

# *Precise parton distributions*

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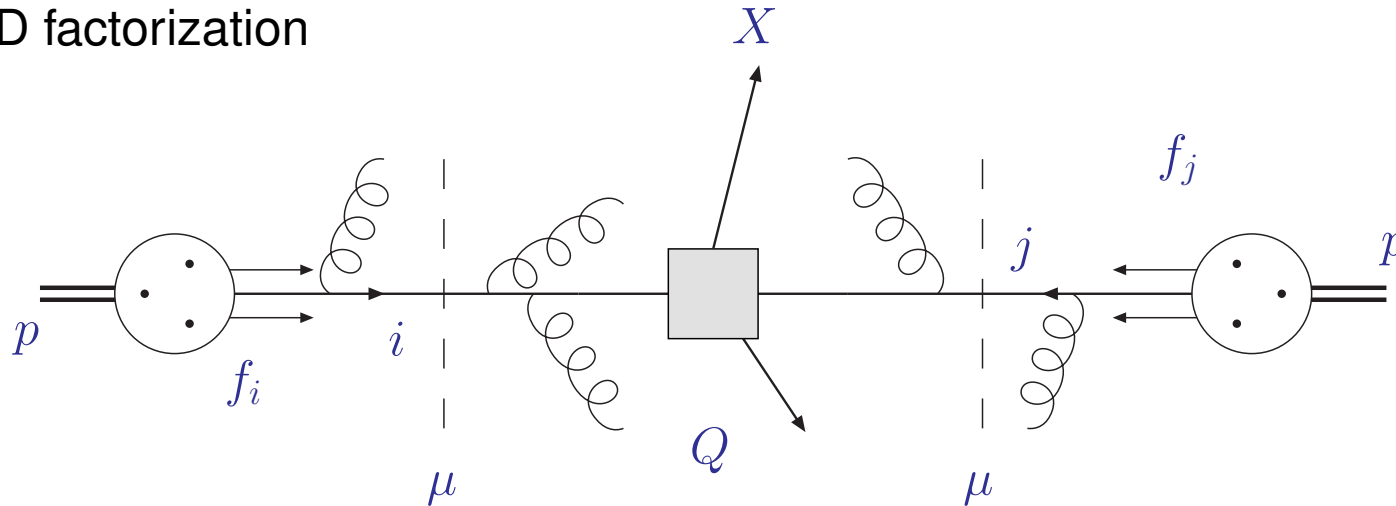
*Universität Hamburg & DESY, Zeuthen*

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Workshop *Future Physics with HERA Data for Current and Planned Experiments*, Hamburg, Nov 12, 2014

