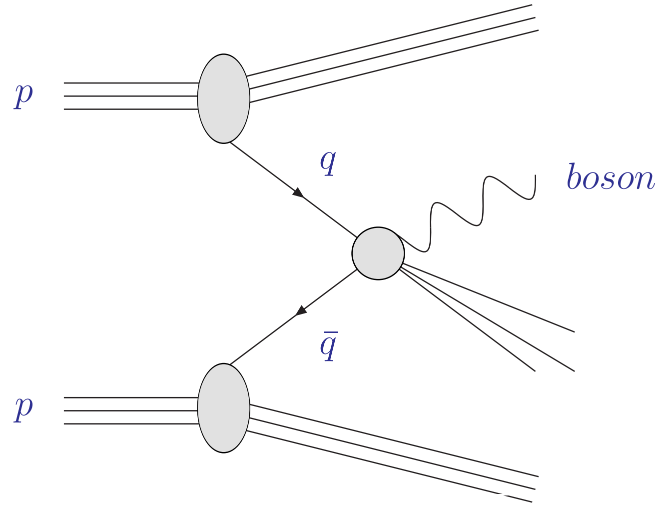
$$q^{\pm}_{\mathrm{ns},ik} \;=\; q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \quad \text{flavour asymmetries}$$

$$q^{\mathrm{v}}_{\mathrm{ns}} \;=\; \sum_{r=1}^{n_f} (q_r - \bar{q}_r) \quad \text{total valence distribution}$$

- New colour factor in $P^{(2)\mathrm{s}}_{\mathrm{ns}}$
  $d^{abc}d_{abc}/n_c = (n_c^2 - 1)(n_c^2 - 4)/n_c^2$

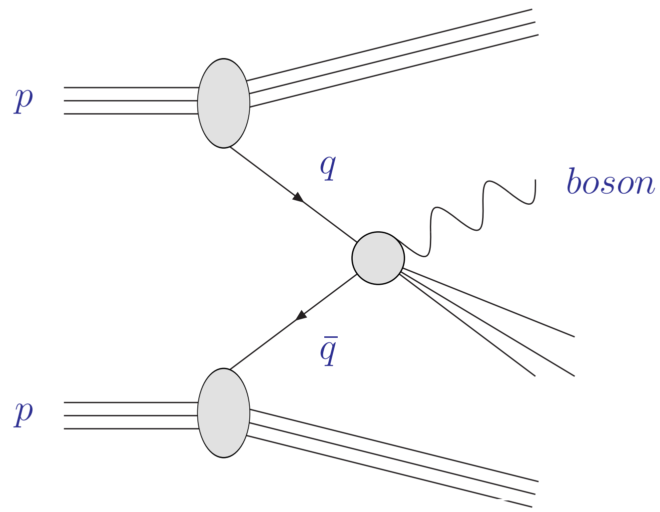


Vector boson production



- Kinematical variables (inclusive)
  - energy (cms) $s = Q^2$ (time-like)
  - scaling variable $x = M^2_{W^\pm/Z}/s$

Vector boson production



- Kinematical variables (inclusive)
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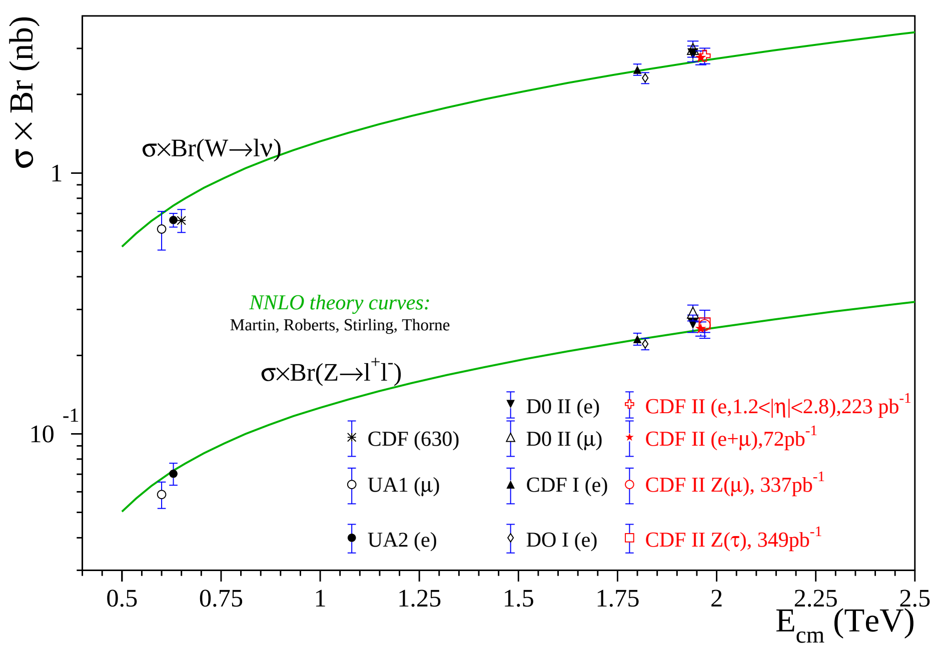
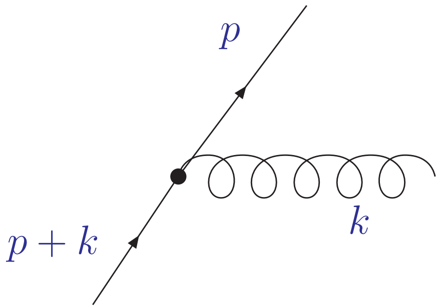


- 20 years of measurements of $W^\pm$ and Z cross sections at hadron colliders

Universal aspects of radiative corrections

- Recall perturbative QCD:
  - calculation of observables as series in $\alpha_s \ll 1$
  - but: large logarithmic corrections, $\ln(\dots) \gg 1$
    double logarithms (Sudakov)
- Soft/Collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft θ_{qg} = 0 collinear



$$\frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

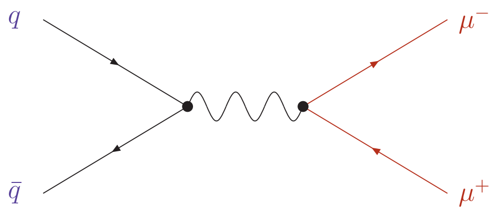
$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g \, d\theta_{qg} \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \ln^2(\dots)$$

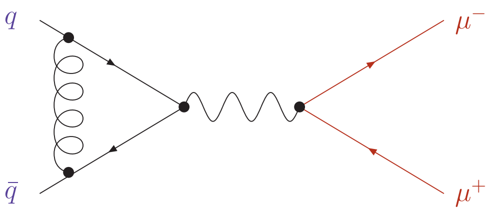
- Improved perturbation theory: resum logarithms to all orders

Sudakov logarithms in cross sections

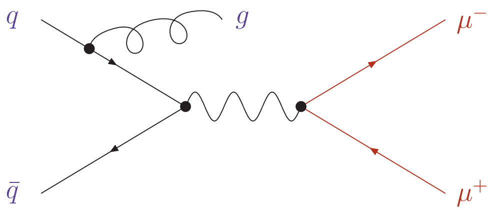
- Intuitive aspects of higher order corrections (e.g. Drell-Yan)
  - lowest order, elastic



  - first order correction
    $virtual < 0$ (elastic)



  - first order correction
    $Brems > 0$ (inelastic)

