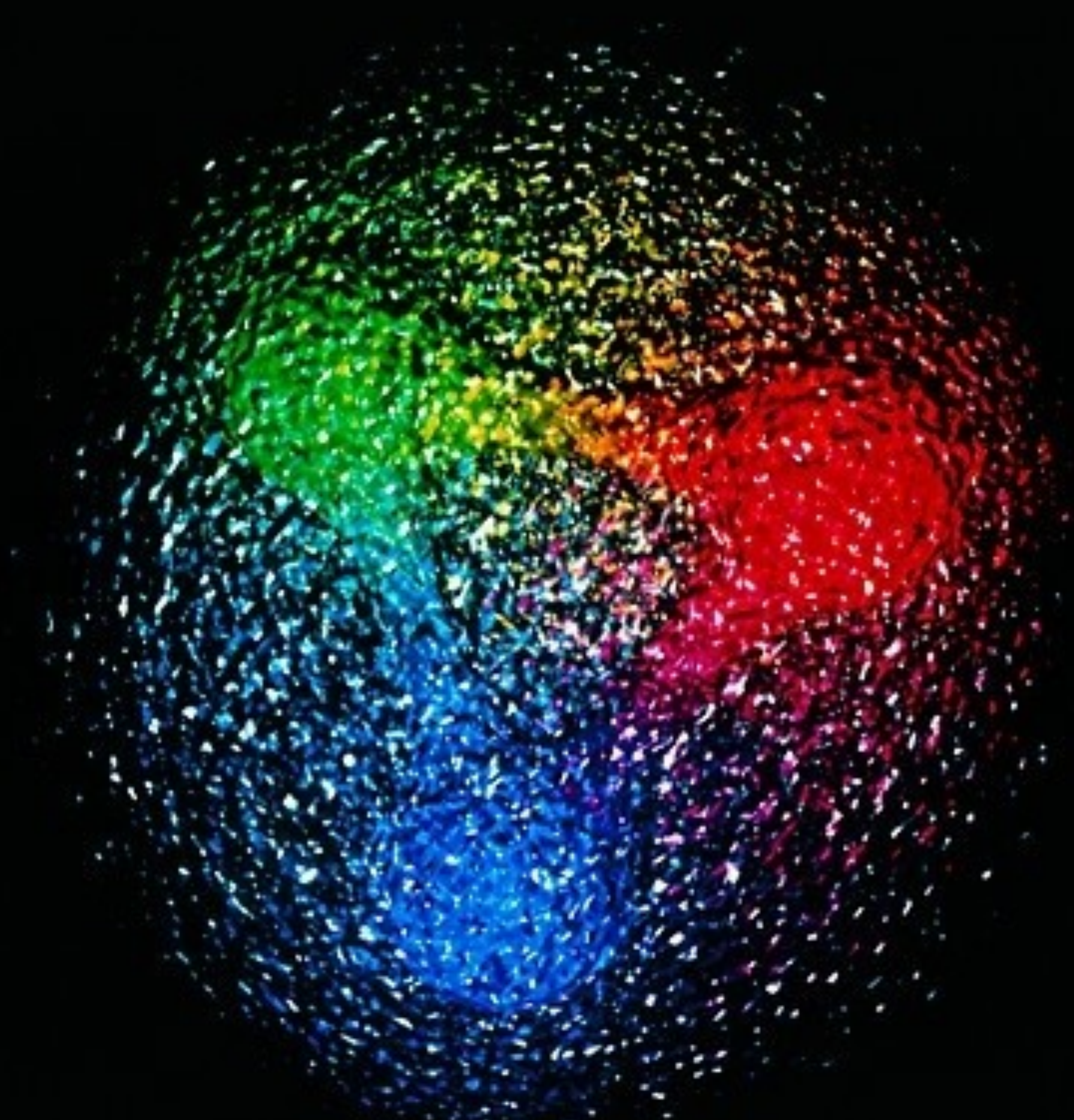
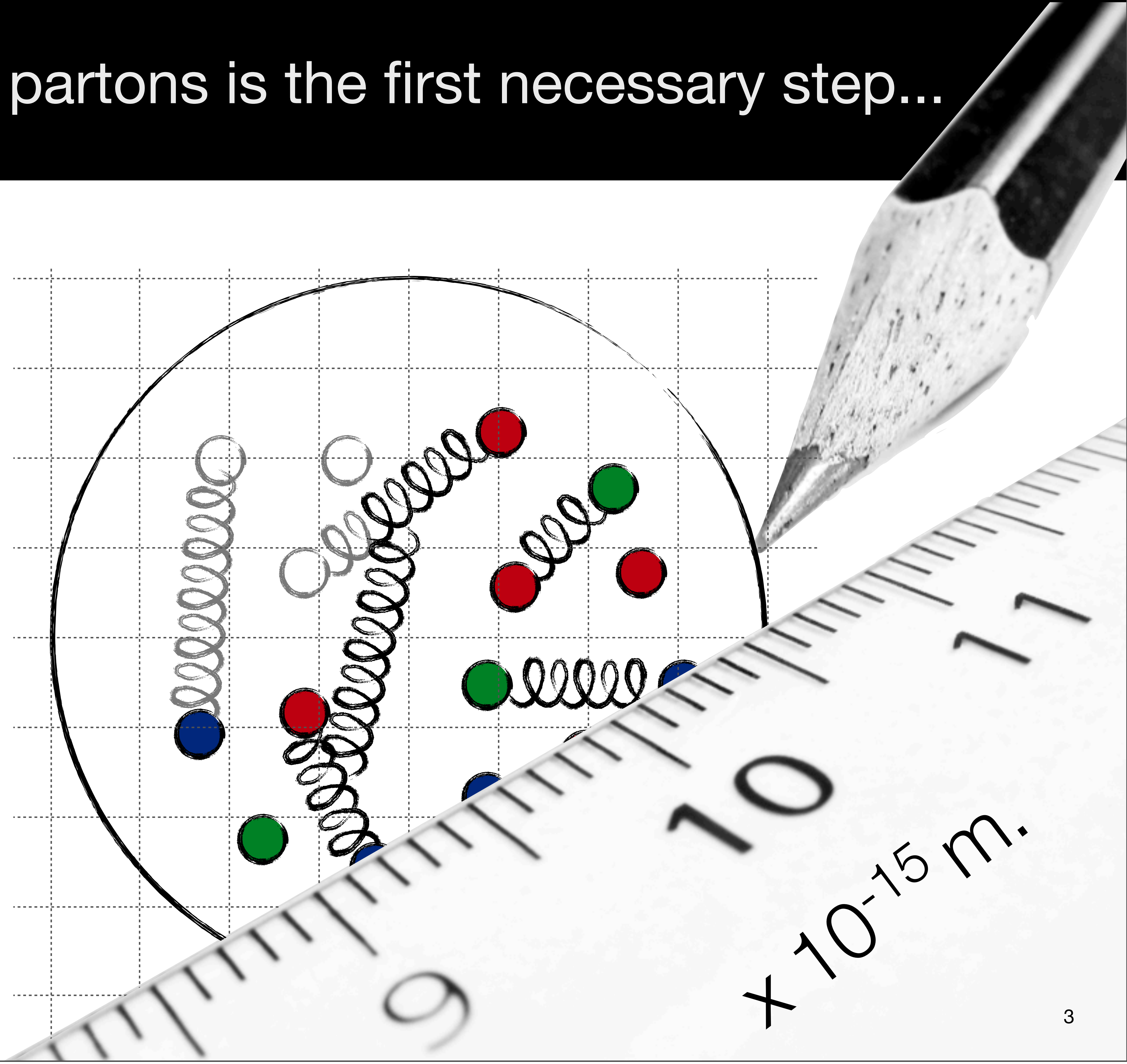
Inclusive and semi-inclusive spin physics

Alessandro Bacchetta

What is the structure of the proton?



Mapping partons is the first necessary step...



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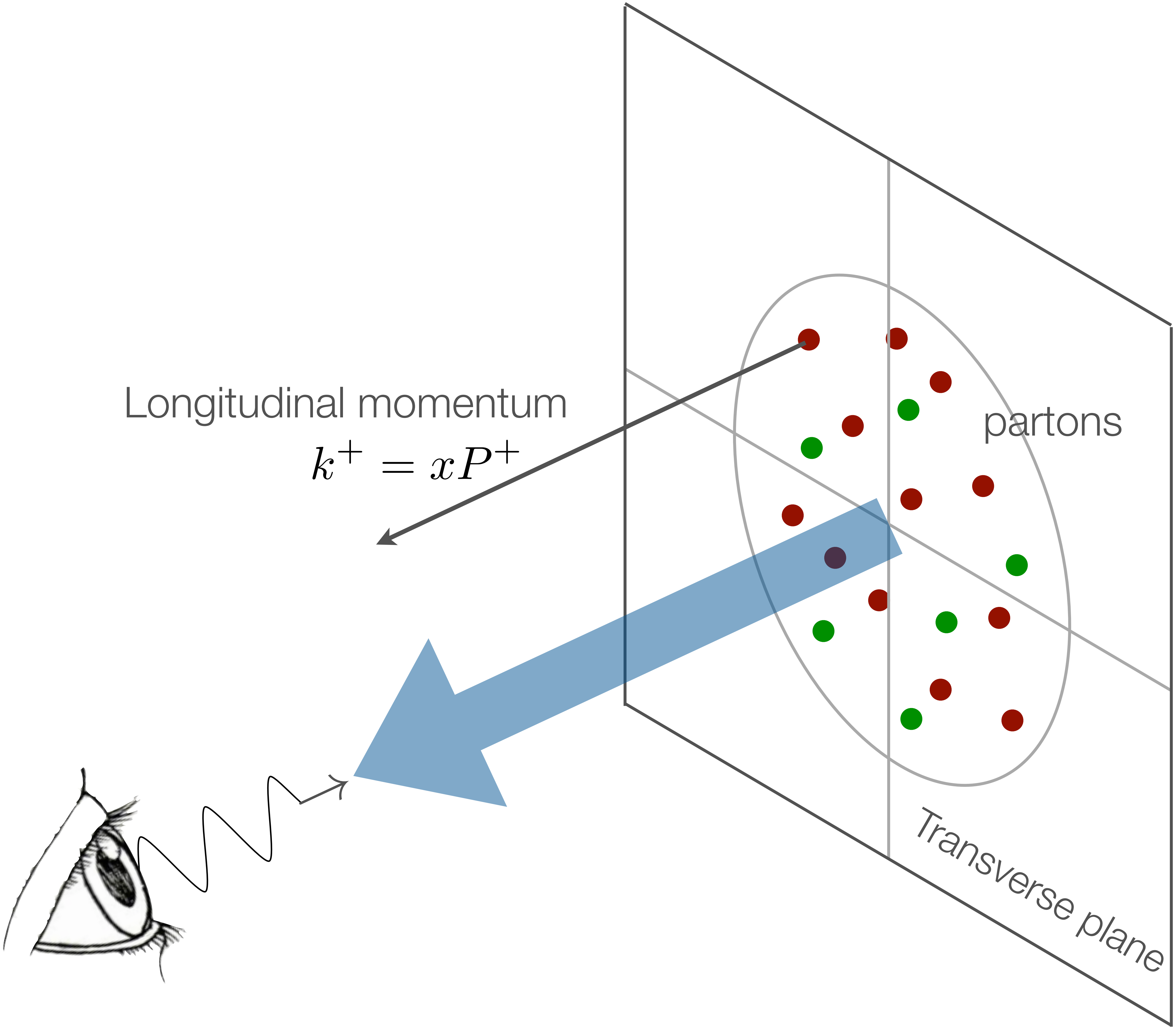
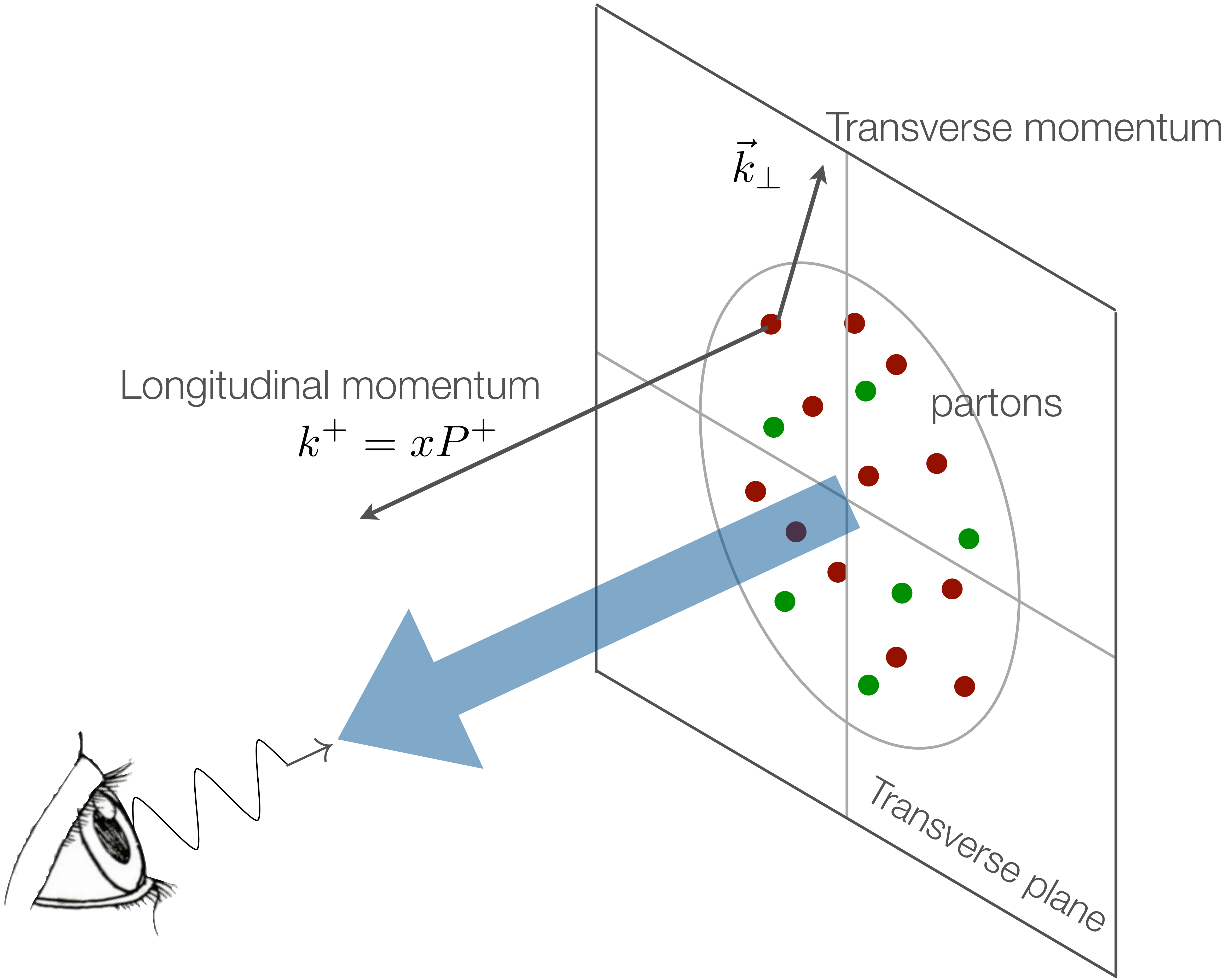
- Measure what we cannot calculate with QCD
- Use to make predictions in hadronic collisions


Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane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$\vec{k}_\perp$
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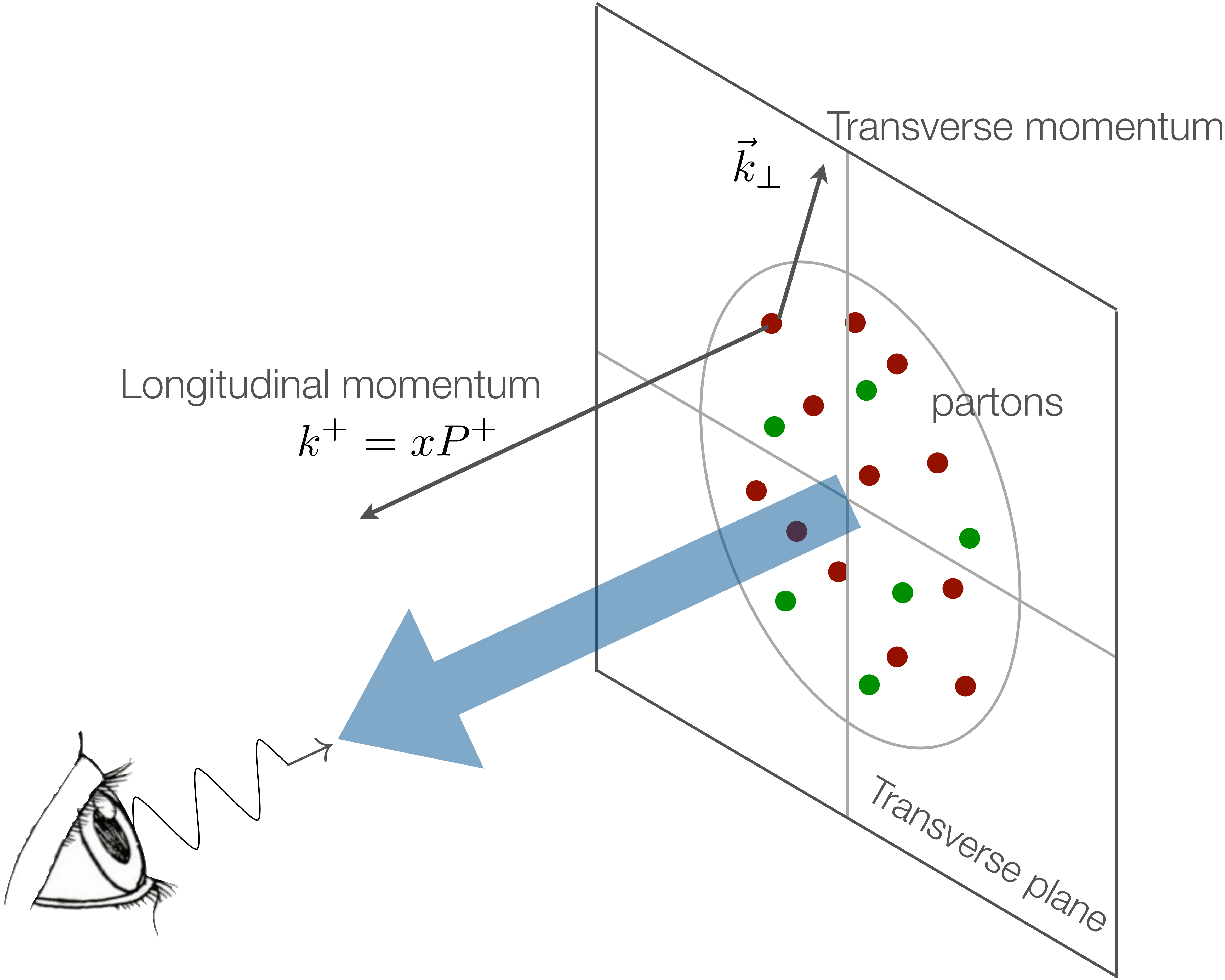
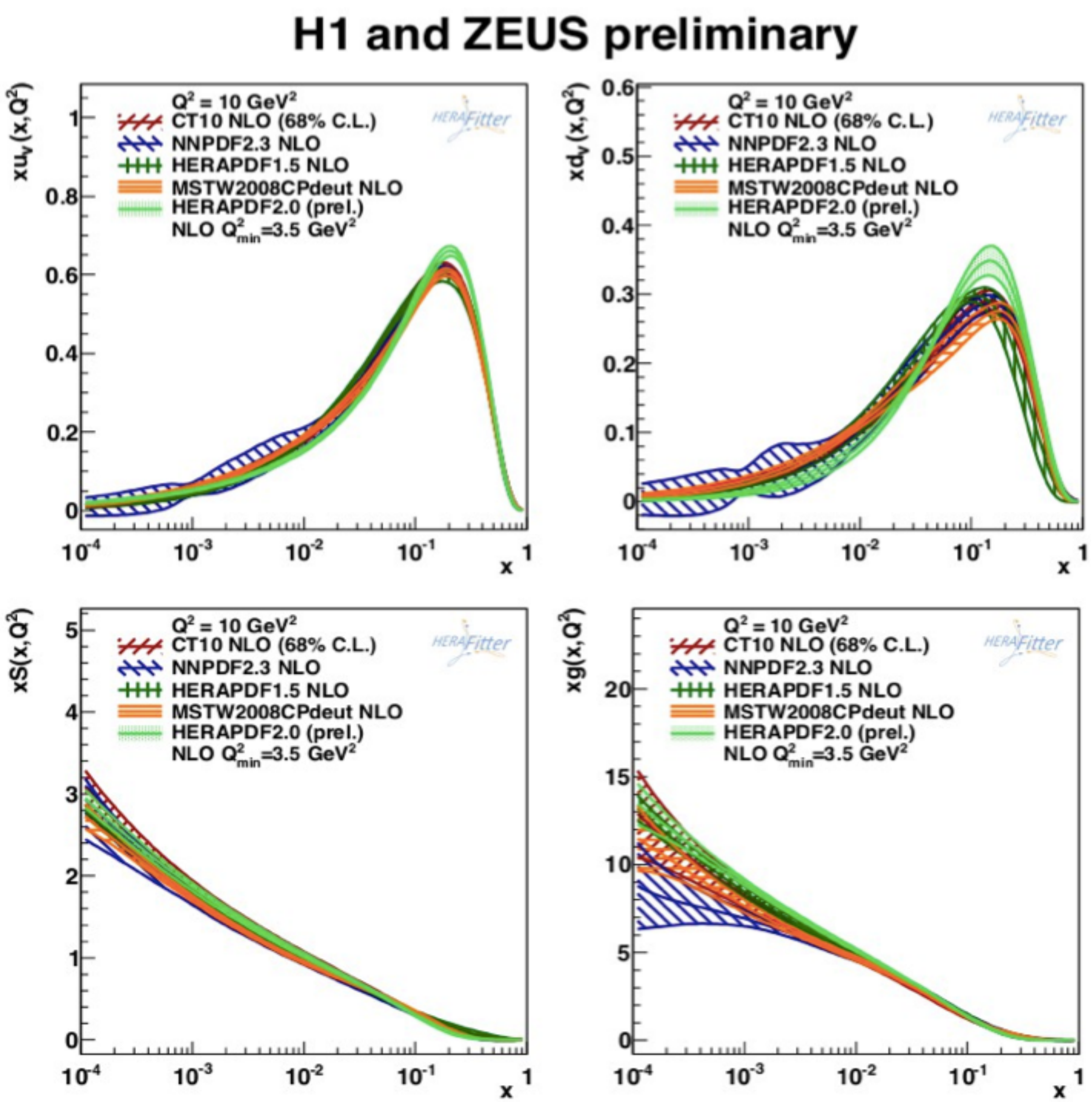


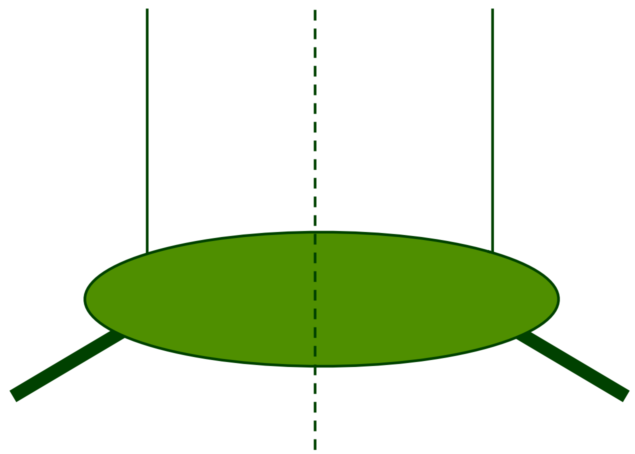
for spatial dimensions, see talk by P. Kroll

1D mapping (momentum space)



Standard PDFs
typically accessible in inclusive DIS

1D mapping with spin: already a lot of fun



quark pol.

nucleon pol.

	U	L	T
U	f_1		
L		g_{1L}	
T			h_1

Twist-2 PDFs

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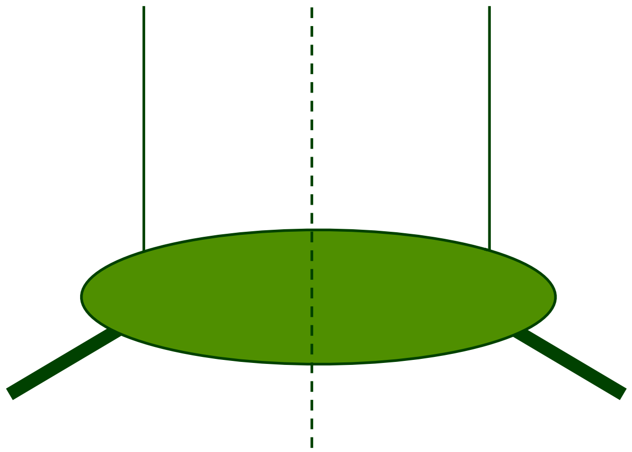

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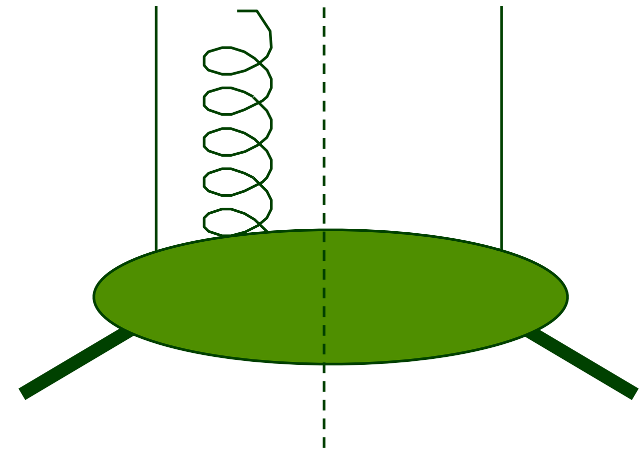
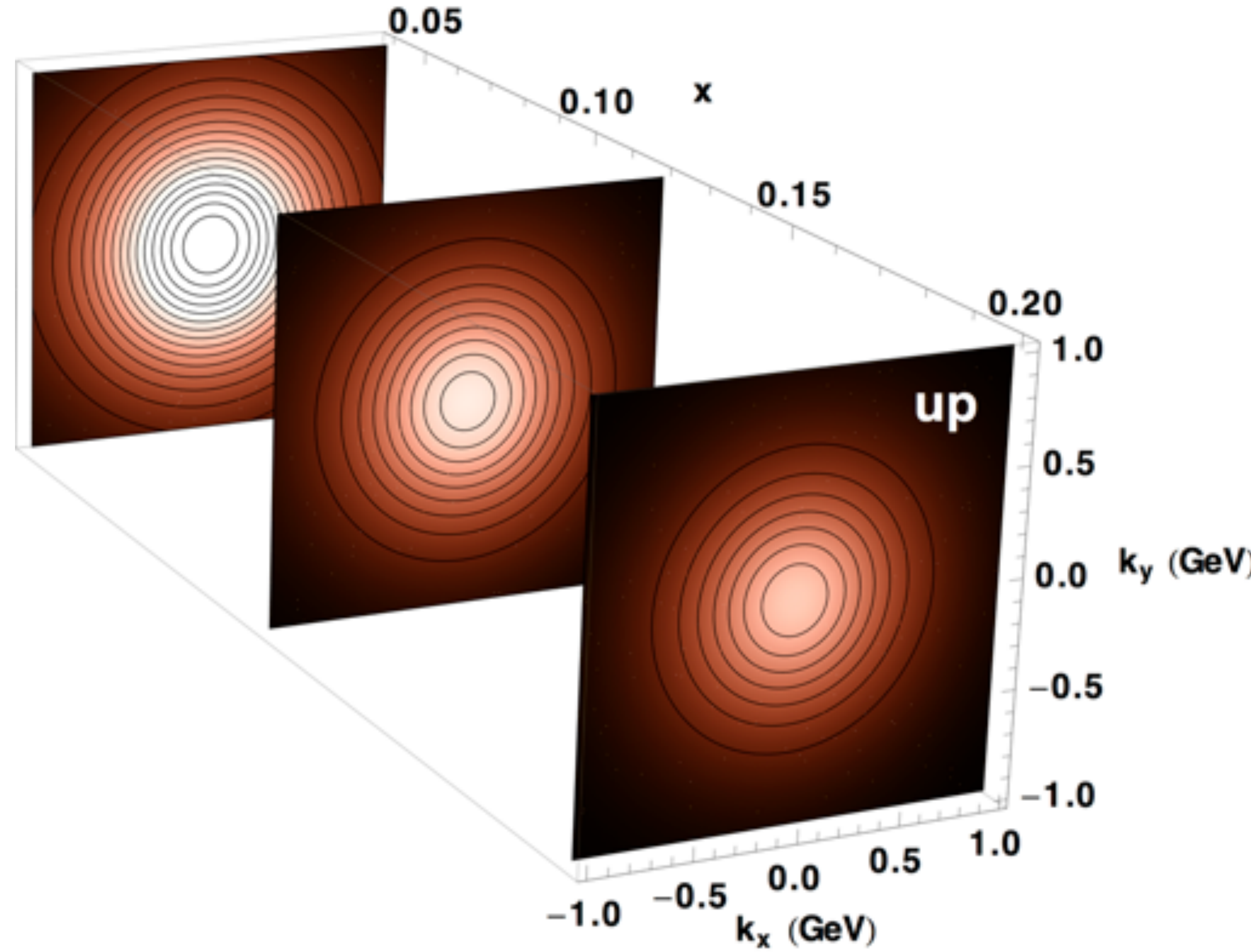


nucleon pol.

	U	L	T
U			e
L			h_L
T		g_T	

Twist-3 PDFs

3D mapping (momentum space)



Transverse-momentum distributions (TMDs) typically accessible in semi-inclusive DIS

3D mapping with spin

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

3D mapping with spin

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Twist-2 TMDs

Sivers TMD


HERMES truly pioneered
the field of 3D mapping

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(even though it was not foreseen in original plans!)

From the HERMES Technical Design Report (93)

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All “collinear” physics, no mentioning of 3D!

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Inclusive DIS

$$\frac{d\sigma}{dx\,dy\,d\phi_S} = \frac{2\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \epsilon\,F_{UU,L}$$

$$+ S_{\parallel}\lambda_e\sqrt{1-\varepsilon^2}\,F_{LL} + |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}$$

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naming according to AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

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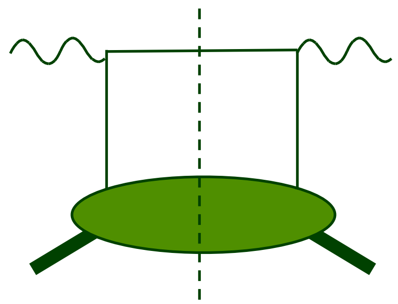
$$F_{UU,T} = 2x_B F_1,$$

$$F_{UU,L} = (1+\gamma^2)F_2 - 2x_B F_1,$$

$$F_{LL} = 2x_B\,\big(g_1 - \gamma^2 g_2\big),$$

$$F_{LT}^{\cos\phi_S} = -2x_B\gamma\,\big(g_1 + g_2\big)$$

Inclusive structure functions: parton model



$$F_{UU,T} = x \sum_a e_a^2 \, f_1^a(x),$$

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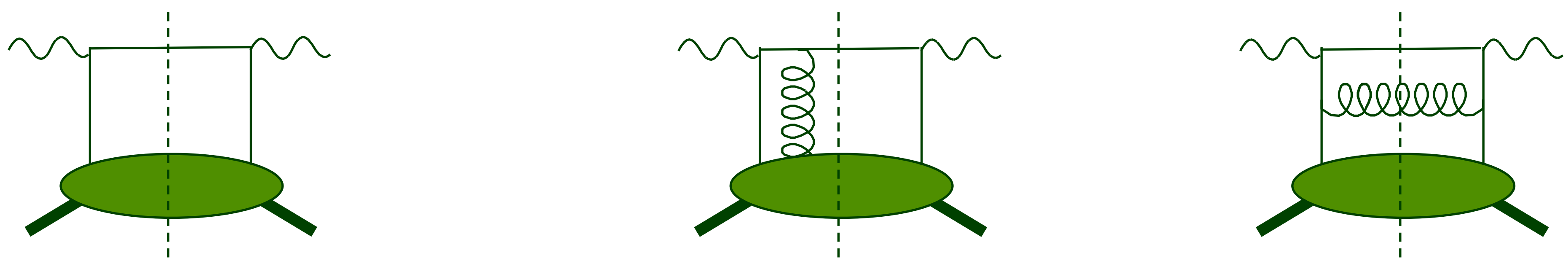
$$F_{LT}^{\cos\phi_S} = -x \sum_a e_a^2 \, \frac{2M}{Q} \, x g_T^a(x)$$ (in Wandzura-Wilczek approximation)

Inclusive structure functions: QCD corrections



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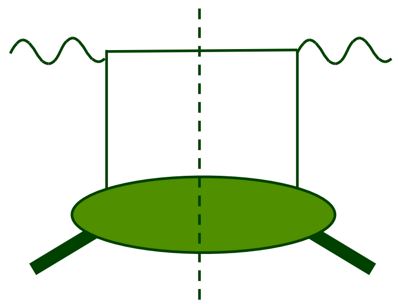

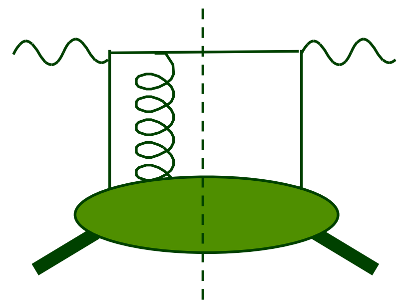

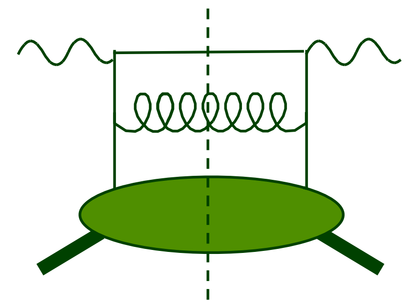

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evolution, gauge links, quark-gluon correlators...

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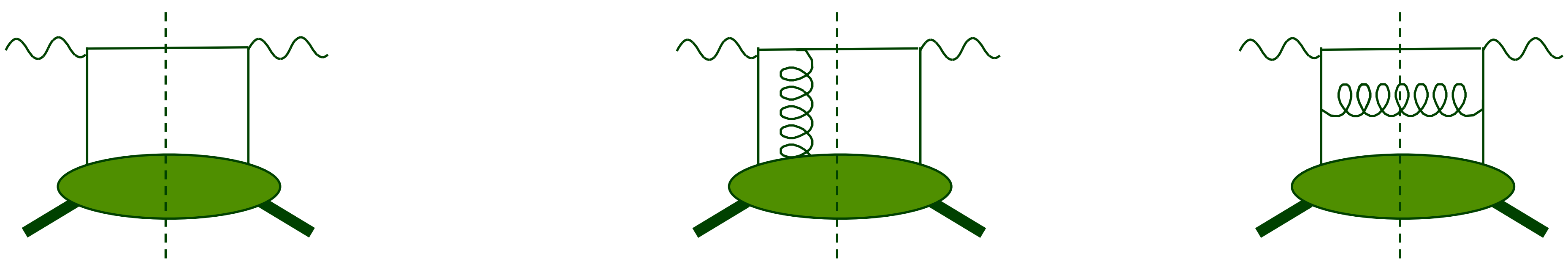

NLO

$$F_{UU,T} = x \sum_a e_a^2 \, f_1^a(x;Q^2), \qquad F_{UU,T} = x \sum_a e_a^2 \left\{ f_1^a(x;Q^2) + \frac{\alpha_S}{2\pi} \left(C^a \otimes f_1^a + C^g \otimes f_1^g \right) \right\},$$

$$F_{UU,L} = 0, \qquad F_{UU,L} = x \sum_a e_a^2 \, \frac{\alpha_S}{2\pi} \left(C_L^a \otimes f_1^a + C_L^g \otimes f_1^g \right),$$

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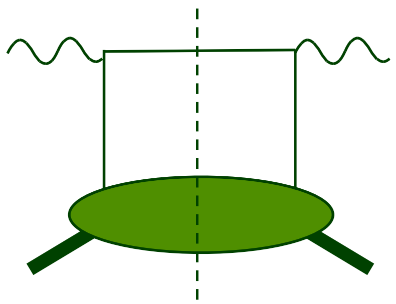
What has been measured?



