Accidental Inflation from Kähler Uplifting

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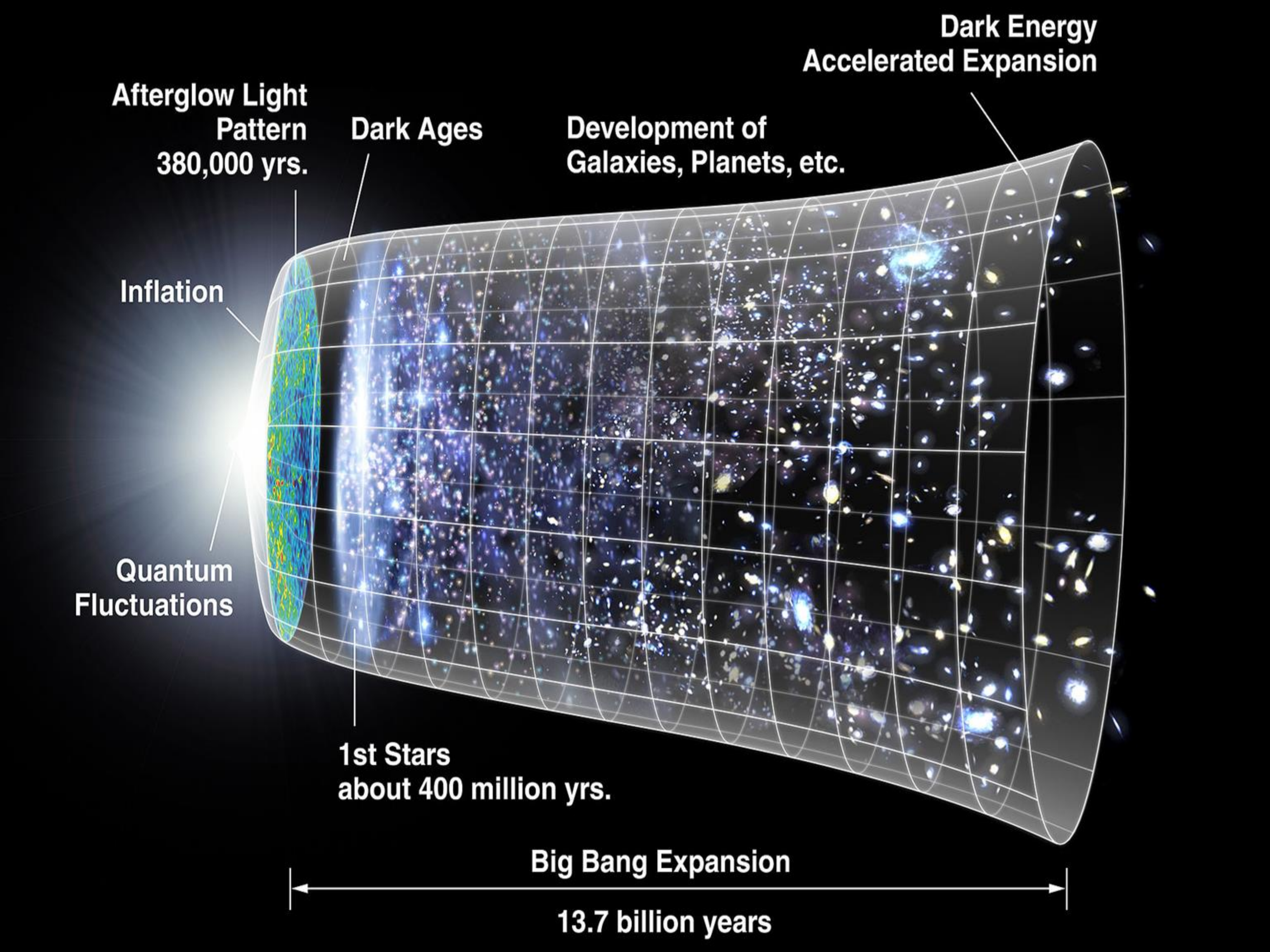
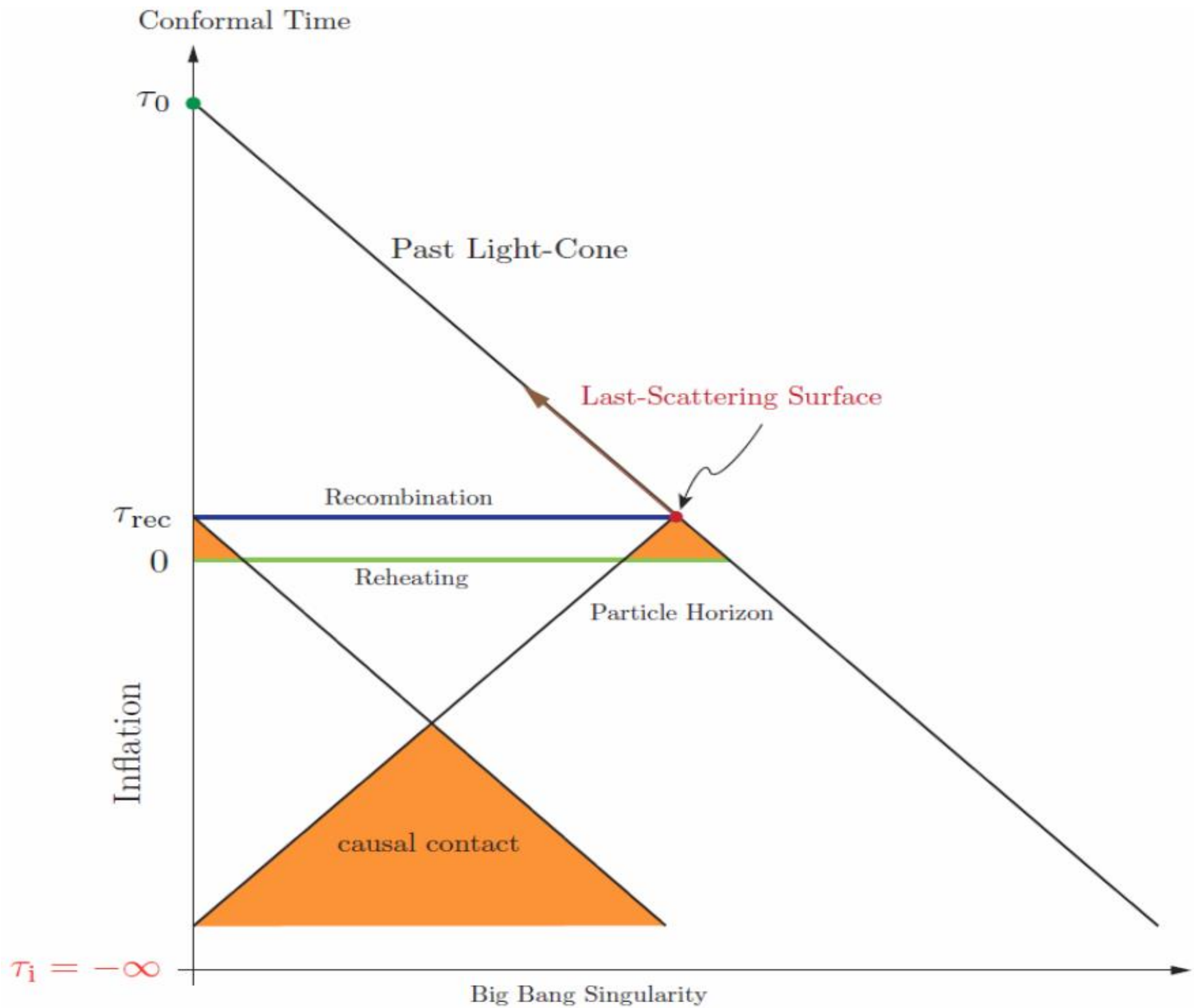
Outline

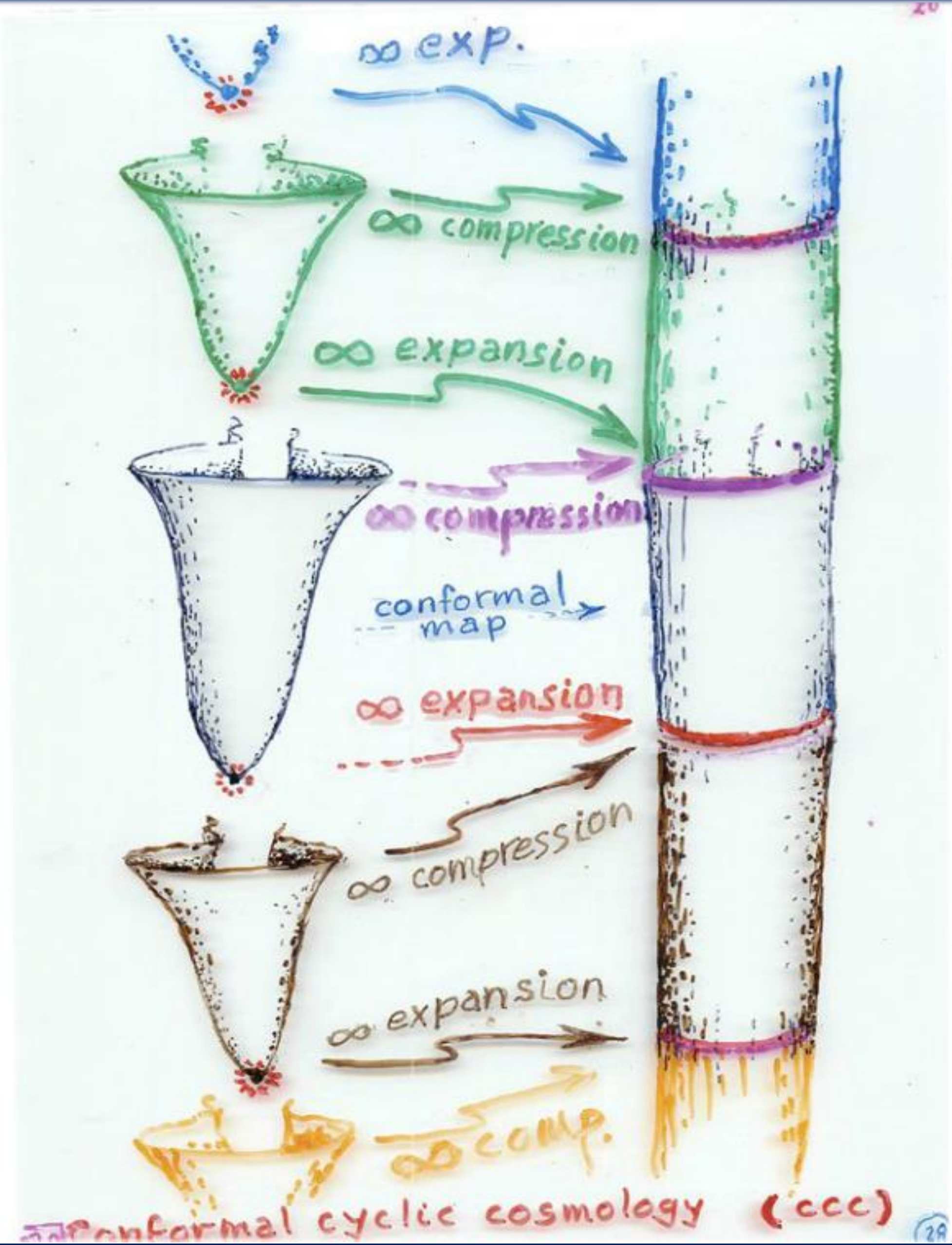
- Introduction/Background
- Kähler Moduli Stabilization
- Kähler Moduli Inflation, Dynamics & Potentials Hunting
- Conclusion/Future Work