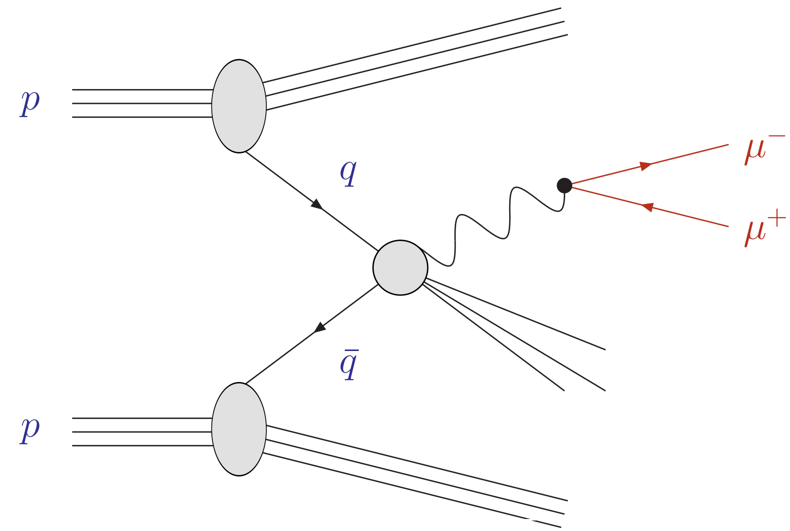


- at threshold for $\mu^+\mu^-$-creation
  - strong Sudakov-supression inelastic tendency

$$\sigma \sim \exp\left[-\alpha_s \ln^2(1 - 4m_\mu^2/s)\right]$$

  - universal factor for parton splittings (leading log accuracy)
    modelling of MC parton showers

- Hadronic reaction $p\bar{p}$:
  - recall master equation $\sigma_{pp\to\mu^+\mu^-} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to\mu^+\mu^-}$
  - initial partons: also Sudakov-supressed

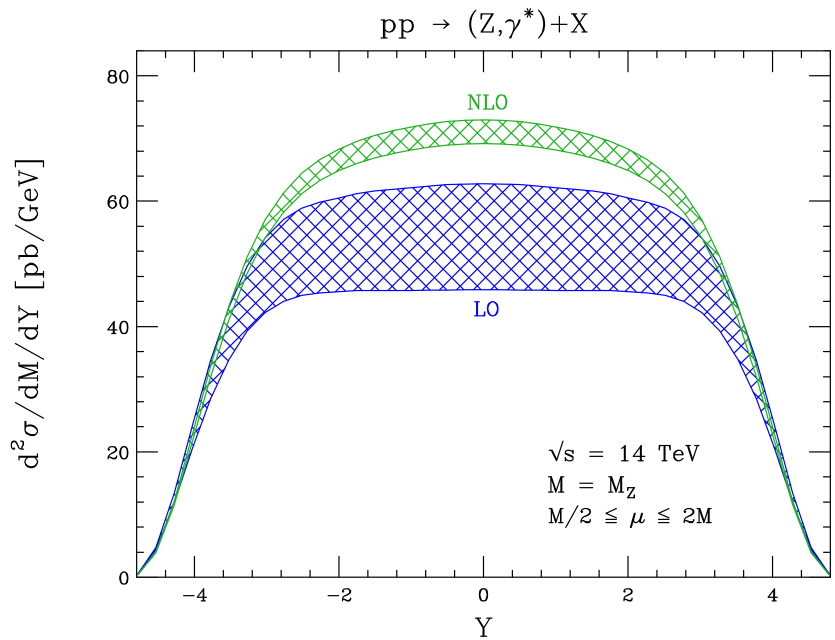


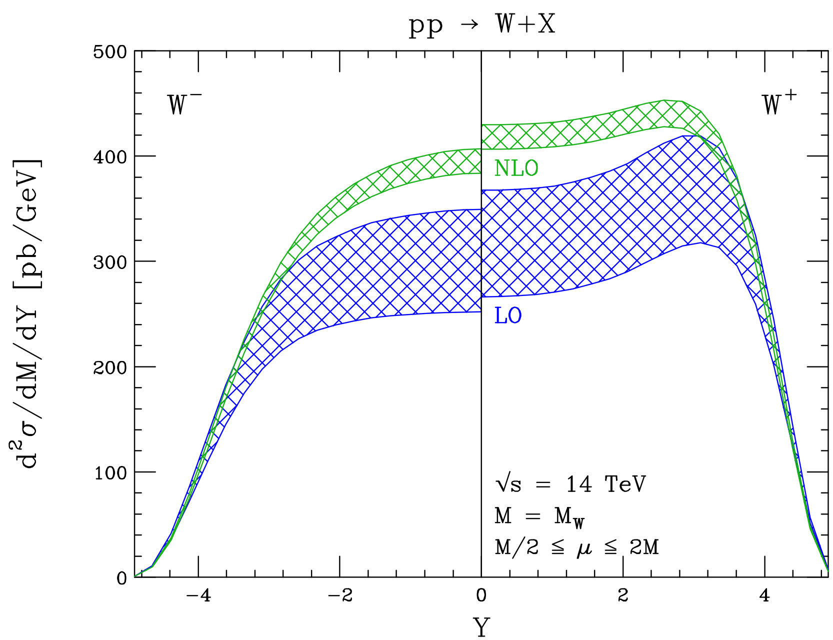
- Parton cross section $\hat{\sigma}_{ij\to\mu^+\mu^-}$
  - Sudakov-enhancement after mass factorization

$$\hat{\sigma}_{ij\to\mu^+\mu^-} = \frac{\sigma_{pp\to\mu^+\mu^-}}{f_i \otimes f_j} = \frac{\mathrm{e}^{-\alpha_s \ln^2(\dots)}}{\left(\mathrm{e}^{-\alpha_s \ln^2(\dots)}\right)^2} = \mathrm{e}^{+\alpha_s \ln^2(\dots)}$$

  - large double logarithms

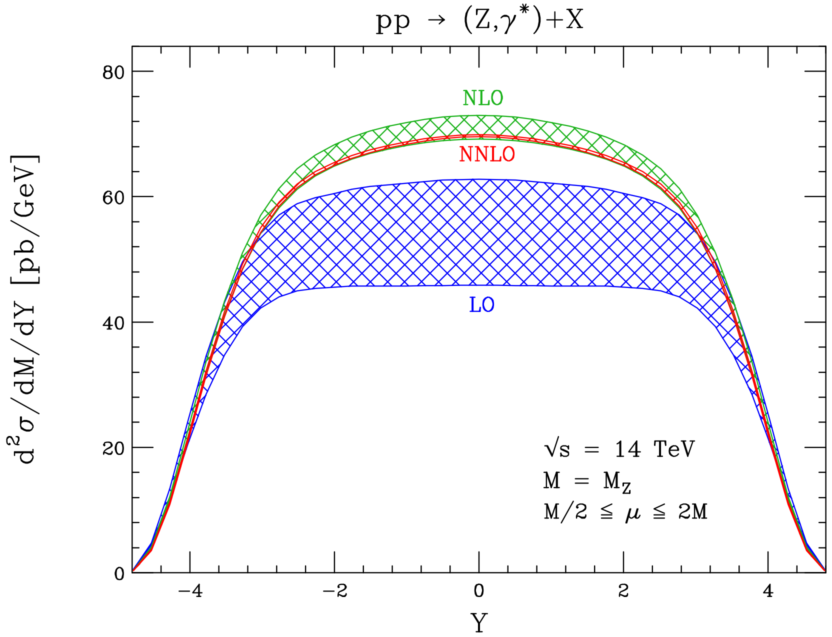
Differential distributions (rapidity)