# QCD factorization

- QCD factorization

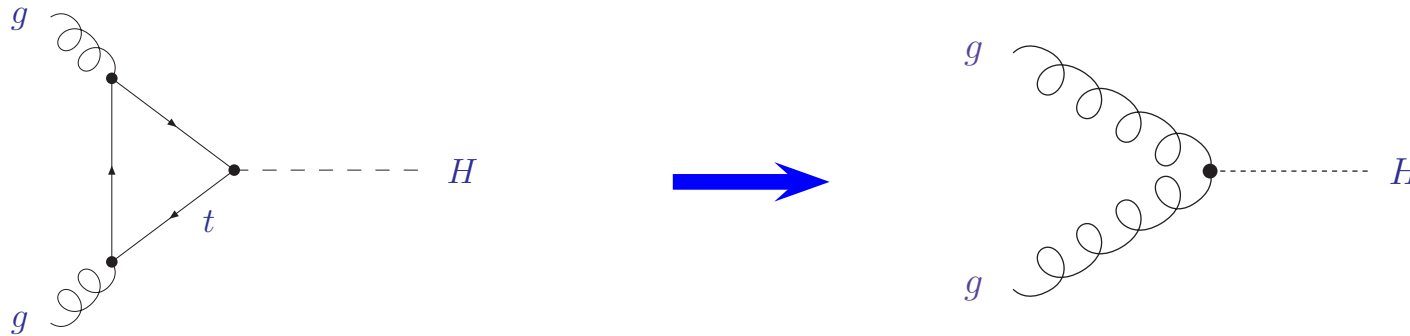


$$\sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2)$$

- Hard parton cross section  $\hat{\sigma}_{ij \rightarrow X}$  calculable in perturbation theory
  - known to NLO, NNLO, ... ( $\mathcal{O}(\text{few}\%)$  theory uncertainty)
- Non-perturbative parameters: parton distribution functions  $f_i$ , strong coupling  $\alpha_s$ , particle masses  $m_X$ 
  - known from global fits to exp. data, lattice computations, ...