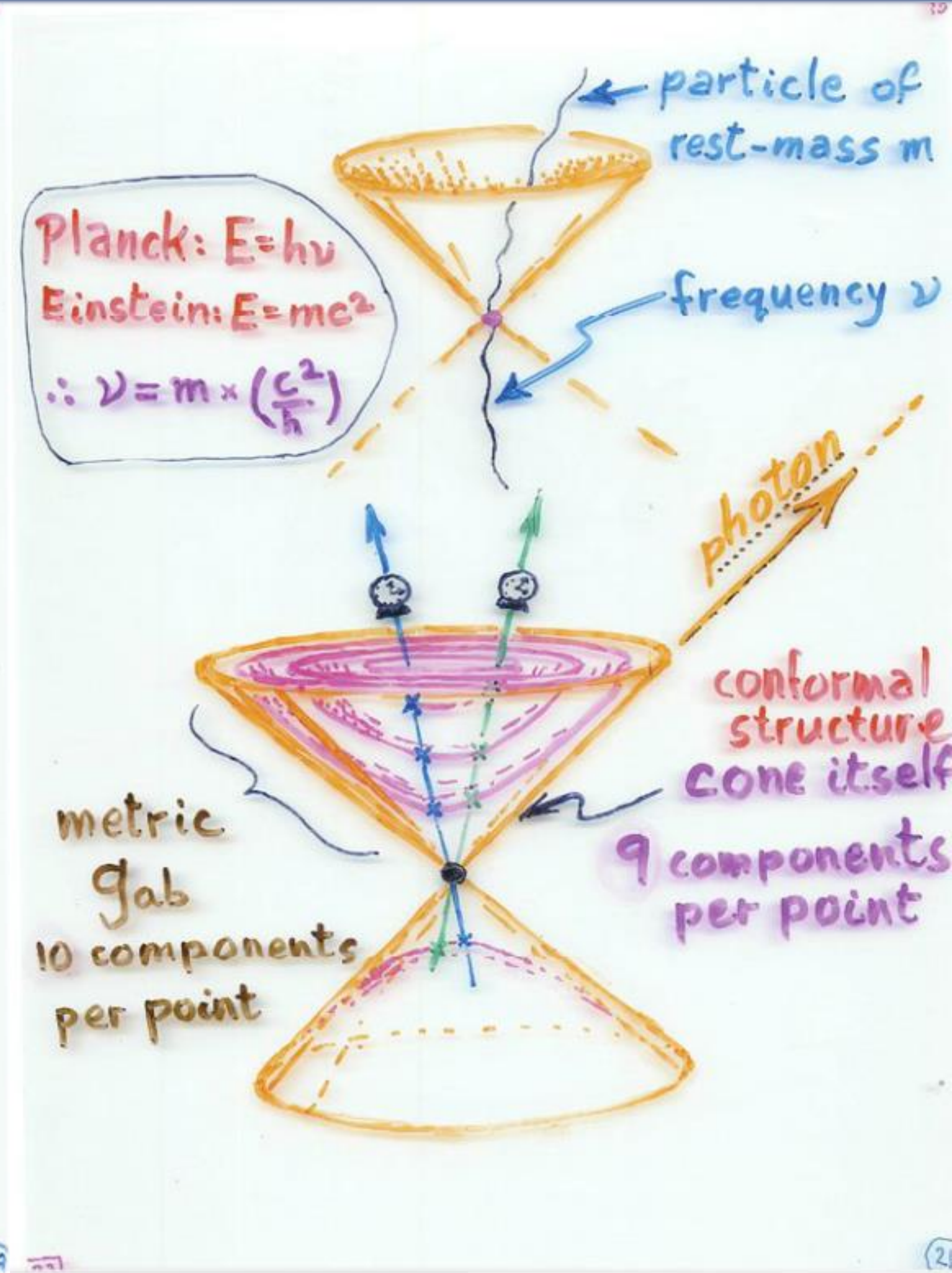

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

The Horizon Problem & Inflation



Alternative Proposal: Conformal Cyclic Cosmology





Model Independent Predictions

Tensor power fixes the Hubble rate at 'horizon-crossing' $k = a(t_\star)H(t_\star)$:

$$\Delta_t^2 = \frac{2}{\pi^2}\frac{H_\star^2}{M_{pl}^2}$$

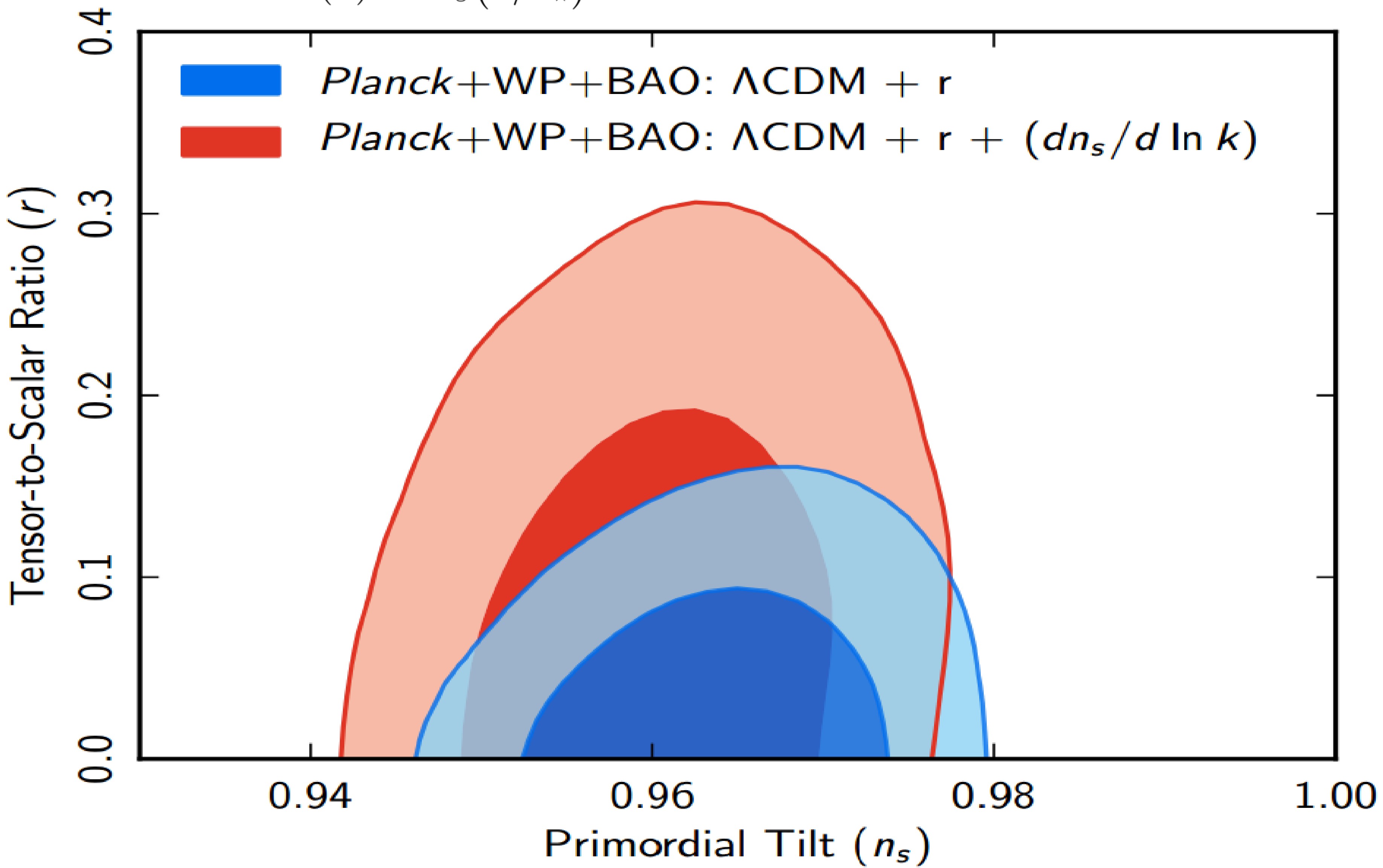
Measuring r fixes the energy scale of inflation:

$$r \equiv \frac{\Delta_t^2(k)}{\Delta_s^2(k)} \quad , \qquad \Delta_s^2 \approx 10^{-9}$$

$$\Delta_t^2 \propto H^2 \approx V \quad \Rightarrow \quad V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\ \mathrm{GeV}$$

Model Dependent Predictions

$$\mathcal{P}(k) = A_s(k/k_\star)^{n_s(k_\star)-1+\frac{1}{2}dn_s/d\ln k\ln(k/k_\star)+\ldots}$$