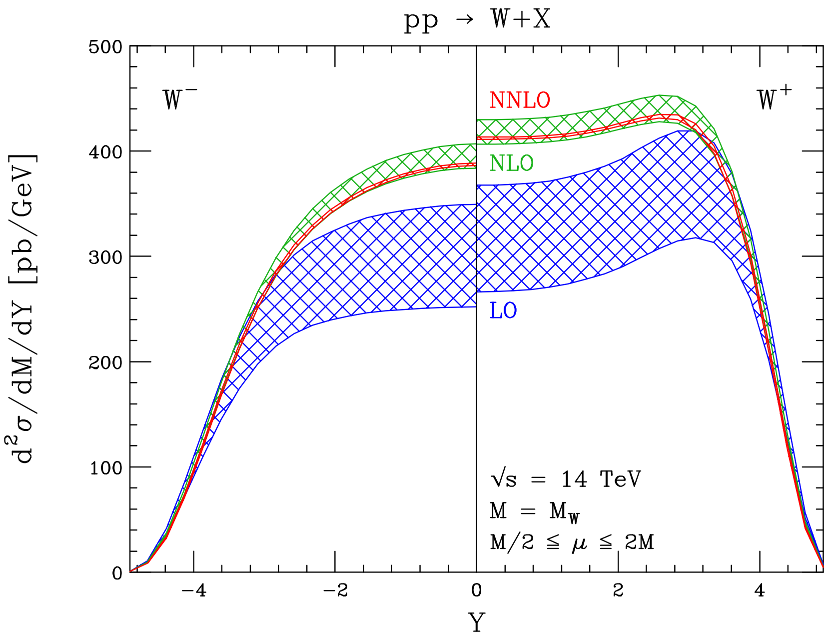




- $W^{\pm}, Z$-boson rapidity distribution with scale variation $m_{W,Z}/2 \leq \mu \leq 2m_{W,Z}$
  Anastasiou, Petriello, Melnikov '05

Differential distributions (rapidity)



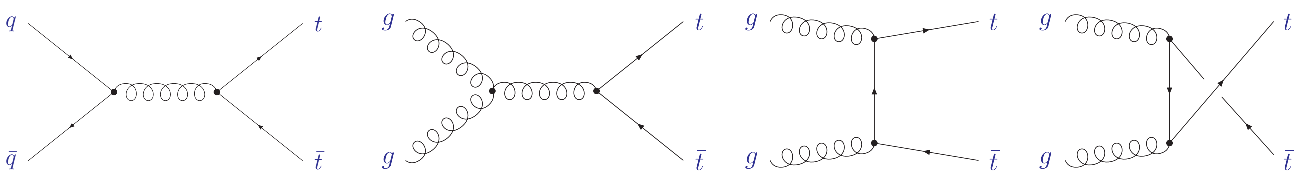


- $W^{\pm}, Z$-boson rapidity distribution with scale variation $m_{W,Z}/2 \leq \mu \leq 2m_{W,Z}$
  Anastasiou, Petriello, Melnikov ‘05
- Reduction of theoretical uncertainties (renormalization / factorization scale) to level of 1% in NNLO QCD analysis
  Dissertori ‘05

Top quark production

- Leading order Feynman diagrams

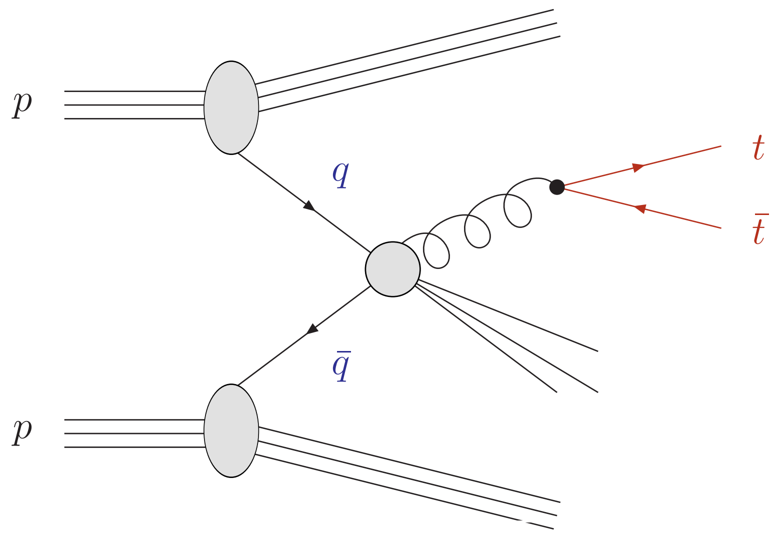
$$\mathrm{q} + \bar{\mathrm{q}} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$
$$\mathrm{g} + \mathrm{g} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$



- NLO in QCD Nason, Dawson, Ellis '88; Beenakker, Smith, van Neerven '89; Mangano, Nason, Ridolfi '92; Bernreuther, Brandenburg, Si, Uwer '04; ...
  - accurate to $\mathcal{O}(15\%)$ at LHC
- Much activity towards higher orders in QCD
  - small-mass limit $m^2 \ll s, t, u$ for two-loop virtual corrections to $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ and $\mathrm{gg} \to \mathrm{t}\bar{\mathrm{t}}$ S.M., Czakon, Mitov '07
  - full mass dependence for two-loop virtual $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ Czakon '08
  - analytic two-loop fermionic corrections for $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ Bonciani, Ferroglia, Gehrmann, Maitre, Studerus '08
  - one-loop squared terms (NLO $\times$ NLO) Anastasiou, Mert Aybat '08; Kniehl, Merebashvili, Körner, Rogal '08

- Proton-proton

$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$



- Recall Drell-Yan process:
  parton cross section Sudakov enhanced close to threshold $s \simeq 4m^2$
  - Sudakov-type logarithms $\ln(\beta)$ with velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$ at n^{th}-order

$$\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$$

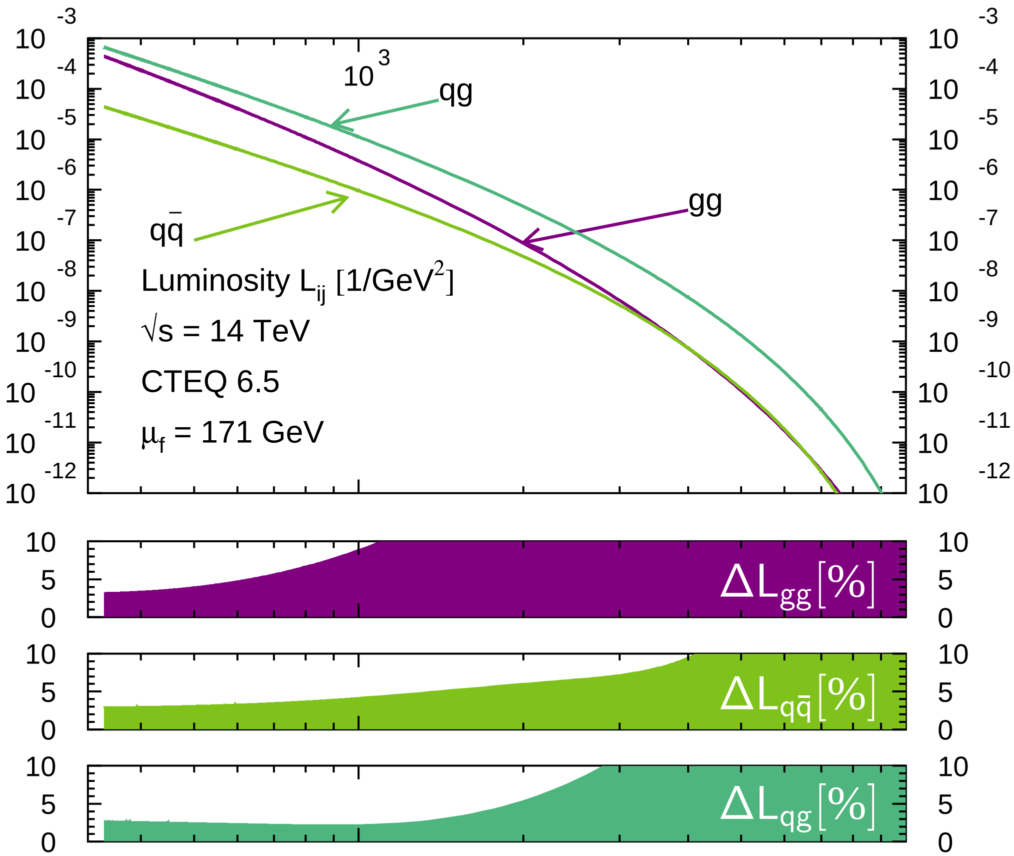
  - resummation in Mellin space (renormalization group equation)