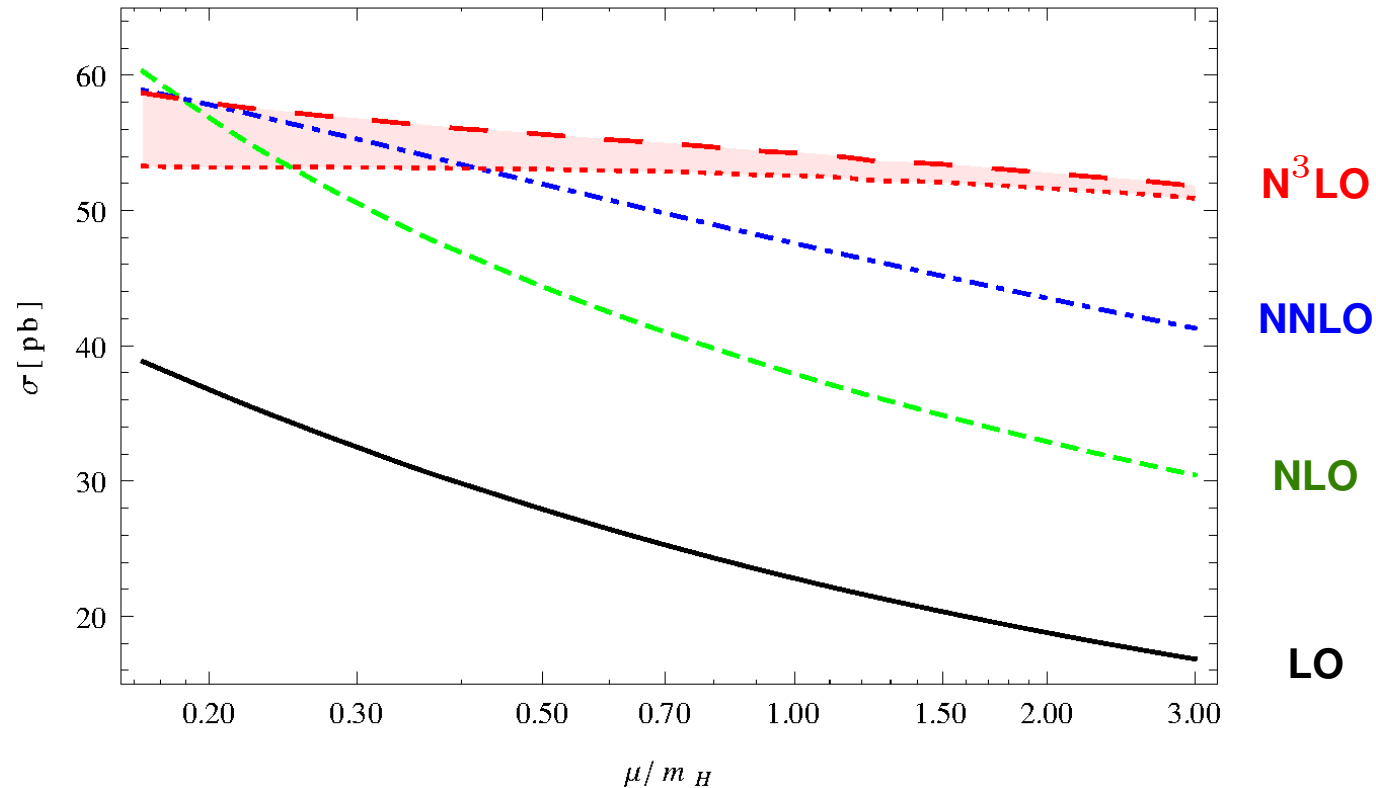
# Higgs production in $gg$ -fusion

## Effective theory



- Hard scattering cross section  $\hat{\sigma}_{ij \rightarrow H+X}$  dominated by gluon-gluon fusion
  - typically treated in effective theory in limit  $m_t \rightarrow \infty$ ; Lagrangian
$$\mathcal{L} = -\frac{1}{4} \frac{H}{v} C_H G^{\mu\nu a} G_{\mu\nu}^a$$
- QCD corrections significant
  - NNLO corrections still large  
Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement with soft N<sup>3</sup>LO corrections S.M., Vogt '05; Laenen, Magnea '05:  
in  $x$ -space '+'-distributions  $\alpha_s^k \ln^{2k-1}(1-x)/(1-x)$
  - NNLL resummation Catani, de Florian, Grazzini, Nason '03; Ahrens et al. '10; [...];  
Ahmed, Mahakhud, Rana, Ravindran '14
  - soft-virtual corrections completed recently:  $\delta(1-x)$ -term at N<sup>3</sup>LO  
Anastasiou et al.'14

# Perturbation theory at work



- Apparent convergence of perturbative expansion
  - improvement with soft-virtual **N<sup>3</sup>LO** corrections (all powers of  $\ln^k N$ ) supplemented by  $\ln^k N/N$  corrections from physical kernels  
de Florian, Mazzitelli, S.M., Vogt '14
- Perturbative stability under renormalization scale variation
  - point of minimal sensitivity around  $\mu = m_H/2$
  - overall theoretical uncertainty  $\Delta\sigma(H)$  less than 5%

# Dependence on parton distributions

- NNLO cross section  $\sigma(gg \rightarrow H + X)$  at LHC with uncertainties:

$$\sigma(H) + \Delta\sigma(\text{scale}) + \Delta\sigma(\text{PDF} + \alpha_s)$$

	LHC7	LHC8	LHC14
ABM11	13.23 <sup>+1.35</sup> <sub>-1.31</sub> <sup>+0.30</sup> <sub>-0.30</sub>	16.99 <sup>+1.69</sup> <sub>-1.63</sub> <sup>+0.37</sup> <sub>-0.37</sub>	44.68 <sup>+4.02</sup> <sub>-3.78</sub> <sup>+0.85</sup> <sub>-0.85</sub>
ABM12	13.28 <sup>+1.35</sup> <sub>-1.32</sub> <sup>+0.31</sup> <sub>-0.31</sub>	17.05 <sup>+1.68</sup> <sub>-1.64</sub> <sup>+0.39</sup> <sub>-0.39</sub>	44.81 <sup>+4.01</sup> <sub>-3.80</sub> <sup>+0.94</sup> <sub>-0.94</sub>
HiggsXSWG	15.13 <sup>+1.07</sup> <sub>-1.18</sub> <sup>+1.15</sup> <sub>-1.07</sub>	19.27 <sup>+1.39</sup> <sub>-1.50</sub> <sup>+1.45</sup> <sub>-1.33</sub>	49.85 <sup>+6.08</sup> <sub>-4.19</sub> <sup>+3.69</sup> <sub>-3.09</sub>

- Comparison for PDF sets at NNLO
  - ABM11, ABM12 [Alekhin, Blümlein, S.M. '13](#) and HiggsXSWG (used by ATLAS and CMS, relying on MSTW08 [Martin, Stirling, Thorne, Watt '09](#) and with soft gluon resummation)
- Largest differences in predictions from PDFs and value of  $\alpha_s$ 
  - ABM central values significantly lower by some 11 – 14%
- PDF and  $\alpha_s$  differences significantly larger than residual theory uncertainty due to incomplete N<sup>3</sup>LO QCD corrections

# Non-perturbative parameters

## Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparison to experimental data
  - masses of heavy quarks  $m_c, m_b, m_t$
  - parton distribution functions  $f_i(x, \mu^2)$
  - strong coupling constant  $\alpha_s(M_Z)$

## Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - renormalization and factorization scales  $\mu_R, \mu_F$
  - chosen scheme (e.g.  $\overline{MS}$  scheme)
  - ...