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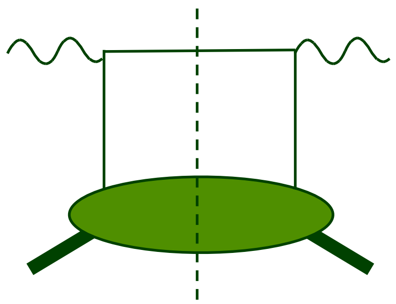

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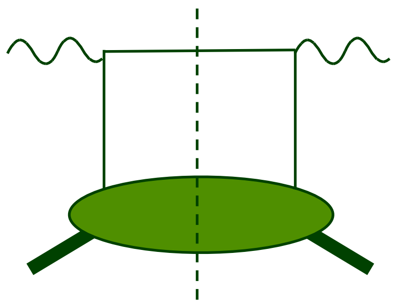

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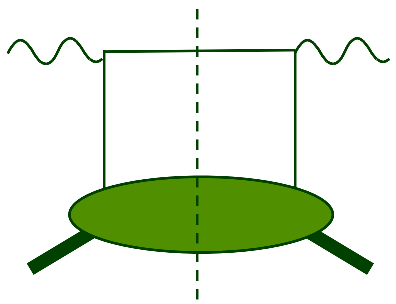

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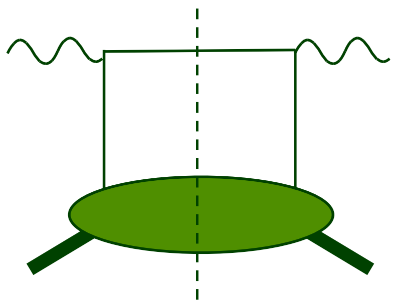

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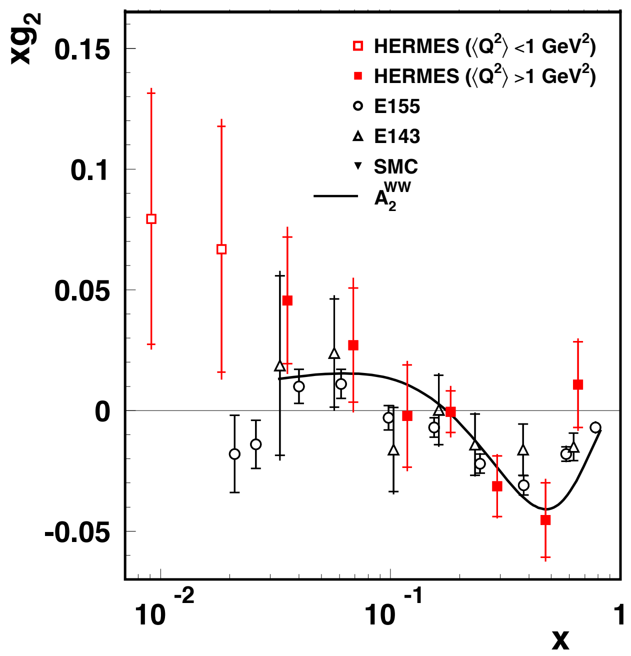
HERMES also looked for unexpected sinϕ_S modulation
PLB 682 (2010)

What can you get out?

Airapetian et al., EPJ C72 (2012)



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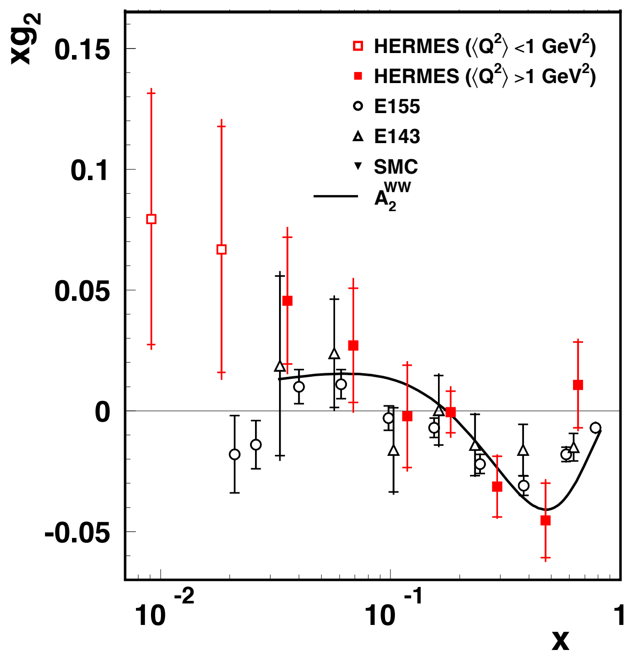

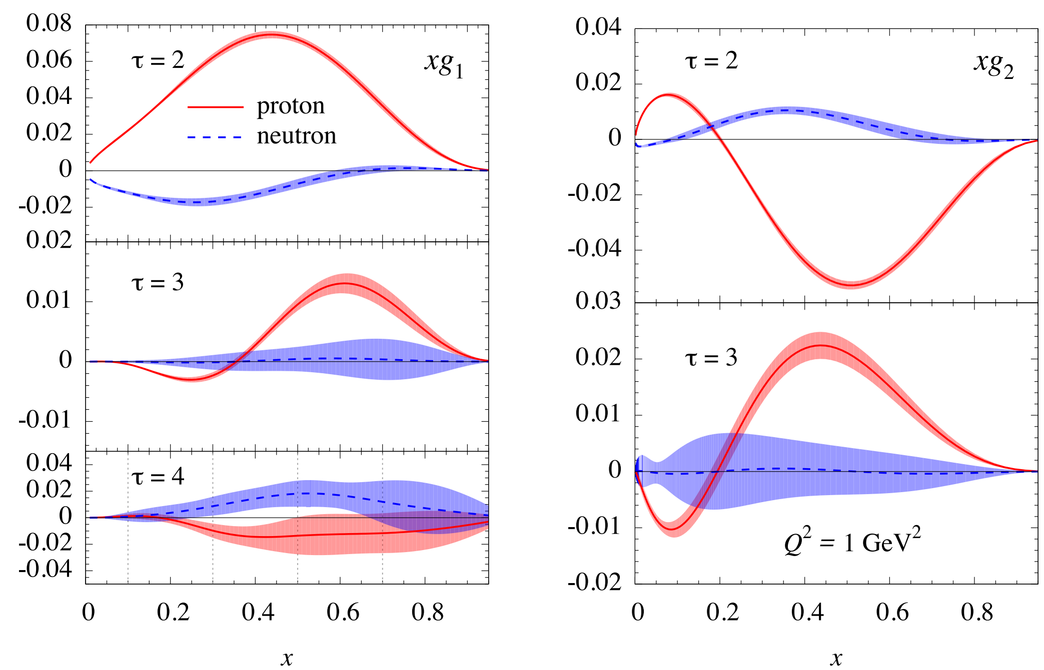

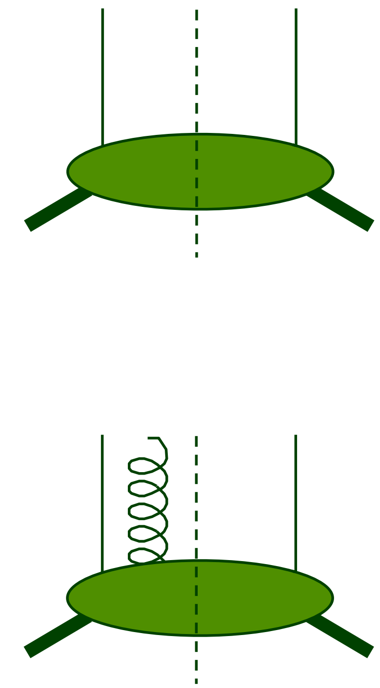

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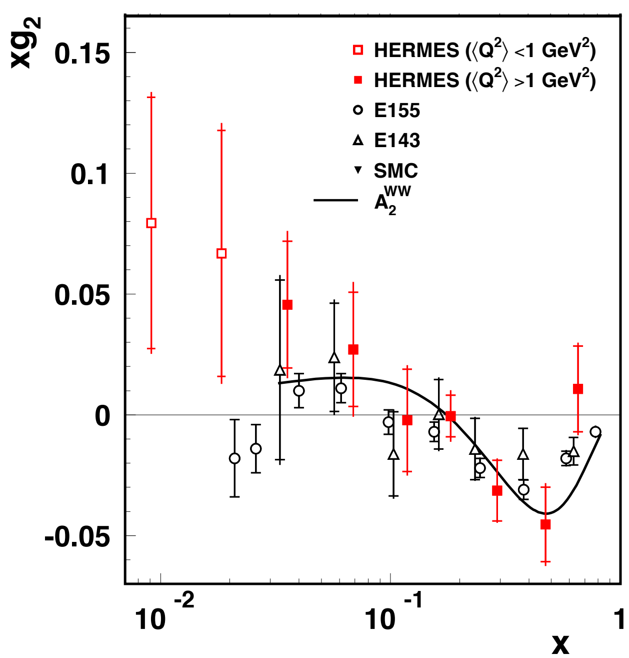

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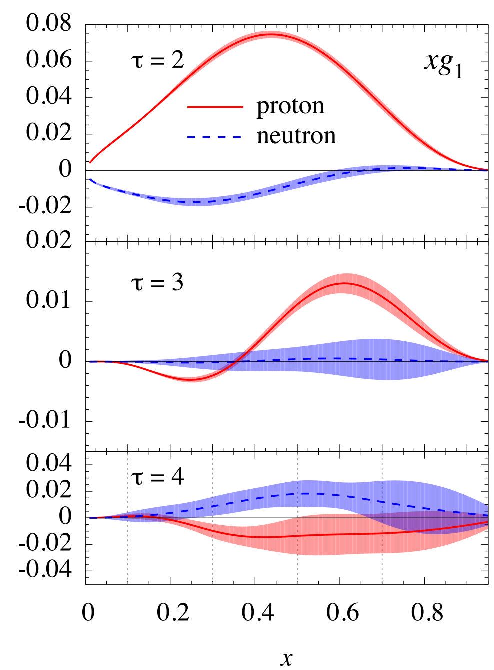
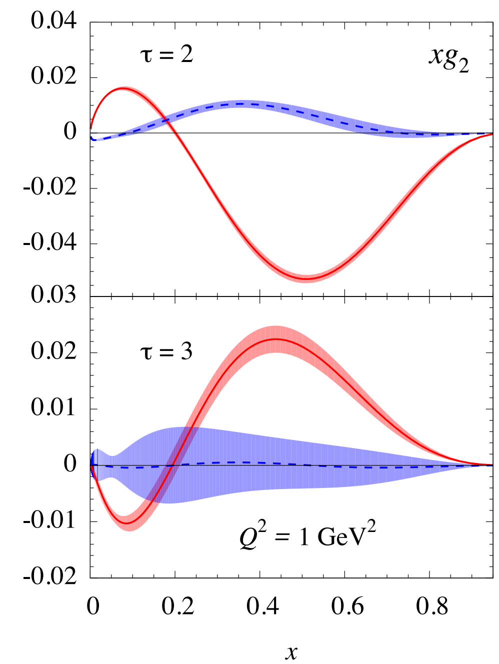
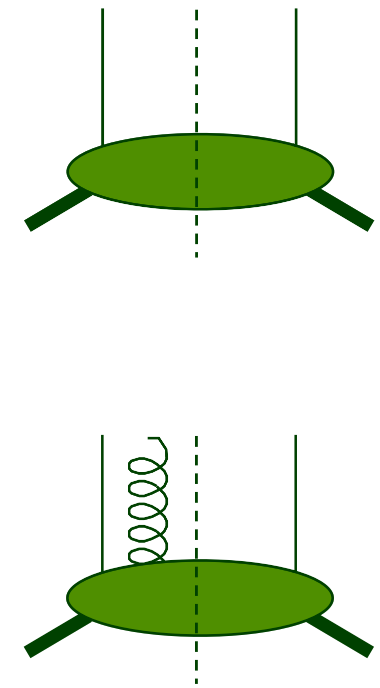




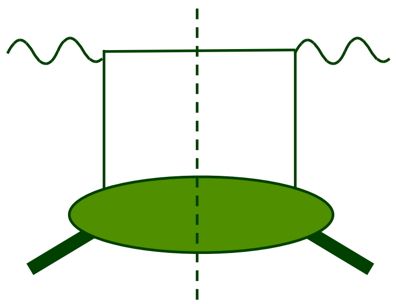


Involvement of quark-gluon correlations is challenging but stimulating

see, e.g., talk by L. Motyka

Higher precision for twist-2 PDF fits requires knowledge of twist-3

Is there still something to be done?



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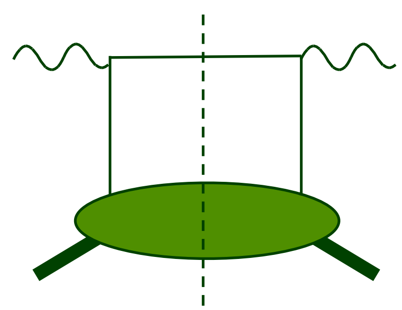

Maybe look for
parity violating structure functions?

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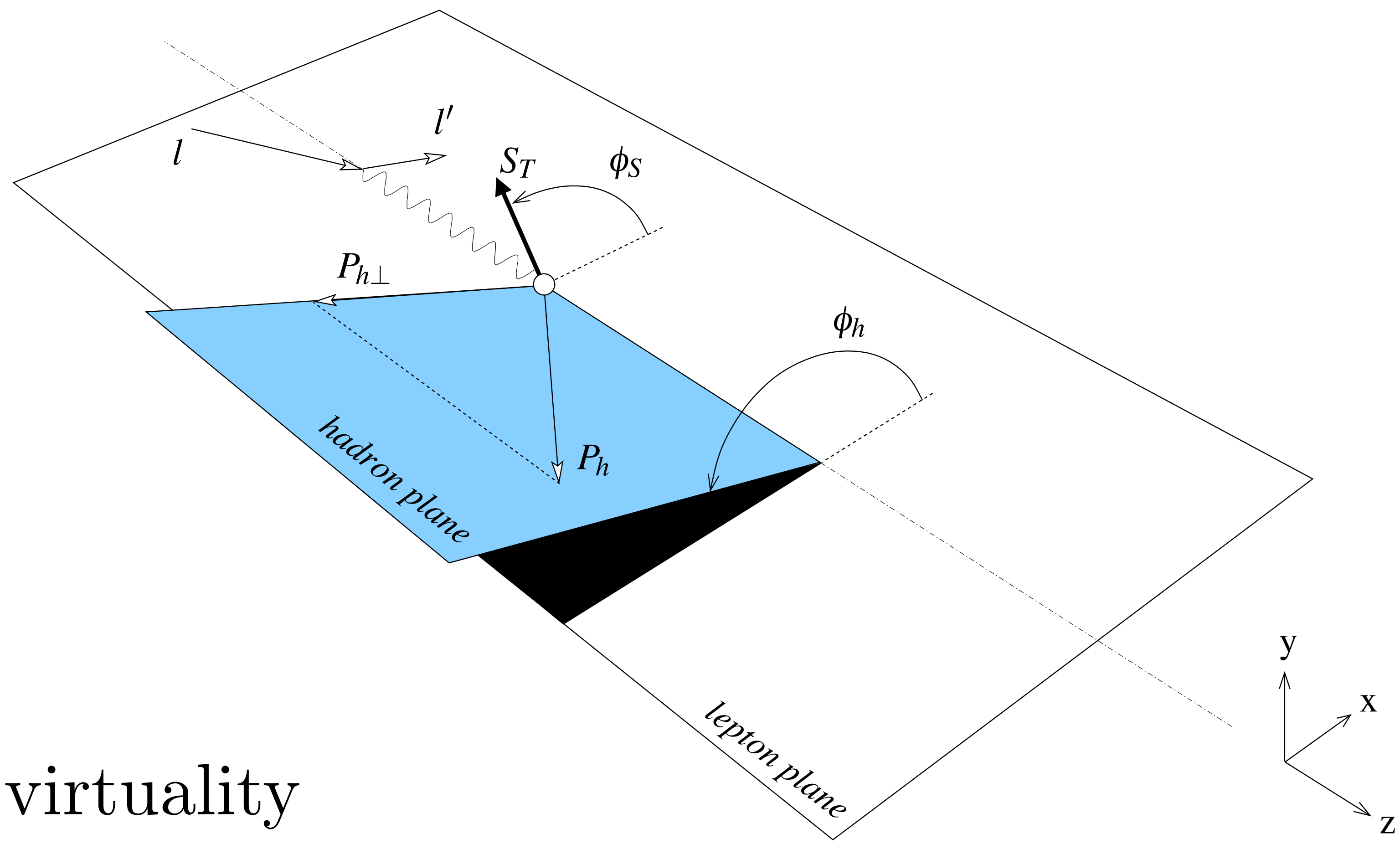
Semi-inclusive DIS

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$

$$x = \frac{Q^2}{2\,P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z = \frac{P\cdot P_h}{P\cdot q}.$$



Q = photon virtuality

Λ_{QCD} = hadronic scale

$P_{h\perp}$ = hadron transverse momentum

Integrated over transverse momentum

$$\frac{d\sigma}{dx\,dy\,d\phi_S dz} = \frac{2\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \epsilon\,F_{UU,L} + S_\parallel \lambda_e\,\sqrt{1-\varepsilon^2}\;F_{LL}$$

$$+\,|\boldsymbol{S}_\perp|\,\sqrt{2\,\varepsilon(1+\varepsilon)}\;\sin\phi_S\;F_{UT}^{\sin\phi_S} + |\boldsymbol{S}_\perp|\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\;\cos\phi_S\;F_{LT}^{\cos\phi_S}\Big\}$$

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$F_{UU,T}(x, z, Q^2)$

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$$F_{UU,L} = 0,$$

$$F_{LL} = x\sum_a e_a^2\,g_1^a(x)\,D_1^a(z),$$

$$F_{UT}^{\sin\phi_S} = -x\sum_a e_a^2\,\frac{2M_h}{Q}\,h_1^a(x)\,\frac{\tilde{H}^a(z)}{z},$$

$$F_{LT}^{\cos\phi_S} = -x\sum_a e_a^2\,\frac{2M}{Q}\left(xg_T^a(x)D_1^a(z) + \frac{M_h}{M}\,h_1^a(x)\,\frac{\tilde{E}^a(z)}{z}\right)$$

Integrated over transverse momentum

$F_{UU,T}(x, z, Q^2)$

$$\frac{d\sigma}{dx\,dy\,d\phi_S dz} = \frac{2\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \epsilon\,F_{UU,L} + S_\parallel \lambda_e\,\sqrt{1-\varepsilon^2}\;F_{LL}$$
$$+\,|\boldsymbol{S}_\perp|\,\sqrt{2\,\varepsilon(1+\varepsilon)}\;\sin\phi_S\,F_{UT}^{\sin\phi_S} + |\boldsymbol{S}_\perp|\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\;\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}$$

$$F_{UU,T} = x\sum_a e_a^2\, f_1^a(x)\, D_1^a(z),$$

measured by HERMES

$$F_{UU,L} = 0,$$

$$F_{LL} = x\sum_a e_a^2\, g_1^a(x)\, D_1^a(z),$$

measured by HERMES

$$F_{UT}^{\sin\phi_S} = -x\sum_a e_a^2\,\frac{2M_h}{Q}\,h_1^a(x)\,\frac{\tilde{H}^a(z)}{z},$$

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What can still be measured?

$$\frac{d\sigma}{dx\,dy\,d\phi_S dz} = \frac{2\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \epsilon\,F_{UU,L} + S_\parallel \lambda_e\,\sqrt{1-\varepsilon^2}\;F_{LL}$$
$$+\,|\boldsymbol{S}_\perp|\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + |\boldsymbol{S}_\perp|\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}$$

$$F_{UU,T} = x\sum_a e_a^2\,f_1^a(x)\,D_1^a(z),$$

Still some data to be analyzed (2006/07)
Possibly also π^0, η, p

$$F_{UU,L} = 0,$$

missing (out of HERMES's reach)

$$F_{LL} = x\sum_a e_a^2\,g_1^a(x)\,D_1^a(z),$$

Possibly also π^0, η, p

$$F_{UT}^{\sin\phi_S} = -x\sum_a e_a^2\,\frac{2M_h}{Q}\,h_1^a(x)\,\frac{\tilde{H}^a(z)}{z},$$

missing
related to pp single-spin asymmetries

$$F_{LT}^{\cos\phi_S} = -x\sum_a e_a^2\,\frac{2M}{Q}\left(xg_T^a(x)D_1^a(z) + \frac{M_h}{M}\,h_1^a(x)\,\frac{\tilde{E}^a(z)}{z}\right)$$

missing
included in '93 tech rep!

Not integrated over transverse momentum

$$
\begin{aligned}
&\frac{d\sigma}{dx\, dy\, d\phi_S\, dz\, d\phi_h\, dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\, y\, Q^2}\, \frac{y^2}{2\,(1-\varepsilon)} \Bigg\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} + S_L \Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \Big] \\
&\quad + S_L\, \lambda_e \Big[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \Big] \\
&\quad + S_T \Big[\sin(\phi_h-\phi_S) \Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \Big) + \varepsilon\, \sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\, \sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Big] + S_T \lambda_e \Big[\sqrt{1-\varepsilon^2}\, \cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Big] \Bigg\}
\end{aligned}
$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Not integrated over transverse momentum

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$
\begin{aligned}
&\frac{d\sigma}{dx\, dy\, d\phi_S\, dz\, d\phi_h\, dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\, y\, Q^2} \frac{y^2}{2\,(1-\varepsilon)} \Bigg\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} + S_L \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \right] \\
&\quad + S_L\, \lambda_e \left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \right] \\
&\quad + S_T \Bigg[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Bigg] + S_T \lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Bigg] \Bigg\}
\end{aligned}
$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Not integrated over transverse momentum

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}$ [redacted] $\Big]$

$+ S_L\,\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon}$ [redacted]

$+ S_T\Big[\,\mathrm{si}$ [redacted] $\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$

DON'T PANIC!

$+ \varepsilon\,\mathrm{si}$ [redacted] $\phi_h - \phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] + S_T\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\}$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Unpolarized sector

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$
$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.$$

Unpolarized sector

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

HERMES did almost everything it could

Can still analyze π^0, η and some data from 2006/07

HERMES, PRD 87 (2013) 012010

HERMES, PRD 87 (2013) 074029

Unpolarized sector

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$
$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.$$

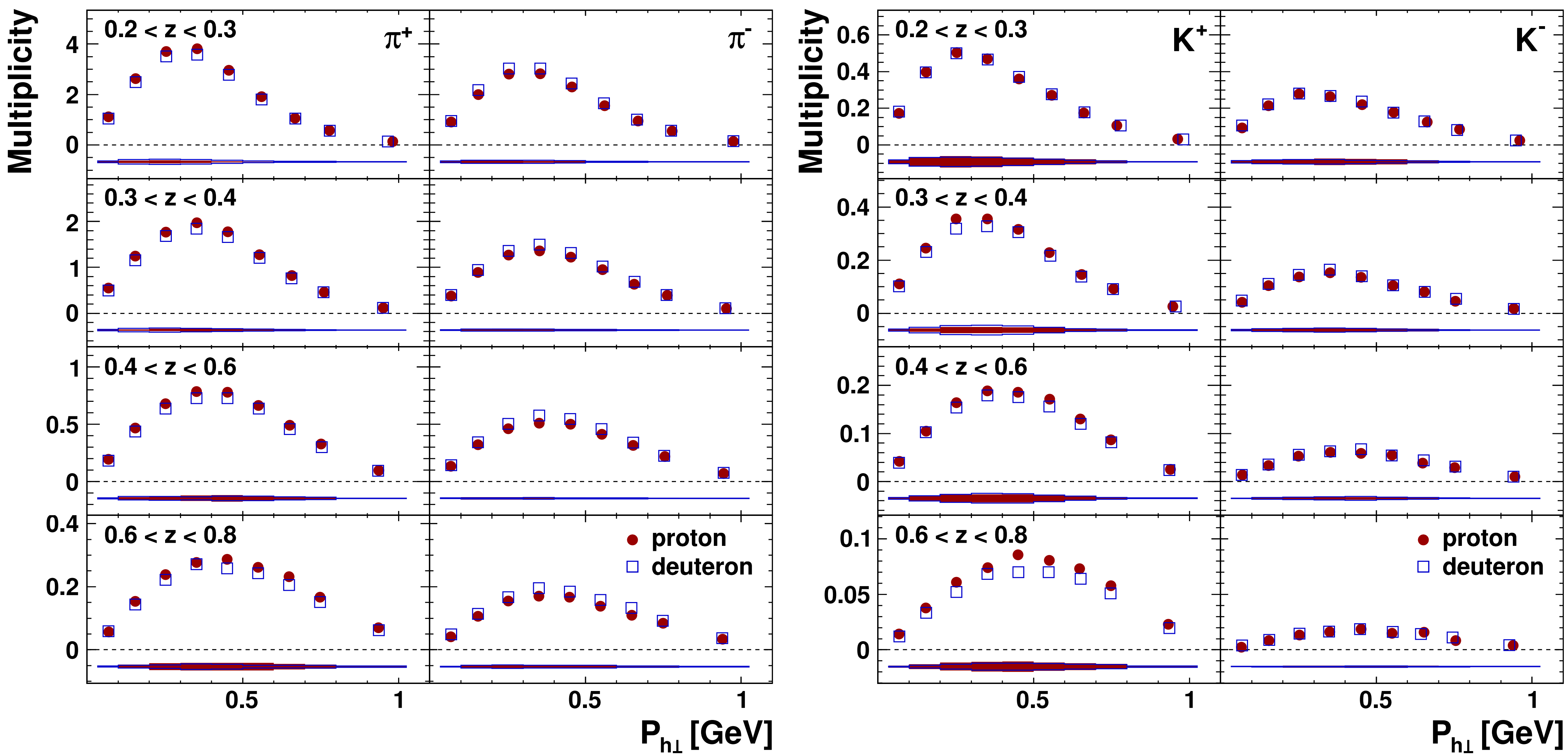
HERMES did almost everything it could
Can still analyze π^0, η and some data from 2006/07

HERMES, PRD 87 (2013) 012010
HERMES, PRD 87 (2013) 074029

All four structure functions could be measured by H1 and ZEUS, with their four-dimensional dependence

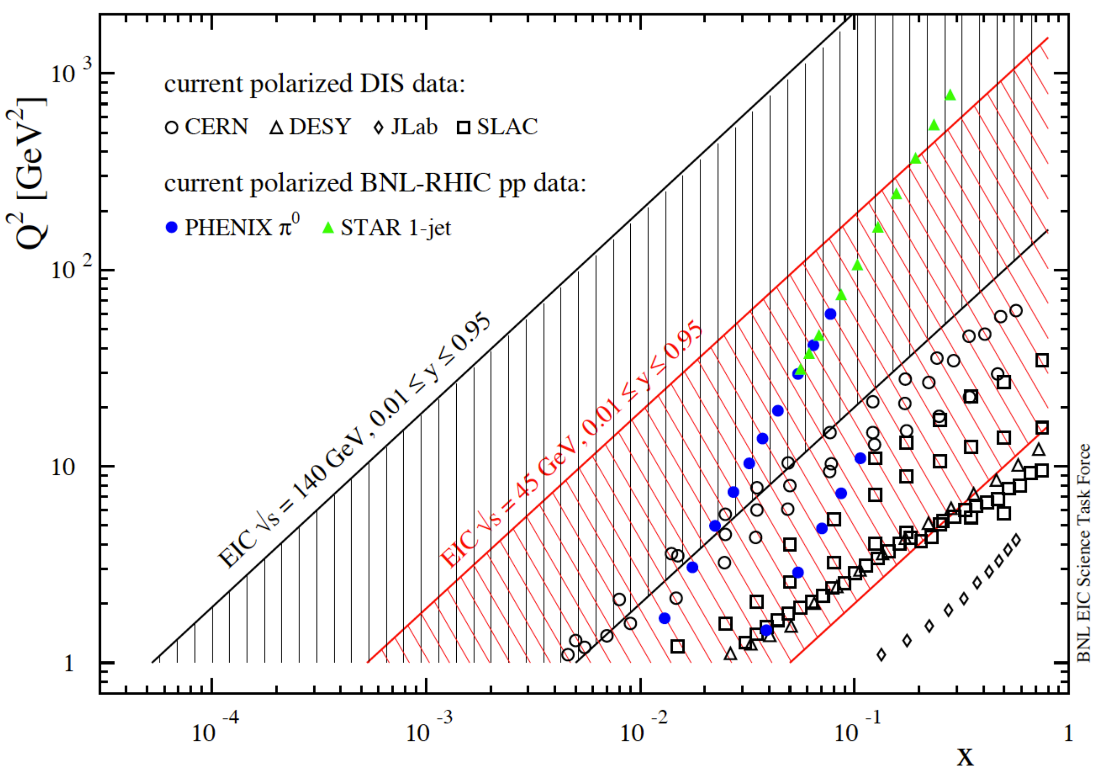
e.g., H1, EPJ C73 (13)
ZEUS, PLB 481 (00)

Transverse-momentum-dependent multiplicities



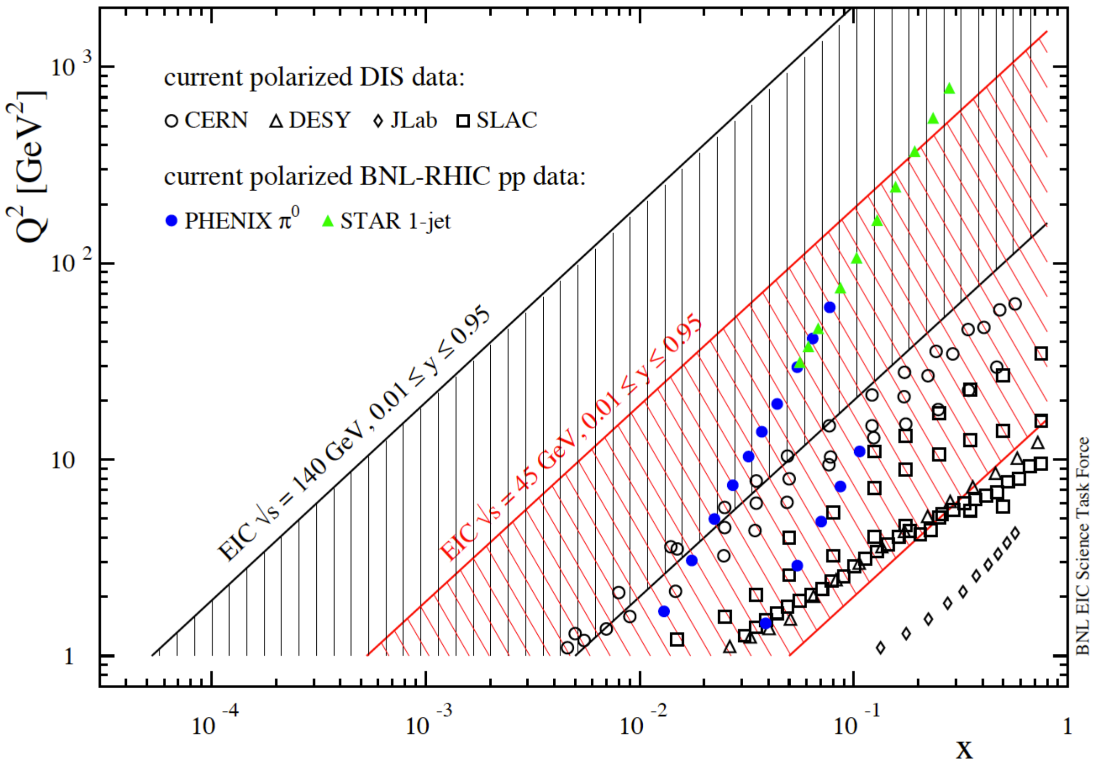
Airapetian et al., PRD 87 (2013)