Model Dependent Predictions

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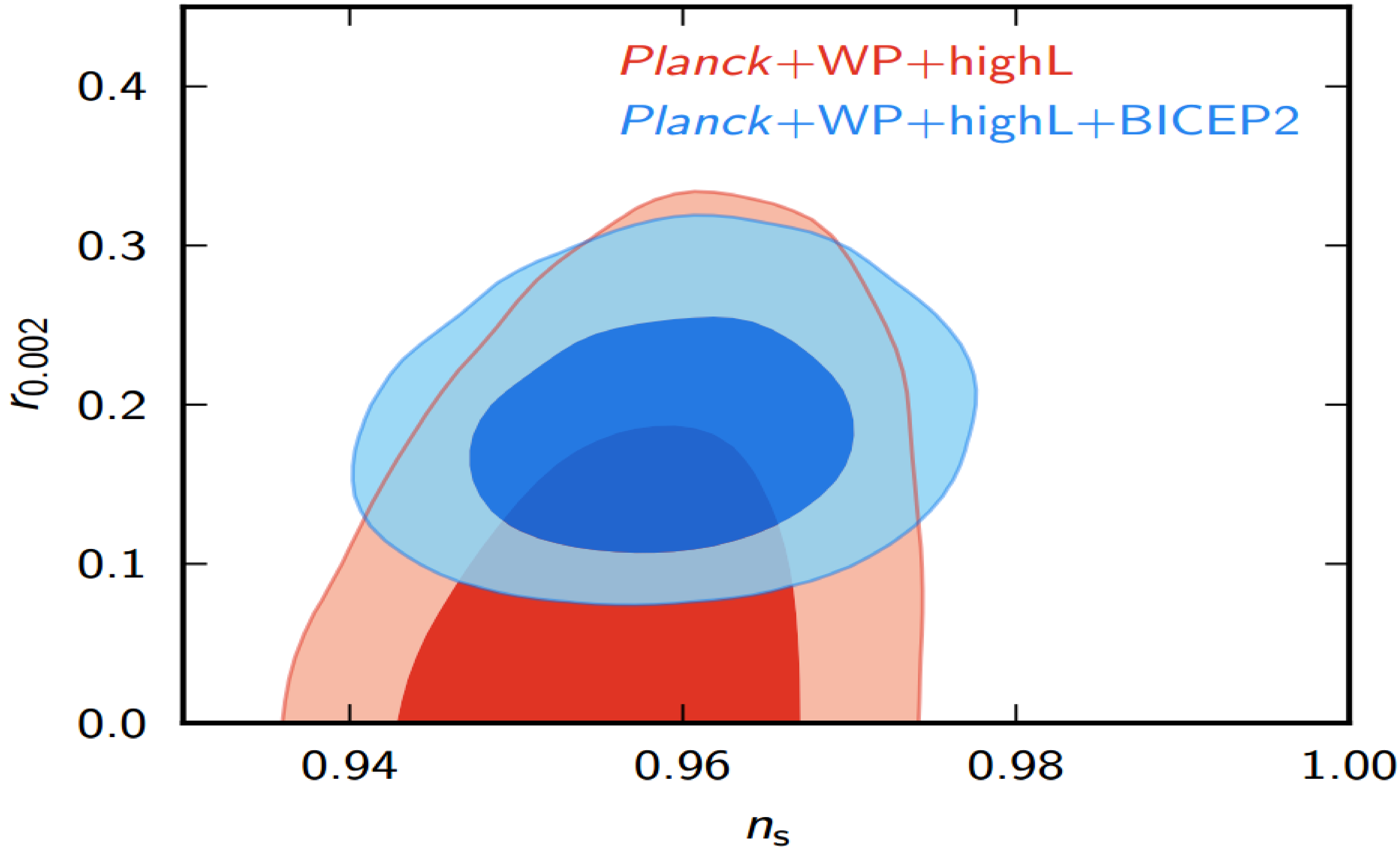


String Inflation Models

String Scenario	n_s	r
$D3/\overline{D3}$ Inflation	$0.966 \leq n_s \leq 0.972$	$r \leq 10^{-5}$
Inflection Point Inflation	$0.92 \leq n_s \leq 0.93$	$r \leq 10^{-6}$
DBI Inflation	$0.93 \leq n_s \leq 0.93$	$r \leq 10^{-7}$
Wilson Line Inflation	$0.96 \leq n_s \leq 0.97$	$r \leq 10^{-10}$
D3/D7 Inflation	$0.95 \leq n_s \leq 0.97$	$10^{-12} \leq r \leq 10^{-5}$
Racetrack Inflation	$0.95 \leq n_s \leq 0.96$	$r \leq 10^{-8}$
N − flation	$0.93 \leq n_s \leq 0.95$	$r \leq 10^{-3}$
Axion Monodromy	$0.97 \leq n_s \leq 0.98$	$0.04 \leq r \leq 0.07$
Kahler Moduli Inflation	$0.96 \leq n_s \leq 0.967$	$r \leq 10^{-10}$
Fibre Inflation	$0.965 \leq n_s \leq 0.97$	$0.0057 \leq r \leq 0.007$
Poly − instanton Inflation	$0.95 \leq n_s \leq 0.97$	$r \leq 10^{-5}$

Burgess et al. (2013), arXiv: 1306.3512

Kähler Moduli Stabilization and Inflation

Kähler potential for Kähler modulus T_i and dilaton S in generic compactifications of type IIB string theory with 3-form fluxes and D7-branes:

$$K = -2\ln\hat{\mathcal{V}} - \ln\left(S+\bar{S}\right)$$

Volume of the internal Calabi–Yau threefold X_3:

$$\hat{\mathcal{V}} = \gamma\left(T+\overline{T}\right)^{\frac{3}{2}}$$

Non-perturbative contributions to the superpotential for stabilization:

$$W = W_0 + \sum_{i=1}^{n} A_i\, e^{-a_i T}, \qquad a_i = \frac{2\pi}{N_i}$$

The full F-term scalar potential after integrating out S, $D_S W = 0$:

$$V = e^K\left(K^{\overline{T}T}\,\overline{D_T W}\,D_T W - 3|W|^2\right)$$

A No-go Theorem for Tree-level Kähler Potential

For any holomorphic superpotential W and Kähler potential with $0 < A \leq 3$:

$$K = -A \ln\left(T + \bar{T}\right)$$

The inflationary potential constructed from:

$$V = e^K \left(K^{\bar{T}T}\overline{D_T W} D_T W - 3|W|^2\right) ,$$

this generically produces an AdS minimum. *Brustein and Steinhardt (1993)*

A SUSY-breaking extremum $D_T W \neq 0$ will always have a steep direction:

$$\eta = \frac{1}{V}\begin{pmatrix} K^{\bar{T}T}V_{T\bar{T}} & K^{\bar{T}T}V_{TT} \\ K^{\bar{T}T}V_{\bar{T}\bar{T}} & K^{\bar{T}T}V_{\bar{T}T} \end{pmatrix}, \qquad \mathrm{Tr}(\eta) \leq -\frac{4}{A}$$

While for viable inflation:

$$|\eta| \leq \mathcal{O}\left(\frac{1}{100}\right)$$

Ben-Dayan et al. (2008), arXiv: 0802.3160

Kähler Uplifting and Necessary Conditions for Inflation/dS Minimum

An interplay between the α'^3 corrections and non-perturbative W:

$$K = -2\ln\left(\hat{\mathcal{V}} + \frac{\hat{\xi}}{2}\right) ,$$

with $\hat{\xi} = \xi(S_0 + \bar{S}_0)^{\frac{3}{2}}$ in terms of the dilaton vev S_0

The full scalar potential after integrating out S, $D_S W = 0$:

$$V = e^K \left(K^{T\overline{T}} \left[W_T \overline{W_T} + (W_T \cdot \overline{W K_T} + \overline{W_T} \cdot W K_T) \right] + 3\hat{\xi} \frac{\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2}{(\hat{\mathcal{V}} - \hat{\xi})(\hat{\xi} + 2\hat{\mathcal{V}})^2} |W|^2 \right)$$

uplifting

A necessary condition for inflation/dS minimum:

$$R = \frac{2}{3} - \frac{35\hat{\xi}}{96\sqrt{2}\,\gamma\, t^{3/2}} < 0$$

Properties of the Scalar Potential

Recall the superpotential W and Kähler uplifted scalar potential V:

$$V = e^K \left(K^{T\overline{T}} \left[W_T \overline{W_T} + (W_T \cdot \overline{W K_T} + \overline{W_T} \cdot W K_T) \right] + \underline{3\hat{\xi} \frac{\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2}{(\hat{\mathcal{V}} - \hat{\xi})(\hat{\xi} + 2\hat{\mathcal{V}})^2} |W|^2} \right)$$

uplifting

$$W = W_0 + \sum_{i=1}^{n} A_i \, e^{-a_i T}$$

V is invariant under the following scalings:

$$\hat{\xi} \longrightarrow \lambda^{\frac{3}{2}} \hat{\xi}, \quad a_i \longrightarrow \frac{a_i}{\lambda}, \quad A_i \longrightarrow \lambda^{\frac{3}{2}} A_i, \quad W_0 \longrightarrow \lambda^{\frac{3}{2}} W_0, \quad t \longrightarrow \lambda t$$

The axionic direction at $\tau = 0$ is always an extremum.

In particular, with 1 condensate, it is a minimum:

$$V_{\tau\tau} = -\frac{a^3 A e^{-at} W_0}{2\gamma^2 t^2} > 0 \quad \text{if} \quad W_0 < 0$$

dS Minimum with One Condensate

The potential can be approximated in the large volume limit $\hat{\mathcal{V}} \gg \hat{\xi}$ and the validity of the non-perturbative superpotential $|W_0| \gg Ae^{-at}$:

$$V(t,\tau) \simeq \frac{-W_0 a^3 A}{2\gamma^2}\left[\frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}\cos(a\tau)\right]; \qquad C = \frac{-27 W_0 \hat{\xi} a^{3/2}}{64\sqrt{2}\gamma A},$$

where $x = at$.

A meta-stable minimum at $t \approx 40$ can be achieved with:

$$a = \frac{2\pi}{100}, \quad W_0 = -37.73, \quad A = 1, \quad \gamma = \frac{\sqrt{3}}{2\sqrt{5}}, \quad \hat{\xi} = 7.98$$

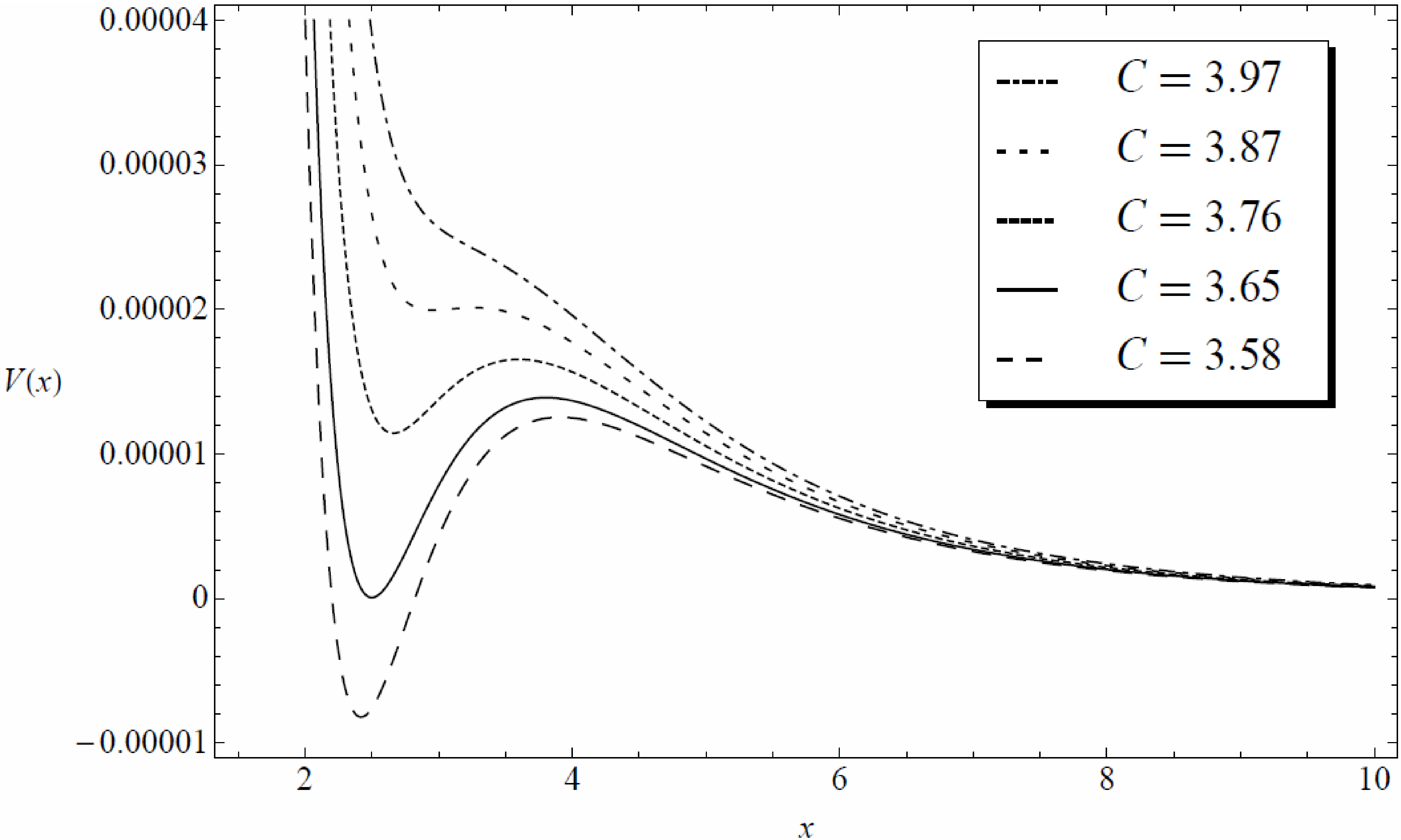
The value of C determines the vacuum energy at the minimum