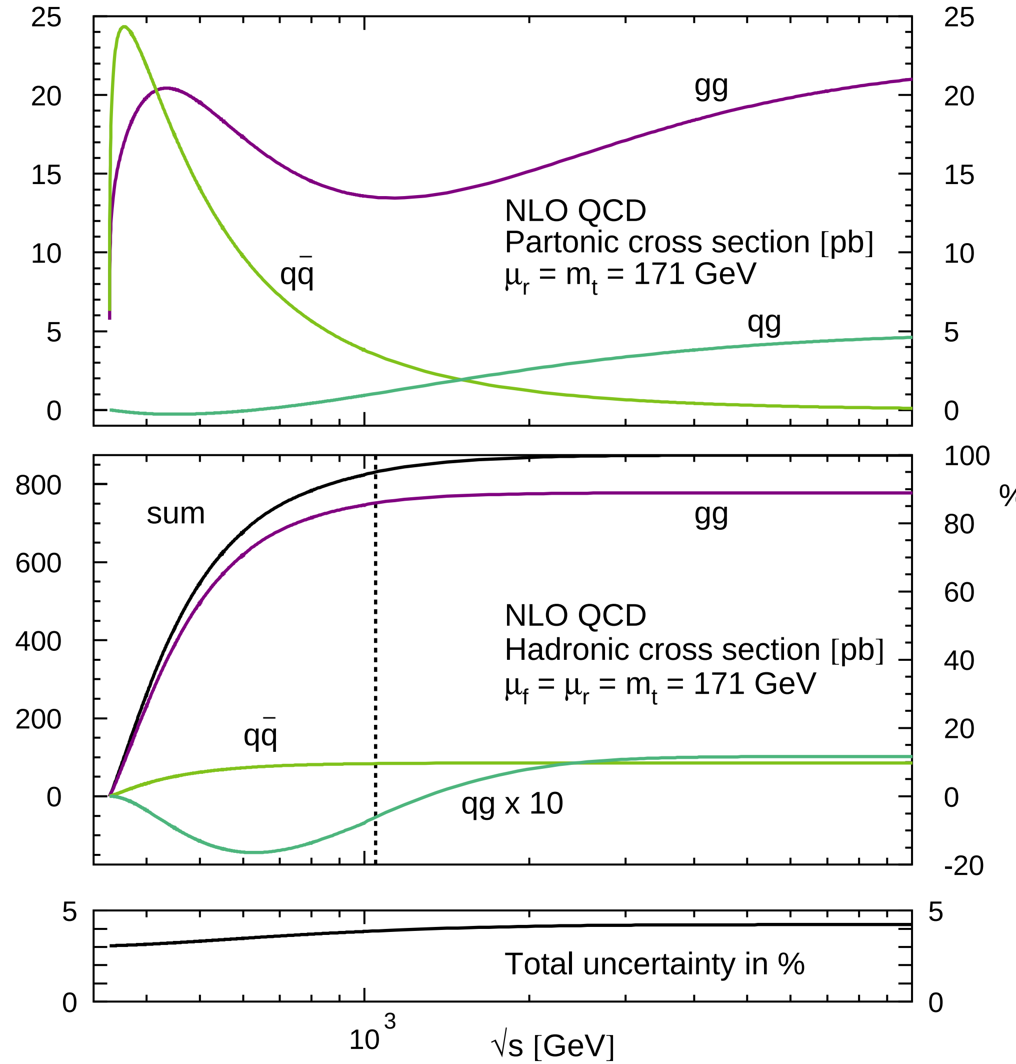
$$\hat{\sigma}_{\mathrm{ij}}^N = (1 + \alpha_s\, g_{01} + \ldots) \cdot \exp\left(G^N\right) + \mathcal{O}(N^{-1} \ln^n N)$$

- Resummed G^N predicts fixed orders in perturbation theory
  - generating functional for towers of large logarithms

Total cross section at LHC

$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$





New results

- NNLO cross section for heavy-quark hadro-production near threshold (all powers of $\ln\beta$ and Coulomb corrections) S.M., Uwer '08
  - e.g. gg-fusion for $n_f = 5$ light flavors at $\mu = m$

$$\hat{\sigma}^{(1)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 96\ln^2\beta - 9.5165\ln\beta + 35.322 + 5.1698\frac{1}{\beta} \right\}$$

$$\hat{\sigma}^{(2)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 4608\ln^4\beta - 1894.9\ln^3\beta + \left(-3.4811 + 496.30\frac{1}{\beta}\right)\ln^2\beta \right.$$
$$\left. + \left(3144.4 + 321.17\frac{1}{\beta}\right)\ln\beta + 68.547\frac{1}{\beta^2} - 196.93\frac{1}{\beta} + C^{(2)}_{gg} \right\}$$

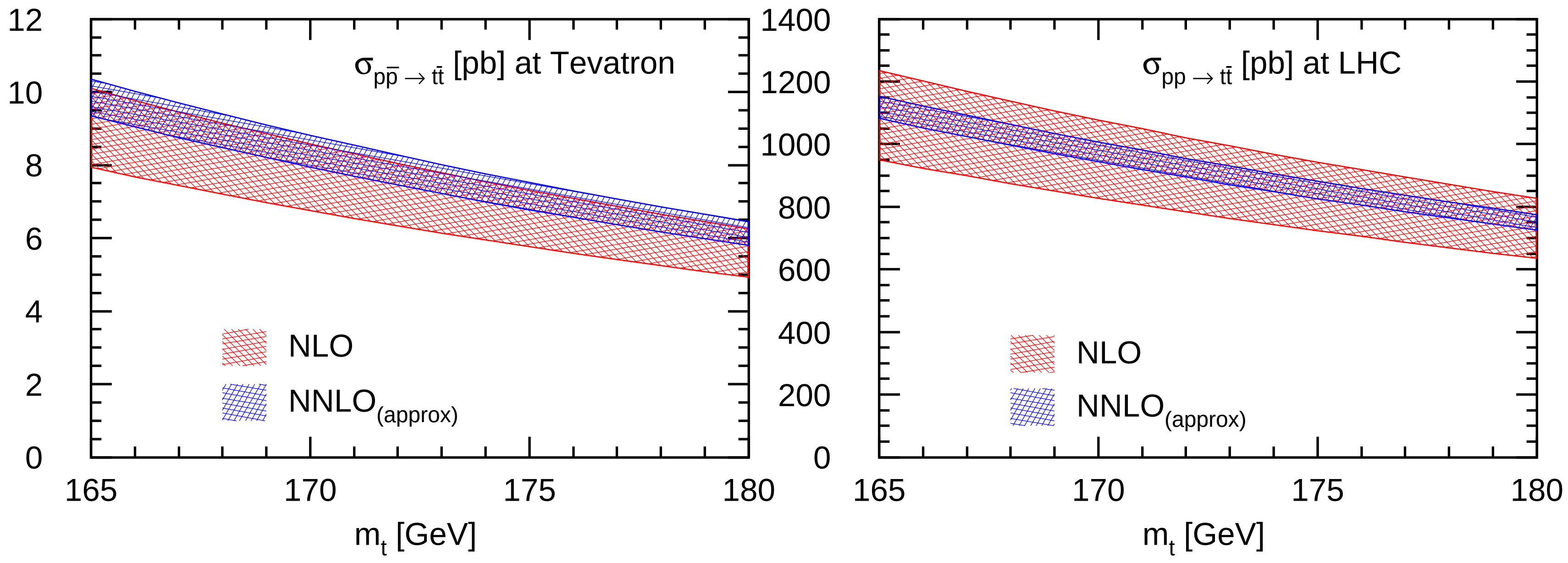
- Add all scale dependent terms
  - $\ln(\mu/m)$-terms exactly known from renormalization group methods