x-Q^2 coverage

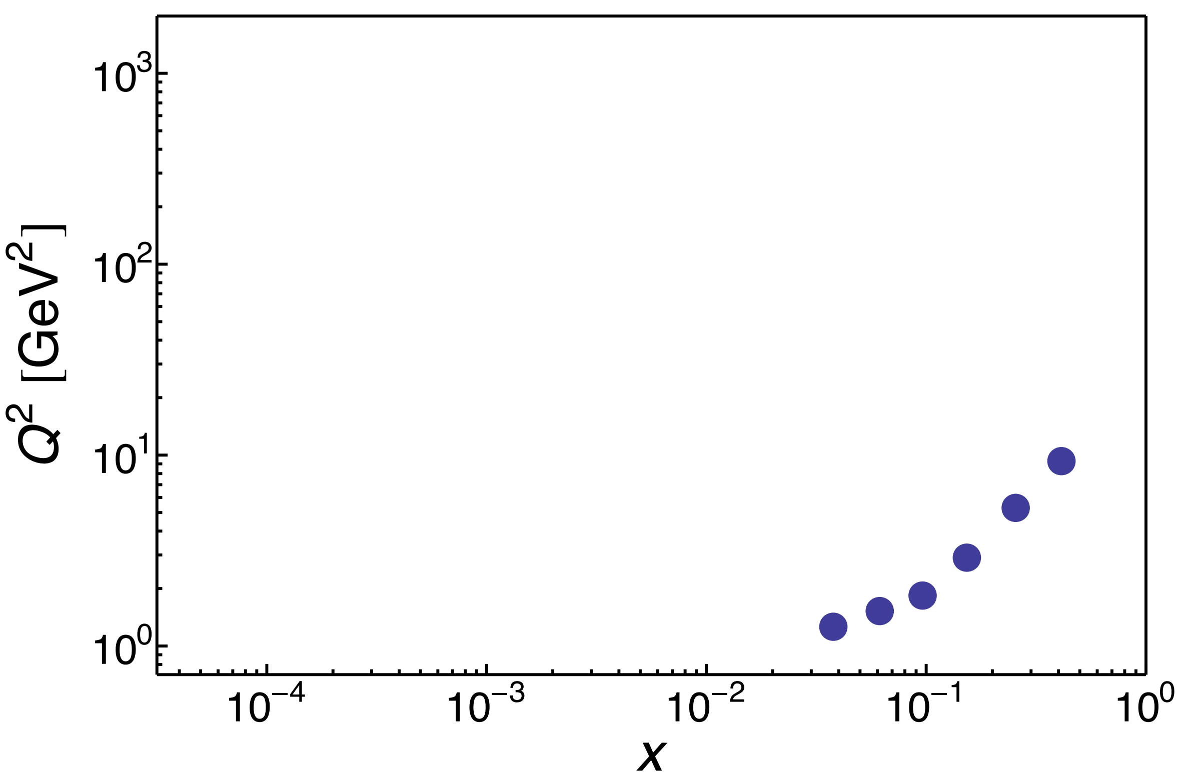


Coverage of polarized collinear measurements

x-Q^2 coverage

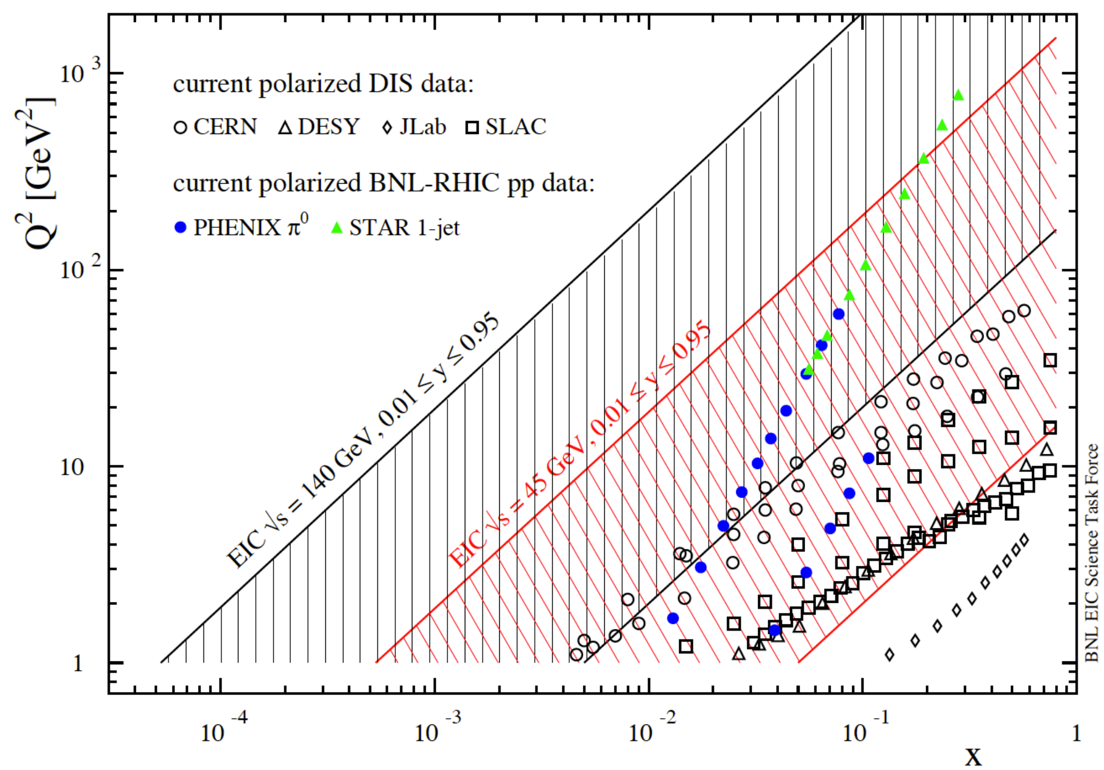


Coverage of polarized collinear measurements

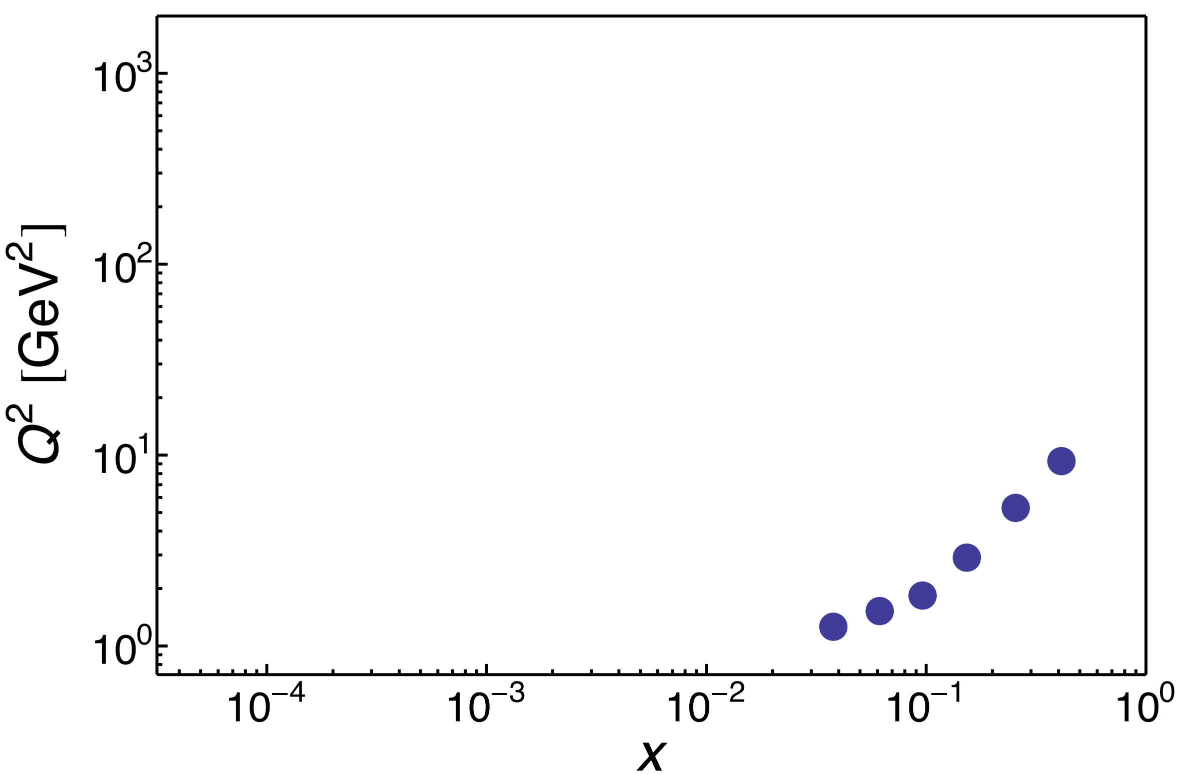


Coverage of HERMES transverse-momentum dependent measurements.
The coverage is limited, x-Q^2 are correlated, but:

x-Q^2 coverage



Coverage of polarized collinear measurements



Coverage of HERMES transverse-momentum dependent measurements.
The coverage is limited, x-Q^2 are correlated, but:

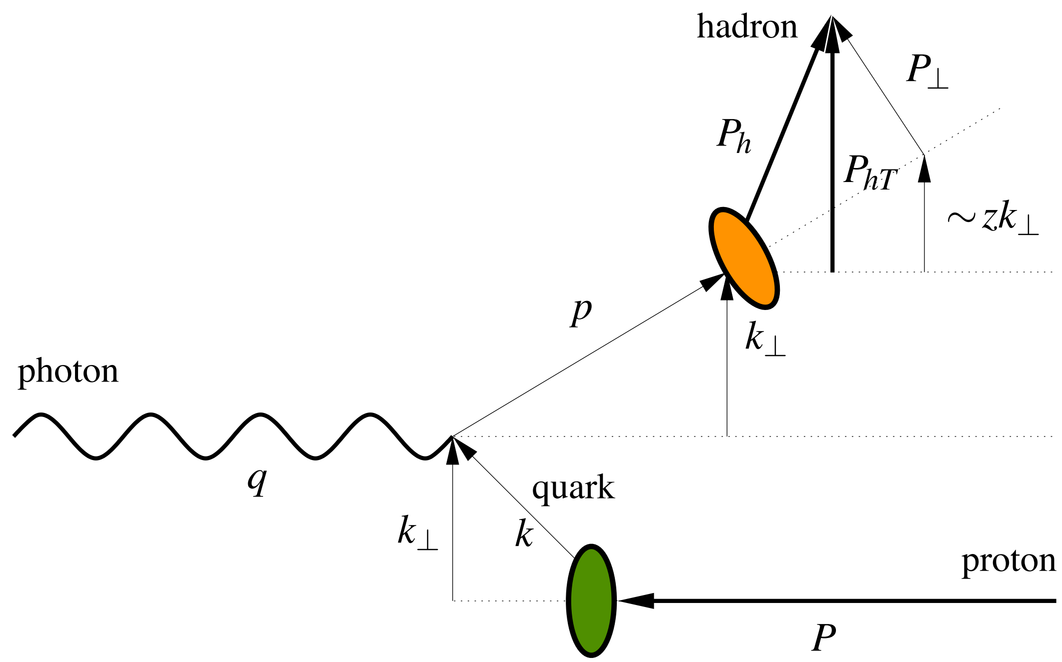
6 bins in x, 8 bins in z,

7 bins in P_{hT},

2 targets, 4 final-state hadrons,

= 2688 data points

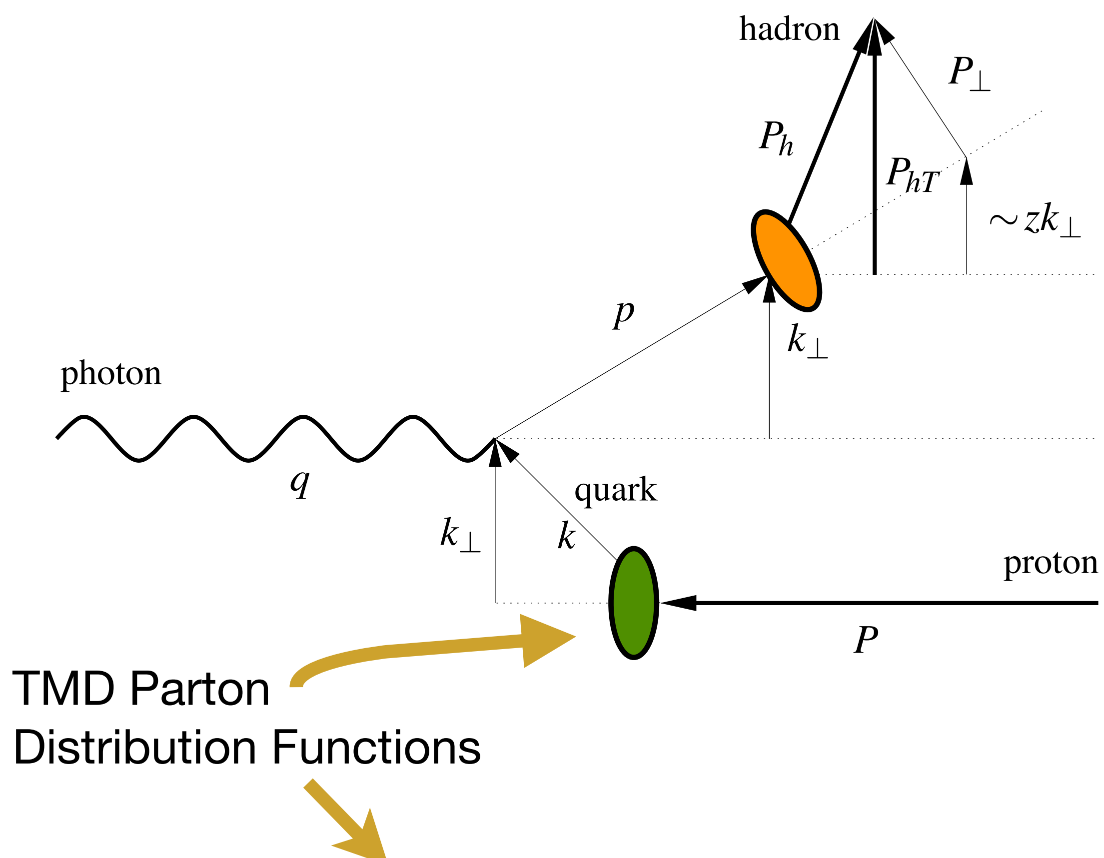
What can you get out?



"Parton model"

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = \sum_a \int d\boldsymbol{k}_\perp \, d\boldsymbol{P}_\perp \, f_1^a\big(x,\boldsymbol{k}_\perp^2\big)\, D_1^{a\to h}\big(z,\boldsymbol{P}_\perp^2\big)\, \delta\big(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp\big) + \mathcal{O}\big(M^2/Q^2\big)$$

What can you get out?



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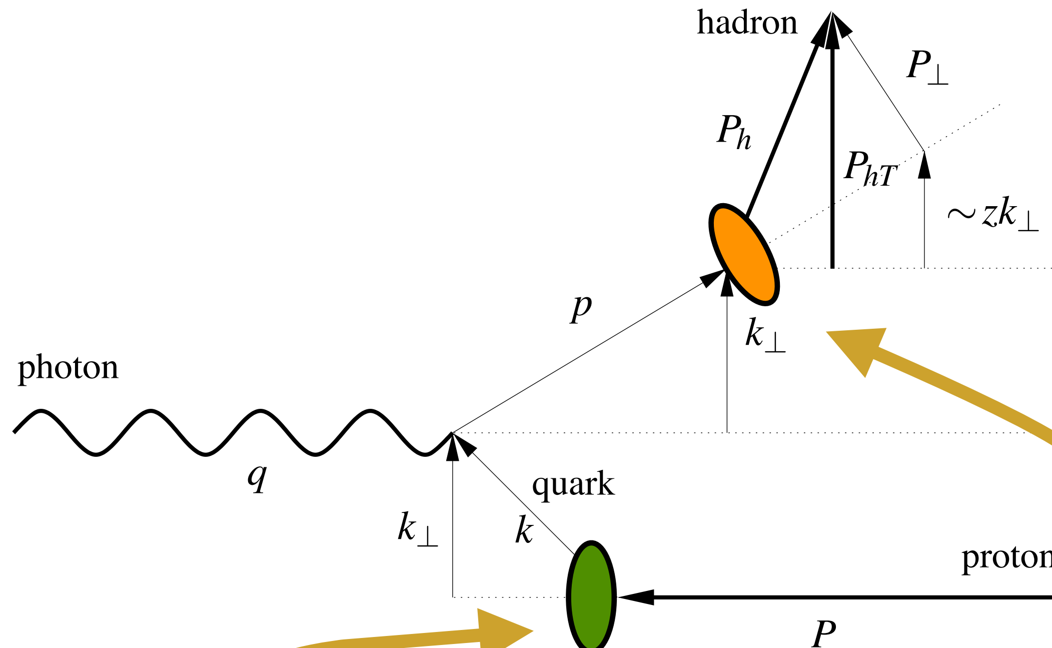


TMD Parton
Distribution Functions

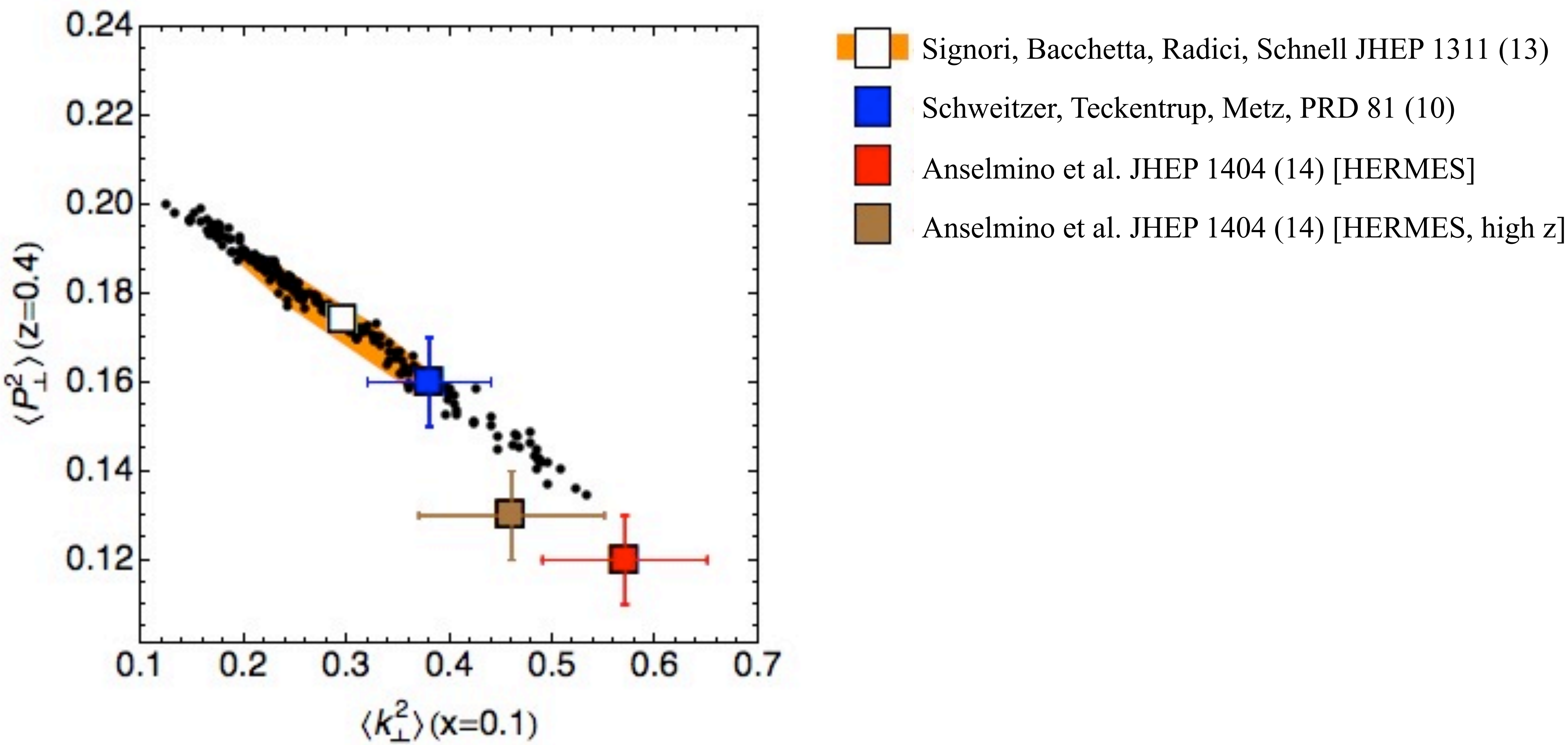
TMD Parton
Fragmentation Functions

"Parton model"

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2) = \sum_a \int d\boldsymbol{k}_\perp \, d\boldsymbol{P}_\perp \, f_1^a\big(x, \boldsymbol{k}_\perp^2\big) \, D_1^{a\to h}\big(z, \boldsymbol{P}_\perp^2\big) \, \delta\big(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp\big) + \mathcal{O}\big(M^2/Q^2\big)$$

Outcome of extraction of unpolarized TMDs

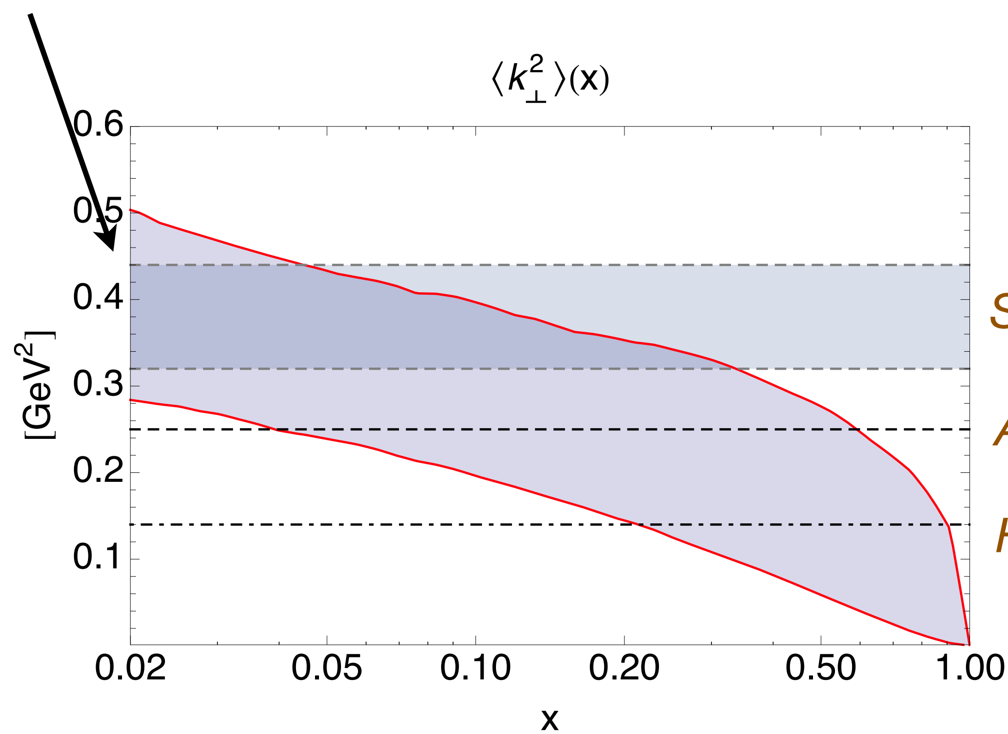
Transverse momentum in FFs



Transverse momentum in PDFs

Behavior with x

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)



Schweitzer, Teckentrup, Metz, PRD 81 (2010)

Anselmino et al., PRD 71 (2005)

HERMES gmc_trans

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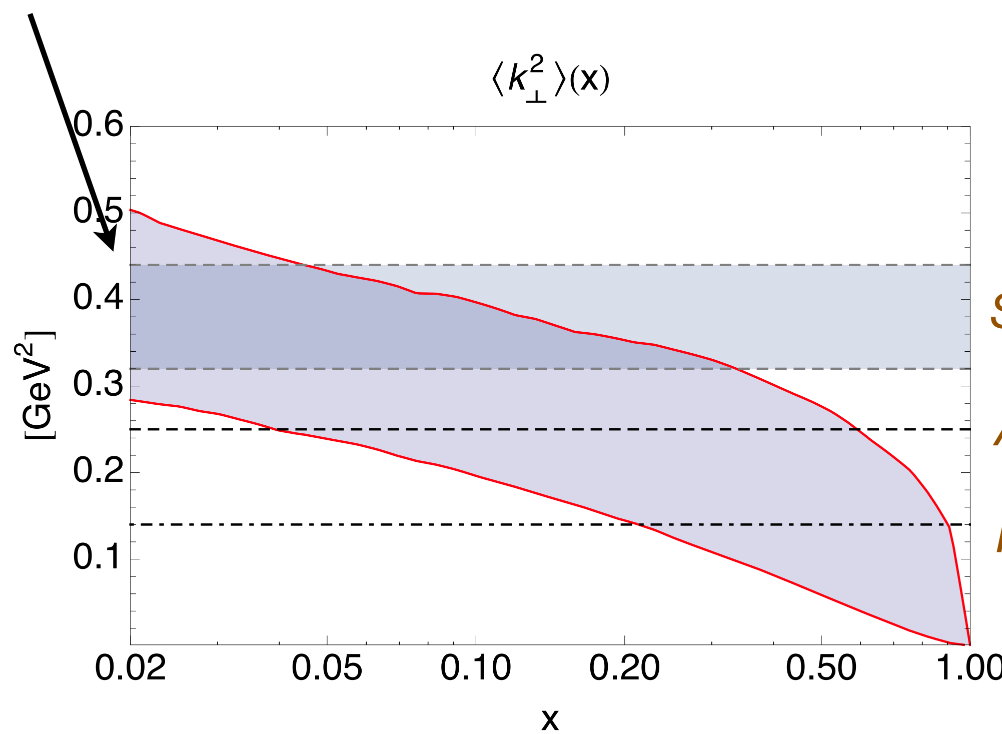

Schweitzer, Teckentrup, Metz, PRD 81 (2010)

Anselmino et al., PRD 71 (2005)

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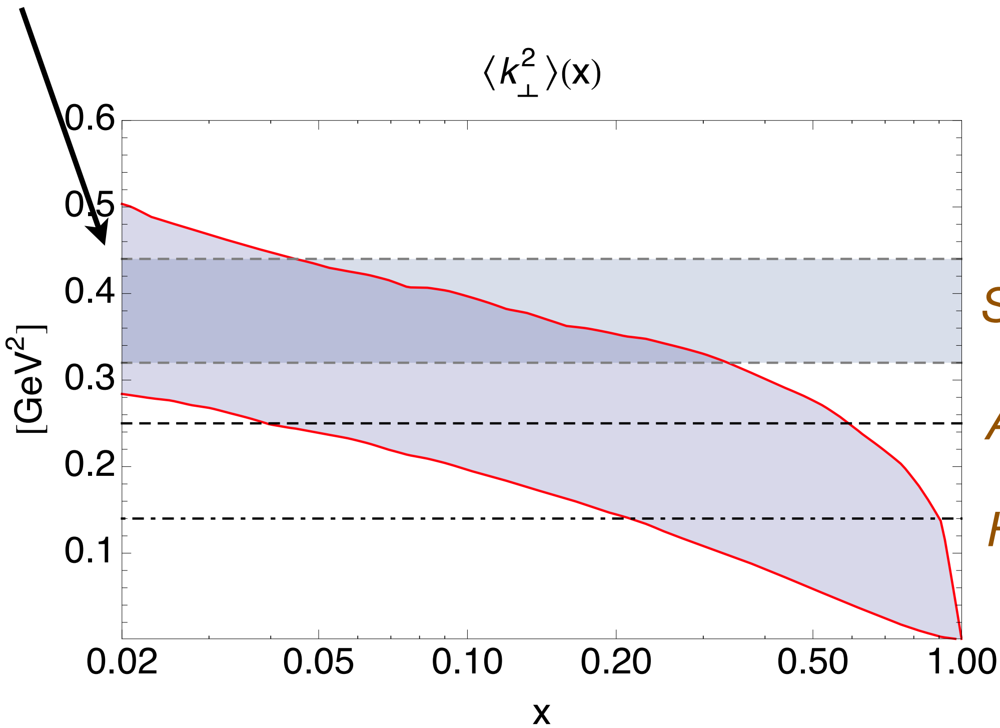
Still difficult to say, but possibly a widening at lower x

Behavior with x

Signori, Bacchetta, Radici, Schnell JHEP 1311 (13)



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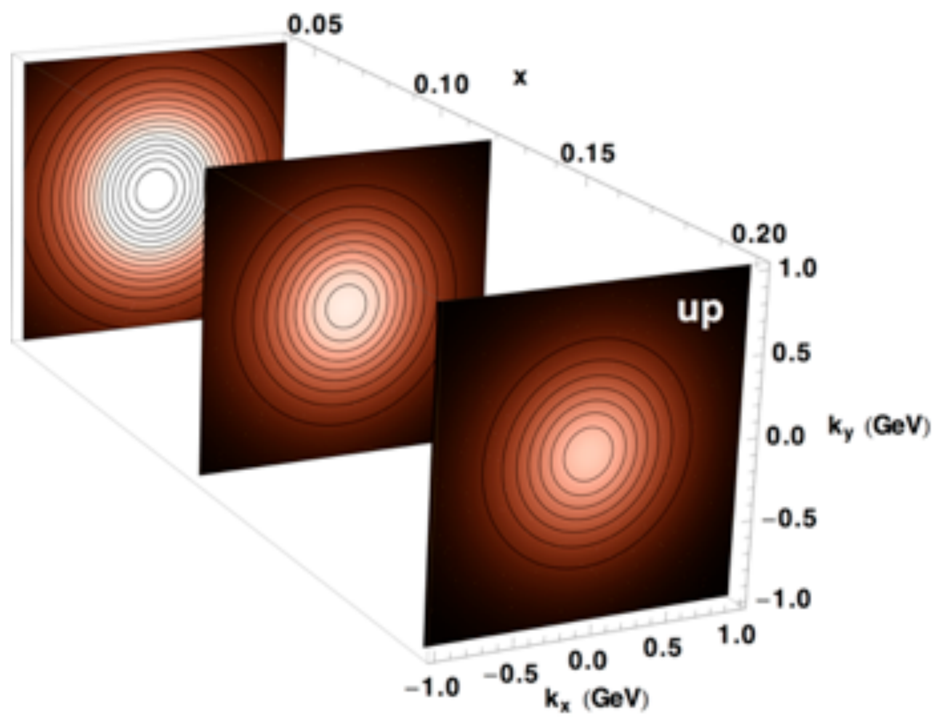
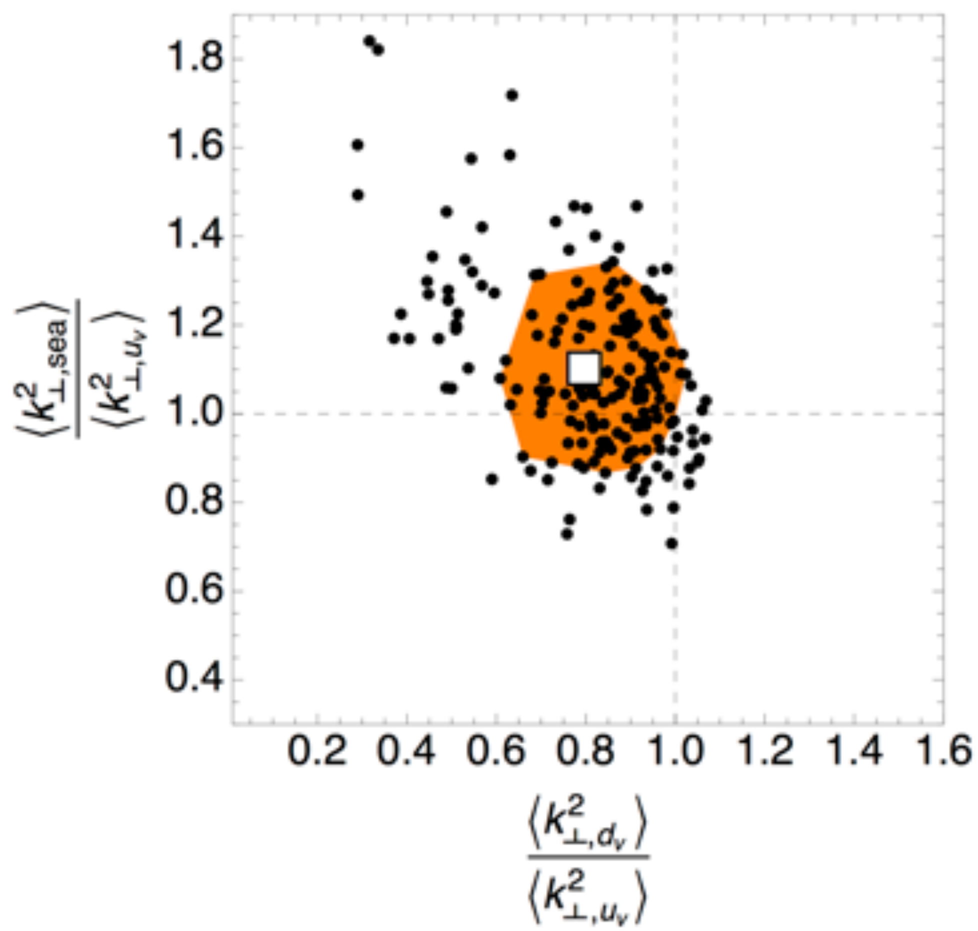


Flavor dependence of unpolarized TMDs

Ratio of width of sea / width of up valence



Ratio width of down valence/
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Flavor dependence of unpolarized TMDs

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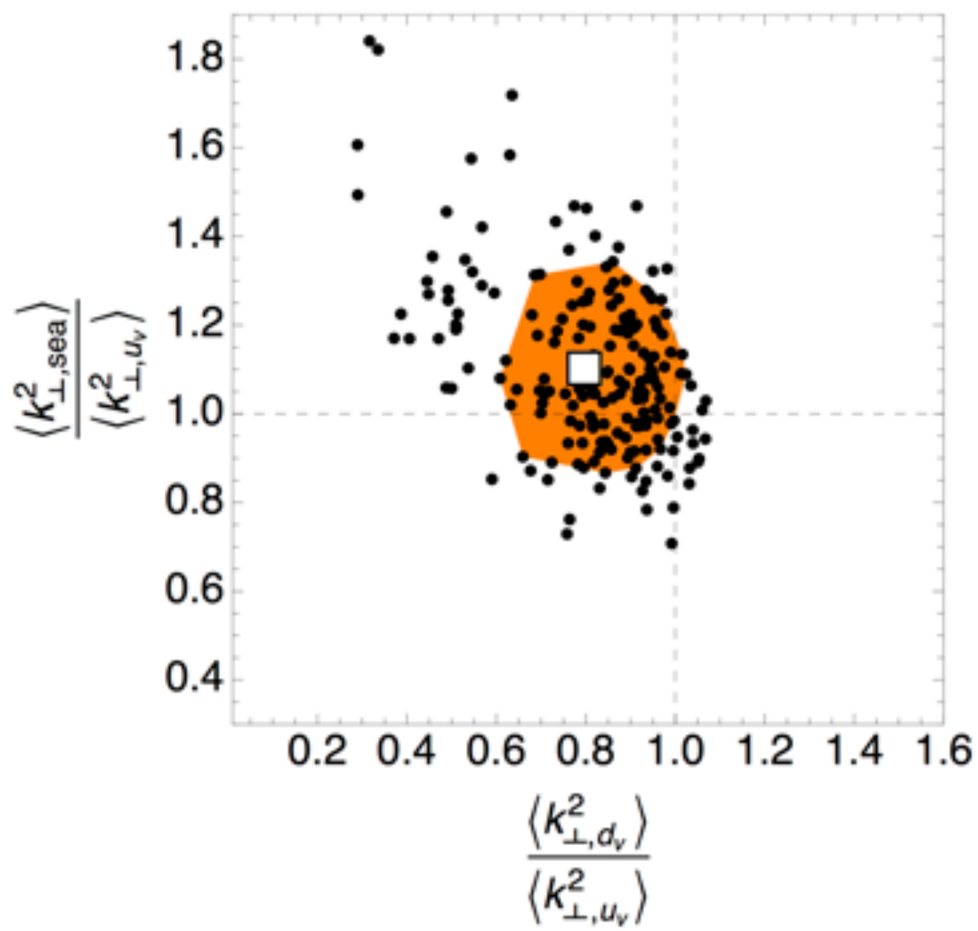

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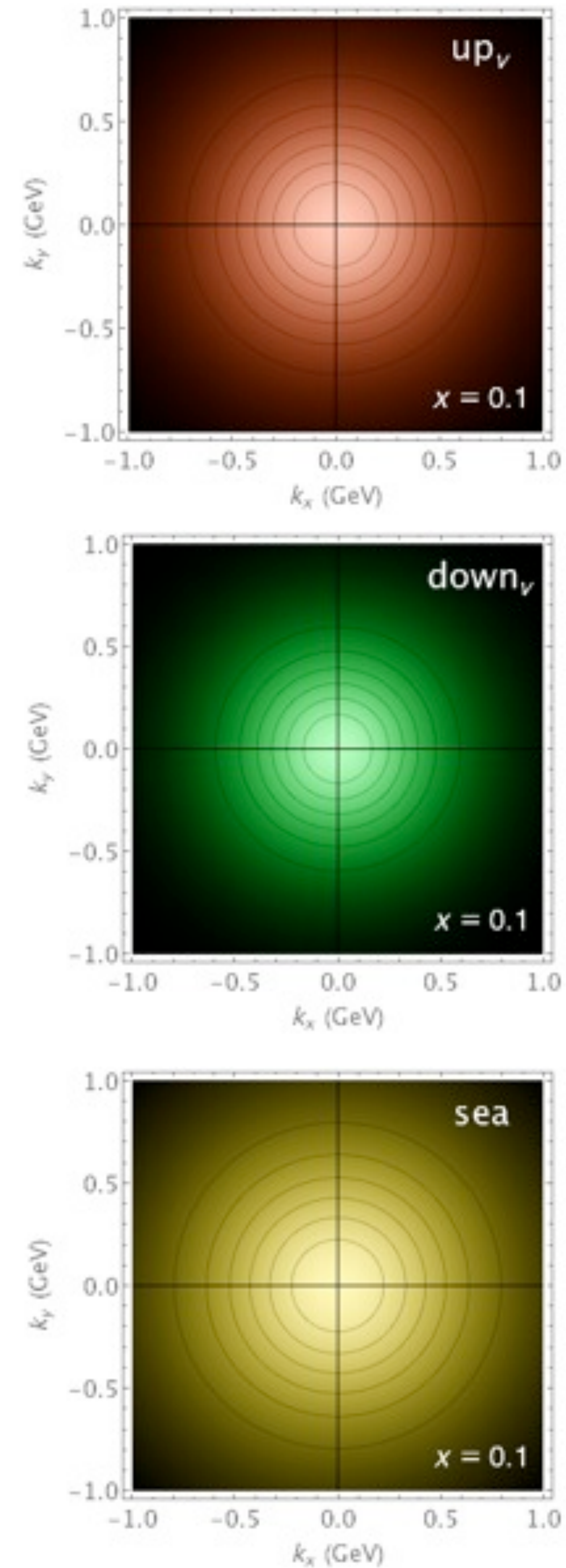