Continuity implies a region flat enough for inflation must exist

A second condensate is necessary for both inflation and dS minimum

Rummel and Westphal (2011), arXiv: 1107.2115

dS Minimum with One Condensate



Rummel and Westphal (2011), arXiv: 1107.2115

Inflation with Two Condensates

We consider a second gaugino condensate on the same four-cycle:

$$W = W_0 + Ae^{-aT} + Be^{-bT}$$

Accidental Inflation:

· Single-field / Two-field

· $\Delta\phi \leq 0.36\ M_p$

· Right-rolling Inflection Point

· Sensitive to initial conditions

· Extreme fine-tuning

· Negligible r

· $\alpha \sim \mathcal{O}(10^{-3})$

Hilltop Inflation by Deflation (IBD):

· Single-field

· $\Delta\phi \leq 0.45\ M_p$

· Left-rolling Hilltop, no overshooting

· Insensitive to initial conditions

· Less fine-tuning

· Enhanced r

· Enhanced α

Generalized Slow-roll Dynamics

Modified Lagrangian:

$$\mathcal{L} = a^3 \left(K_{T\bar{T}} \, (\dot{t}^2 + \dot{\tau}^2) - V(t,\tau) \right)$$

Field equations of motion:

$$\ddot{t} + 3H\dot{t} + \Gamma^T_{TT}(t)(\dot{t}^2 - \dot{\tau}^2) + \frac{K^{T\overline{T}}(t)}{2}\partial_t V = 0$$

$$\ddot{\tau} + 3H\dot{\tau} + 2\Gamma^T_{TT}(t)\dot{t}\dot{\tau} + \frac{K^{T\overline{T}}(t)}{2}\partial_\tau V = 0$$

Friedmann equations:

$$3H^2 = K_{T\overline{T}}|\dot{T}|^2 + V$$

$$\dot{H} = -K_{T\overline{T}}(\dot{t}^2 + \dot{\tau}^2)$$

Generalized Slow-roll Dynamics

Slow-roll parameters:

$$\epsilon = \frac{K^{i\bar{j}}\partial_i V \partial_{\bar{j}} V}{V^2}$$

$$\eta = min\left\{ \mathrm{EV} \begin{pmatrix} K^{i\bar{m}} N_{\bar{m}j} & K^{i\bar{m}} N_{\bar{m}\bar{j}} \\ K^{\bar{i}m} N_{mj} & K^{\bar{i}m} N_{m\bar{j}} \end{pmatrix} \right\}$$

$$N_{i\bar{j}} = \frac{\partial_i \partial_{\bar{j}} V}{V}, \quad N_{ij} = \frac{\partial_i \partial_j V - \Gamma^l_{ij} \partial_l V}{V}, \quad \Gamma^l_{ij} = K^{l\bar{n}} \partial_j \partial_i \partial_{\bar{n}} K$$

Number of e-folds:

$$3H\dot{t} \simeq -\frac{K^{T\overline{T}}}{2} \partial_t V(t,\tau) \quad \Rightarrow \quad N \approx \frac{1}{2} \int_{t_{\mathrm{end}}}^{t} \frac{\sqrt{K_{tt}}}{\sqrt{\epsilon}} \, \mathrm{d}t$$

$K_{tt} \ll 1$ in large volume scenarios

Slow-roll Observables

Canonically normalized slow-roll parameters & predictions:

$$\epsilon = \frac{1}{2}\frac{V'^2}{V^2}\,, \quad \eta = \frac{V''}{V^2}\,, \quad \xi^2 = \frac{V'V'''}{V^2}\,, \quad \varpi^3 = \frac{V'^2V''''}{V^3}$$

$$n_s = 1 + \frac{\mathrm{d}\ln P(k)}{\mathrm{d}\ln k} \approx 1 + 2\eta - 6\epsilon$$

$$r = 16\epsilon$$

$$\alpha = \frac{\mathrm{d}n_s}{\mathrm{d}\ln k} \approx 16\epsilon\eta - 24\epsilon^2 - 2\xi^2$$

$$\beta = \frac{\mathrm{d}^2 n_s}{\mathrm{d}\ln k^2} \approx -192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 - 24\epsilon\xi^2 + 2\eta\xi^2 + 2\varpi^3$$

If running α is significant $\geq \mathcal{O}(0.01)$, with $C = -0.73$:

$$n_s = 1 + 2\eta - 6\epsilon + 2\left[\frac{1}{3}\eta^2 + (8C-1)\epsilon\eta - \left(\frac{5}{3} - C\right)\epsilon^2 - \left(C - \frac{1}{3}\right)\xi^2\right]$$

Spectral Distortions as a Probe for Inflation

Scalar power spectrum & its slow-roll approximation:

$$\mathcal{P}(k) = A_s (k/k_\star)^{n_s(k_\star)-1+\frac{1}{2}dn_s/d\ln k \ln(k/k_\star)+\dots} \approx \frac{1}{24\pi^2}\frac{V}{\epsilon}$$

Energy stored in small scale density perturbations is released through Silk damping, causing μ and y-type spectral distortions

$$\mu \approx 2.2 \int_{k_{min}}^{\infty} P_\zeta(k)\left[\exp\left(-\frac{\hat{k}}{5400}\right) - \exp\left(-\left[\frac{\hat{k}}{31.6}\right]^2\right)\right] \mathrm{d}\ln k\,,$$

$$y \approx 0.4 \int_{k_{min}}^{\infty} P_\zeta(k) \exp\left(-\left[\frac{\hat{k}}{31.6}\right]^2\right) \mathrm{d}\ln k$$

CMB/LSS observations: $k \leq 1\ \mathrm{Mpc}^{-1}$, Spectral distortions: $1 \leq k \leq 10^4\ \mathrm{Mpc}^{-1}$

Constraining α & β indirectly

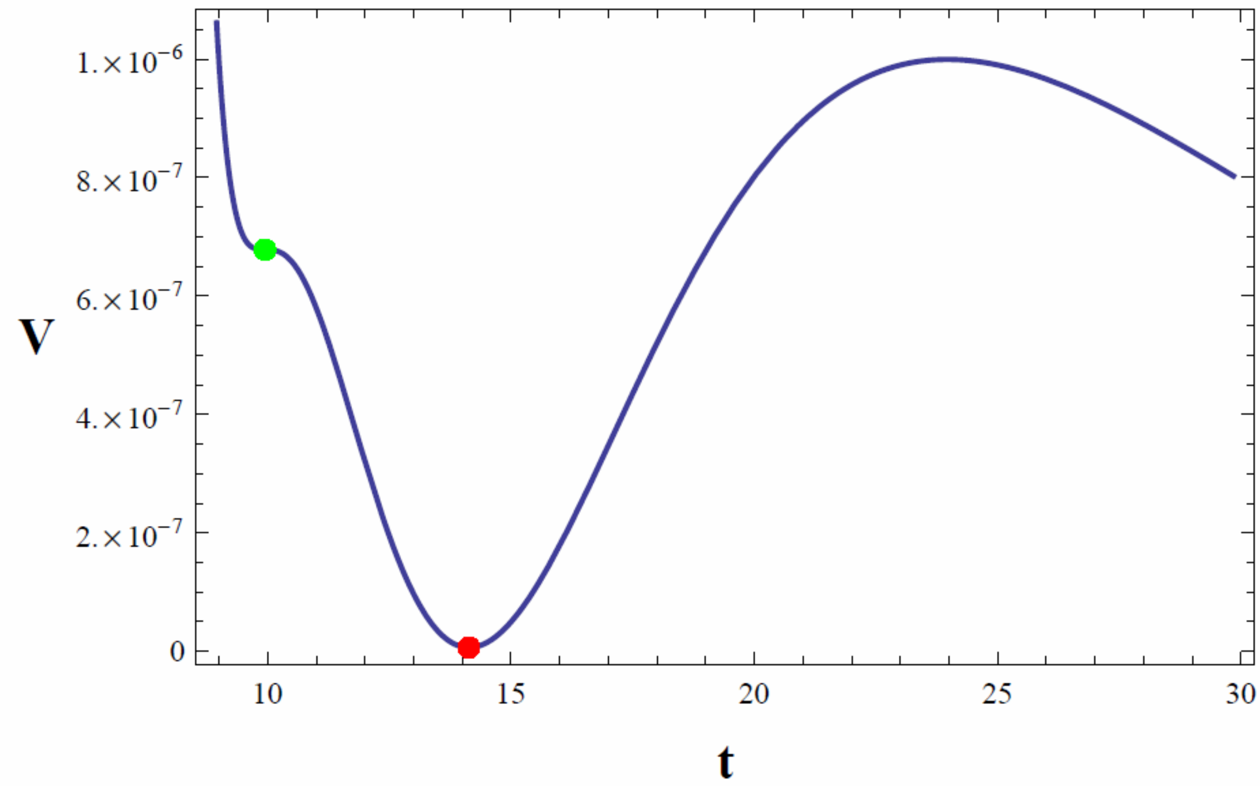
COBE/FIRAS: $|\mu| \leq 9 \cdot 10^{-5}$ & $|y| \leq 1.5 \cdot 10^{-5}$