# Parton distribution fits

## Global PDF fits

- PDF sets currently available
  - ABM12 Alekhin, Blümlein, S.M. '13
  - CT10 Gao et al. '13
  - HERAPDF (v1.5) H1 & ZEUS Coll. '11
  - JR09 Jimenez-Delgado, Reya '09
  - MSTW Martin, Stirling, Thorne, Watt '09
  - NNPDF (v3.0) Ball et al. '14

## Iterative cycle of PDF fits

- i) check of compatibility of new data set with available world data
- ii) study of potential constraints due to addition of new data set to fit
- iii) perform high precision measurement of the non-perturbative parameters
  - parton distributions
  - strong coupling  $\alpha_s(M_Z)$
  - heavy quark masses

# LHC measurements

## General remarks

- QCD corrections important
  - require theory predictions to NNLO accuracy
- PDF fits with 3-flavors for DIS, 5-flavors for LHC data (matching from 3 to 5-flavors)
  - QCD evolution over large range

## Benchmark processes

- Complete NNLO QCD corrections available for
  - $W^\pm$ - and  $Z$ -boson production  
Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - top-quark hadro-production Czakon, Fiedler, Mitov '13
- Jet data from Tevatron and LHC
  - QCD corrections only NLO known
  - possible impact of jet definition and algorithm
  - ongoing effort towards NNLO (corrections expected to be as big as  $\mathcal{O}(15 - 20\%)$ ) Gehrmann-De Ridder, Gehrmann, Glover, Pires '13