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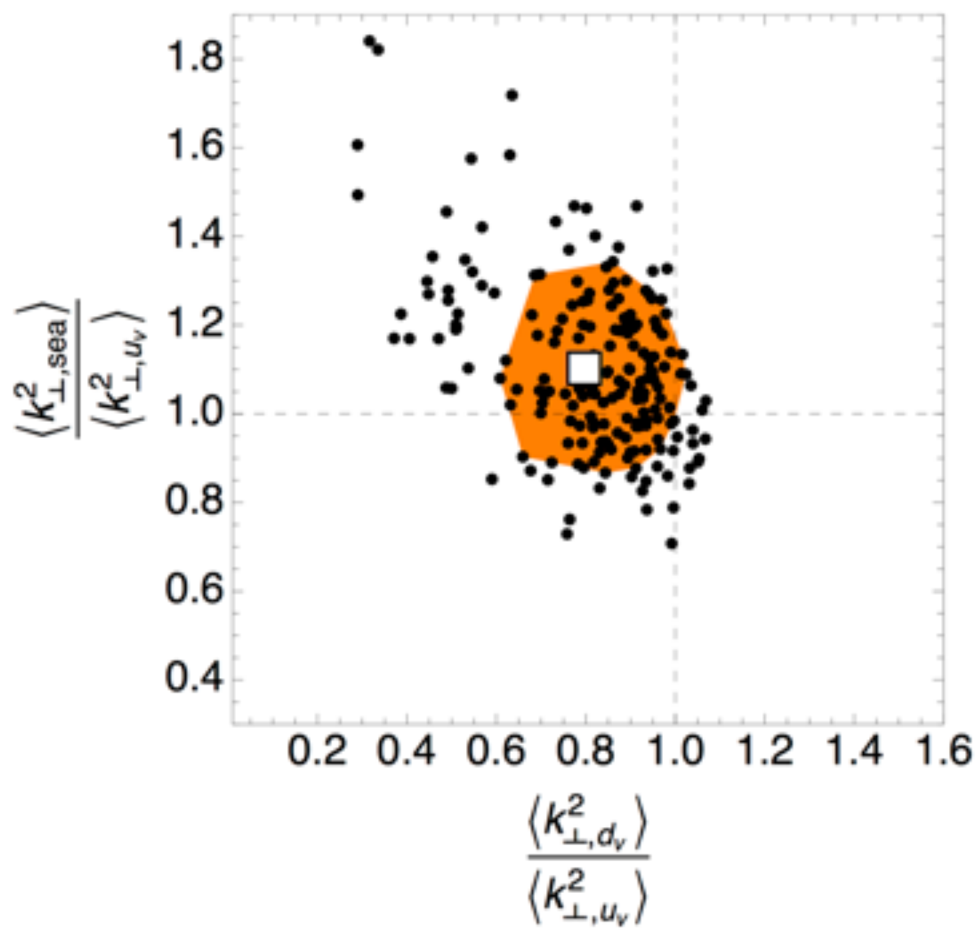

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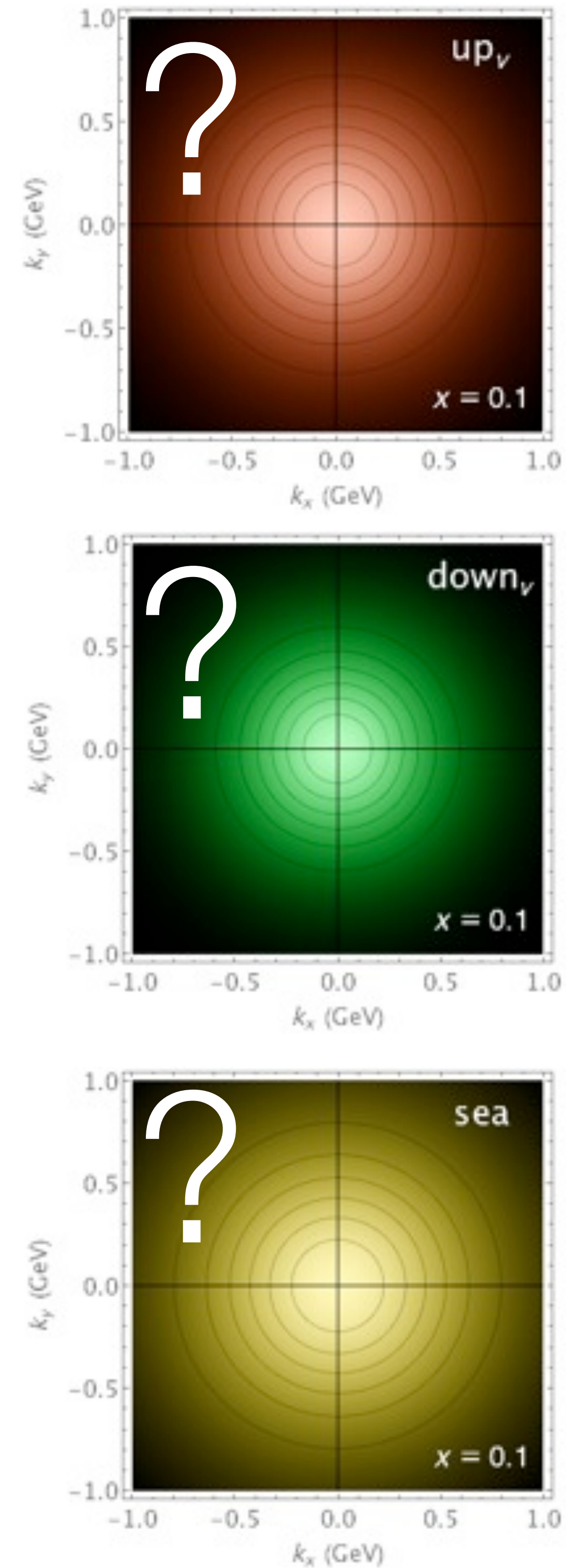
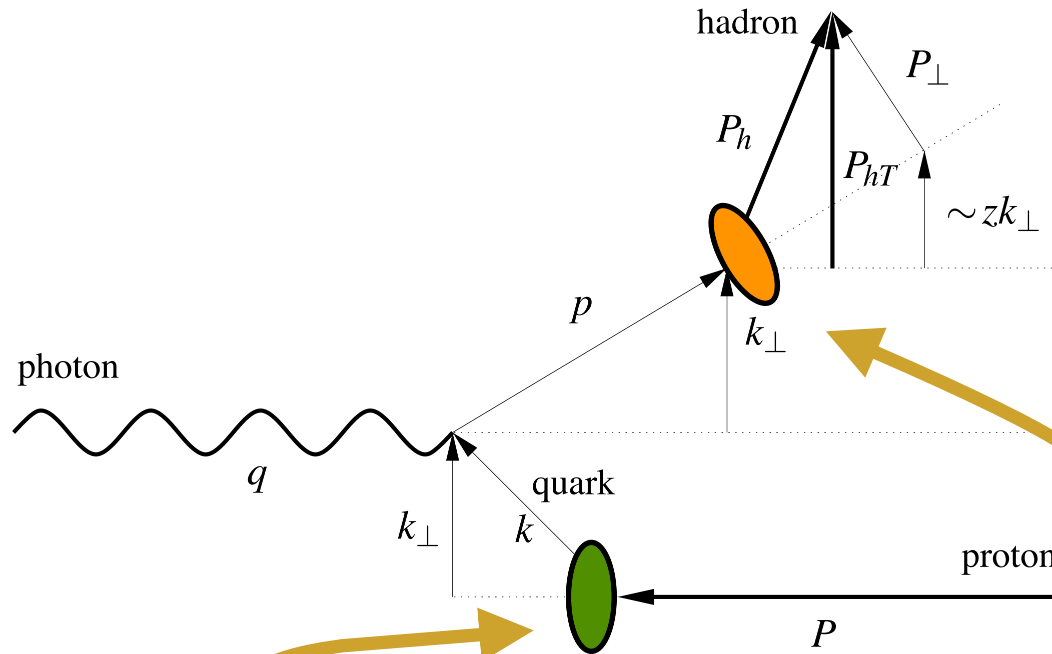


Transverse-momentum convolutions



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Distribution Functions

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Fragmentation Functions

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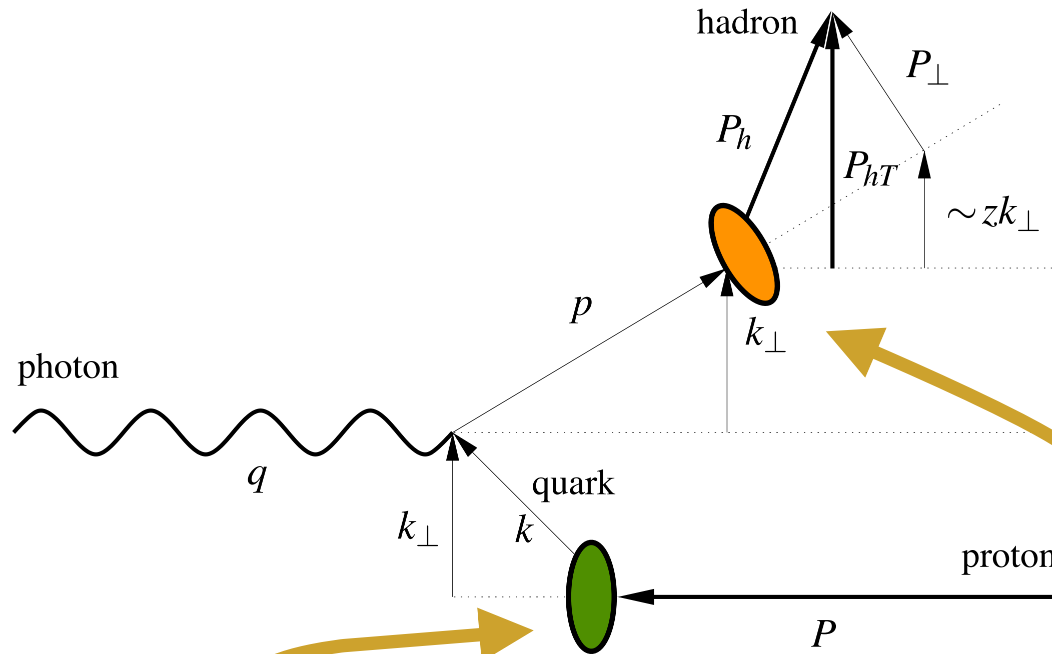
Transverse-momentum convolutions



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With QCD corrections

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = \sum_a \mathcal{H}_{UU,T}^a(Q^2;\mu^2) \int d\boldsymbol{k}_\perp \, d\boldsymbol{P}_\perp \, f_1^a\big(x,\boldsymbol{k}_\perp^2;\mu^2\big) \, D_1^{a\to h}\big(z,\boldsymbol{P}_\perp^2;\mu^2\big) \, \delta\big(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp\big)$$
$$+ Y_{UU,T}\big(Q^2,\boldsymbol{P}_{hT}^2\big) + \mathcal{O}\big(M^2/Q^2\big)$$

Evolution of TMDs

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

Rogers, Aybat, PRD 83 (11)
Collins, “Foundations of Perturbative QCD” (11)

other schemes are possible, e.g.,
Echevarria, Idilbi, Schaefer, Scimemi, EPJ C73 (13)

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collinear PDF

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collinear PDF

pQCD

Rogers, Aybat, PRD 83 (11)
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Echevarria, Idilbi, Schaefer, Scimemi, EPJ C73 (13)

Evolution of TMDs

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

collinear PDF

nonperturbative part
of evolution

pQCD

Rogers, Aybat, PRD 83 (11)
Collins, "Foundations of Perturbative QCD" (11)

other schemes are possible, e.g.,
Echevarria, Idilbi, Schaefer, Scimemi, EPJ C73 (13)

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collinear PDF

pQCD

nonperturbative part of evolution

nonperturbative part of TMD

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

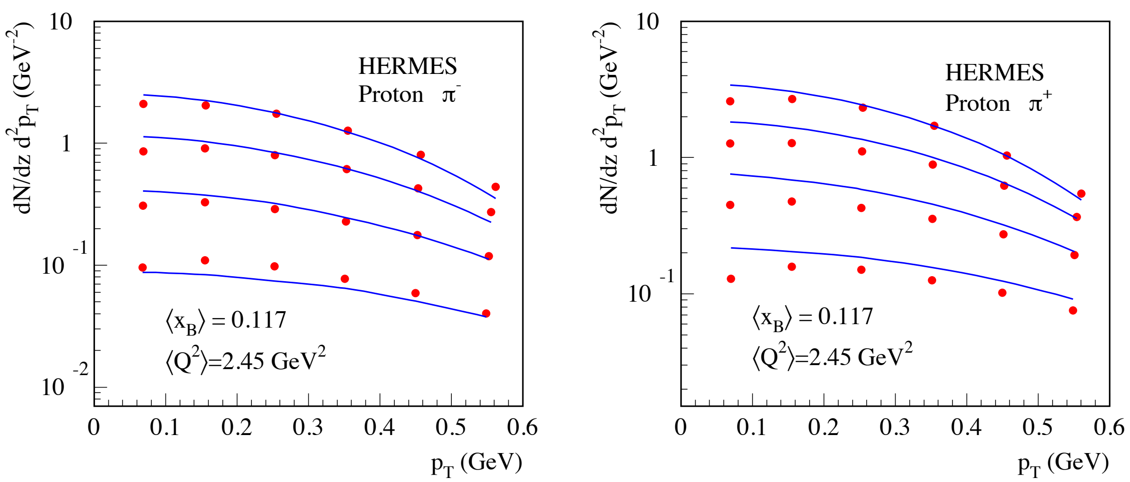
$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_*$$

Rogers, Aybat, PRD 83 (11)
Collins, "Foundations of Perturbative QCD" (11)

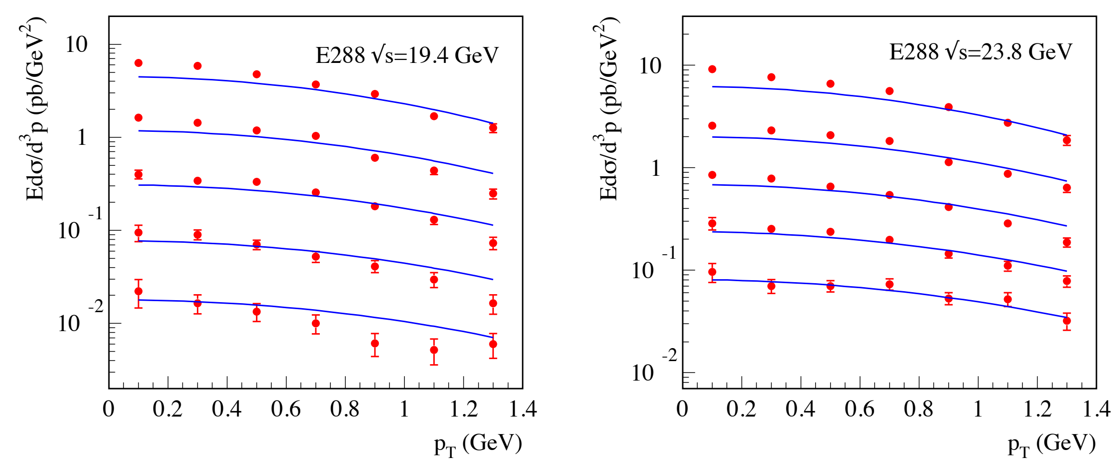
other schemes are possible, e.g.,
Echevarria, Idilbi, Schaefer, Scimemi, EPJ C73 (13)

First attempts to use TMD evolution

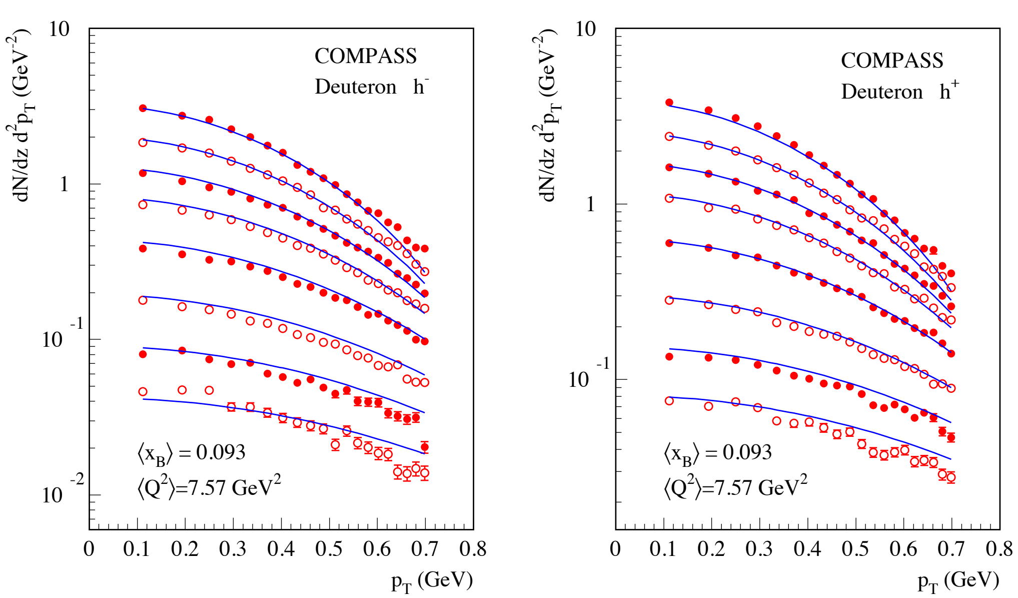
SIDIS



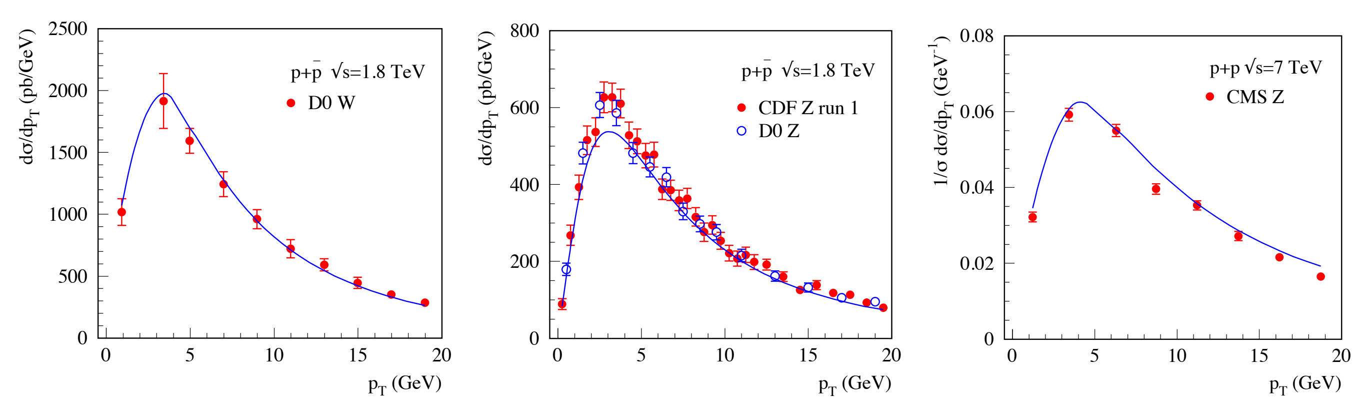
DRELL-YAN



SIDIS



W AND Z PRODUCTION



Echevarria, Idilbi, Kang, Vitev, PRD 89 (14)
for Drell-Yan, see also D'Alesio, Echevarria, Melis, Scimemi, arXiv:1407.3311


“In children’s shoes”
There is a nice and long way ahead of us to reach the level of PDFs

Polarized sector

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

Polarized sector

$$
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&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)} \bigg\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} + S_L \bigg[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \bigg] \\
&\quad + S_L\,\lambda_e \bigg[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, F_{LL}^{\cos\phi_h} \bigg] \\
&\quad + S_T \bigg[\sin(\phi_h-\phi_S) \Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \Big) + \varepsilon\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \bigg] + S_T\lambda_e \bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \bigg] \bigg\}
\end{aligned}
$$

Multidimensional analyses of all of them under way, but still need to be published

Polarized sector

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
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&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad \left. + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}\right. \\
&\qquad \left. + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\Bigg\}
\end{aligned}
$$

Sivers structure function

$f_{1T}^{\perp} \otimes D_1$

Polarized sector

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
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&\quad + S_T\bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
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\end{aligned}
$$

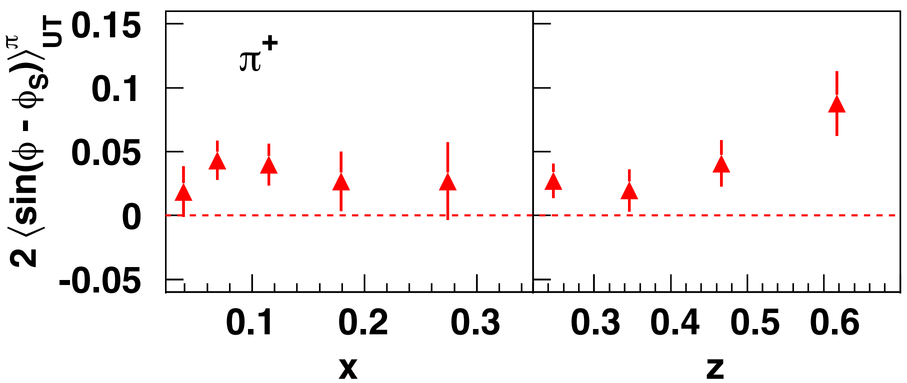
Sivers structure function

$$f_{1T}^{\perp} \otimes D_1$$

Collins structure function

$$h_1 \otimes H_1^{\perp}$$

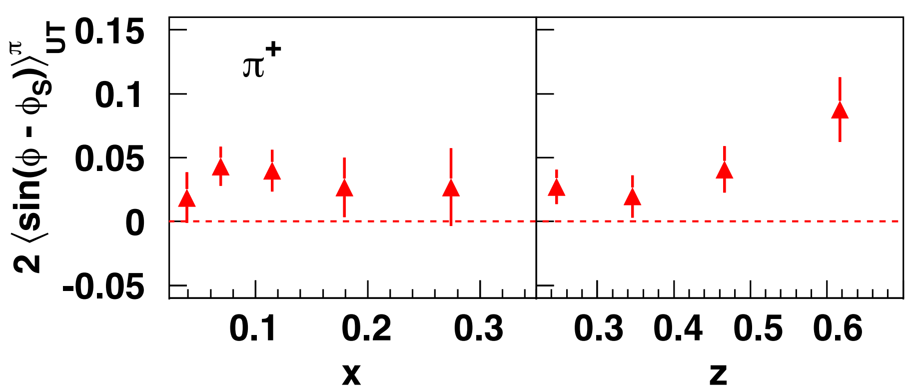
Sivers asymmetry



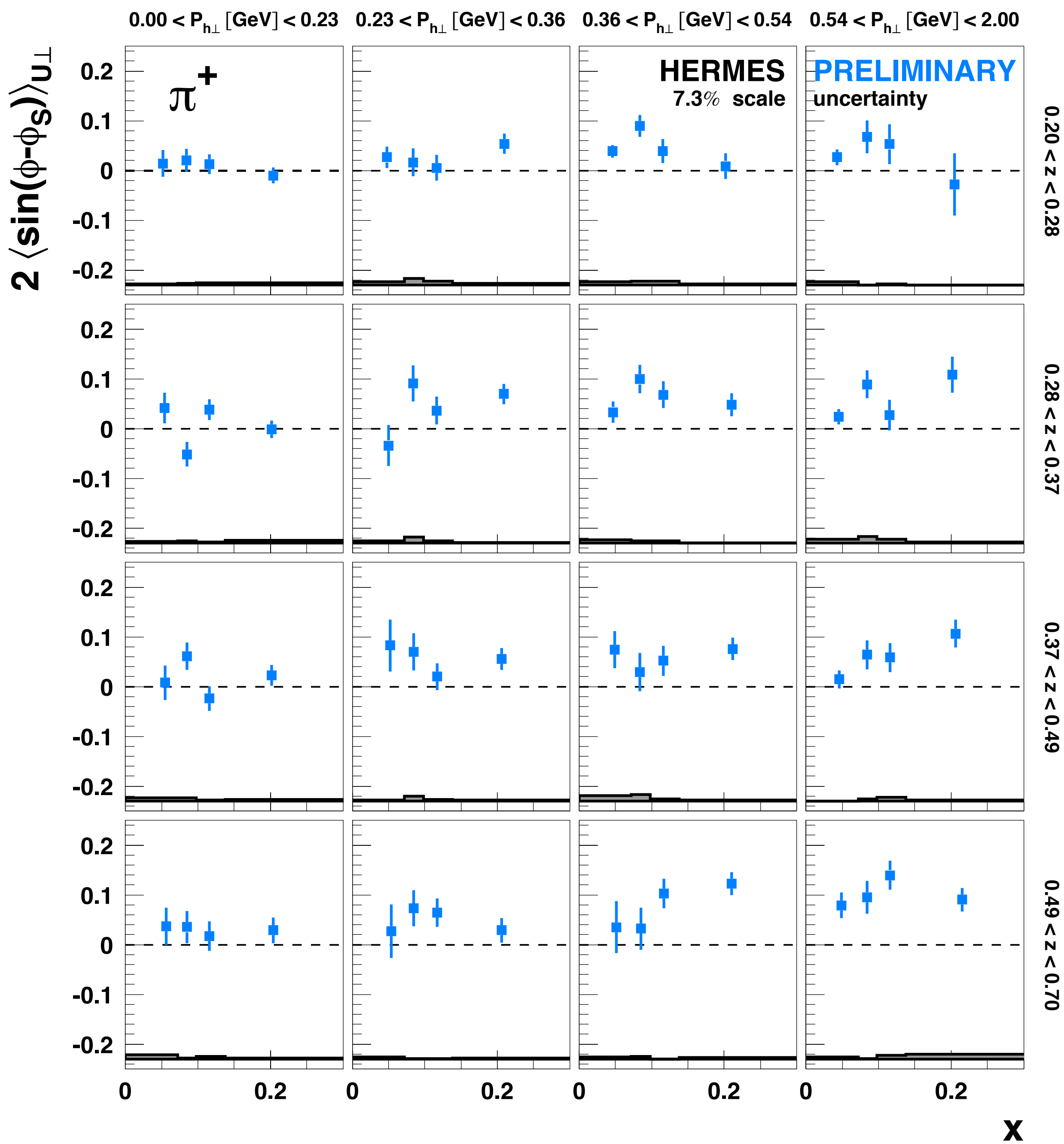
HERMES, PRL 94 (2005)

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$

Sivers asymmetry



HERMES, PRL 94 (2005)

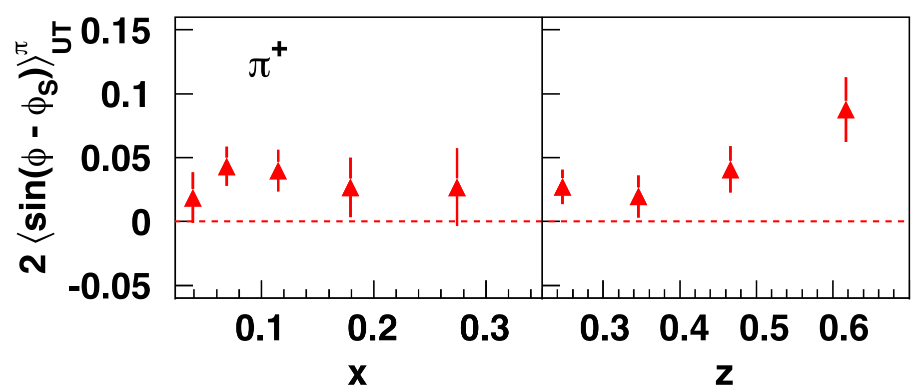


new results 2014 (talk by C. van Hulse)

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$

Sivers asymmetry

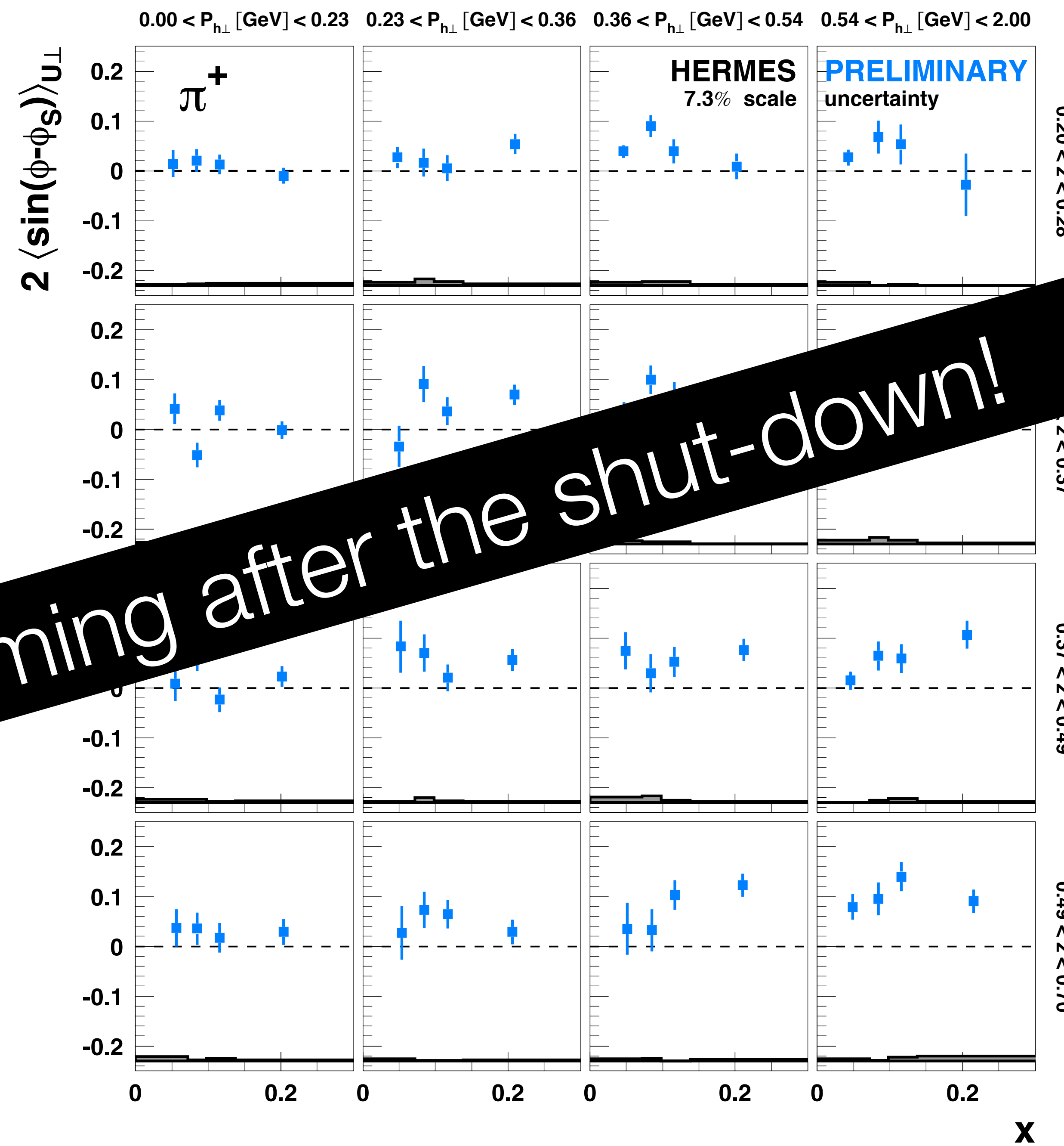
hermes



HERMES, PRL 94 (2005)

Amazing progress... coming after the shut-down!

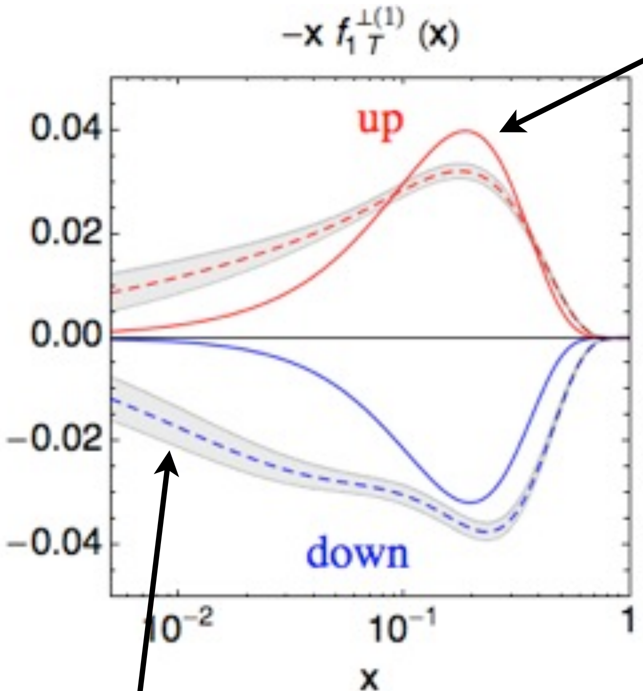
$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$



new results 2014 (talk by C. van Hulse)

What can you get out?