Typical small-field slow-roll inflation: $\mu \sim \mathcal{O}(10^{-8})$, $y \sim \mathcal{O}(10^{-9})$

Chluba et al. (2012), arXiv: 1203.2681

Accidental Inflation: Single-field Model

$$a = \frac{2\pi}{12}\,, \quad b = \frac{2\pi}{41}\,, \quad W_0 = -7.73118337\,, \quad A = 1\,, \quad B = 0.1598\,, \quad \hat{\xi} = 3.95\,, \quad \gamma = 1$$



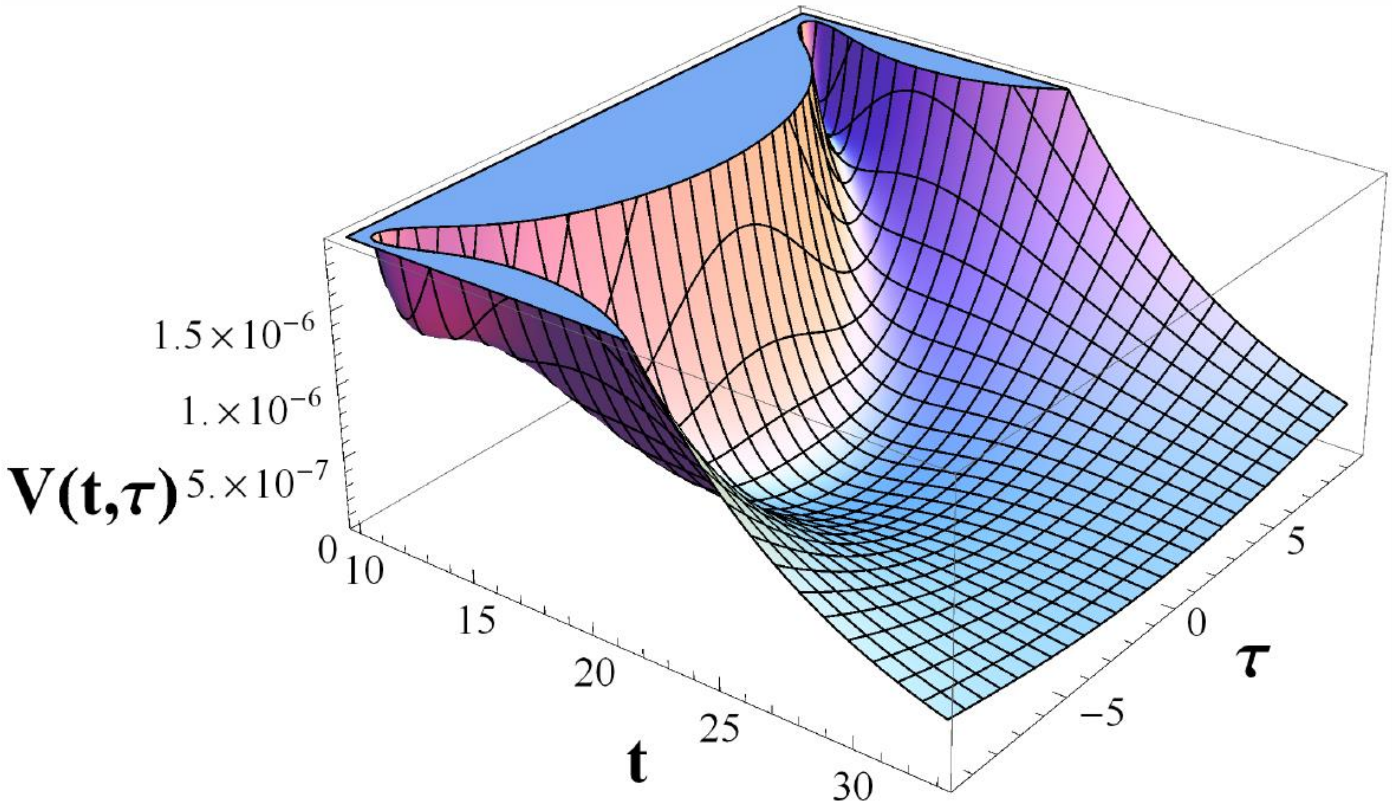
$$(t_{CMB}, \tau_{CMB}) = (9.947364,\, 0); \quad \Delta\phi = 0.196; \quad \eta_{CMB} = (-5.77 \cdot 10^{-3},\, 3644)$$

$$n_s = 0.974,\ r = 1.15 \cdot 10^{-11},\ \alpha = -2.24 \cdot 10^{-3},\ \beta = -2.91 \cdot 10^{-5},\ \mu = 1.74 \cdot 10^{-8},\ y = 2.44 \cdot 10^{-9}$$

$$t_{min} = 14.1362, \ \ m_{3/2}^2 = e^K |W|^2/(V_{CMB})^{1/4} = 1.76$$

Accidental Inflation: Two-field Model

$$a = \frac{2\pi}{30}\,, \quad b = \frac{2\pi}{29}\,, \quad W_0 = -1.8041652\,, \quad A = 1\,, \quad B = -1.031703\,, \quad \hat{\xi} = 1.3\,, \quad \gamma = \sqrt{\frac{3}{20}}$$



$$(t_{CMB}, \tau_{CMB}) = (12.4436, \pm 8.06801); \quad \Delta\phi = 0.357; \quad \eta_{CMB} = (-4.42\cdot 10^{-6}, \underline{177})_{\text{heavy}}$$

$$n_s = 0.963,\ r = 3.69\cdot 10^{-10},\ \alpha = -1.93\cdot 10^{-3},\ \beta = -3.35\cdot 10^{-5},\ \mu = 2.36\cdot 10^{-8},\ y = 2.99\cdot 10^{-9}$$

$$t_{min} = 20.5917,\ \ m_{3/2}^2 = e^K |W|^2/(V_{CMB})^{1/4} = 0.52$$

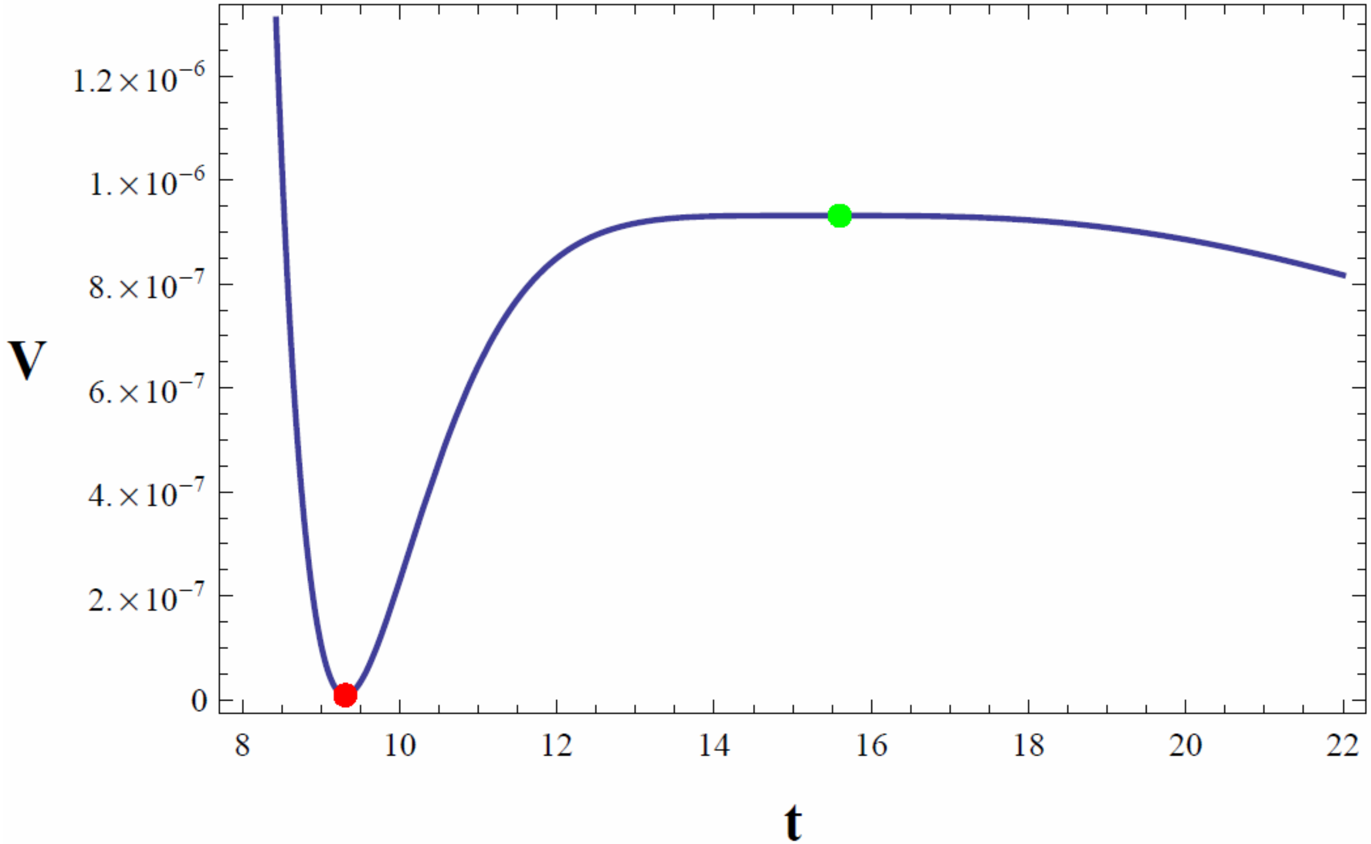
Hilltop IBD: Inflation by Deflation

Hilltop IBD:

$$a = \frac{2\pi}{12}\,, \quad b = \frac{2\pi}{37}\,, \quad W_0 = -3.876\,, \quad A = 1\,, \quad B = 0.161636\,, \quad \hat{\xi} = 6.7\,, \quad \gamma = 1$$

Hilltop IBD Enhanced r, α:

$$a = \frac{2\pi}{12}\,, \quad b = \frac{2\pi}{37}\,, \quad W_0 = -3.817176\,, \quad A = 1\,, \quad B = 0.161158\,, \quad \hat{\xi} = 6.77\,, \quad \gamma = 1$$