# Analysis choices

## Data considered in the fit (ABM PDFs as example)

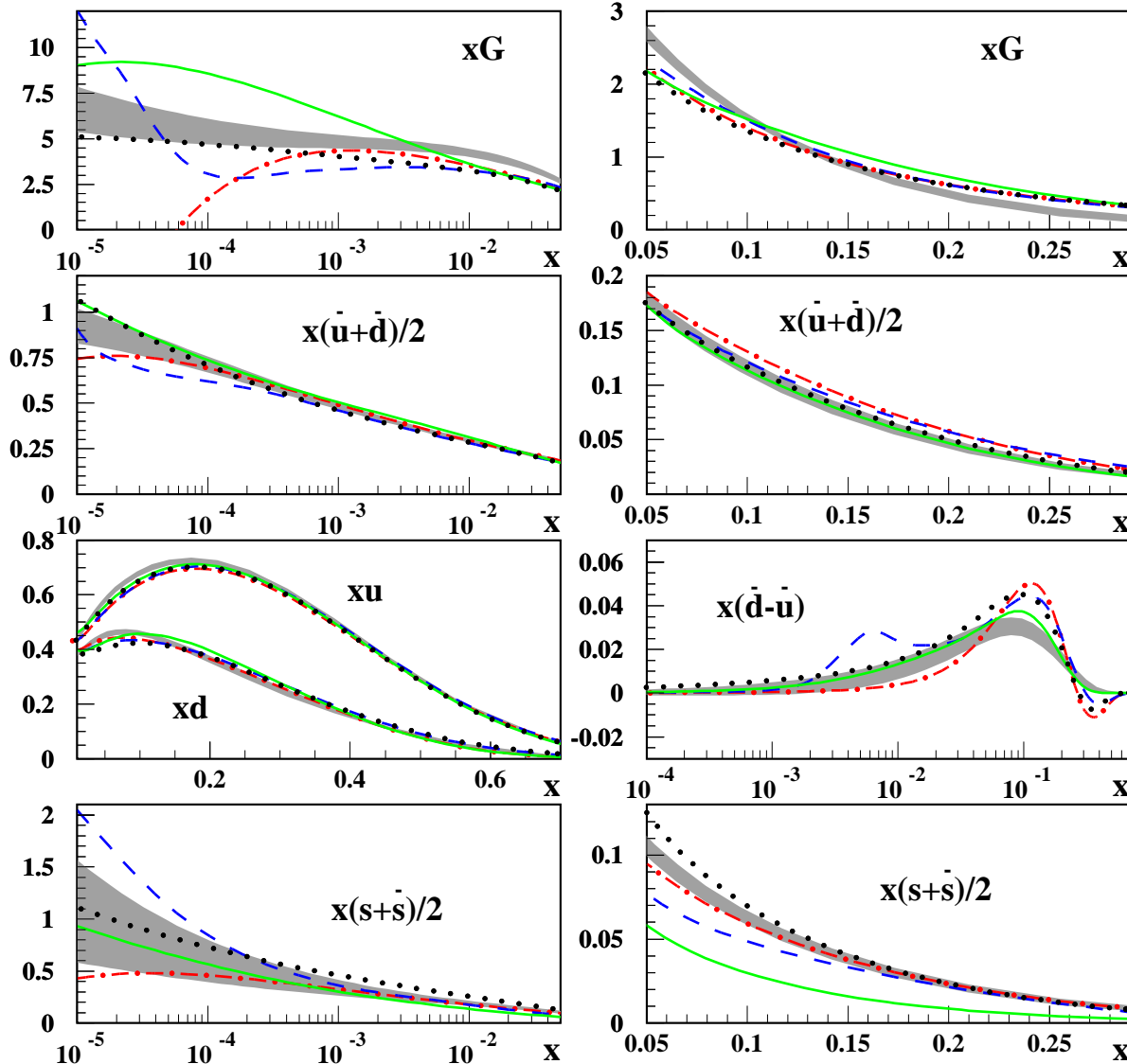
- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC
  - semi-inclusive DIS charm production data (HERA)
  - Drell-Yan data (fixed target) E-605, E-866
  - neutrino-nucleon DIS data (di-muon production) CCFR/NuTeV
  - LHC data for  $W^\pm$ - and  $Z$ -boson production

## Theory considerations

- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Determination of PDFs and strong coupling constant  $\alpha_s$  to NNLO QCD
- Consistent scheme for treatment of heavy quarks
  - fixed-flavor number scheme for  $n_f = 3, 4, 5$
  - $\overline{\text{MS}}$ -scheme for quark masses and  $\alpha_s$
- Full account of error correlations

# Parton distributions tuned to LHC data

$\mu=2 \text{ GeV}, n_f=4$



- $1\sigma$  band for ABM12 PDFs (NNLO, 4-flavors) at  $\mu = 2 \text{ GeV}$   
Alekhin, Blümlein, S.M.'13
- comparison with:  
JR09 (solid lines),  
MSTW (dashed dots),  
NN23 (dashes) and  
CT10 (dots)
- Observed differences due to differences in theory considerations and analysis procedures

# *Examples*

- Strong coupling constant
- Quark masses
- Quark flavor separation

# Strong coupling constant

- Values of  $\alpha_s(M_Z)$  at NNLO from PDF fits  
(compilation from proceedings of MITP workshop [arXiv:1405.4781](https://arxiv.org/abs/1405.4781))