Upshot

- Best approximation to complete NNLO
- Similar results for new massive colored particles (4th generation quarks, squarks, gluinos, . . .)
  S.M., Uwer '08; S.M., Langenfeld '08

Top-quark pair-production at NNLO

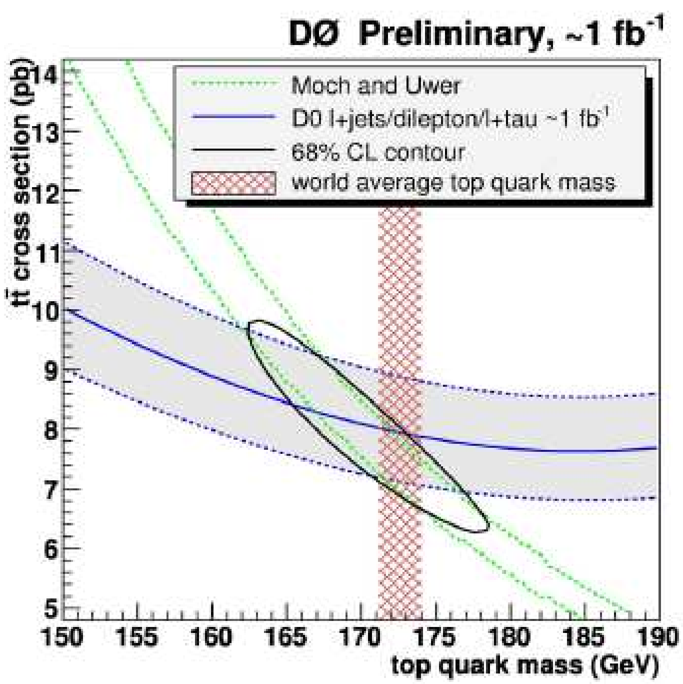
- NLO (with MRST2006 PDF set)
  - scale uncertainty $\mathcal{O}(10\%) \oplus$ PDF uncertainty $\mathcal{O}(5\%)$
- NNLO$_{\rm approx}$ (with MRST2006 PDF set)
  - scale uncertainty $\mathcal{O}(3\%) \oplus$ PDF uncertainty $\mathcal{O}(2\%)$

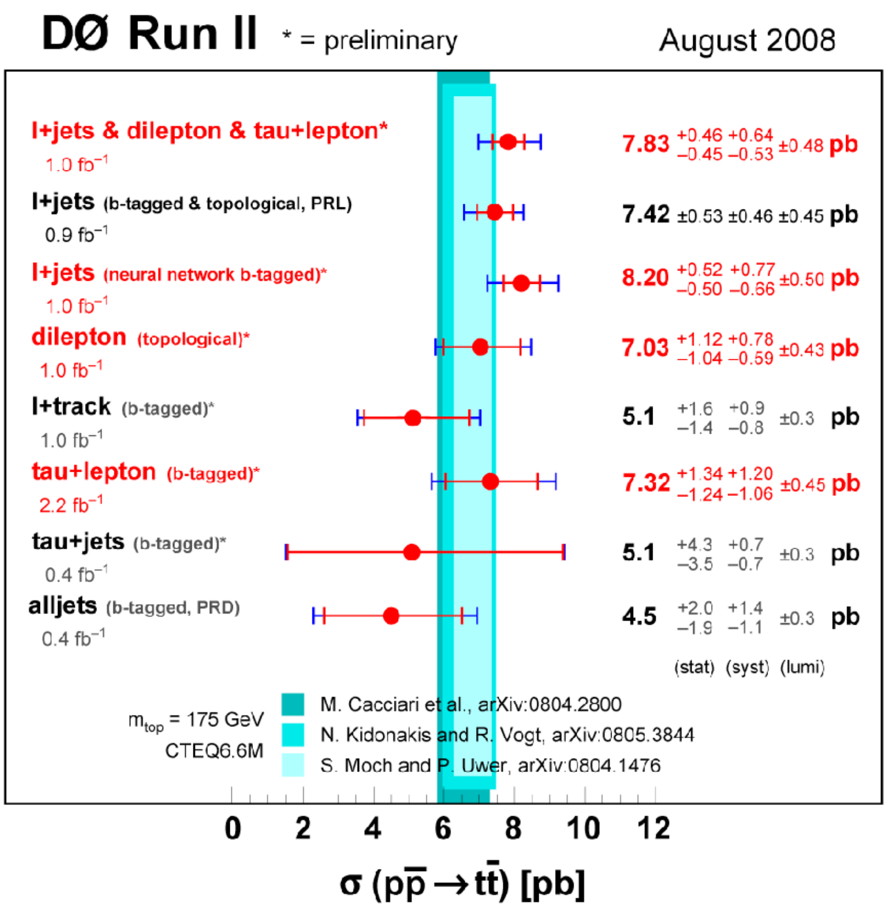


- Theory at NNLO matches anticipated experimental precision $\mathcal{O}(10\%)$

Tevatron analyses

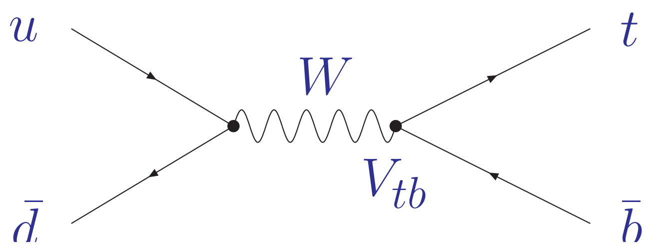
- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- NNLO allows for precision determinations of m_t from total cross section (slope $d\sigma/dm_t$)



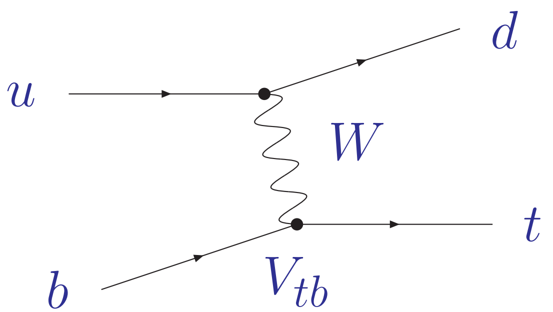


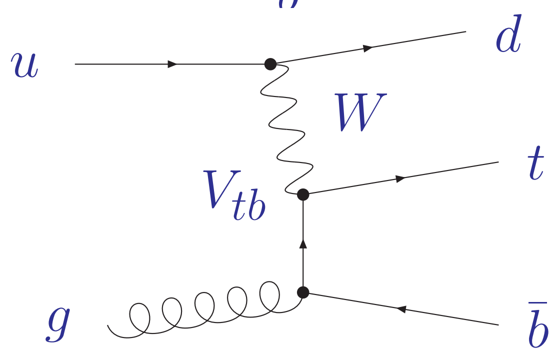
Single top-quark production

- Single-top production allows study of charged-current weak interaction of top quark
  - direct extraction of the CKM-matrix element V_{tb}
  - flagship measurement of Tevatron run II (control QCD bckgrd !)
- s-channel production