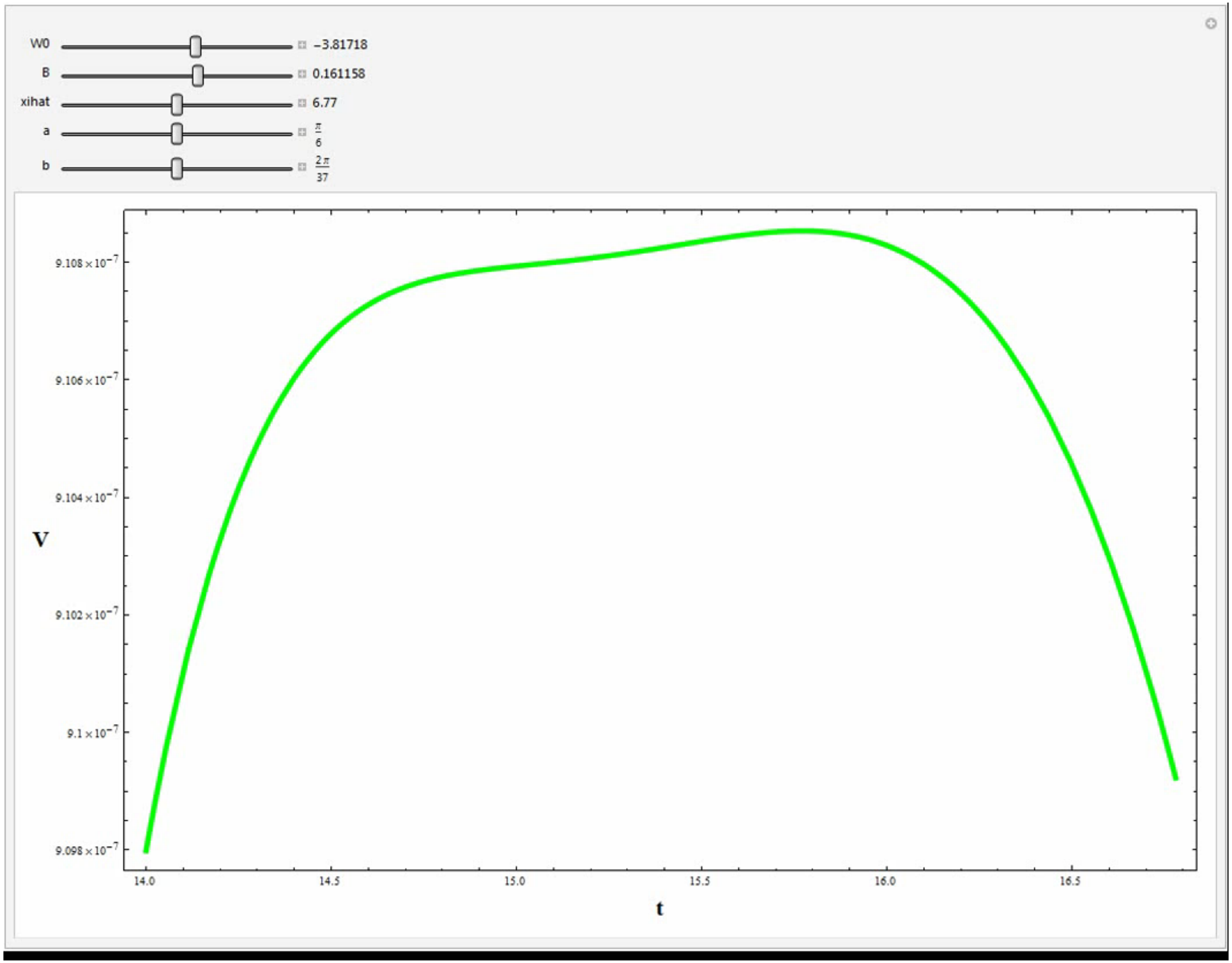


Hilltop IBD: Predictions

Parameter	Hilltop IBD model	Observable running model
n_s	0.965	0.953
r	$5.17 \cdot 10^{-6}$	$1.67 \cdot 10^{-5}$
α	$4.72 \cdot 10^{-3}$	0.012
β	$-1.63 \cdot 10^{-4}$	$-5.32 \cdot 10^{-4}$
μ	$2.02 \cdot 10^{-8}$	$2.79 \cdot 10^{-8}$
y	$2.45 \cdot 10^{-9}$	$2.81 \cdot 10^{-9}$
t_{CMB}	15.4175	15.5058
$\Delta\phi$	0.452	0.445
η_{CMB}	$(-0.0767, 140)$	$(-0.0250, 145)$
$m_{3/2}^2$	1.48	1.46

Hilltop IBD: Properties of the Potential

Two nearby inflection points gives rise to enhanced r. Controlled by a & b

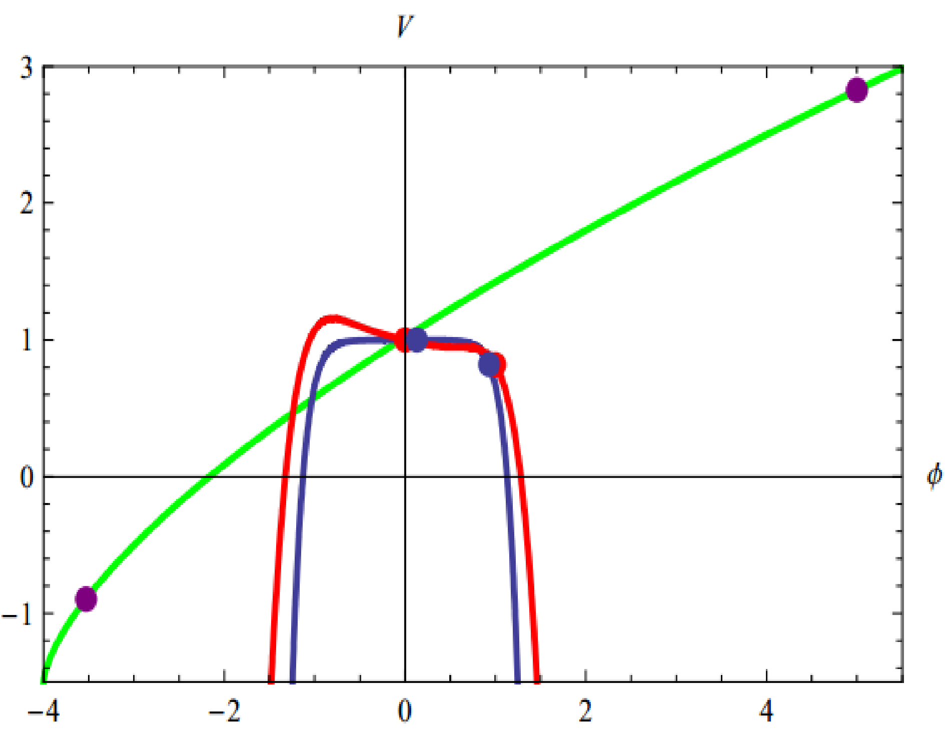


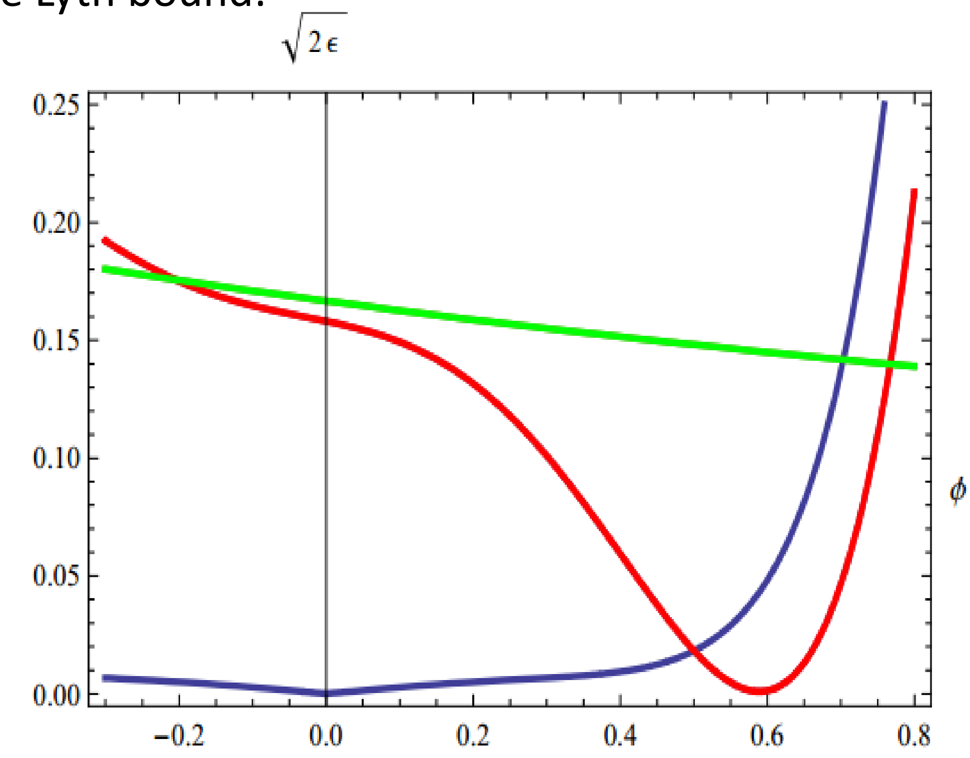
Hilltop IBD: Properties of the Potential

A scalar potential can be locally approximated by a Taylor series in terms of the slow-roll observables:

$$\frac{V(\phi)}{V(0)} = 1 - \sqrt{\frac{r_0}{8}}\phi + \frac{\eta_0}{2}\phi^2 + \frac{\alpha_0}{3\sqrt{2r_0}}\phi^3 + \ldots$$

An additional inflection point provides small-field models with enhanced r, evading the Lyth bound:

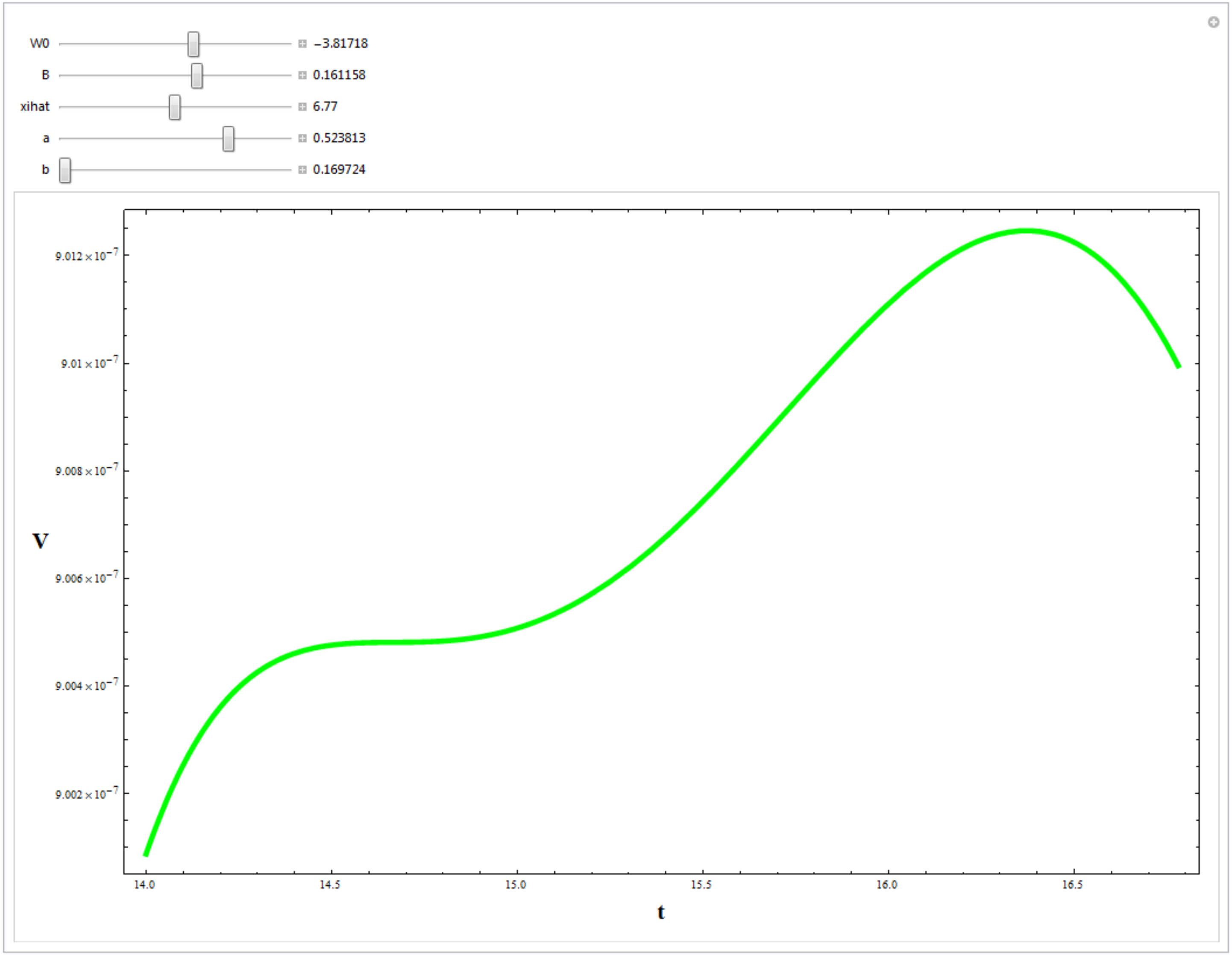




Ido Ben-Dayan and Ram Brustein (2009), arXiv: 0907.2384

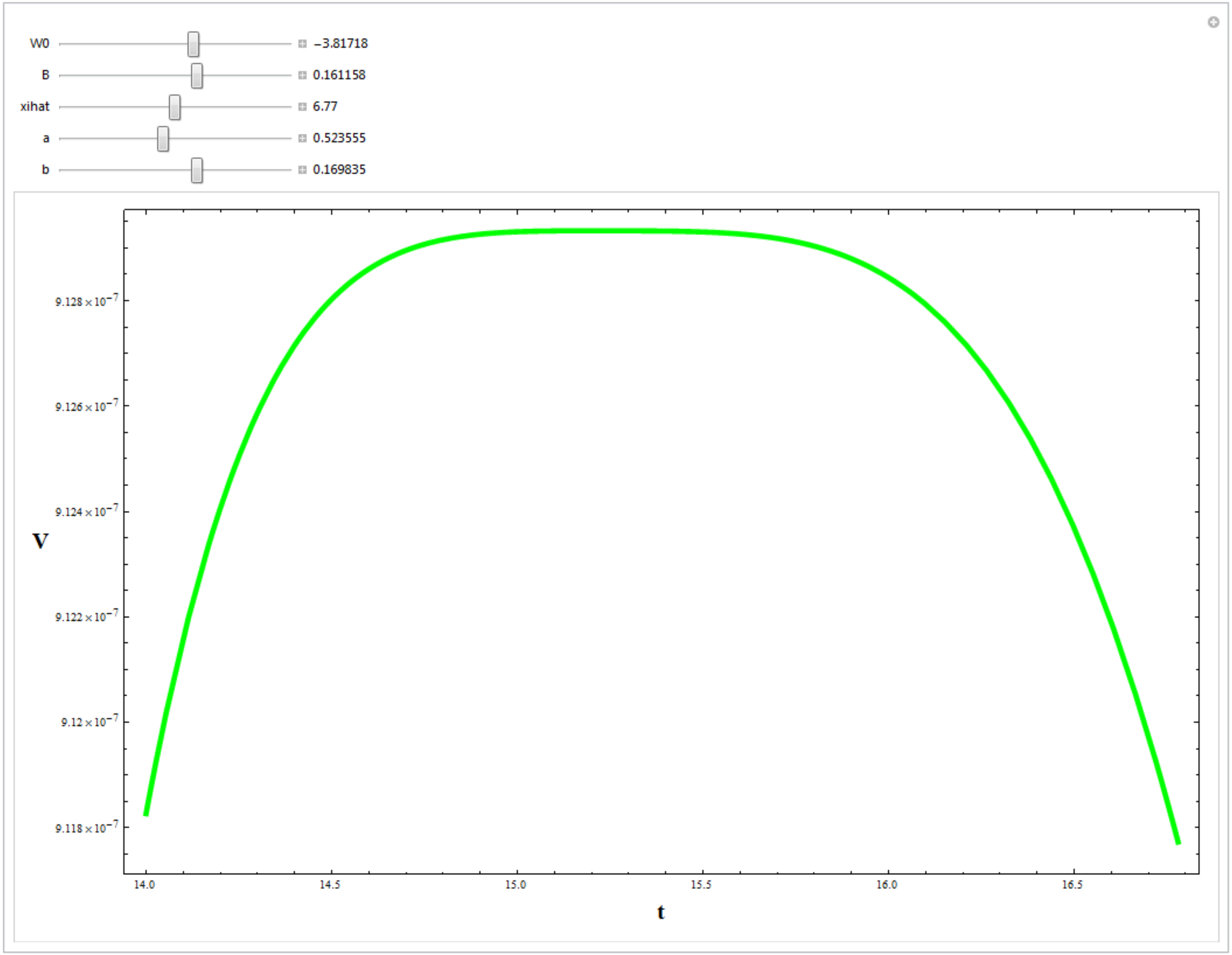
Hilltop IBD: Properties of the Potential

Further enhancing r requires non-integer a & b



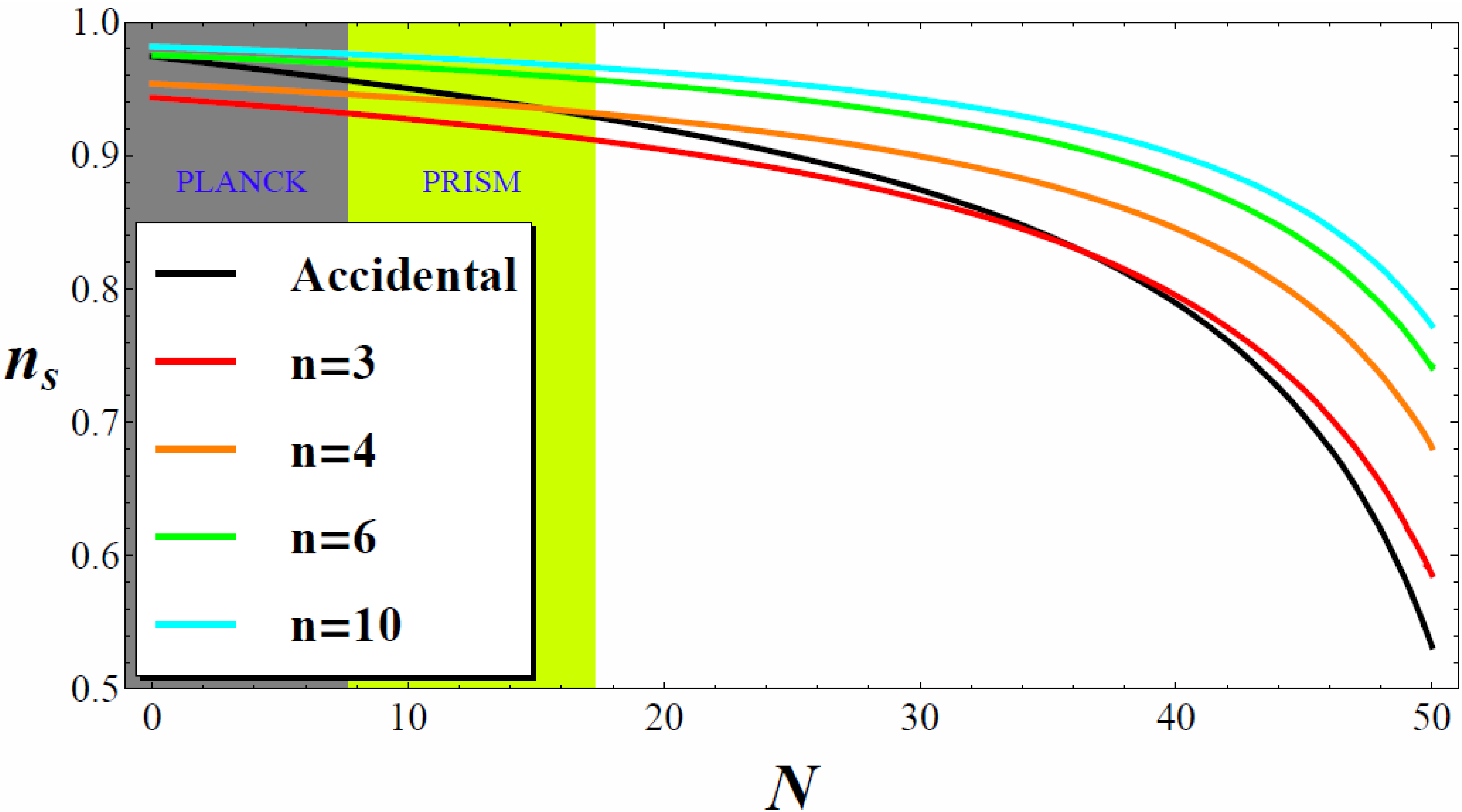
Hilltop IBD: Properties of the Potential

Further flattening the hilltop suitable for eternal inflation requires non-integer a & b