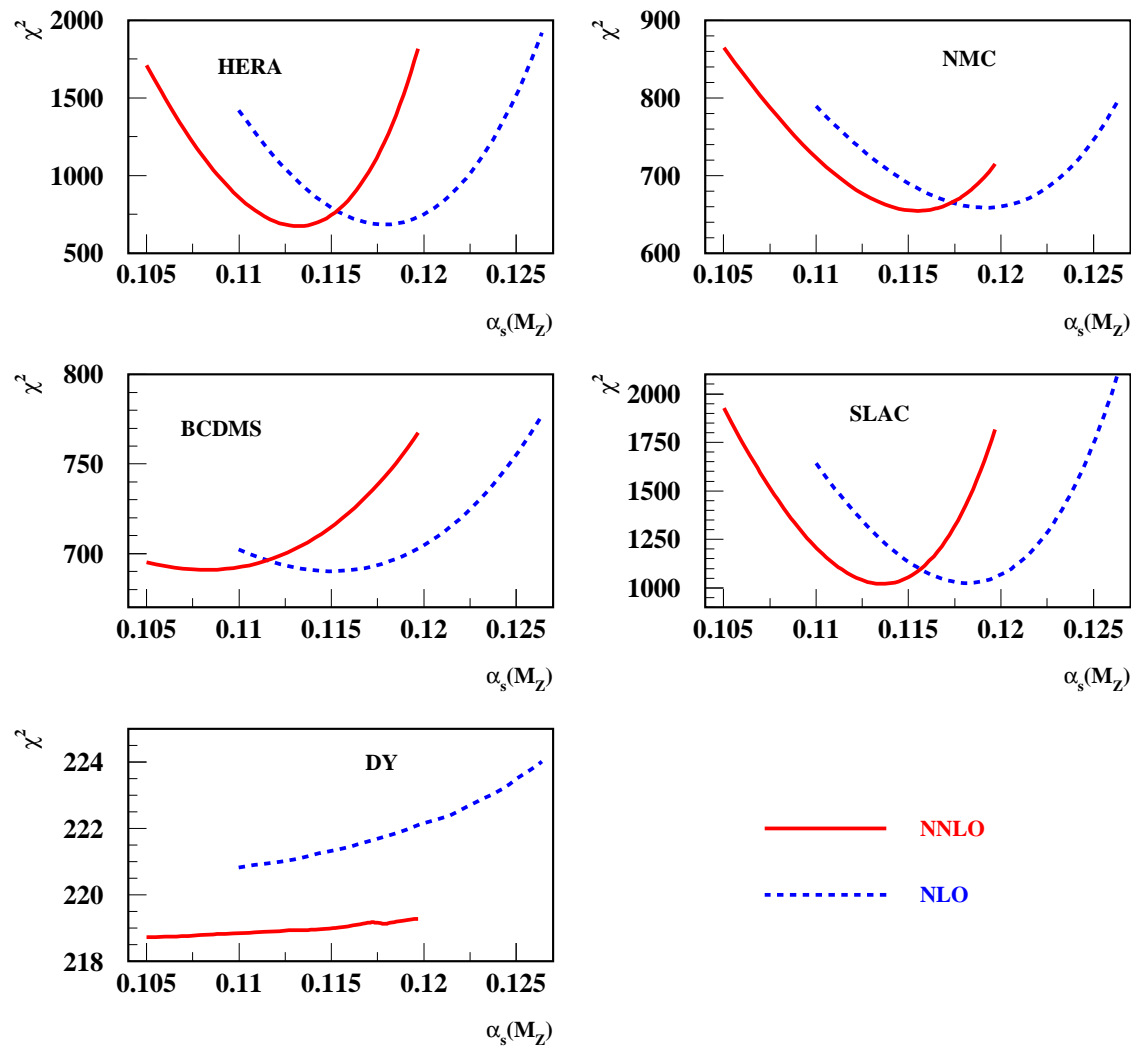
Alekhin [2001]	$0.1143 \pm 0.013$	DIS	Alekhin '01
BBG [2004]	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti '06
GRS	0.112	valence analysis, NNLO	Glück, Reya, Schuck '06
ABKM	$0.1135 \pm 0.0014$	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. '09
JR14	$0.1136 \pm 0.0004$	dynamical approach	Jimenez-Delgado, Reya '14
JR14	$0.1162 \pm 0.0006$	including NLO jets	Jimenez-Delgado, Reya '14
MSTW	$0.1171 \pm 0.0014$		Martin, Stirling, Thorne, Watt '09
Thorne	0.1136	[DIS+DY, HT*] (2014)	Thorne '14
ABM11 <sub>J</sub>	$0.1134 - 0.1149$ $\pm 0.0012$	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. '11
ABM13	$0.1133 \pm 0.0011$		Alekhin, Blümlein, S.M. '13
ABM13	$0.1132 \pm 0.0011$	(without jets)	Alekhin, Blümlein, S.M. '13
CTEQ	0.1159...0.1162		Gao et al. '13
CTEQ	0.1140	(without jets)	Gao et al. '13
NN21	$0.1174 \pm 0.0007$		NNPDF '11