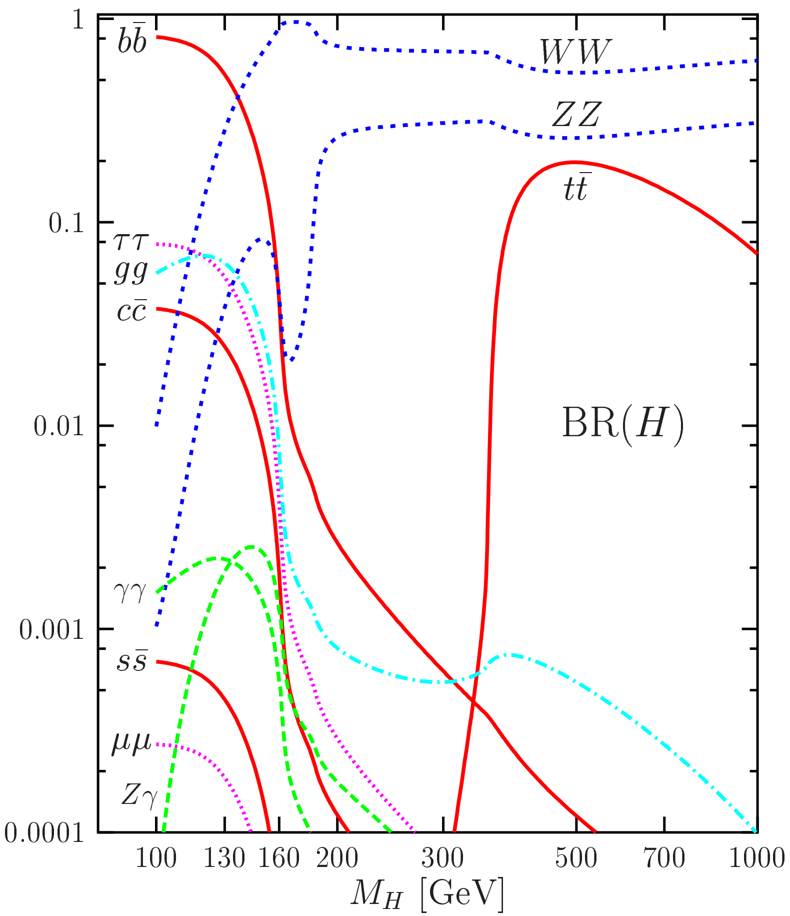
- t-channel production
  - bg-channel at NLO enhanced by gluon luminosity

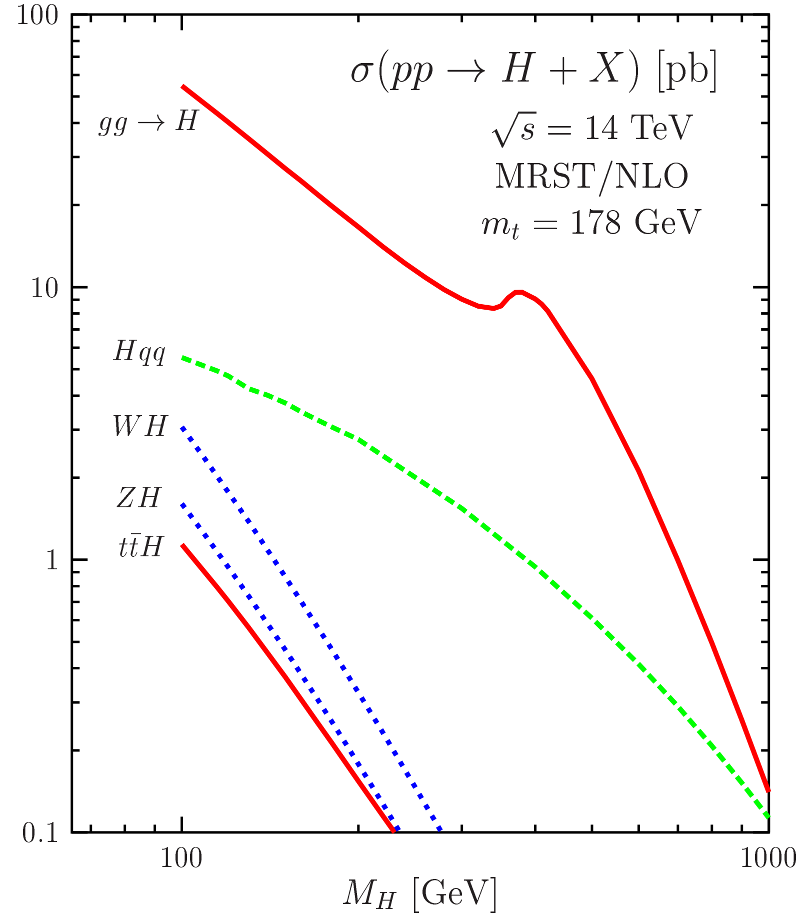




- Large corrections from extensions of Standard Model
  - t-channel: anomalous couplings or flavor changing neutral currents
  - s-channel: charged "top-pion", Kaluza-Klein modes of W or W'-boson

Higgs production at LHC



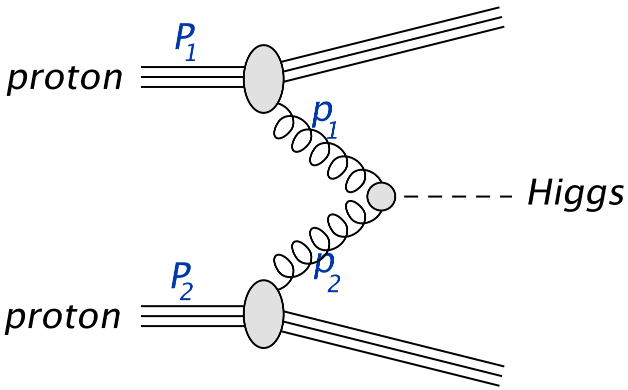


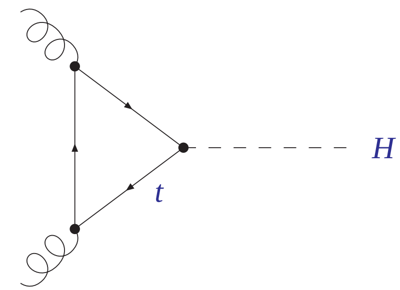
- Standard model Higgs
  - branching ratios for decay (left) and dominant production modes (right)

    Djouadi '05

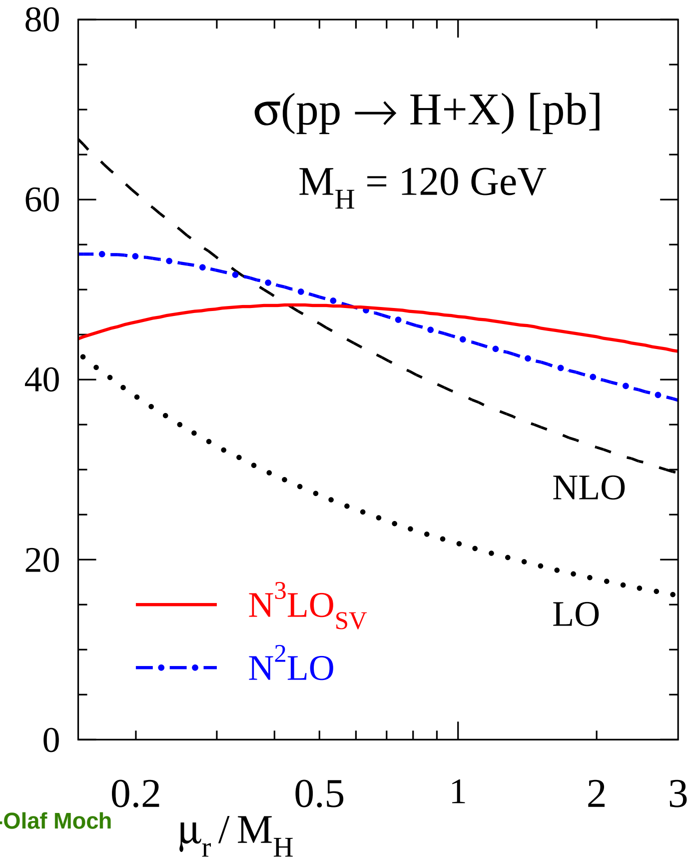
Gluon fusion

- Largest rate for all values of Higgs mass M_H (top-Yukawa coupling)