*Echevarria, Idilbi, Kang, Vitev, PRD 89 (14) [**with TMD evo**]*



*Bacchetta, Radici, PRL 107 (11) [**no TMD evo**]*

ubar

dbar

Extraction of Sivers function

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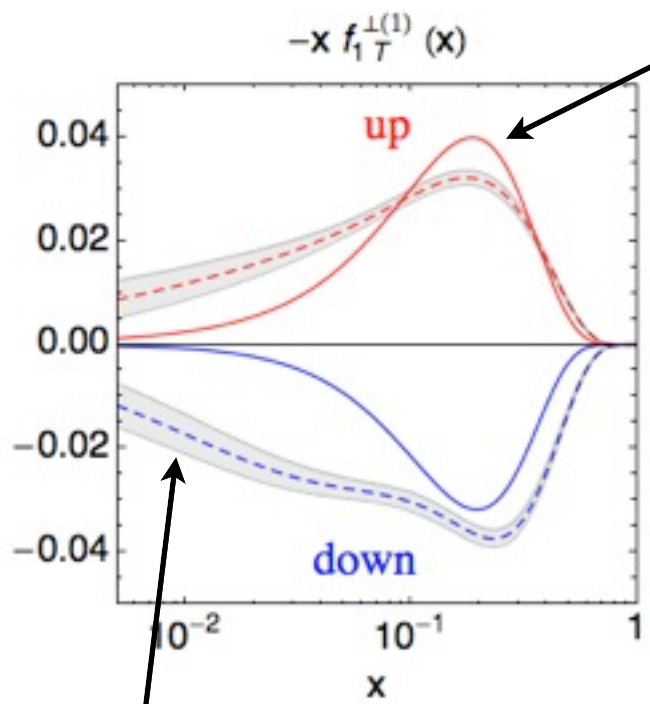
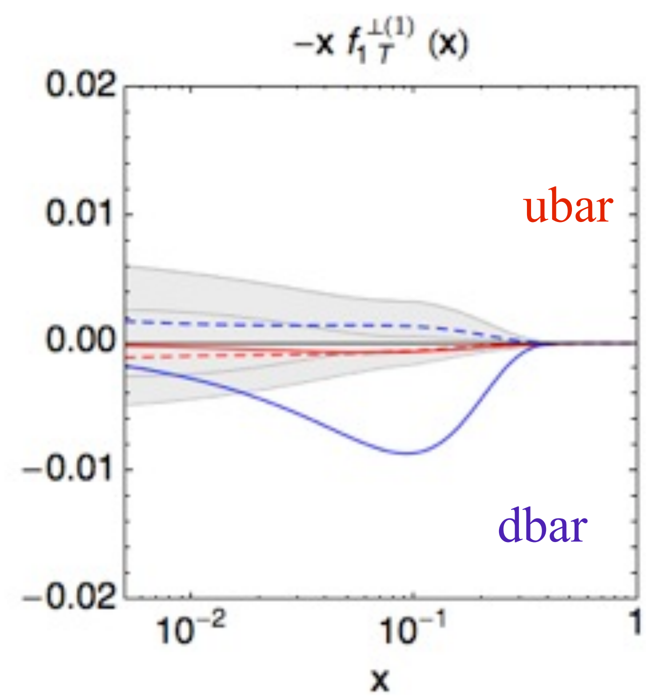
*Echevarria, Idilbi, Kang, Vitev, PRD 89 (14) [**with TMD evo**]*





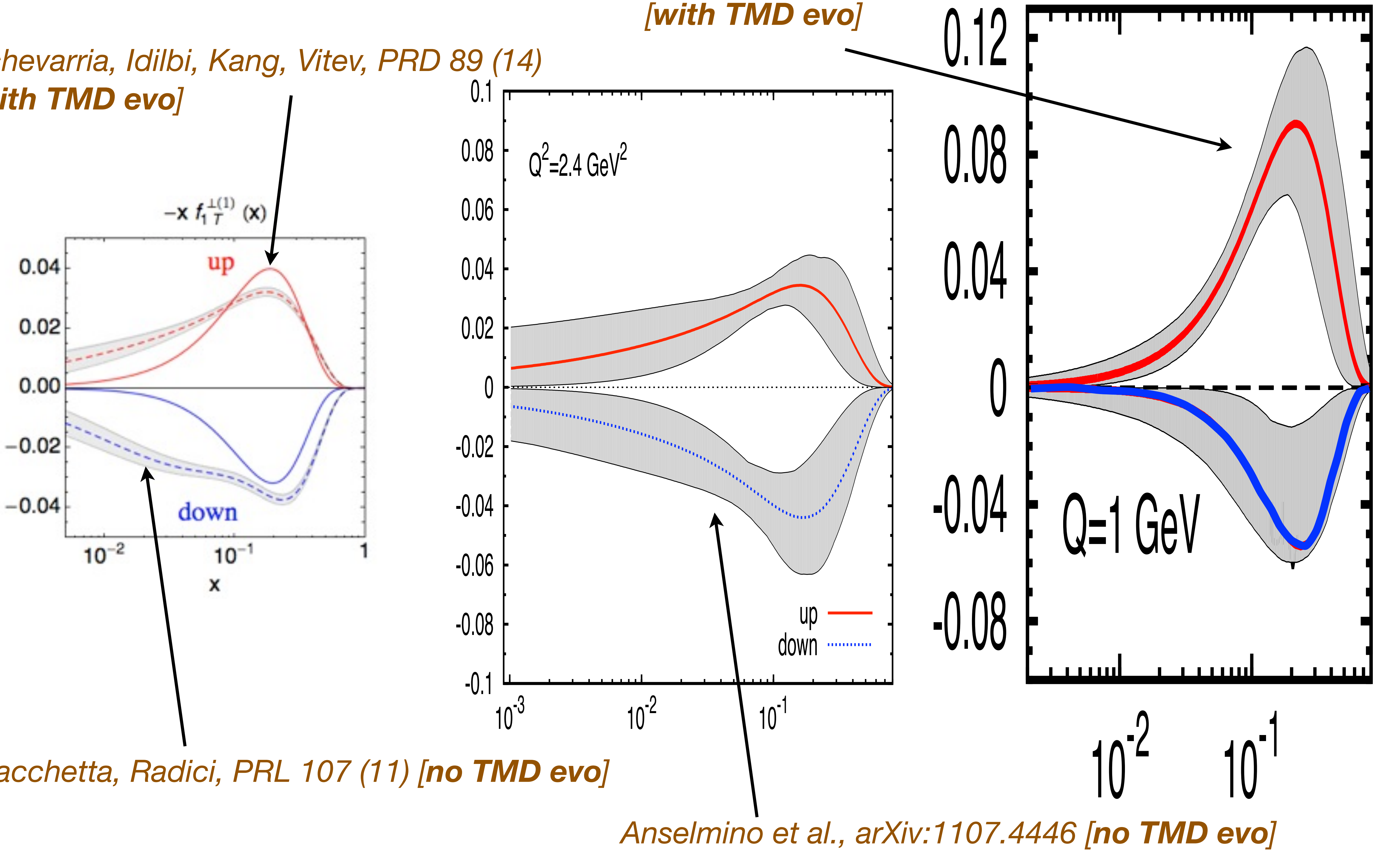
*Bacchetta, Radici, PRL 107 (11) [**no TMD evo**]*

Extraction of Sivers function

What can you get out?

Anselmino, Boglione, Melis, PRD86 (12) *[**with TMD evo**]*

Echevarria, Idilbi, Kang, Vitev, PRD 89 (14) *[**with TMD evo**]*



Bacchetta, Radici, PRL 107 (11) *[**no TMD evo**]*

Anselmino et al., arXiv:1107.4446 *[**no TMD evo**]*

What can you get out?

Anselmino, Boglione, Melis, PRD86 (12) ***[with TMD evo]***

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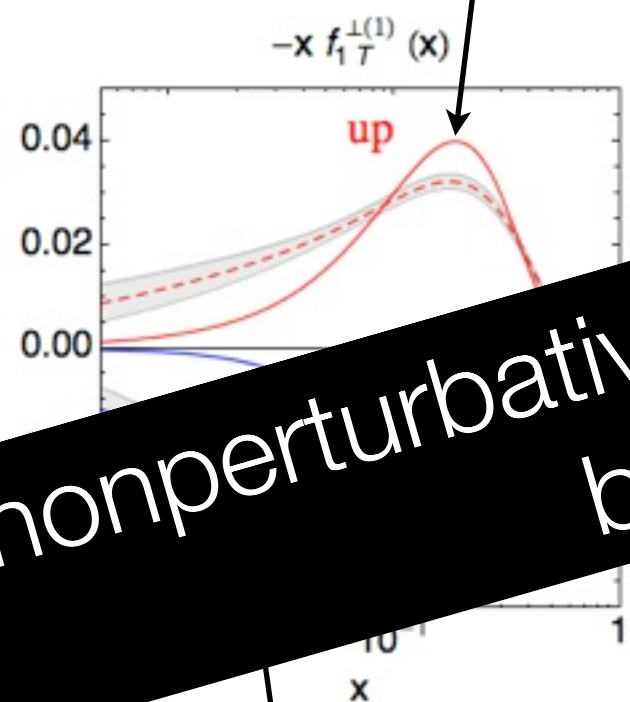
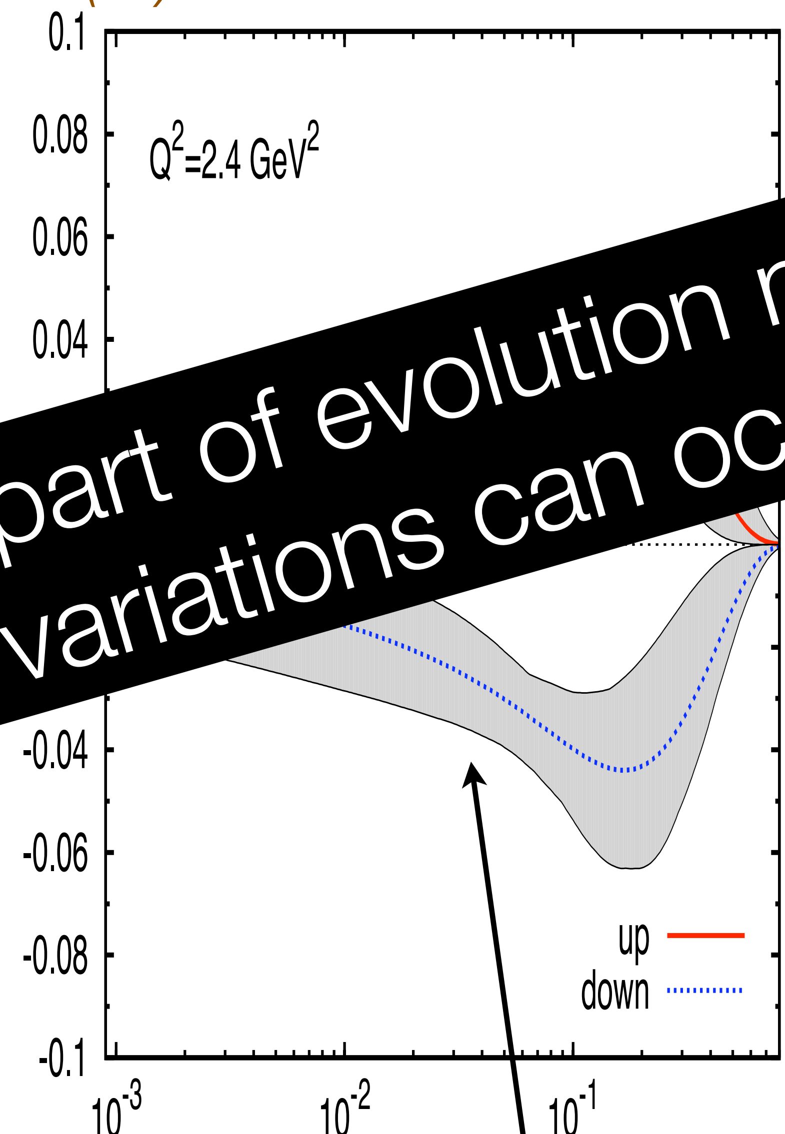
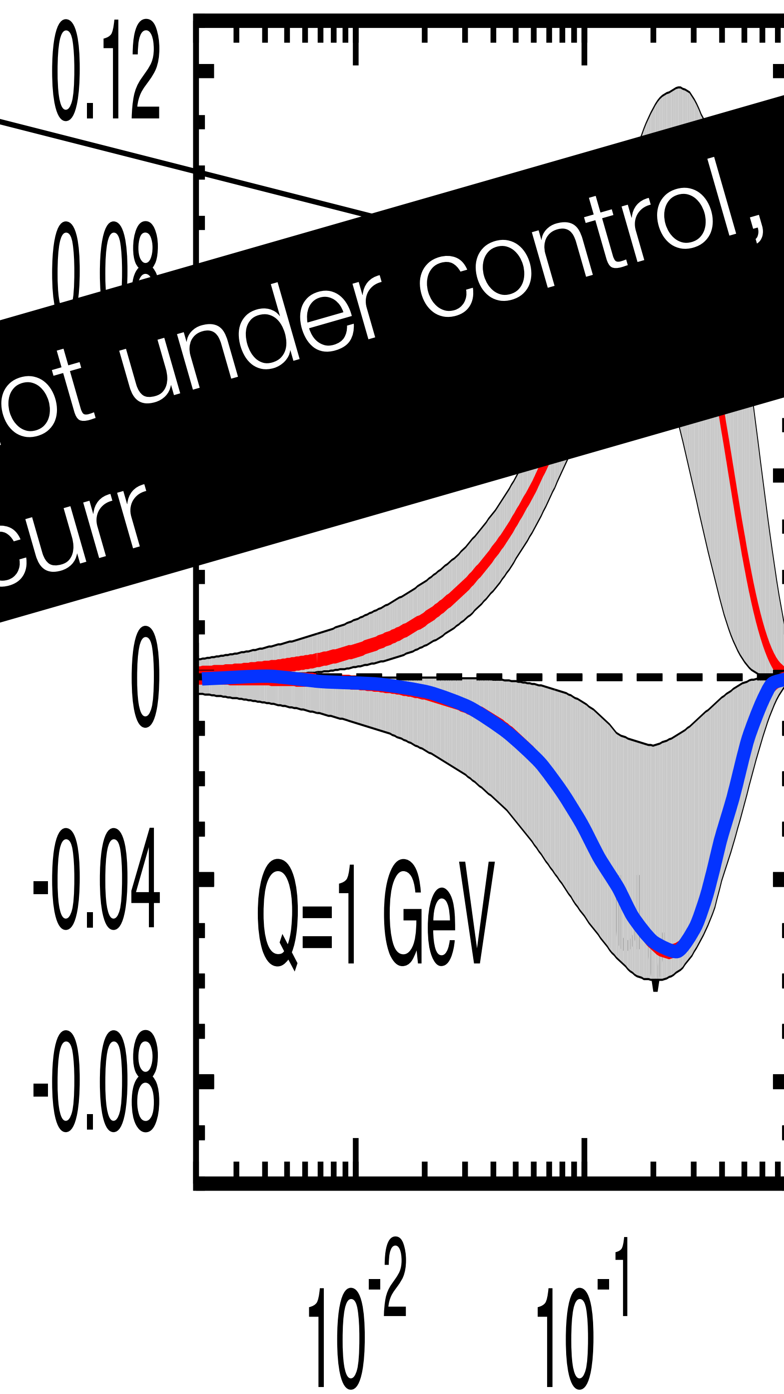




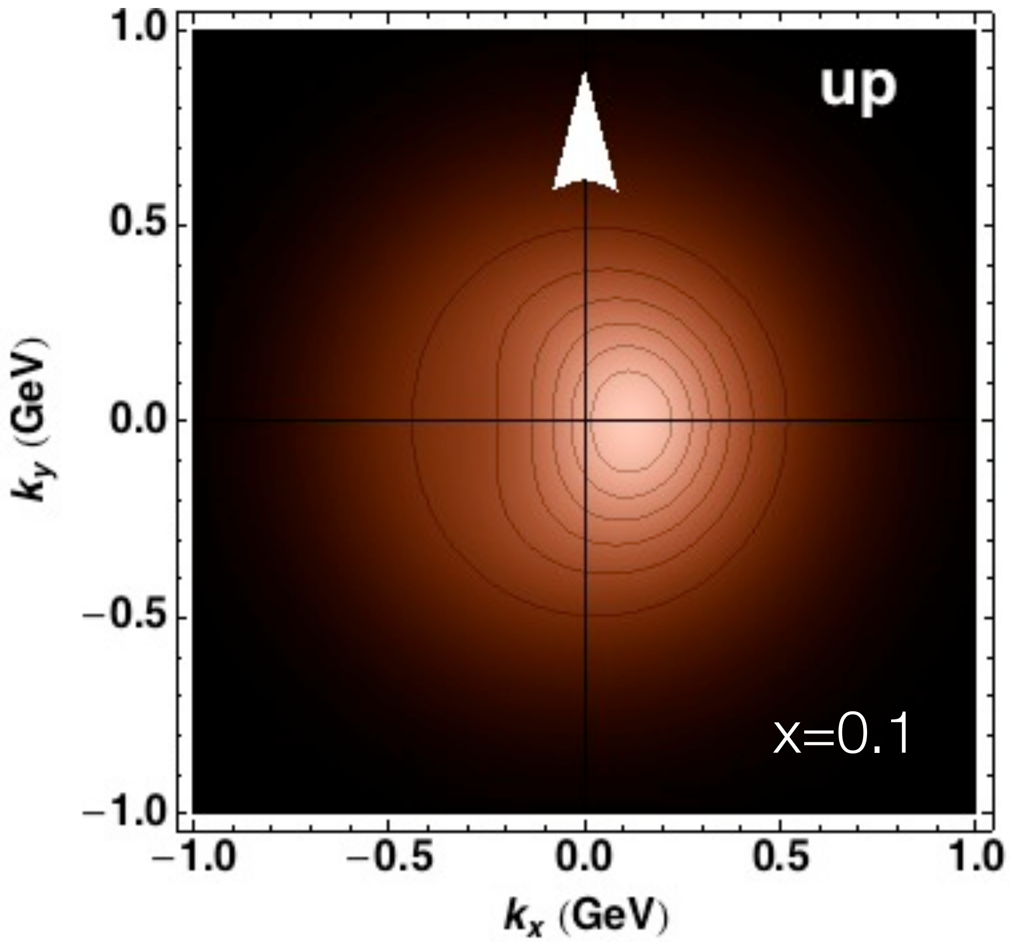


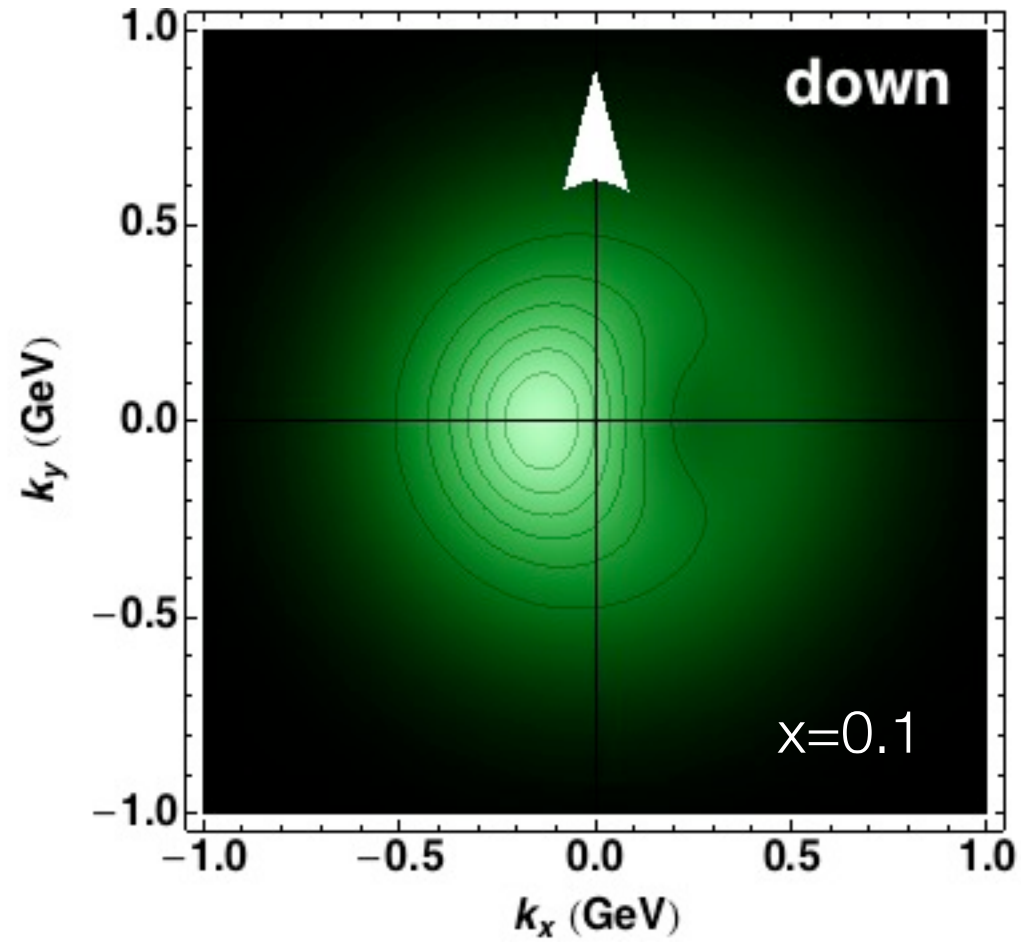
If nonperturbative part of evolution not under control, big variations can occurr

Bacchetta, Radici, PRL 107 (11) ***[no TMD evo]***

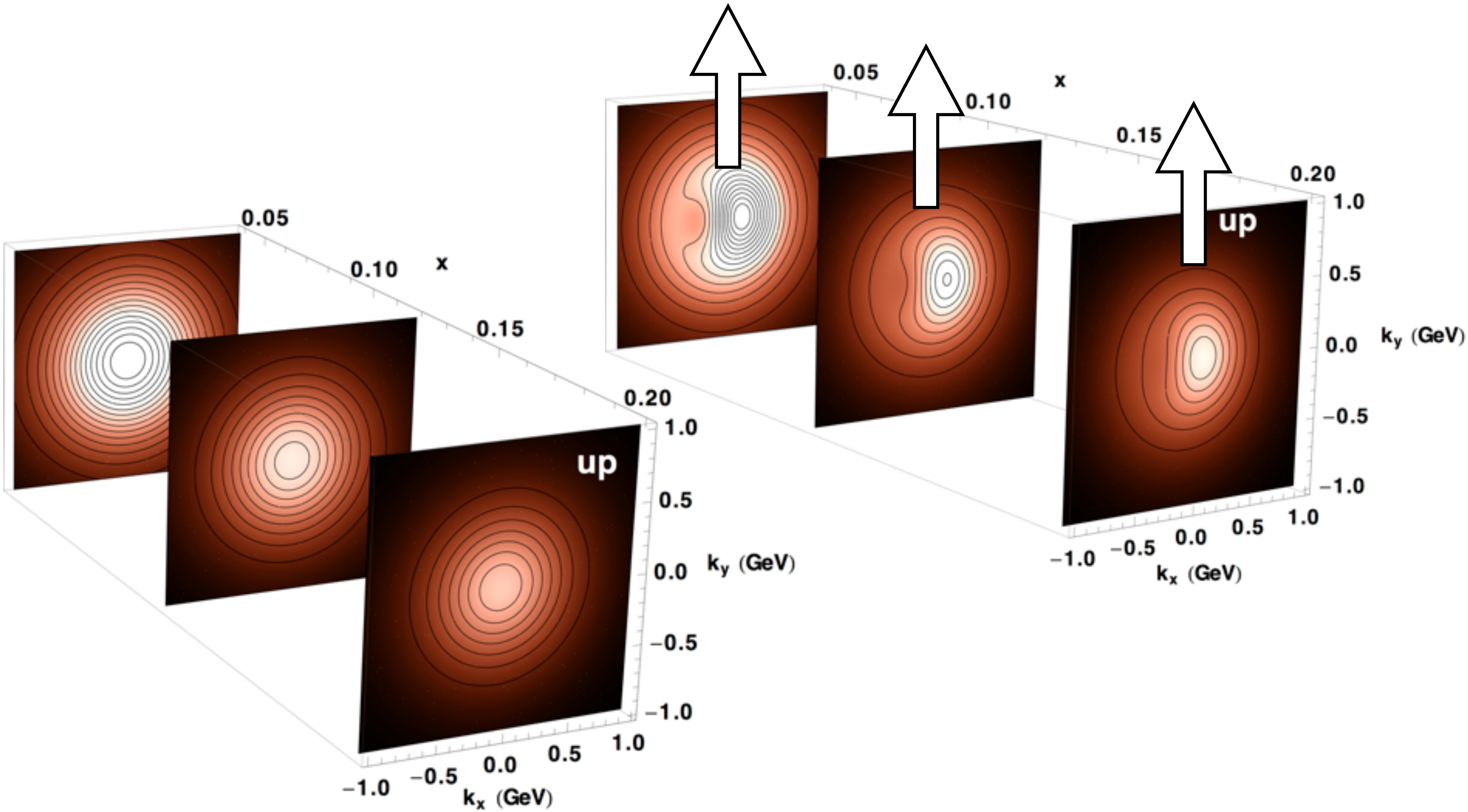
Anselmino et al., arXiv:1107.4446 ***[no TMD evo]***

3D mapping with spin



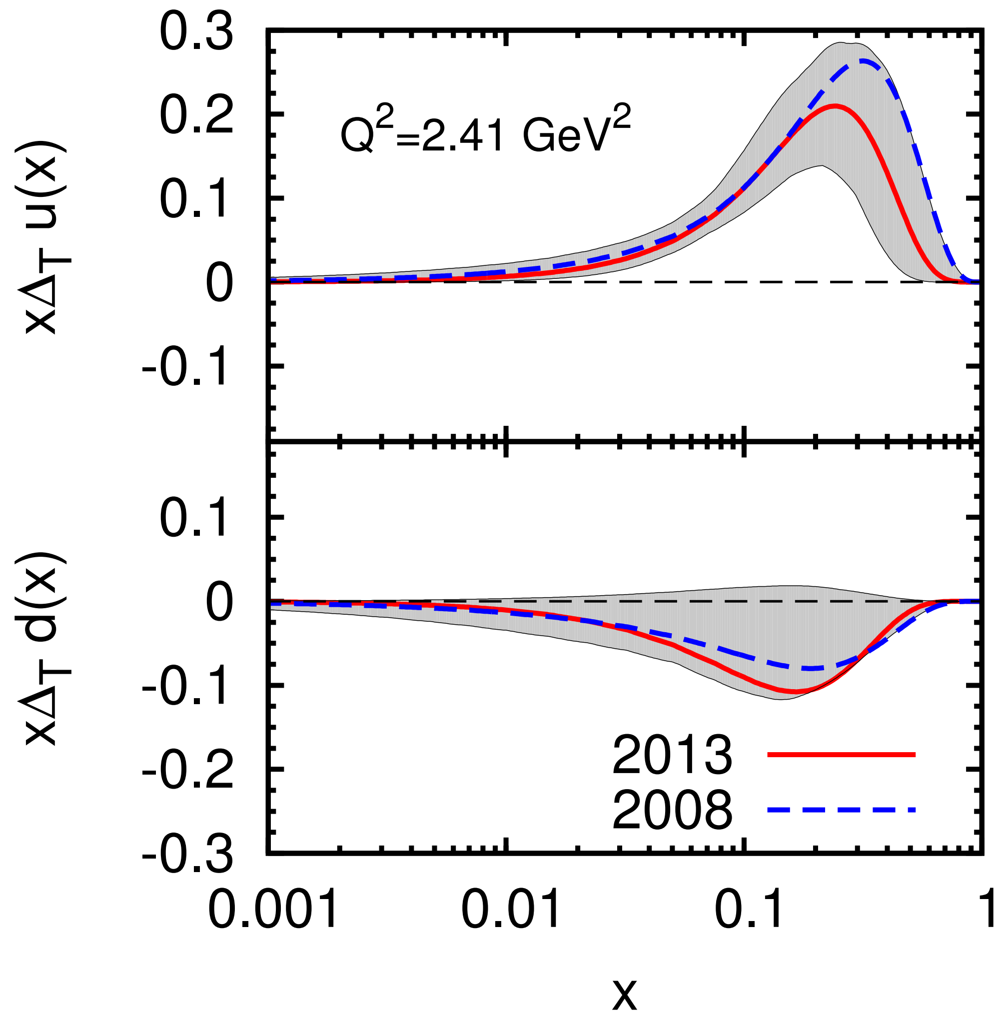


3D mapping with spin

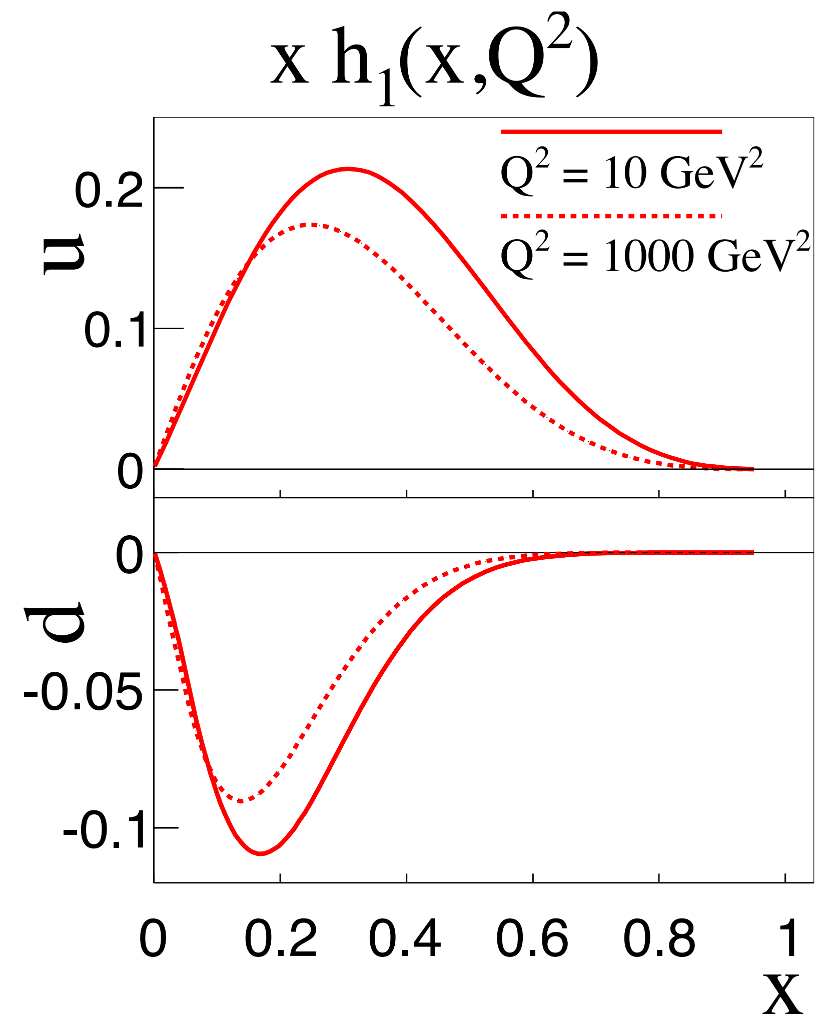


Collins asymmetry and transversity

$$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$$



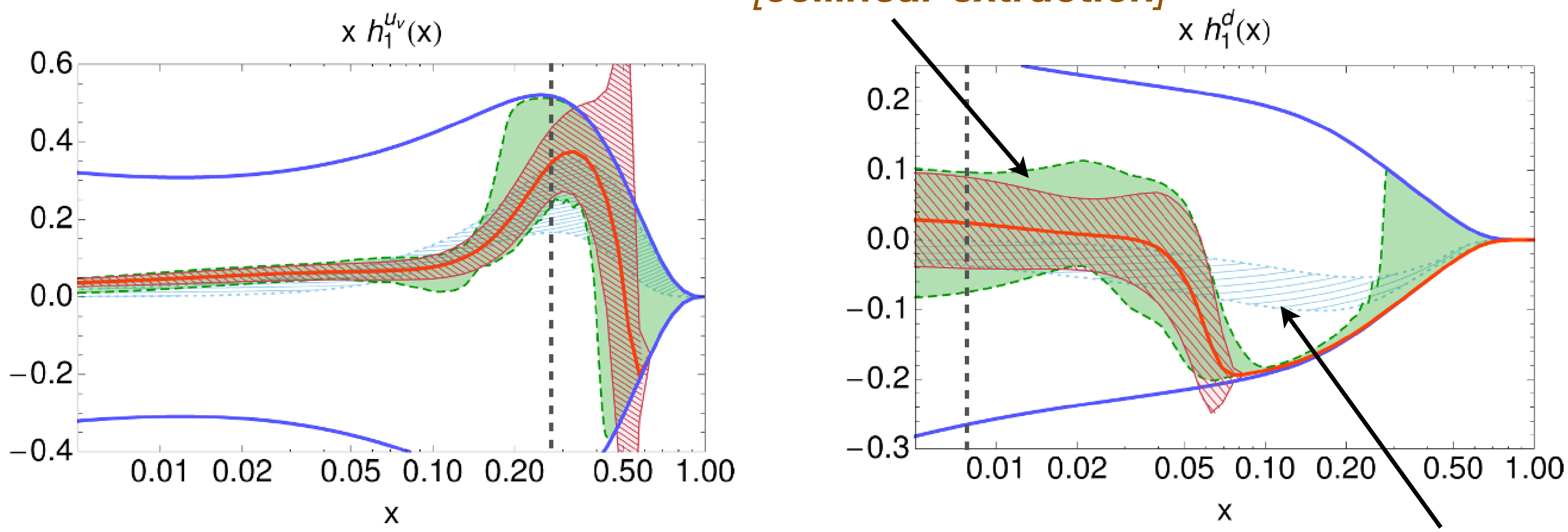
Anselmino et al., PRD87 (2013)
[no TMD evo]



Kang, Prokudin, Sun, Yuan, arXiv:1410.4877
[with TMD evo]

Dihadron DIS production and transversity

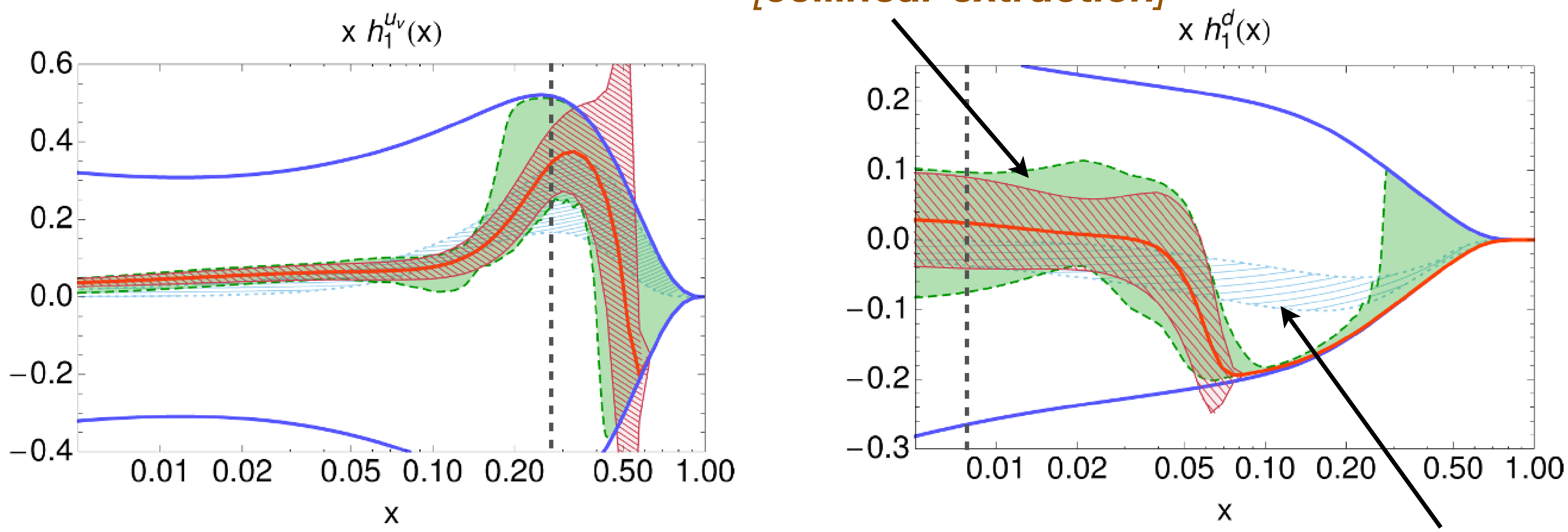
Bacchetta, Courtoy, Radici, JHEP 1303 (2013)
[collinear extraction]



Anselmino et al. 0812.4366
[TMD extraction, no TMD evo]

Dihadron DIS production and transversity

Bacchetta, Courtoy, Radici, JHEP 1303 (2013)
[collinear extraction]