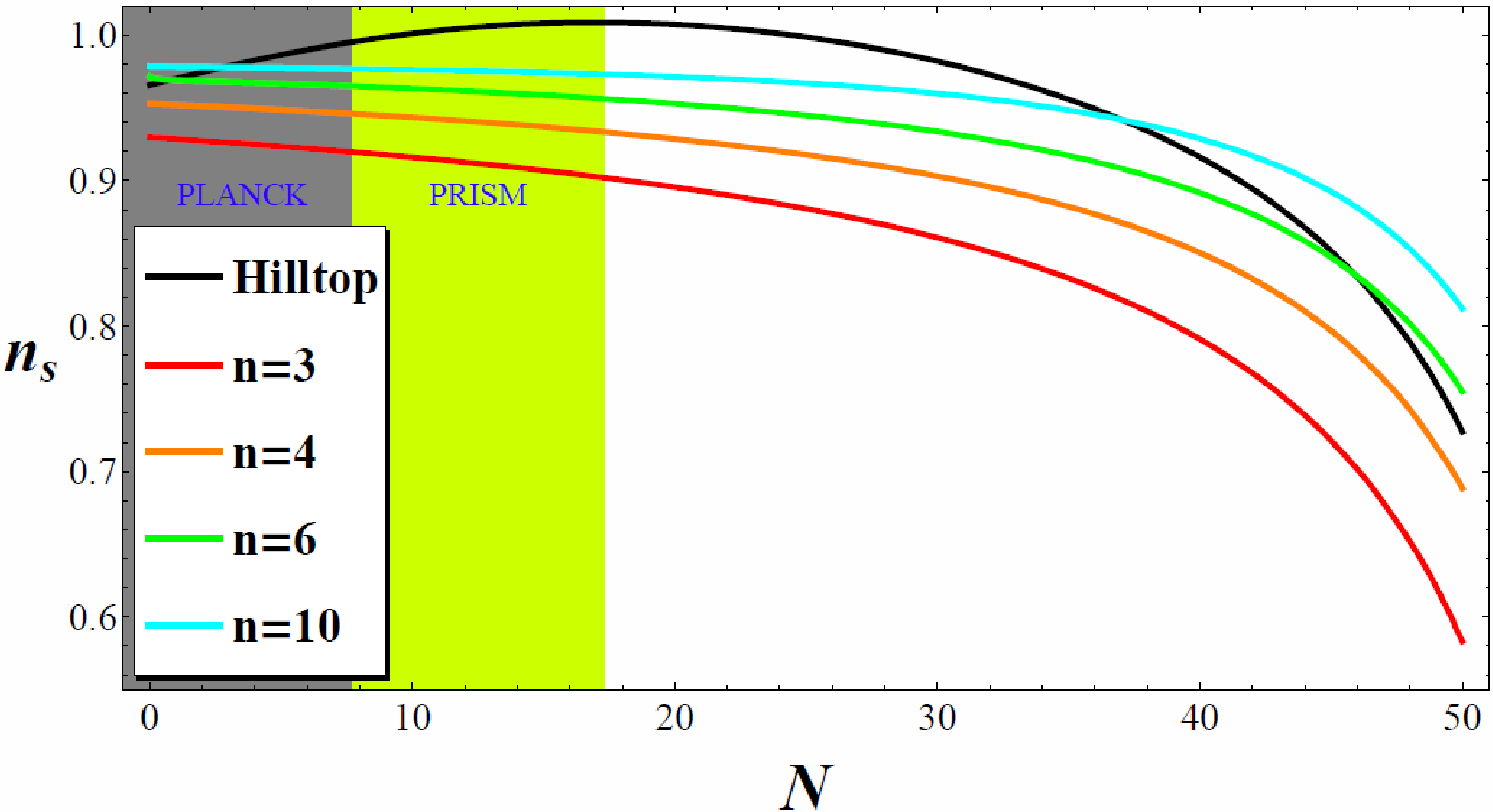
Accidental Inflation: String vs. Field Theory

$$V = 1 - a_1\phi - a_n\phi^n$$



Hilltop IBD: String vs. Field Theory

$$V = 1 + a_2\phi^2 - a_n\phi^n$$



Assessing the Degree of Fine-tuning

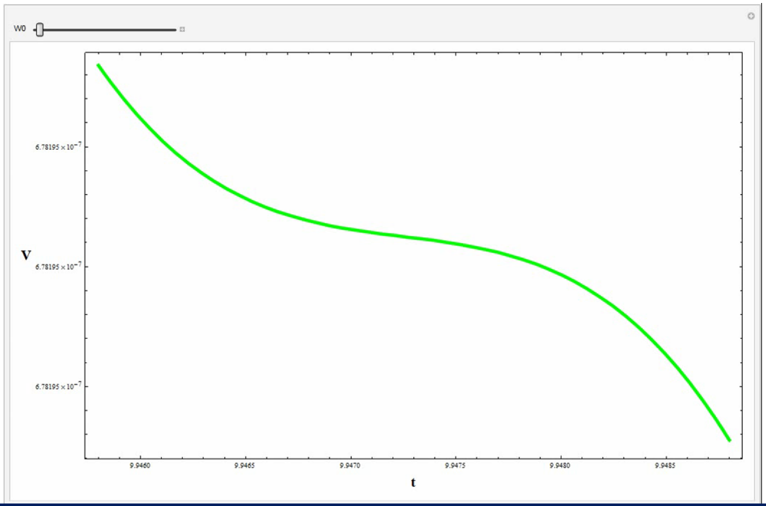
Continuity of the potential suggests us to define:

$$\sigma = \frac{|\Delta W_0|}{|W_0|}$$

Downhill Inflection Point Models:

$$\Delta W_0 = W_0|_{N_{tot}=60} - W_0|_{V'_{CMB}=0}$$

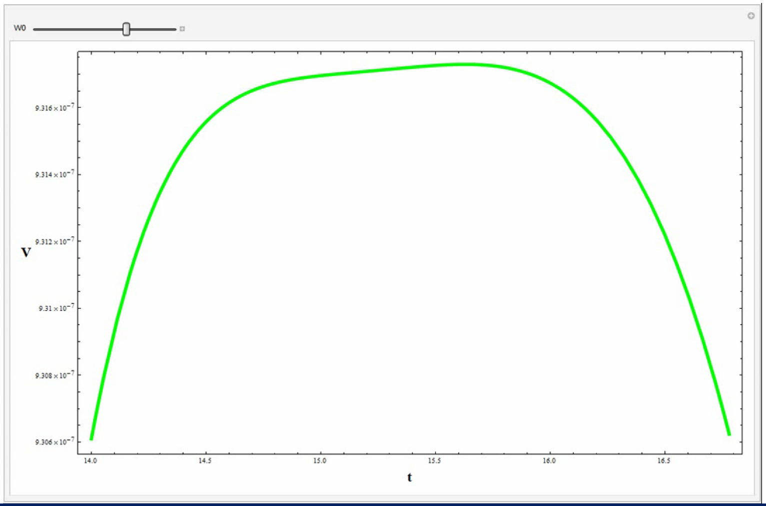
$$\sigma = 10^{-9}$$



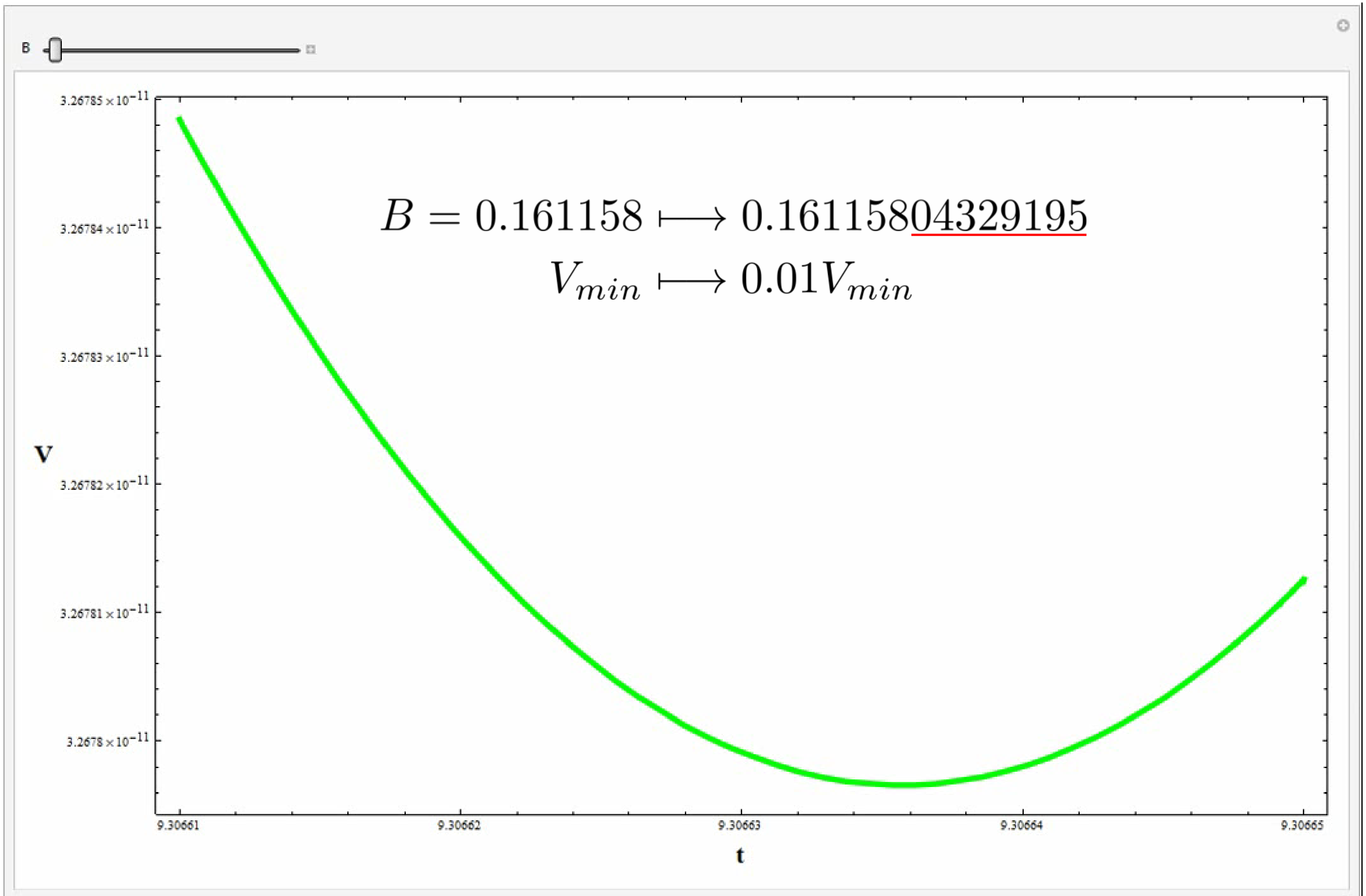
Hilltop IBD Models:

$$\Delta W_0 = W_0|_{n_s=0.9} - W_0|_{V''_{CMB}=0}$$

$$\sigma = 10^{-5}$$



A reasonable Cosmological Constant requires extreme tuning in B



Conclusion & Future Work

- Small-field & effectively single-field
- Reasonable n_s, $r \leq 10^{-5}$, $\alpha \leq 0.012$
- Enhanced scale dependence vs. field theory models
- IBD models evade the problem of overshooting, I.C.; larger r

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- Reducing the gravitino mass
- Tuning the value of CC
- Searching for inflation at larger volume


Thank you