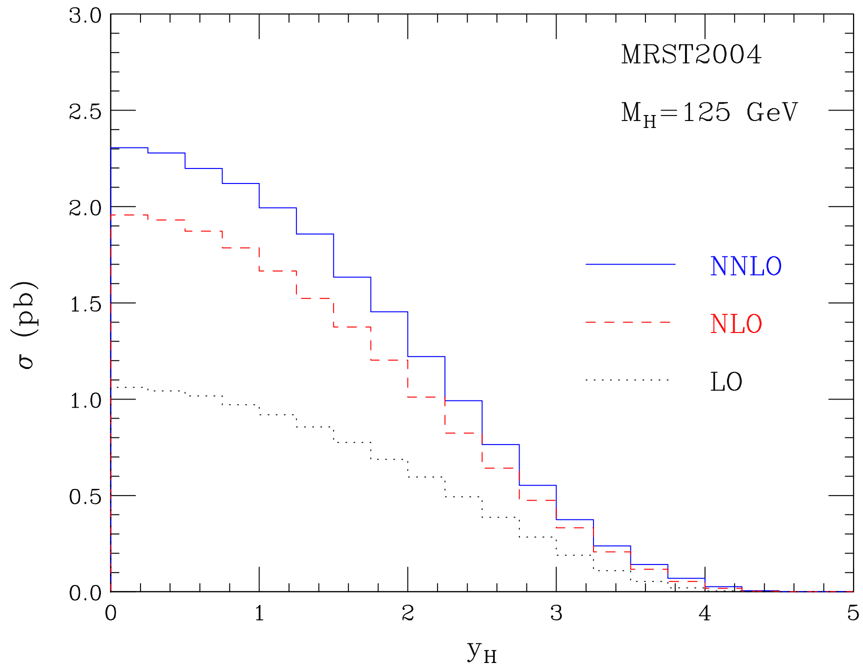
heavy top limit $m_t \to \infty$:
effective gg Higgs vertex

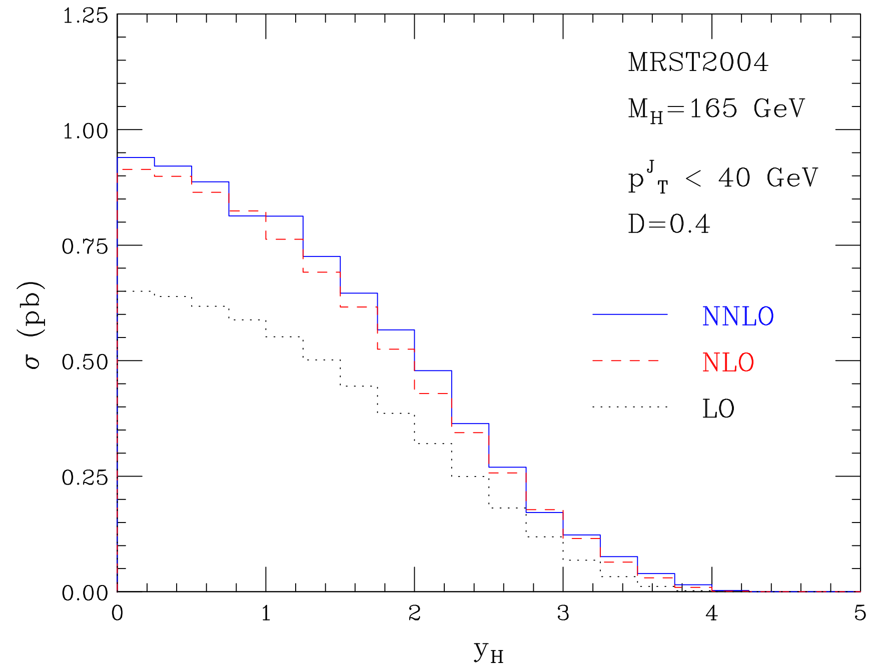


- Total cross section with QCD corrections
- Variation of renormalization scale for Higgs mass $M_H = 120$GeV
  - NNLO corrections
    Harlander, Kilgore ‘02; Anastasiou, Melnikov ‘02; Ravindran, Smith, van Neerven ‘03
  - complete soft N^3LO corrections
    S.M., Vogt ‘05

Differential distributions in gluon fusion

- Bin-integrated Higgs rapidity distribution including decay $H \to \gamma\gamma$
  - QCD corrections up to NNLO Anastasiou, Melnikov, Petriello '05
  - fast parton level Monte Carlo HNNLO Catani, Grazzini '07

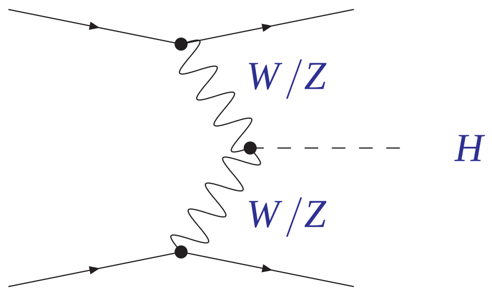


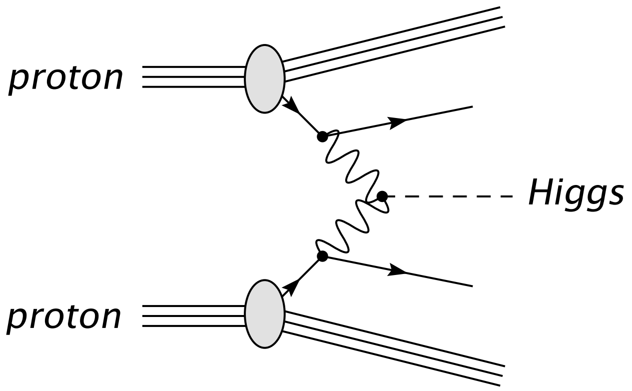


- Impact of kinematical cuts on higher order corrections
  - left: Higgs mass $M_h = 125$ GeV, no cuts on p_t of jets
  - right: Higgs mass $M_h = 165$ GeV and veto on jets with $p_t > 40$ GeV (k_t algorithm for jet reconstruction with jet size $D = 0.4$)

Weak vector-boson fusion

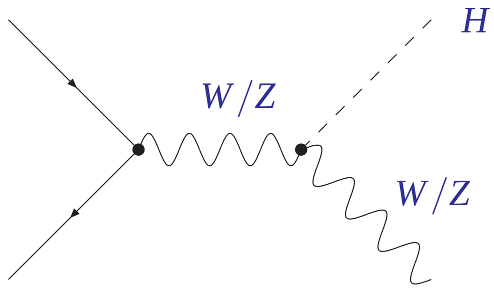
- Channel $qq \to qqH$ (with cuts on jets energies)
- Second largest rate (WWH coupling)
  - mostly dominated by u, d-quarks