Anselmino et al. 0812.4366
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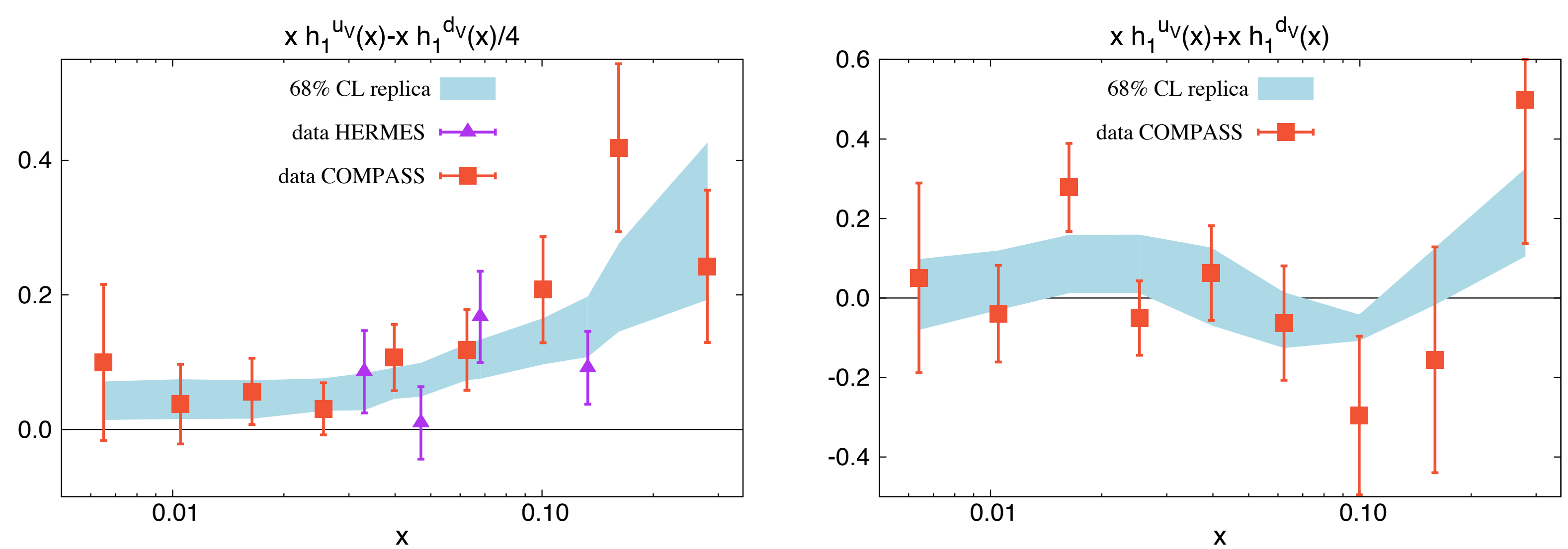
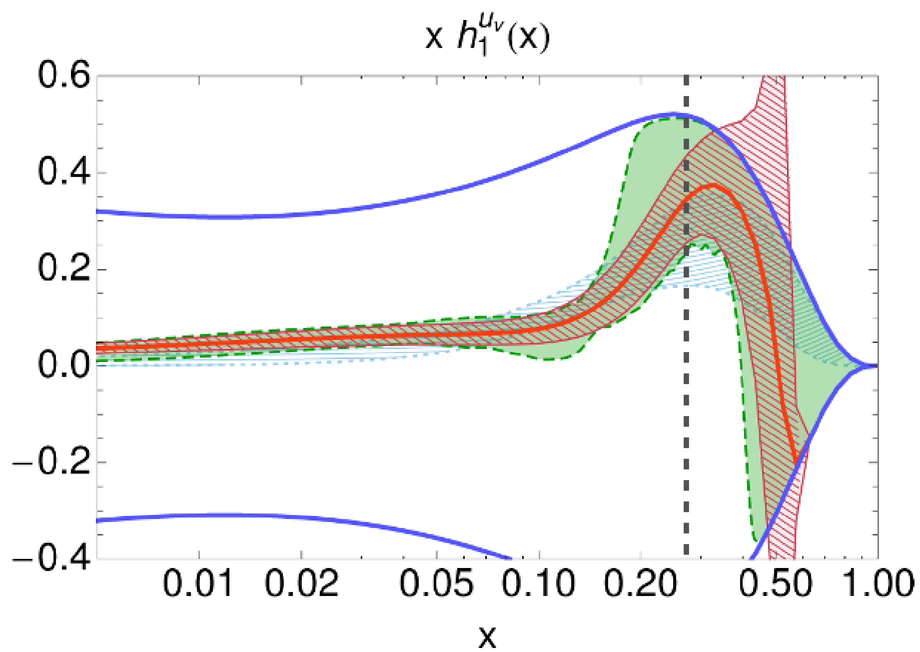
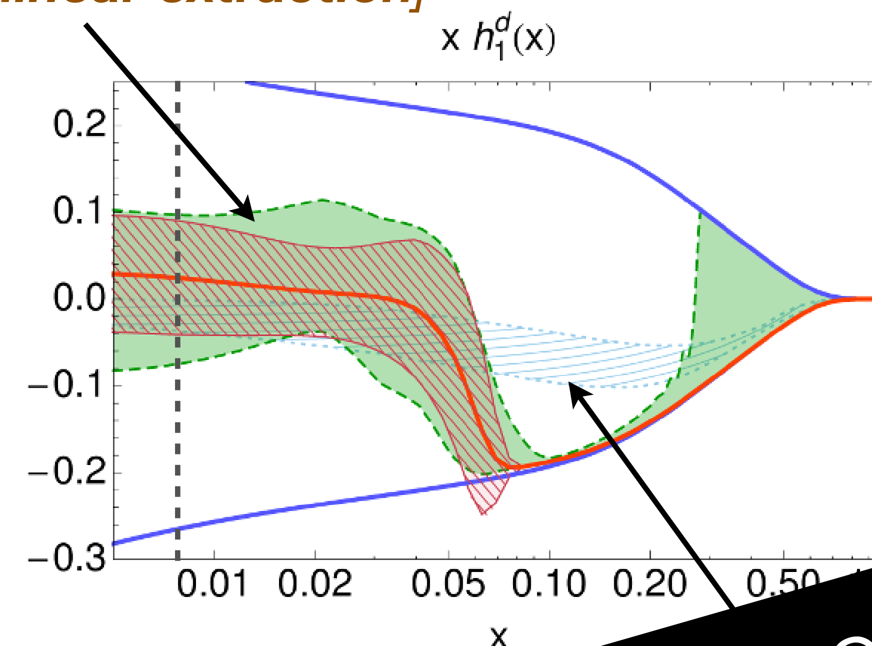


Dihadron DIS production and transversity

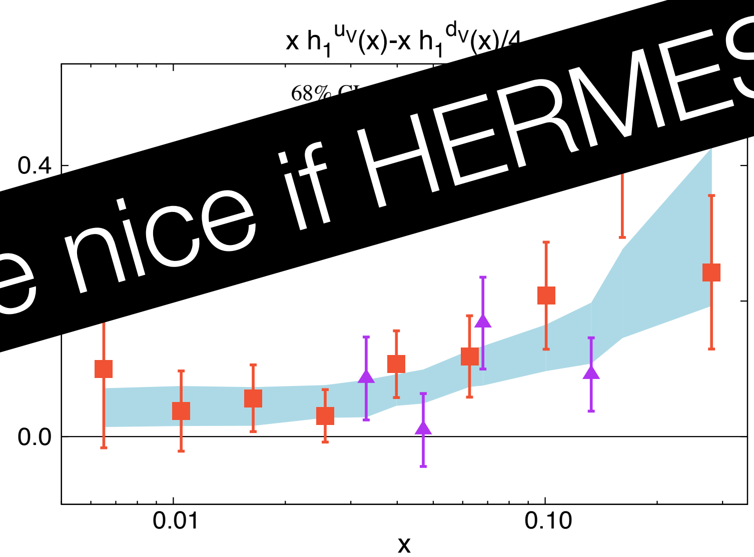
Bacchetta, Courtoy, Radici, JHEP 1303 (2013)
[collinear extraction]

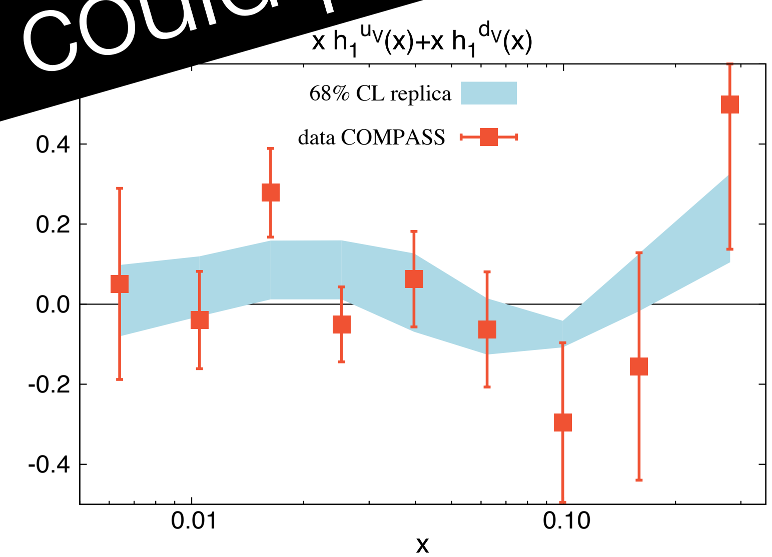




12.4366
xtraction, no TMD evo]

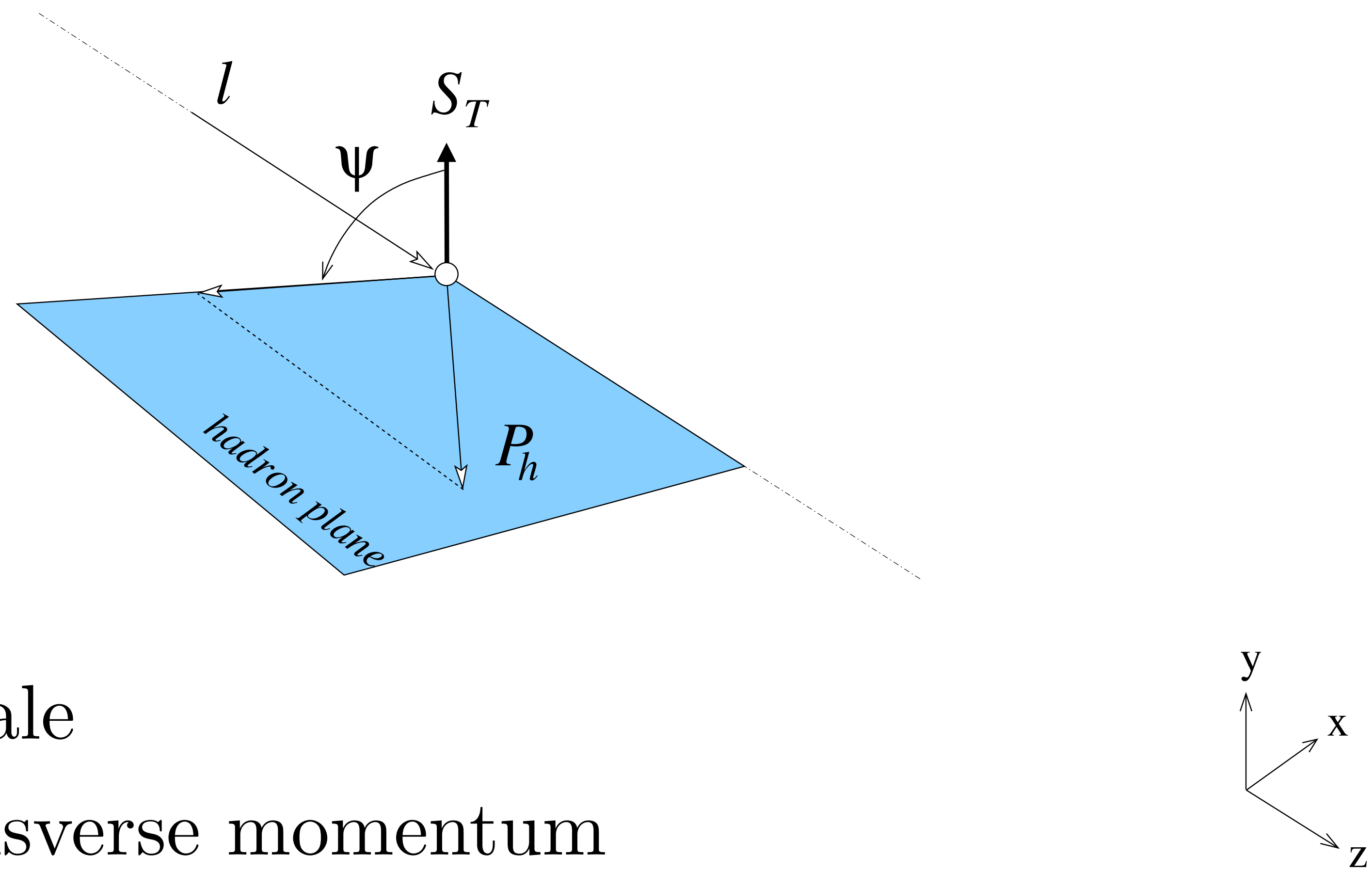
It would be nice if HERMES could provide more data





Inclusive hadron production

$\ell(l) + N(P) \to h(P_h) + X,$

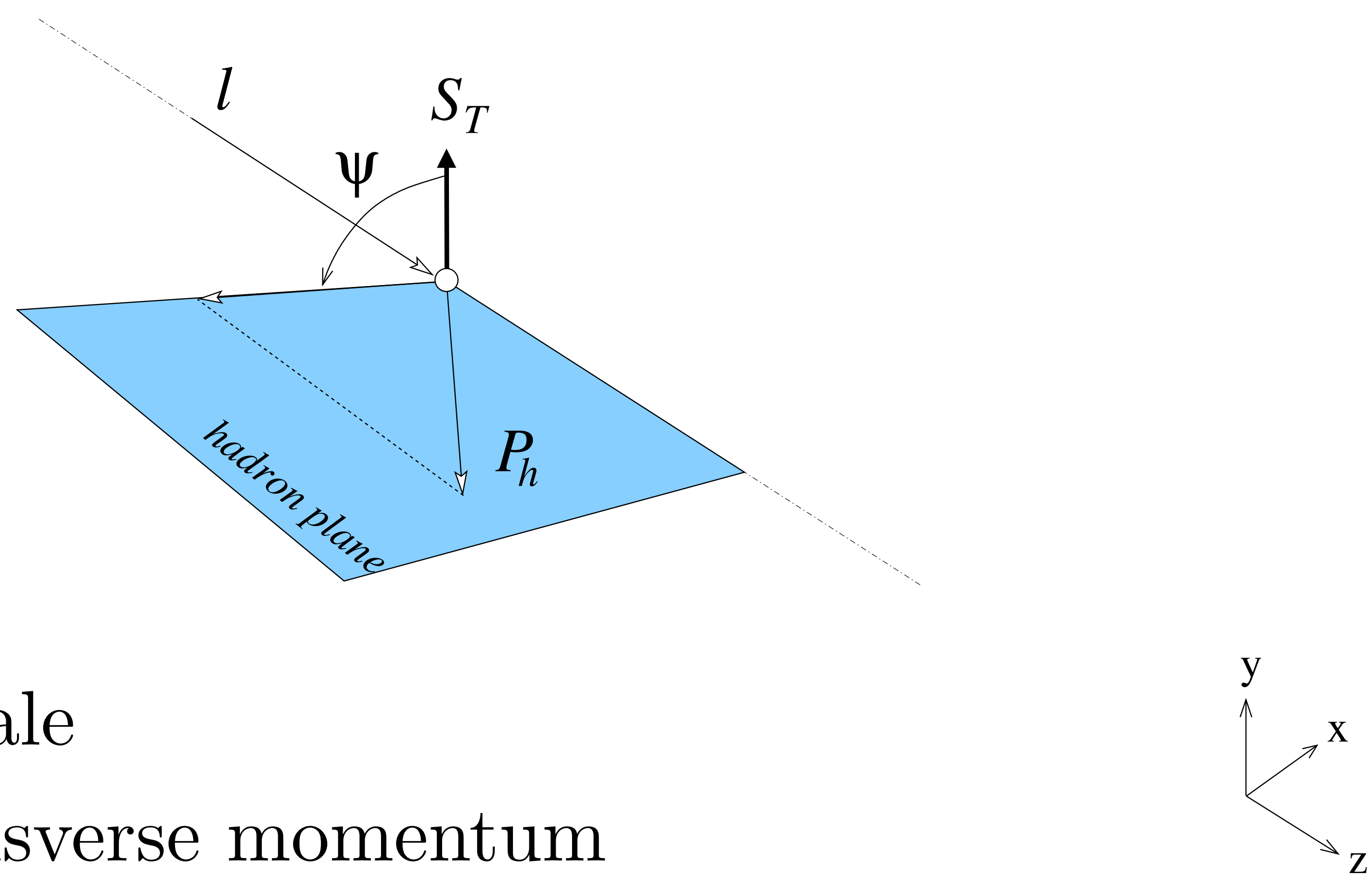


$\Lambda_{\rm QCD}$ = hadronic scale

$P_{h\perp}$ = hadron transverse momentum

Inclusive hadron production

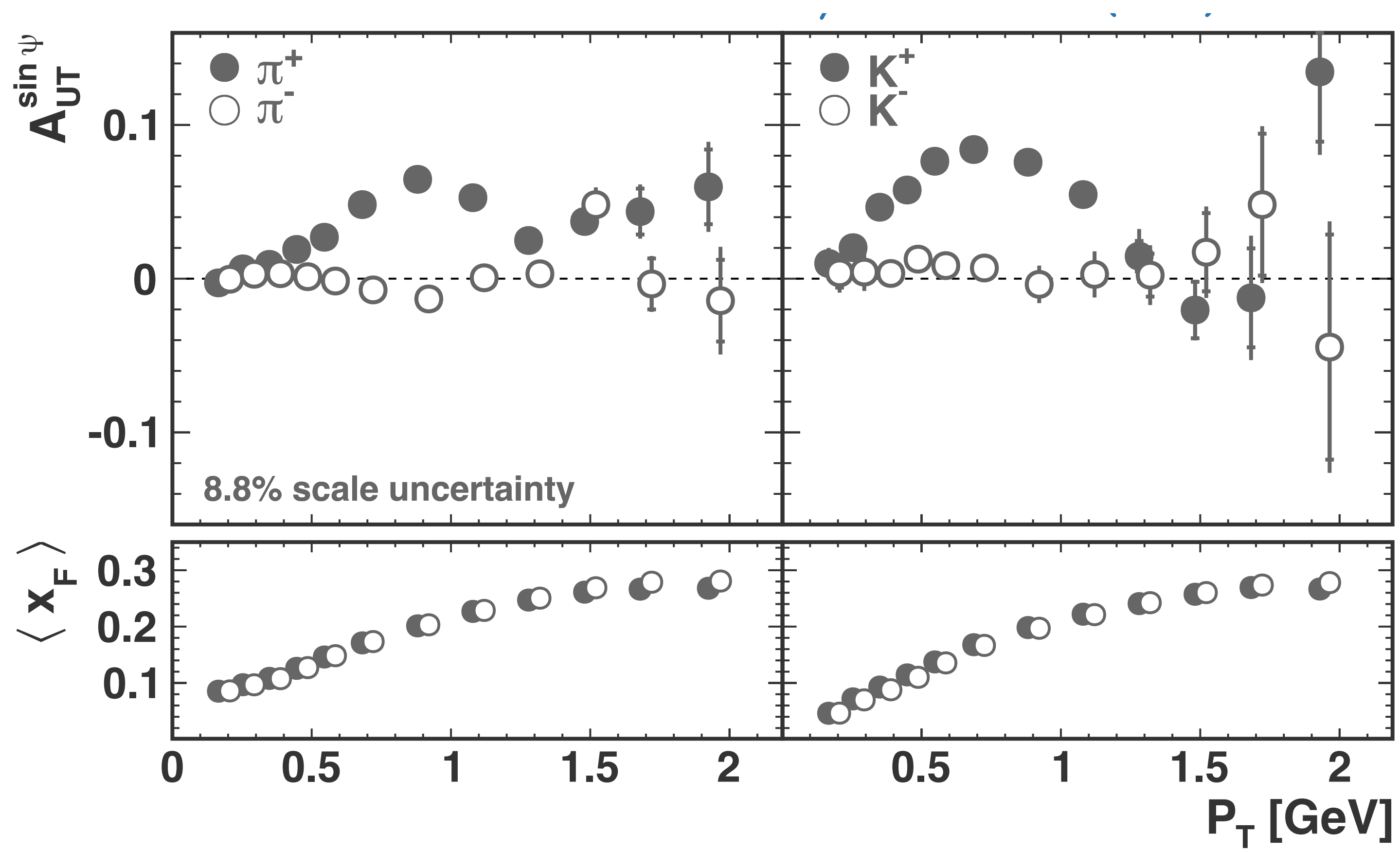
$\ell(l) + N(P) \to h(P_h) + X,$



$\Lambda_{\mathrm{QCD}} = \text{hadronic scale}$

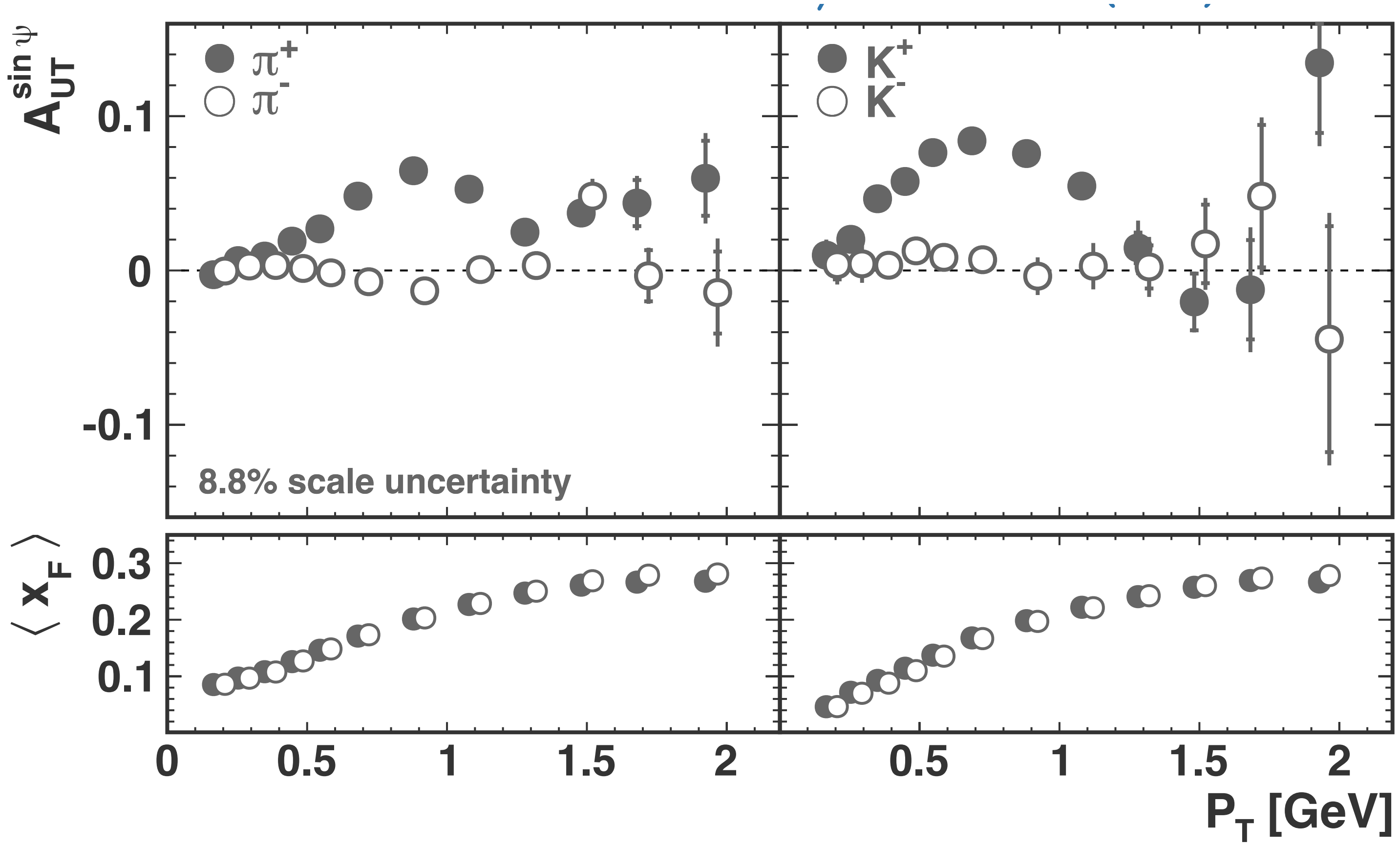
$P_{h\perp} = \text{hadron transverse momentum}$

see talk by C. van Hulse



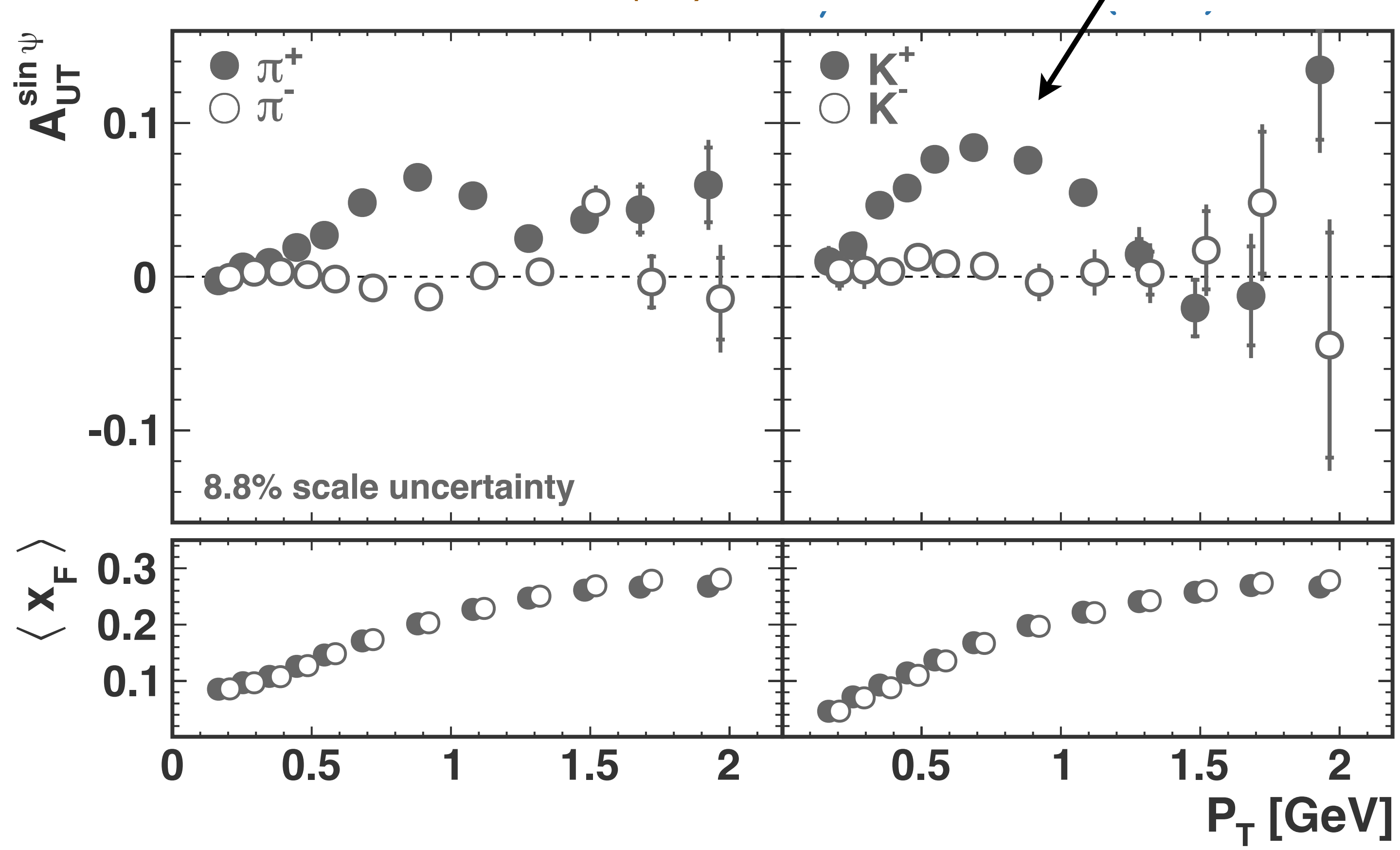
$A_{UT}^{\sin\psi}$
π^+
π^-
K^+
K^-
8.8% scale uncertainty
0.1
0
-0.1
$\langle x_F \rangle$
0.3
0.2
0.1
0
0.5
1
1.5
2
P_T [GeV]

HERMES, PLB 728 (14)



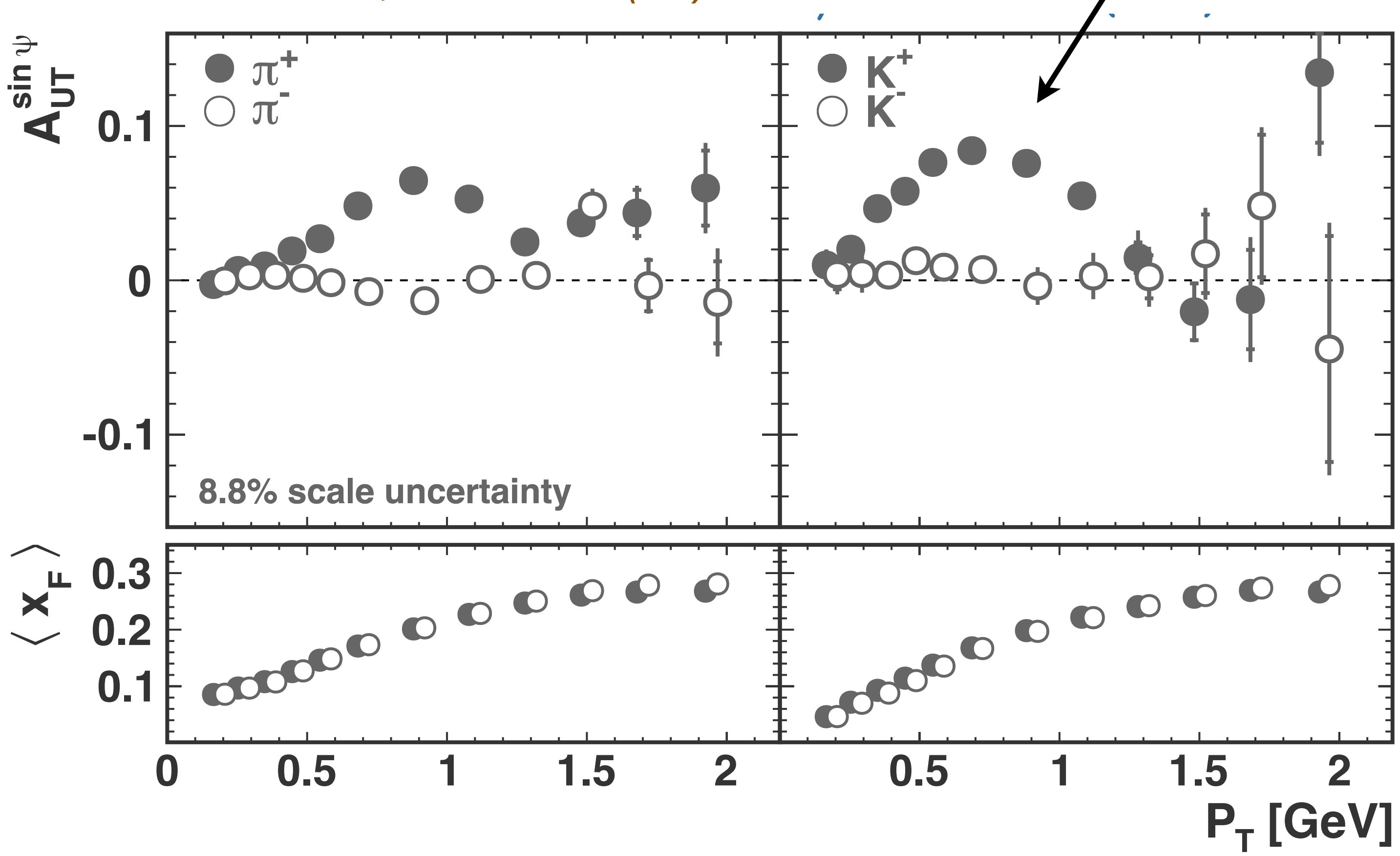
The bulk of the signal has $P_T \sim Q \sim \Lambda_{QCD}$

HERMES, PLB 728 (14)



The bulk of the signal has $P_T \sim Q \sim \Lambda_{QCD}$

HERMES, PLB 728 (14)



Not easy to interpret theoretically (model that extrapolates TMD approach and twist-3 collinear approach?).

Koike, NPA 721 (03)
Anselmino et al., PRD 89 (14)

Conclusions

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Conclusions

- Rich variety of physics topics
- HERMES has pioneered the way into 3 dimensional mapping
- HERMES has done almost everything possible, some important analyses still need to be finalized and published
- The "exploration phase" is almost over, a transition to a precision phase is expected
- COMPASS, experiments at JLab and BNL, and eventually an EIC will bring the investigations forward

Extra material

Polarized sector

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\biggl\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} + S_L\Bigl[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\Bigr] \\
&\quad + S_L\,\lambda_e\Bigl[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, F_{LL}^{\cos\phi_h}\Bigr] \\
&\quad + S_T\Bigl[\sin(\phi_h-\phi_S)\,\Bigl(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)}\Bigr) + \varepsilon\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigr] + S_T\lambda_e\Bigl[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigr]\biggr\}
\end{aligned}
$$

Sivers structure function

$$f_{1T}^{\perp} \otimes D_1$$

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&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
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\end{aligned}
$$

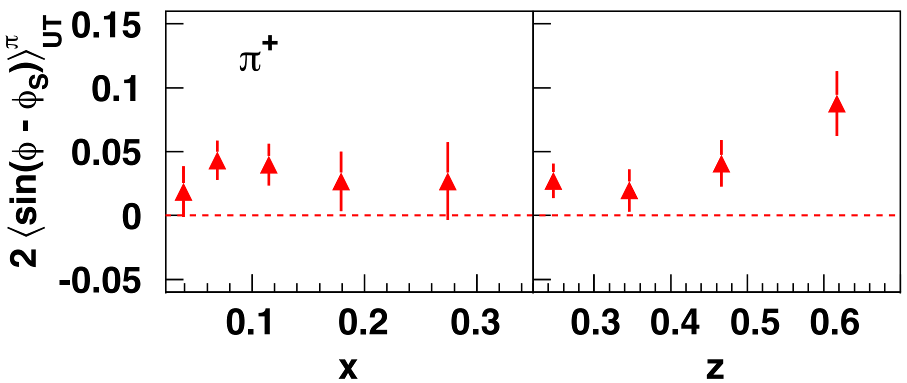
Sivers structure function

$f_{1T}^{\perp} \otimes D_1$

Collins structure function

$h_1 \otimes H_1^{\perp}$

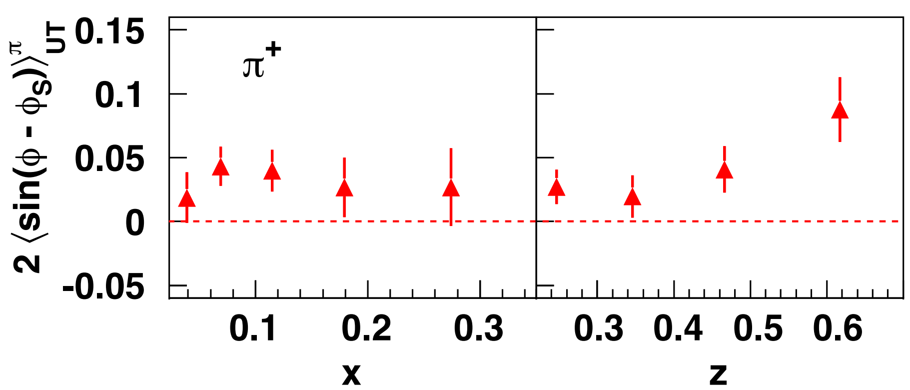
Sivers asymmetry



HERMES, PRL 94 (2005)

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$

Sivers asymmetry



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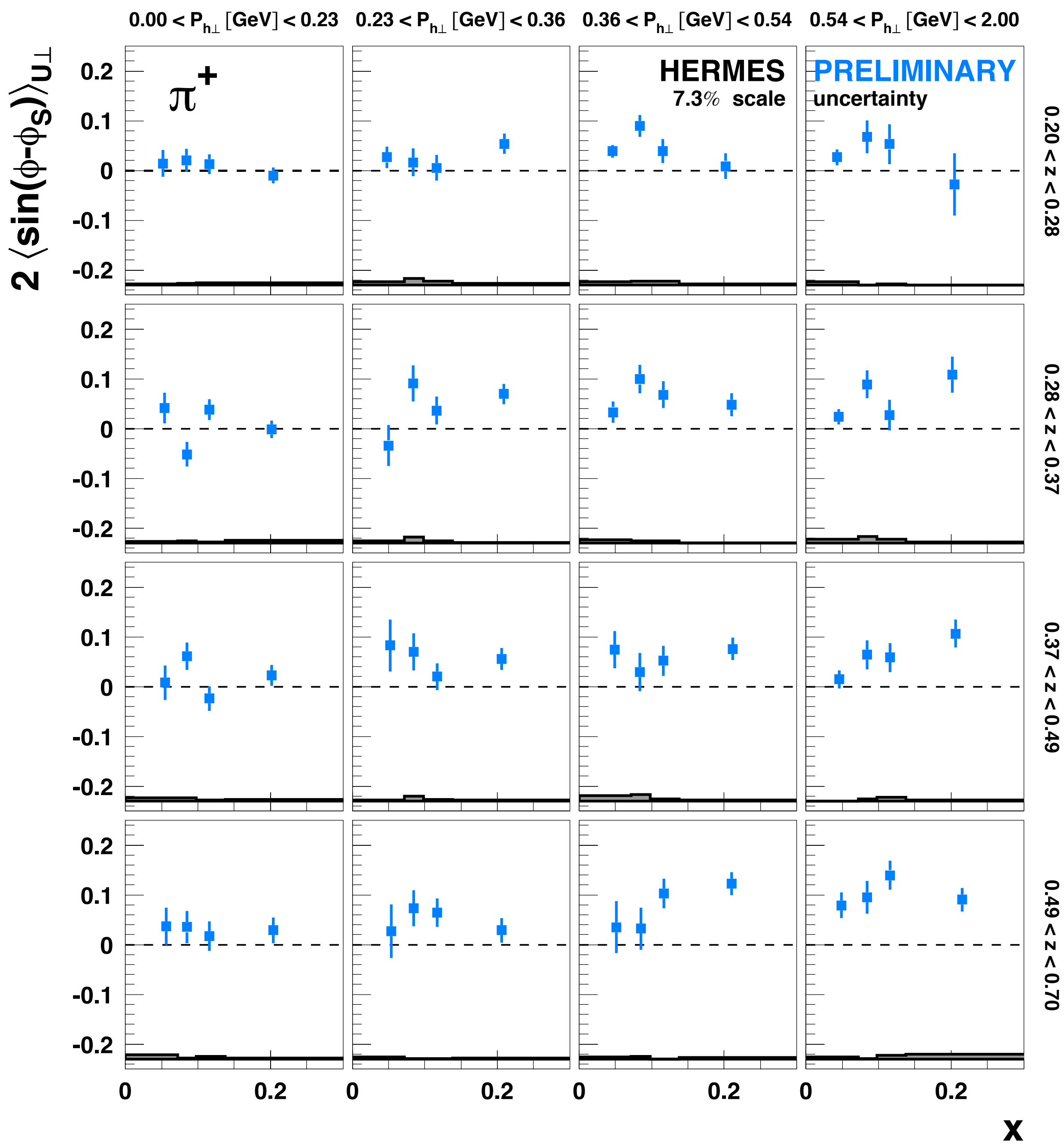

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new results 2014 (talk by C. van Hulse)

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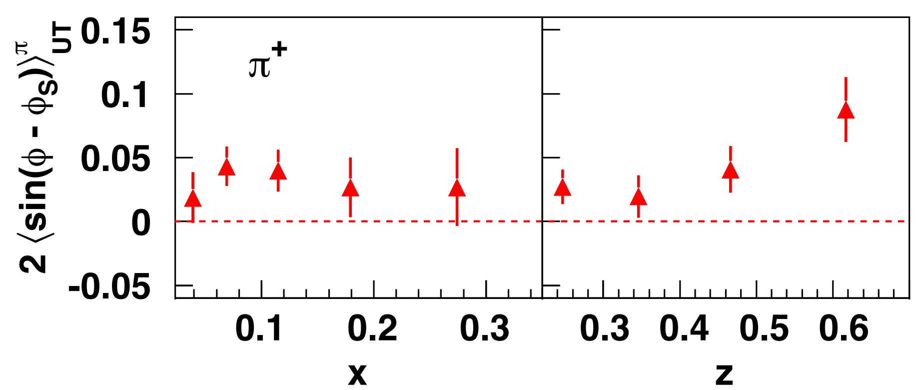
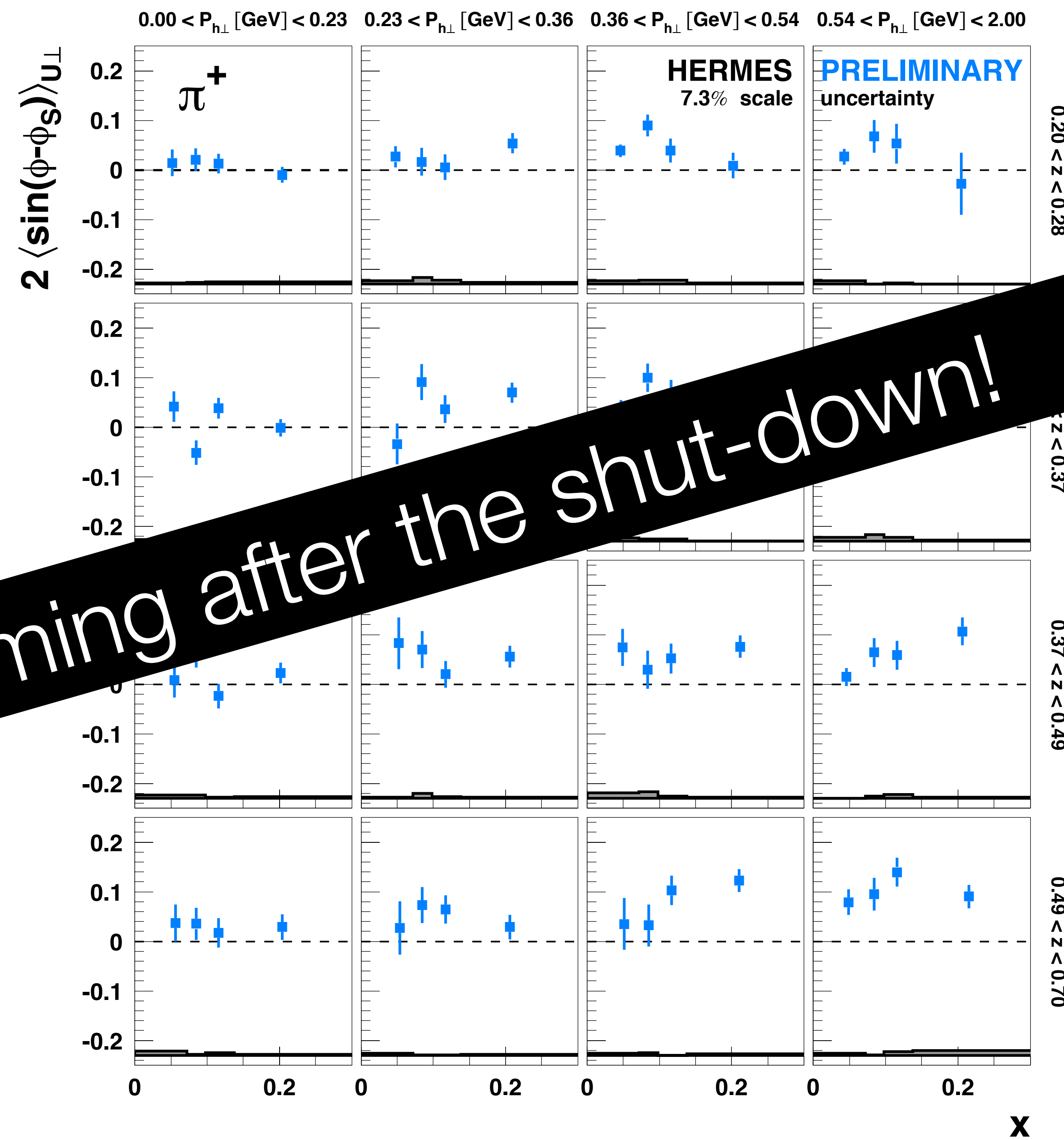


HERMES, PRL 94 (2005)

Amazing progress... coming after the shut-down!



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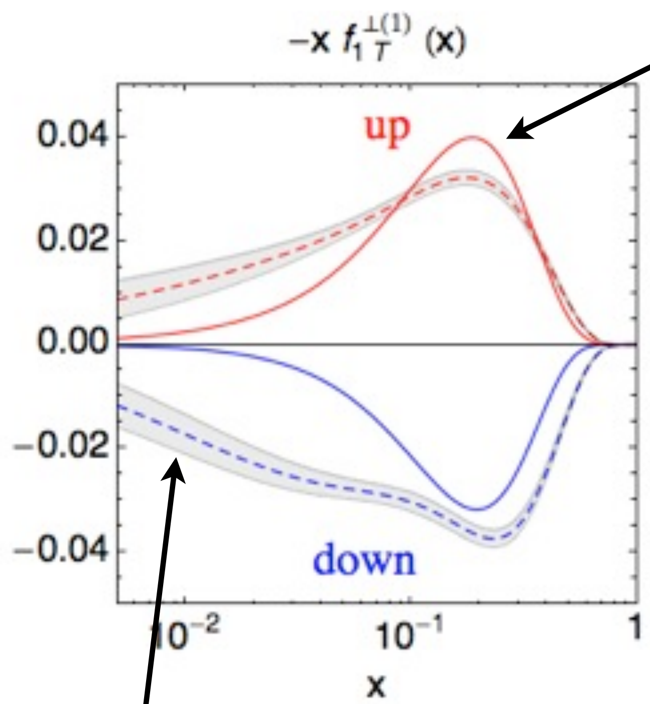
What can you get out?

*Echevarria, Idilbi, Kang, Vitev, PRD 89 (14) [**with TMD evo**]*



*Bacchetta, Radici, PRL 107 (11) [**no TMD evo**]*

ubar

dbar

Extraction of Sivers function

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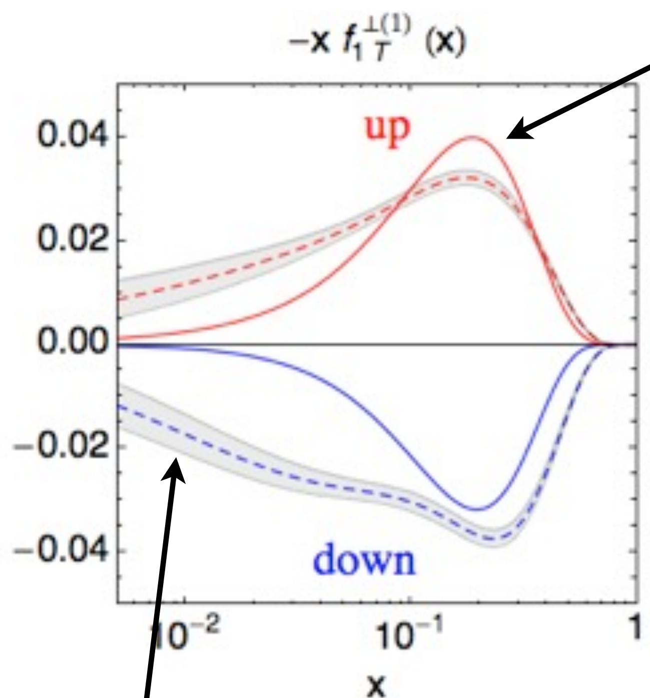

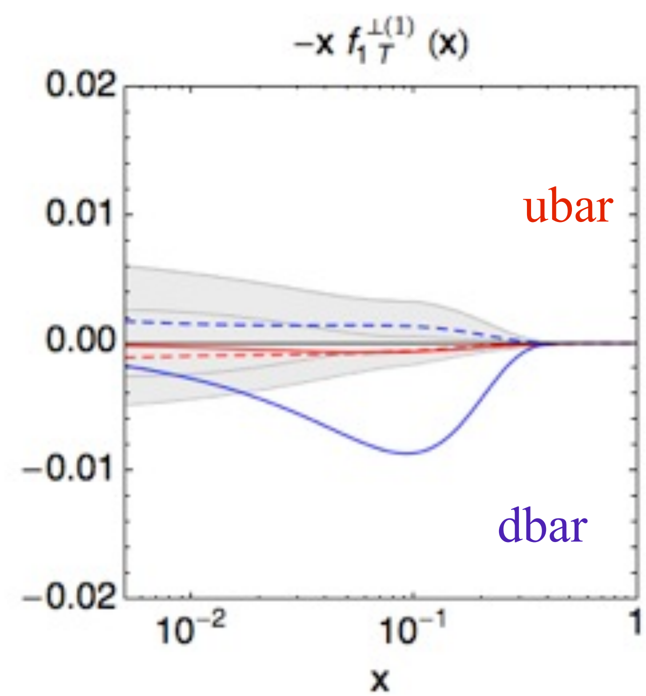

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Extraction of Sivers function

What can you get out?

Anselmino, Boglione, Melis, PRD86 (12)
*[**with TMD evo**]*

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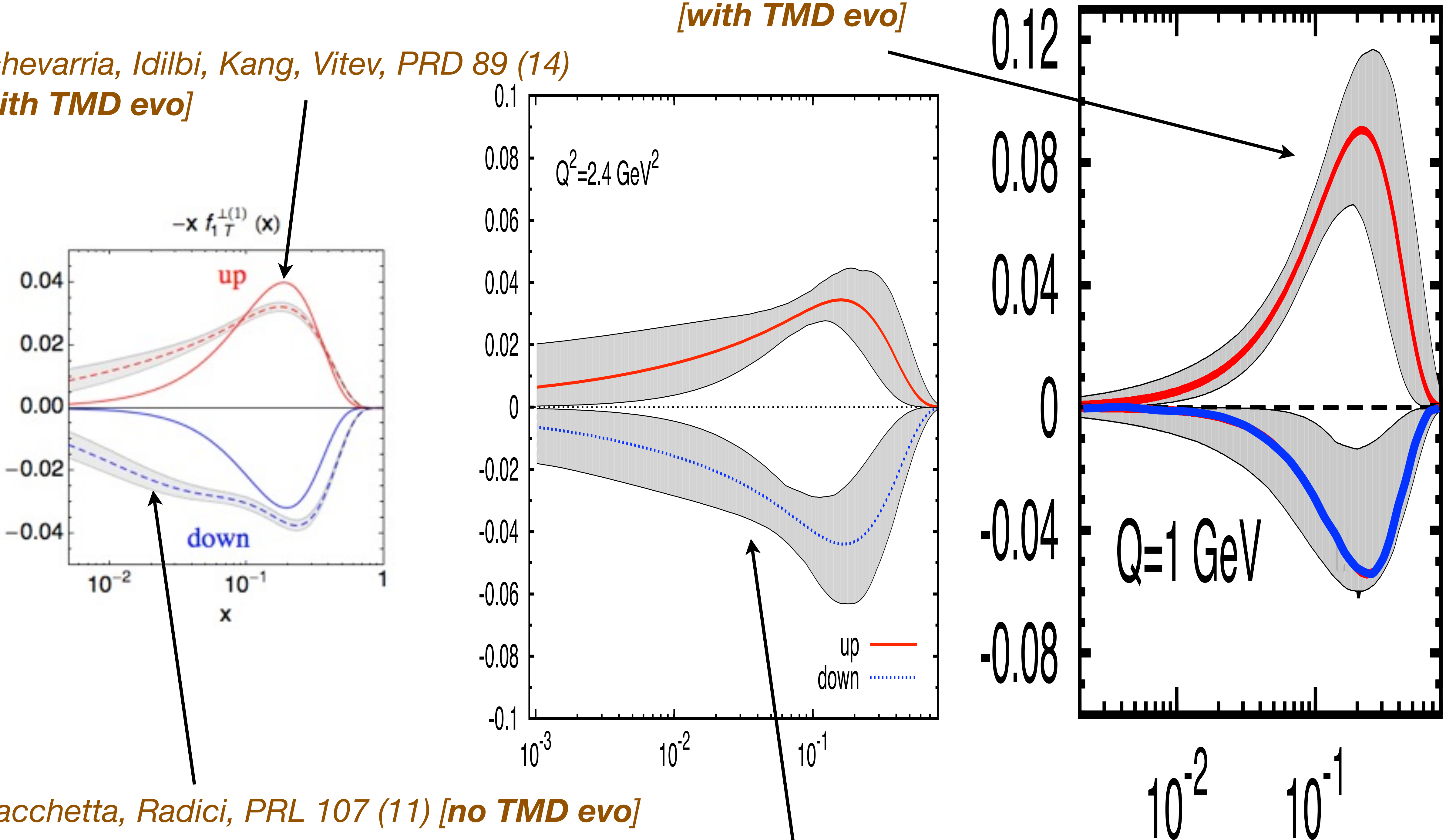
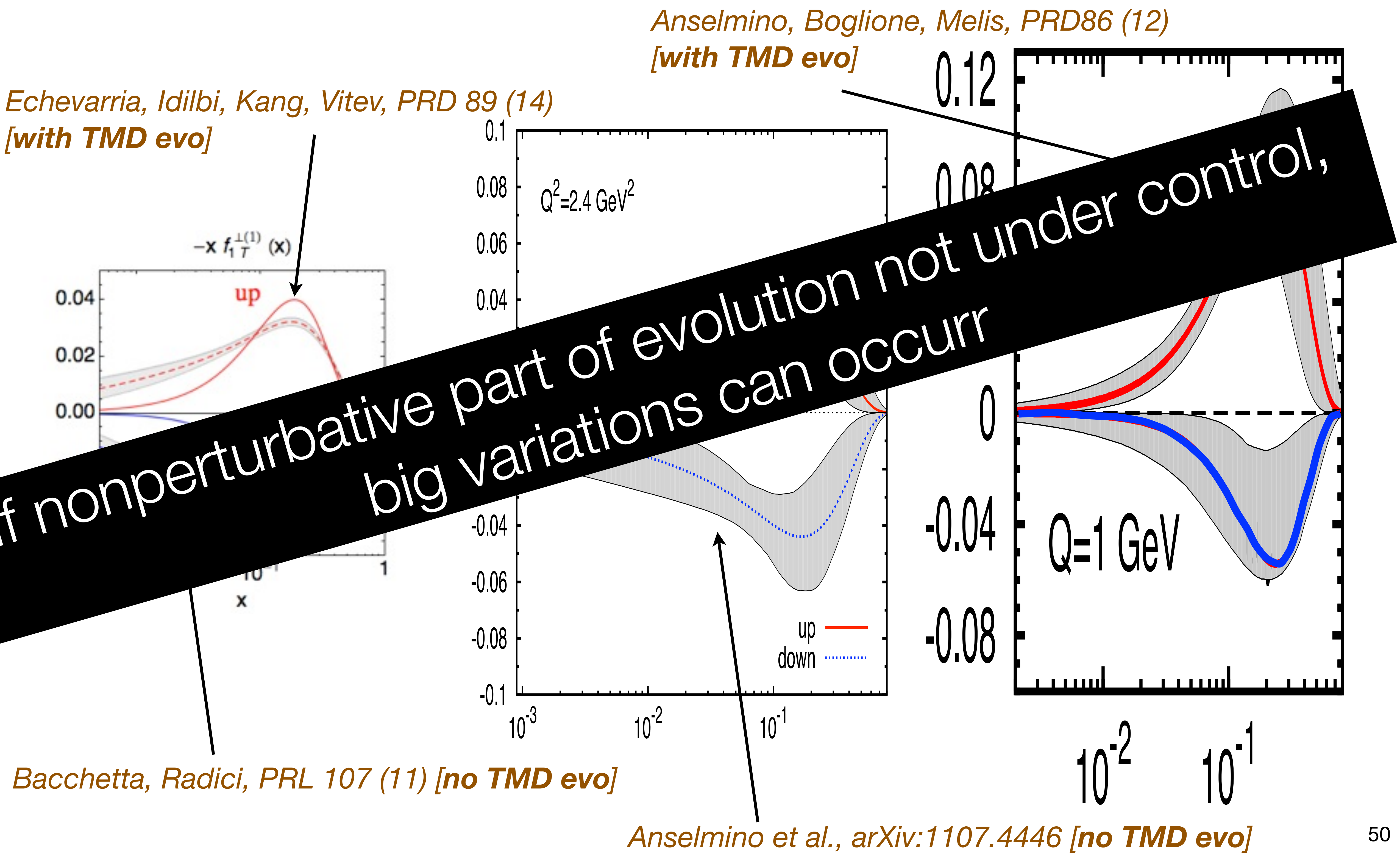


Bacchetta, Radici, PRL 107 (11) [**no TMD evo**]

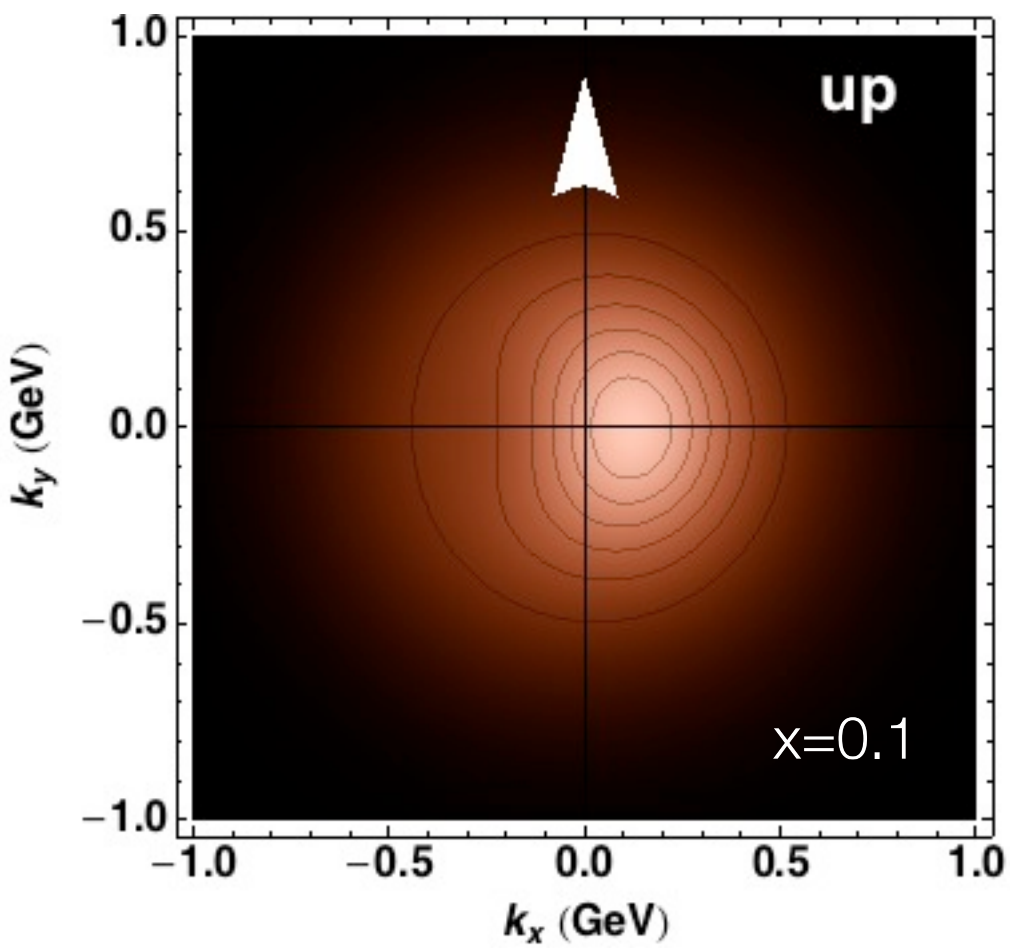
Anselmino et al., arXiv:1107.4446 [**no TMD evo**]

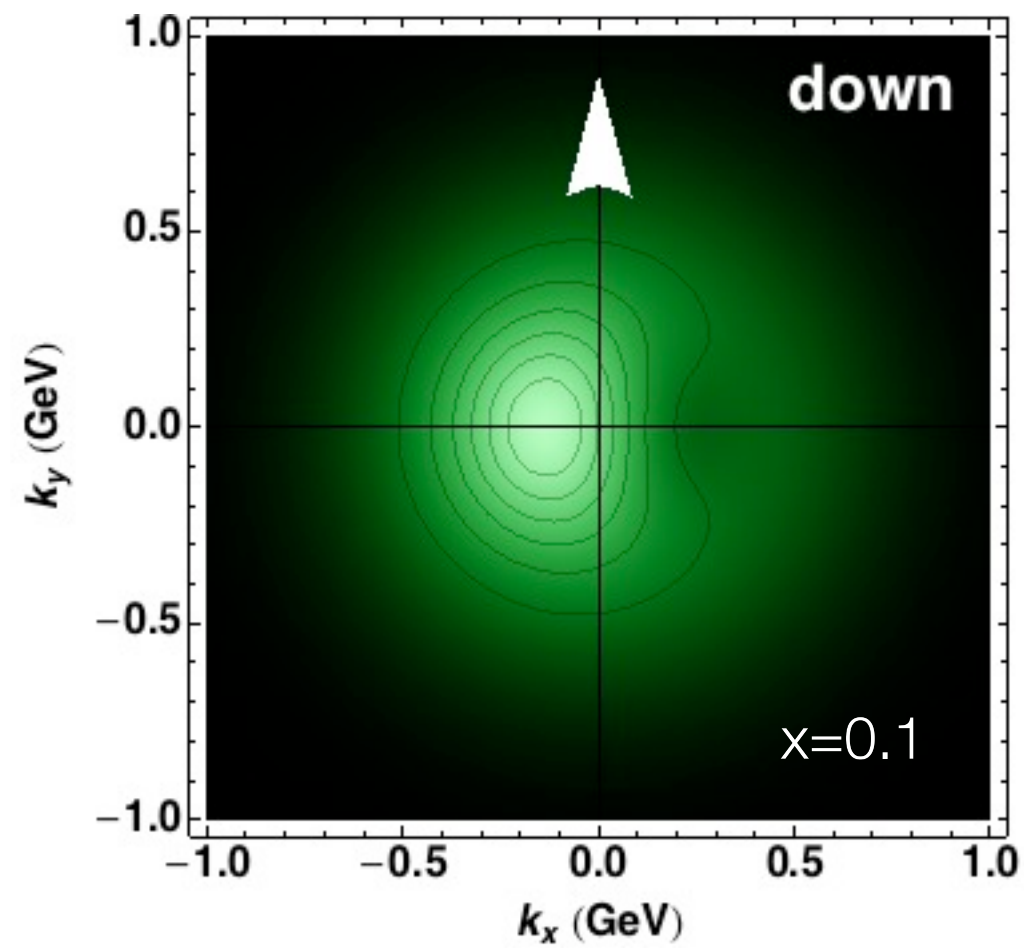
What can you get out?

If nonperturbative part of evolution not under control,
big variations can occurr



3D mapping with spin





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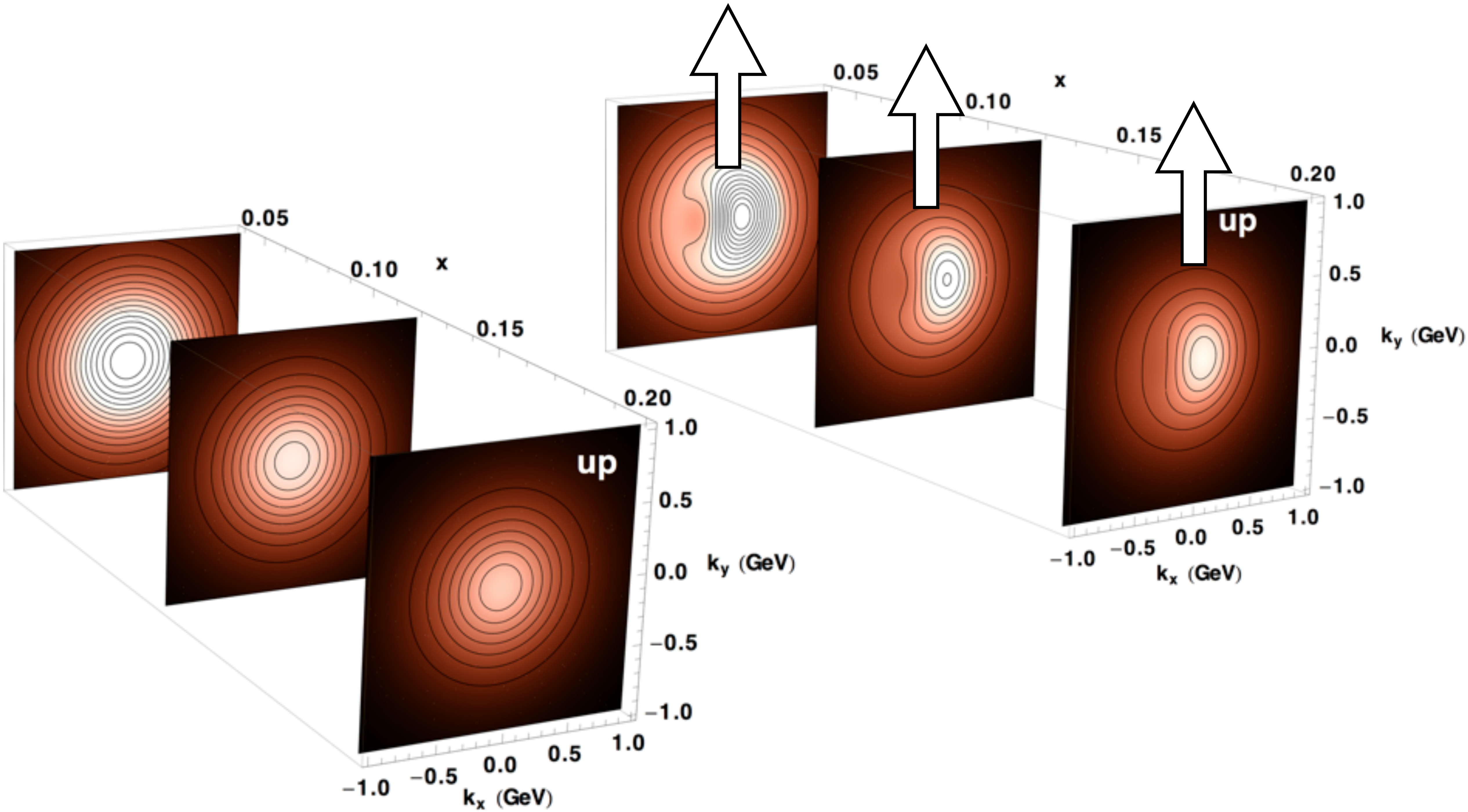
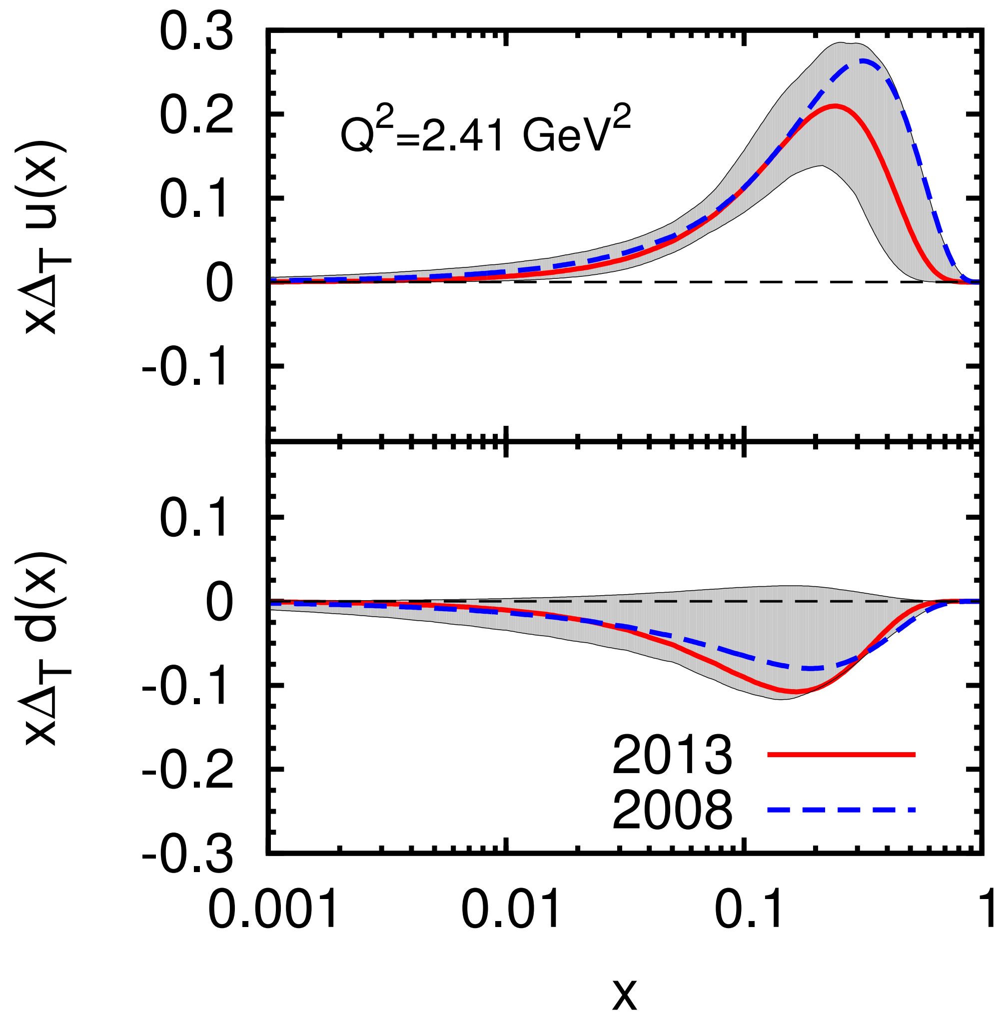
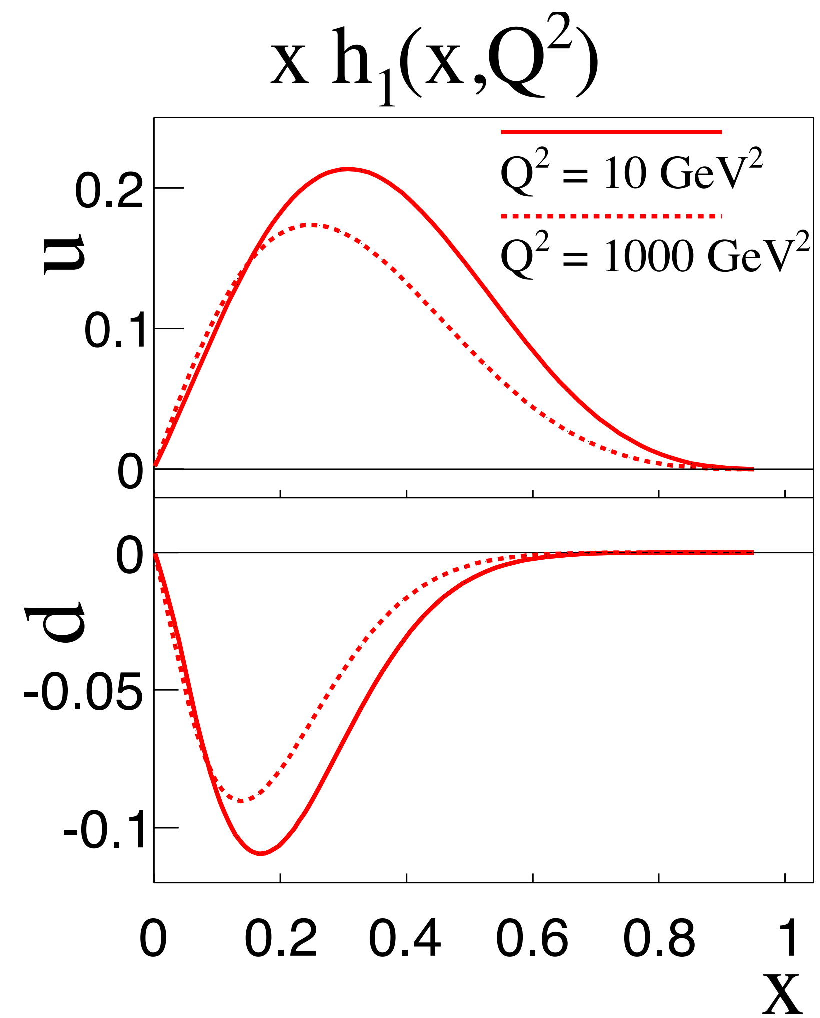