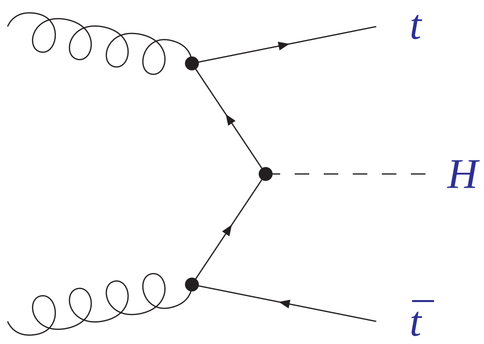
Higgs-strahlung

- Channel $q\bar{q} \to W(Z)H$
- Third largest rate (same couplings as vector boson fusion)



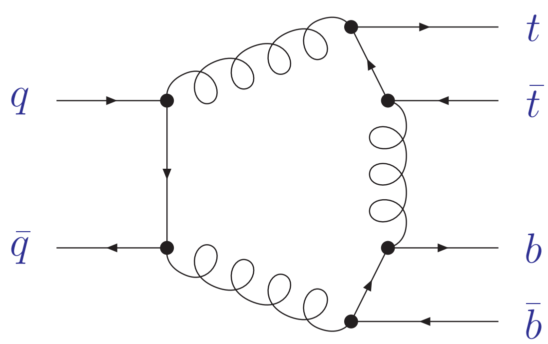
$t\bar{t}$H

- Channel $pp \to t\bar{t}H$
  - discovery channel in low mass region $M_H \lesssim 130$ GeV
  - driven by gluon luminosity, but large SM background $pp \to t\bar{t}H \to t\bar{t}b\bar{b}$



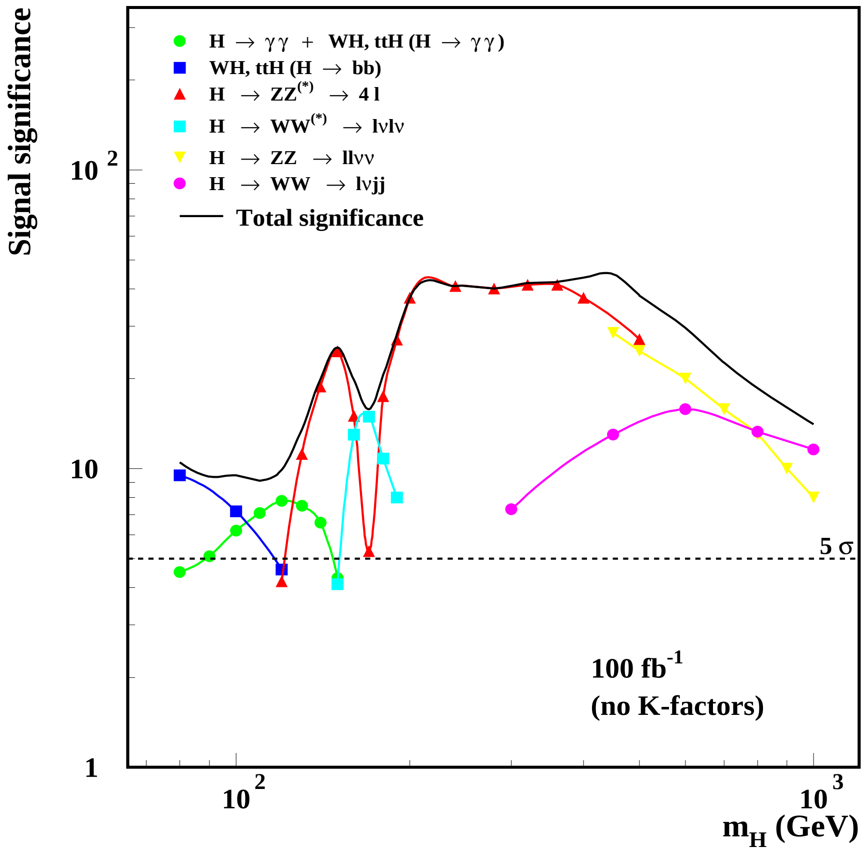
- Main backgrounds for $pp \to t\bar{t}H$
  - combinatorial background from signal (4 b-quarks in final state)
  - $t\bar{t} + 2$ jets, $t\bar{t}b\bar{b}, t\bar{t}Z$
  - complex final states

- **New**: NLO QCD corrections to $q\bar{q} \to t\bar{t}b\bar{b}$
  Denner, Dittmaier, Pozzorini ‘08
  - extremely diffcult hexagon integrals with masses

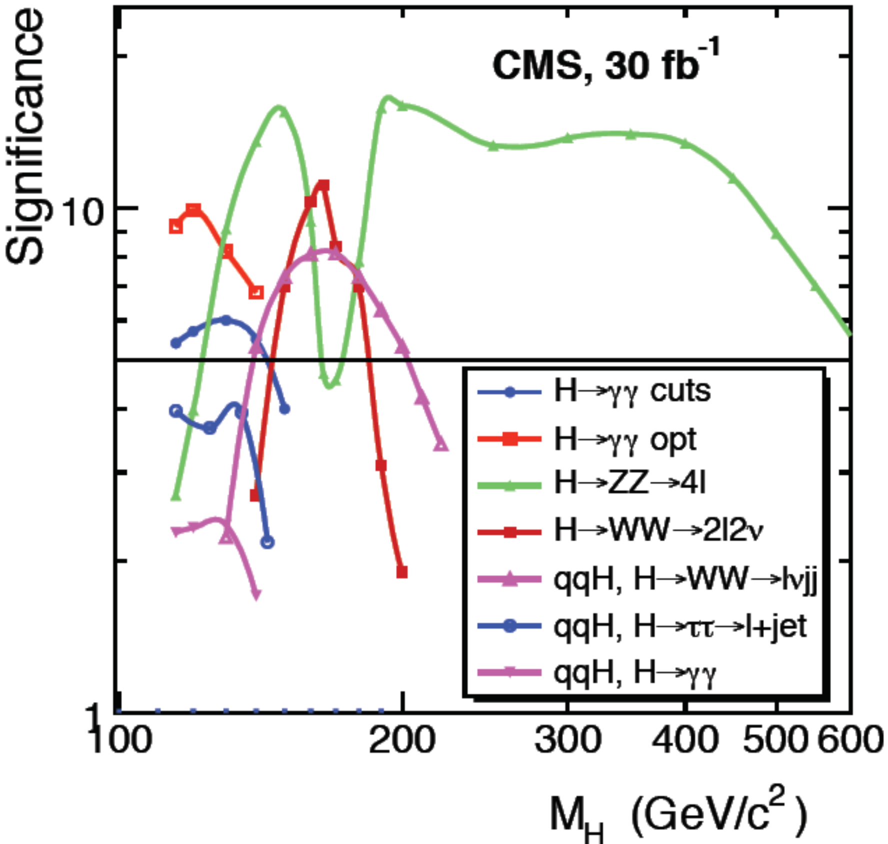


Progress in theory

- Sensitivity for Higgs production at LHC
  - inclusion of higher order theory predictions in new studies
  - e.g. $pp \to t\bar{t}H$ absent in CMS plot



1999



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Summary

Hard QCD

- Hard parton cross section
  - Structure functions in DIS
  - $W^{\pm}$/Z-boson production
  - hadro-production of top quarks
  - Higgs total cross section
- Hadronic final state
  - (multi) jet cross sections
  - jet algorithms and fragmentation of (heavy) quarks
  - parton shower Monte Carlo simulation

Outlook

- QCD tool box ready for LHC challenges
  - however, still much dedicated work to do

Literature

- Review
  - *Expectations at LHC from hard QCD*
    J. Phys. G: Nucl. Part. Phys. **35**, 073001 (2008) [arXiv:0803.0457] [hep-ph].