Collins asymmetry and transversity

$$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$$



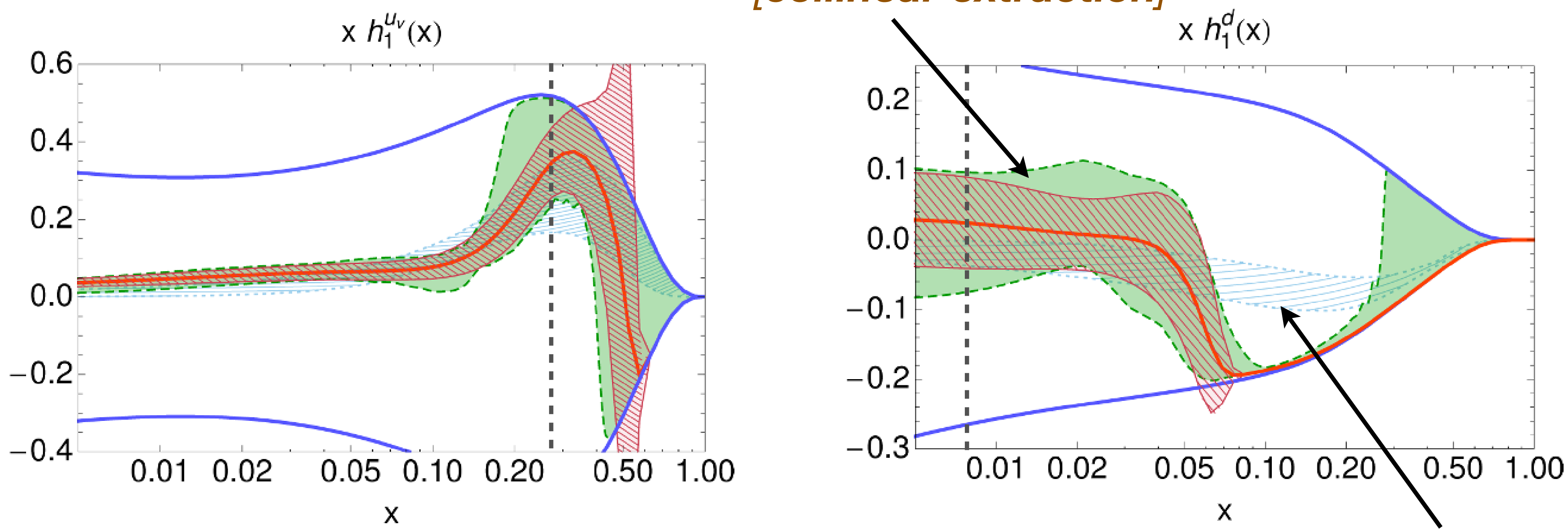
Anselmino et al., PRD87 (2013)
[no TMD evo]



Kang, Prokudin, Sun, Yuan, arXiv:1410.4877
[with TMD evo]

Dihadron DIS production and transversity

Bacchetta, Courtoy, Radici, JHEP 1303 (2013)
[collinear extraction]



Anselmino et al. 0812.4366
[TMD extraction, no TMD evo]

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Bacchetta, Courtoy, Radici, JHEP 1303 (2013)
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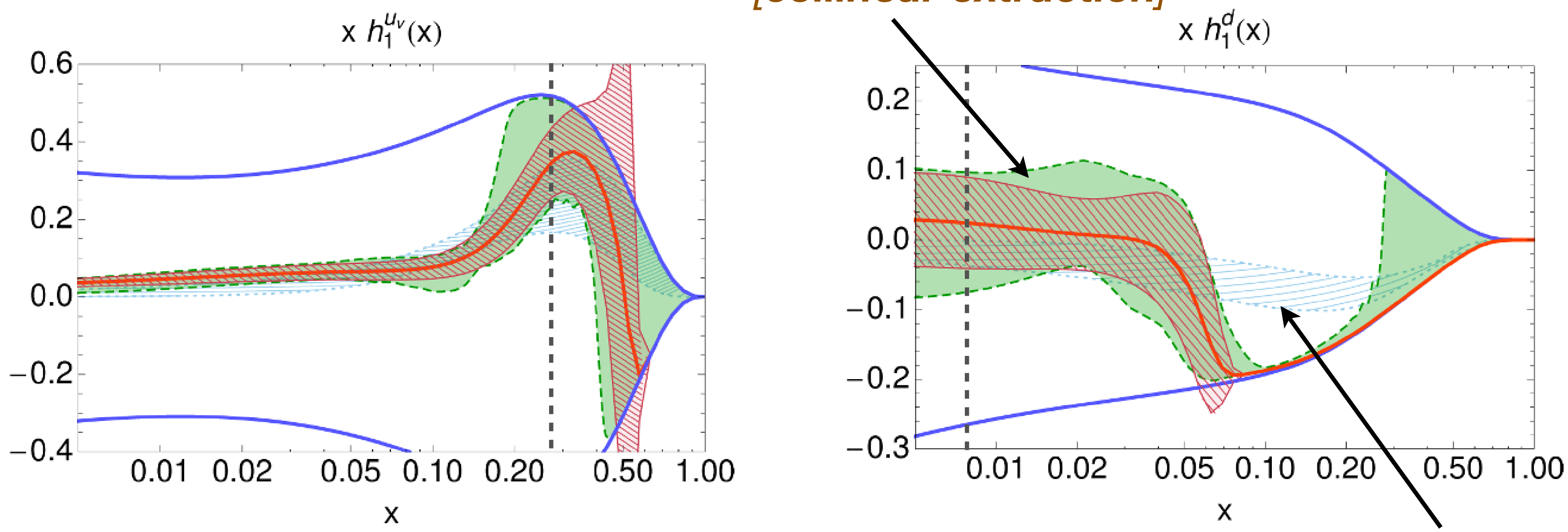

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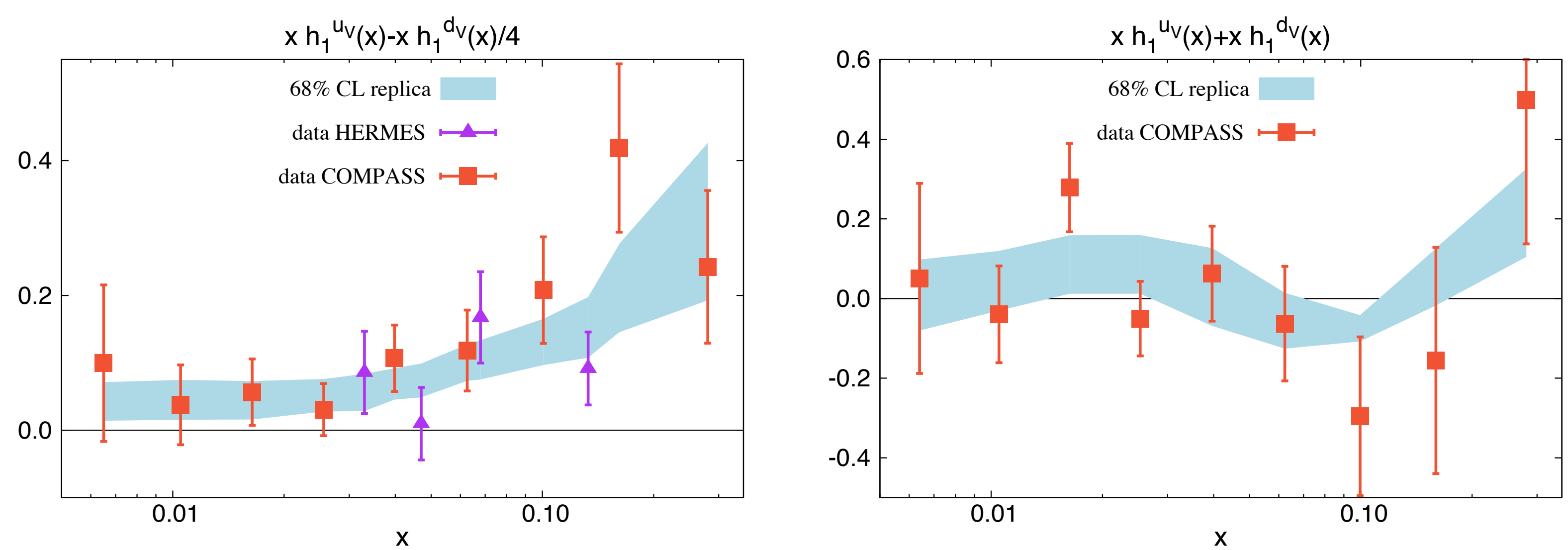

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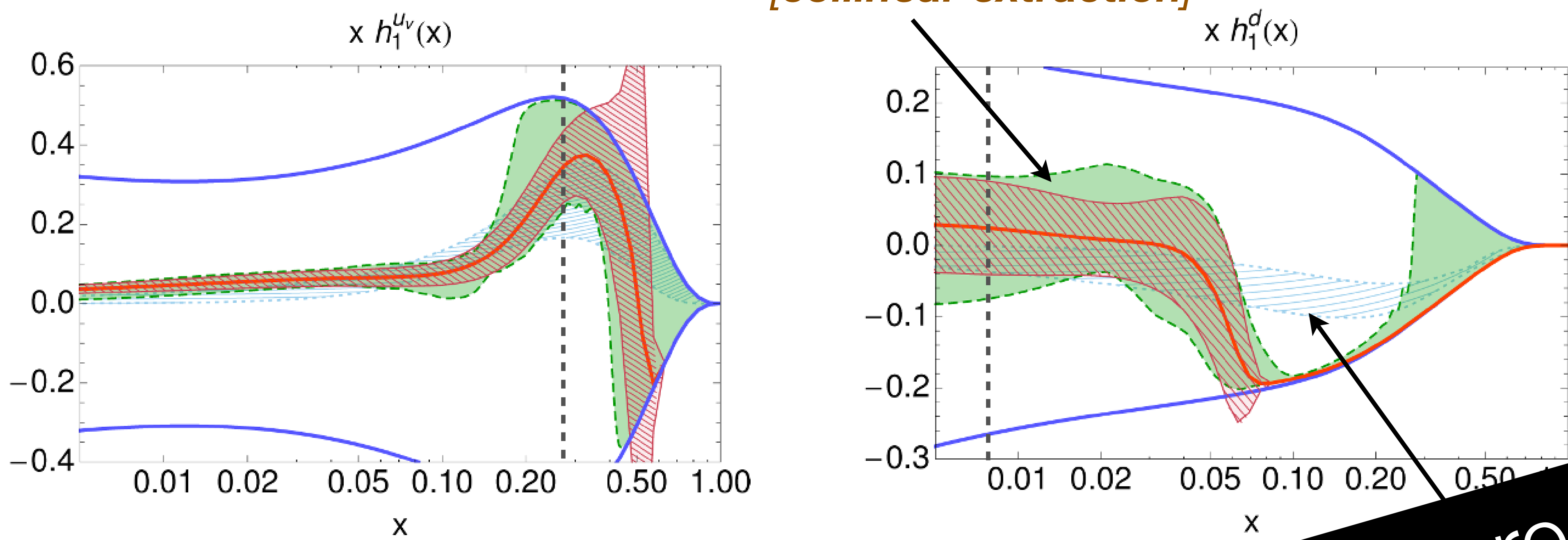
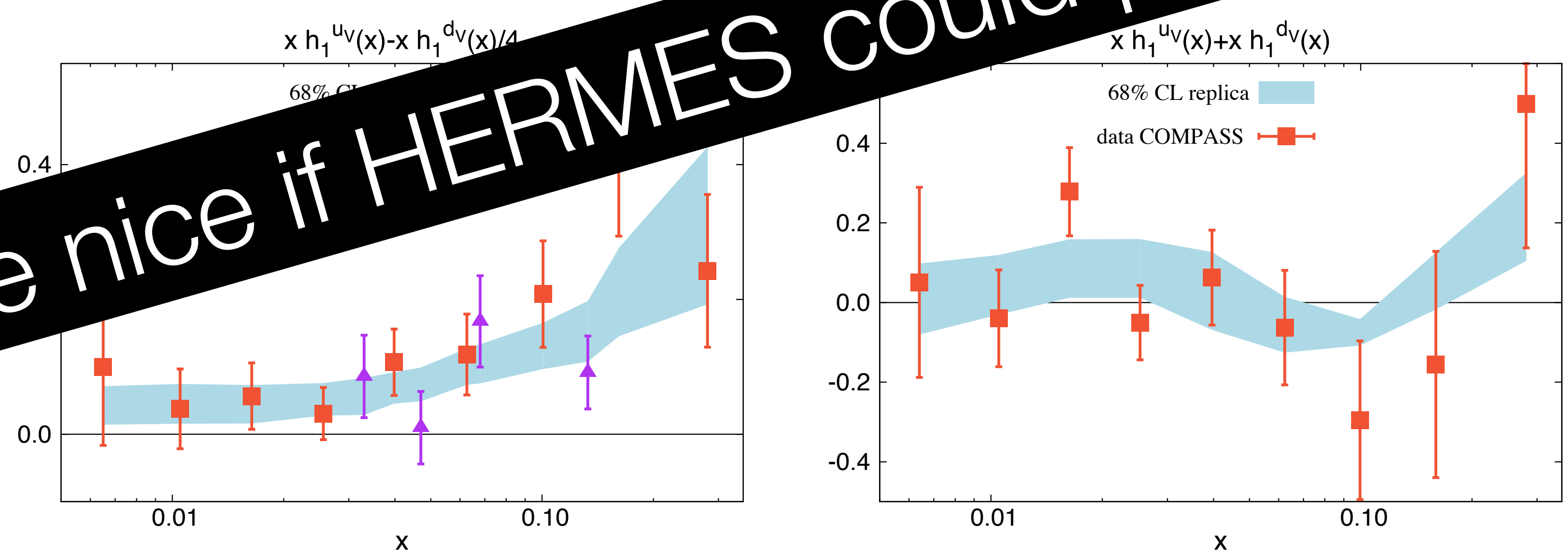


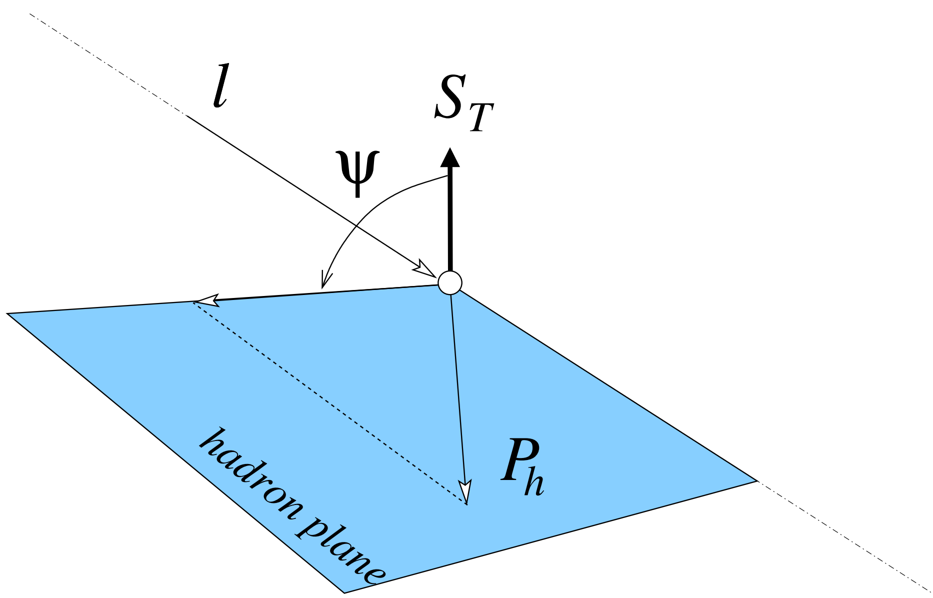
…312.4366
…xtraction, no TMD evo]



It would be nice if HERMES could provide more data

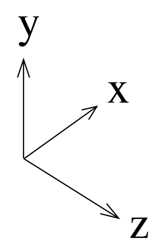
Inclusive hadron production

$$\ell(l) + N(P) \to h(P_h) + X,$$



Λ_{QCD} = hadronic scale

$P_{h\perp}$ = hadron transverse momentum



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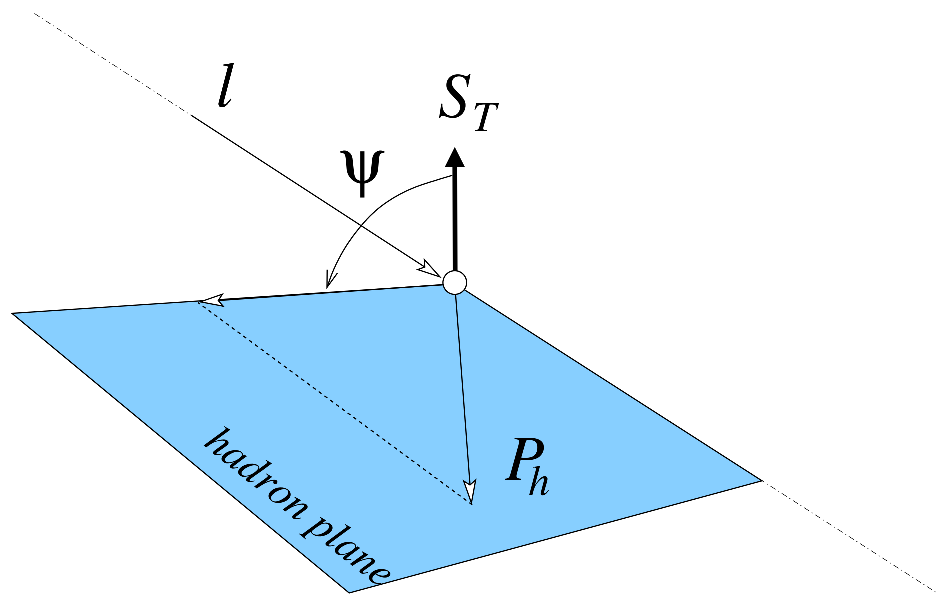

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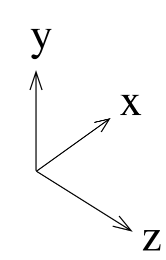
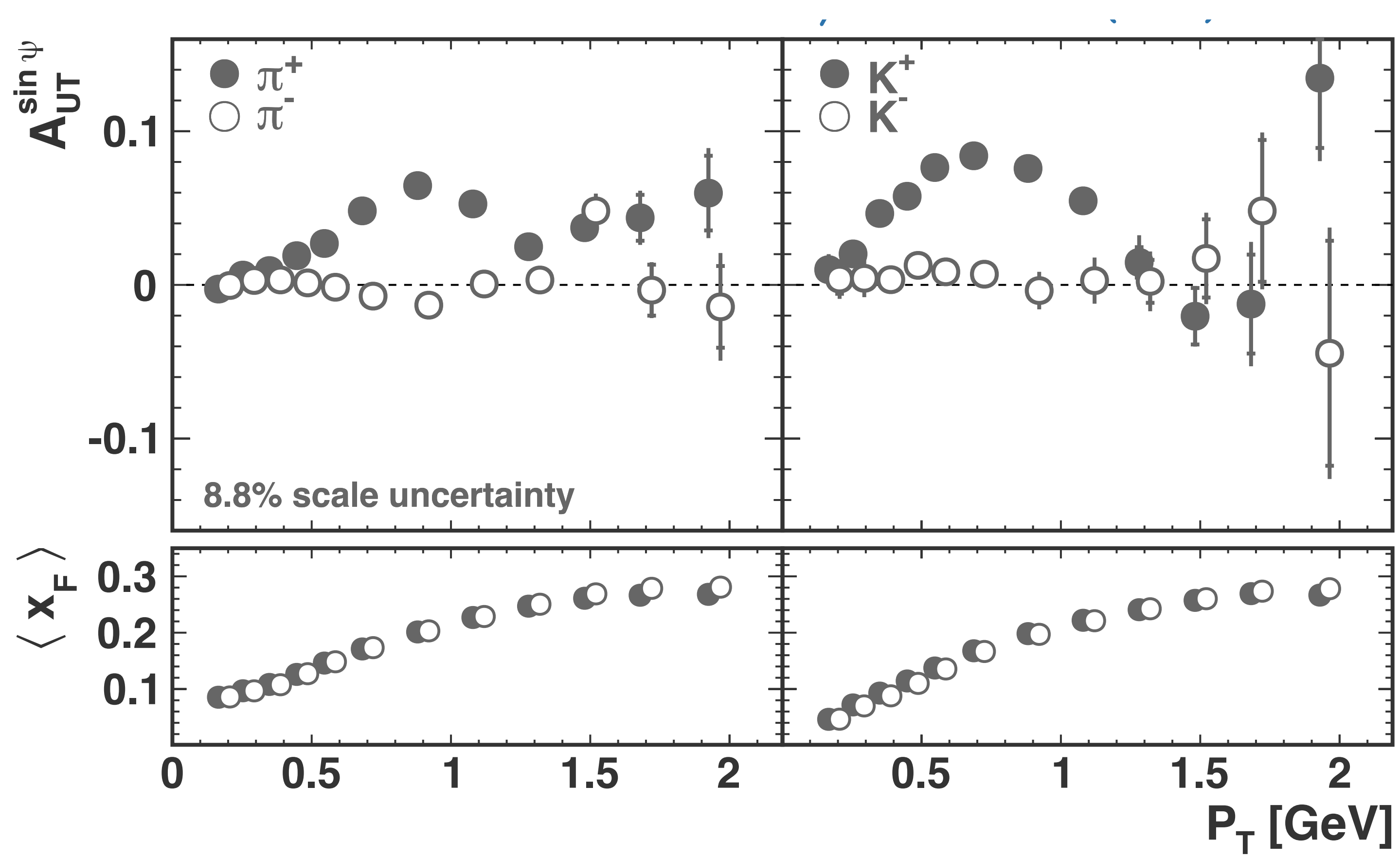
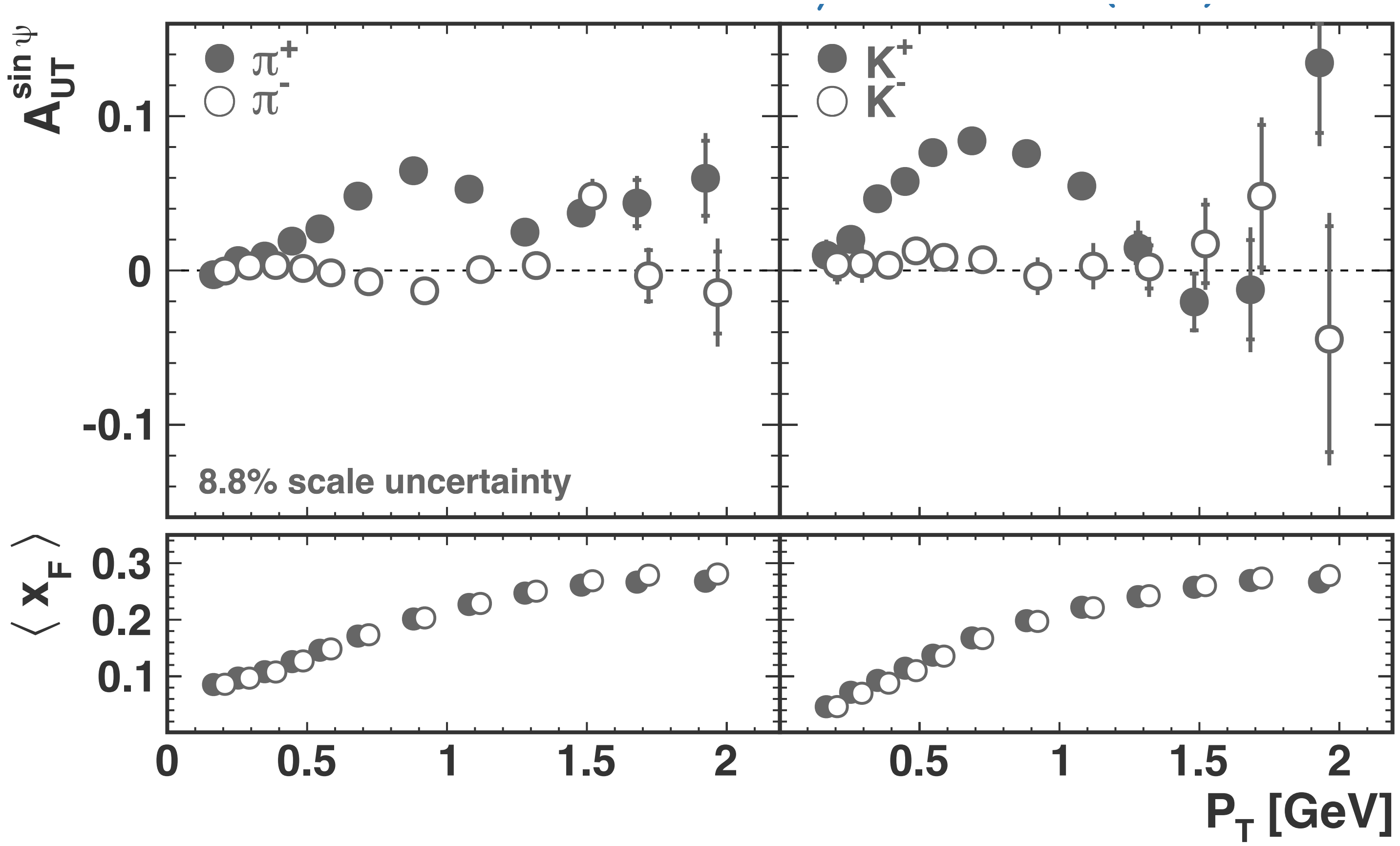


see talk by C. van Hulse


$A_{UT}^{\sin\psi}$
π^+
π^-
K^+
K^-
0.1
0
-0.1
8.8% scale uncertainty
$\langle x_F \rangle$
0.3
0.2
0.1
0
0.5
1
1.5
2
P_T [GeV]

HERMES, PLB 728 (14)



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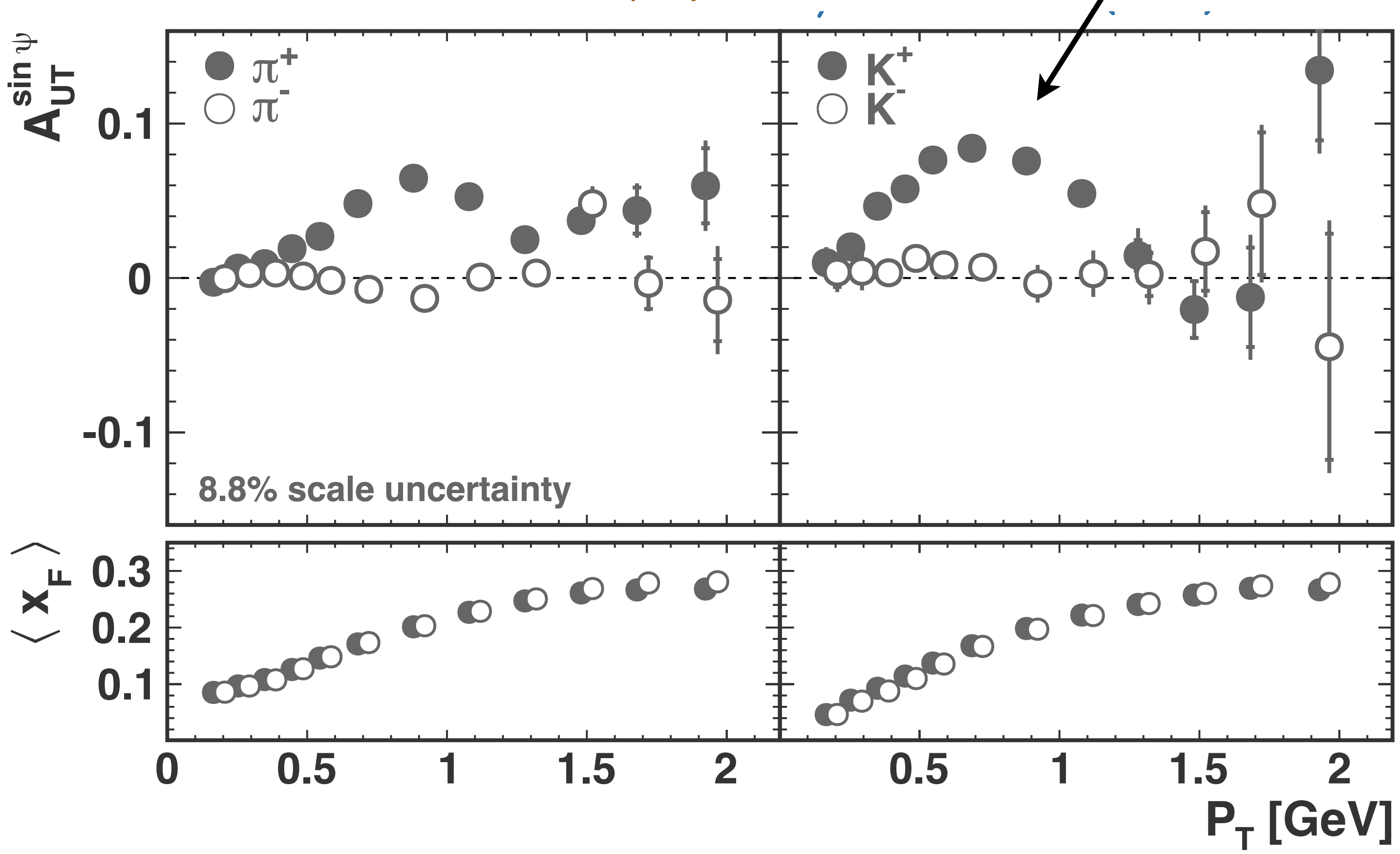

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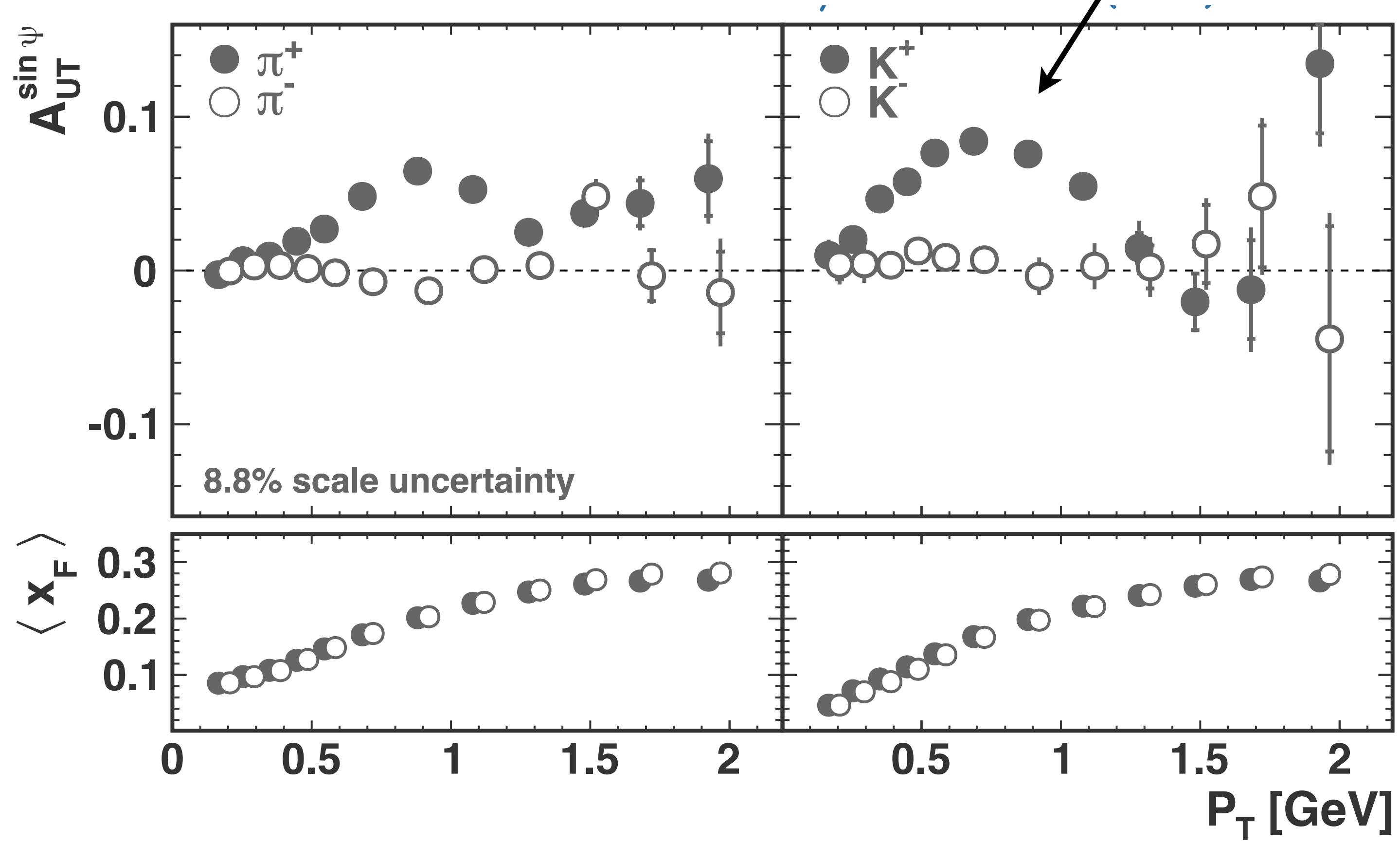

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