- Values of  $\alpha_s(M_Z)$  at NNLO from related measurements and lattice

$e^+e^-$ thrust	$0.1131^{+0.0028}_{-0.0022}$	Gehrmann et al.	<a href="#">arXiv:1210.6945</a>
$e^+e^-$ thrust	$0.1140 \pm 0.0015$	Abbate et al.	<a href="#">arXiv:1204.5746</a>
CMS	$0.1151 \pm 0.0033$	$t\bar{t}$	<a href="#">arXiv:1307.1907</a>
NLO Jets ATLAS	$0.111^{+0.0017}_{-0.0007}$		<a href="#">arXiv:1312.5694</a>
NLO Jets CMS	$0.1148 \pm 0.0055$		<a href="#">arXiv:1312.5694</a>
3-jet rate	$0.1175 \pm 0.0025$	Dissertori et al. 2009	<a href="#">arXiv:0910.4283</a>
Z-decay	$0.1189 \pm 0.0026$	BCK 2008/12 (N <sup>3</sup> LO)	<a href="#">arXiv:1201.5804</a>
$\tau$ decay	$0.1212 \pm 0.0019$	BCK 2008	<a href="#">arXiv:0801.1821</a>
$\tau$ decay	$0.1204 \pm 0.0016$	Pich 2011	<a href="#">arXiv:1110.0016</a>
$\tau$ -decay rate	$0.325 \pm 0.018$ (at $m_\tau$ )	FOTP:	Jamin '13
$\tau$ -decay rate	$0.374 \pm 0.025$ (at $m_\tau$ )	CIPT:	Jamin '13
lattice	$0.1205 \pm 0.0010$	PACS-CS 2009 (2+1 fl.)	<a href="#">arXiv:0906.3906</a>
lattice	$0.1184 \pm 0.0006$	HPQCD 2010	<a href="#">arXiv:1004.4285</a>
lattice	$0.1200 \pm 0.0014$	ETM 2012 (2+1+1 fl.)	<a href="#">arXiv:1201.5770</a>
lattice	$0.1156 \pm 0.0022$	2012 (2+1 fl.)	<a href="#">arXiv:1205.6155</a>
lattice	$0.1130 \pm 0.0010$ ( <i>stat</i> )	RBC-UKQCD (prel., 2014)	UKQCD '14
world average	$0.1184 \pm 0.0007$	(2012)	<a href="#">arXiv:1210.0325</a>

# $\alpha_s$ from DIS and PDFs

ABM11



- Profile of  $\chi^2$  for different data sets in ABM11 PDF fit [Alekhin, Blümlein, S.M. '12](#)

## Origin of differences in $\alpha_s$ values

- Inspection reveals differences in  $\alpha_s$  values due to different physics models and analysis procedures [Alekhin, Blümlein, S.M. '11](#)
  - target mass corrections in DIS cross sections (powers of nucleon mass  $M_N^2/Q^2$ )
  - higher twist in DIS cross sections
 
$$F_2^{\text{ht}} = F_2 + ht^{(4)}(x)/Q^2 + ht^{(6)}(x)/Q^4 + \dots$$
  - error correlations among DIS data sets
- Check with variants of [ABM](#) analysis
  - no higher twist etc. reproduce larger  $\alpha_s$  values

	$\alpha_s$ at NNLO	target mass corr.	higher twist	error correl.
ABM11	$0.1134 \pm 0.0011$	yes	yes	yes
NNPDF21	$0.1166 \pm 0.0008$	yes	no	yes
MSTW	$0.1171 \pm 0.0014$	no	no	no

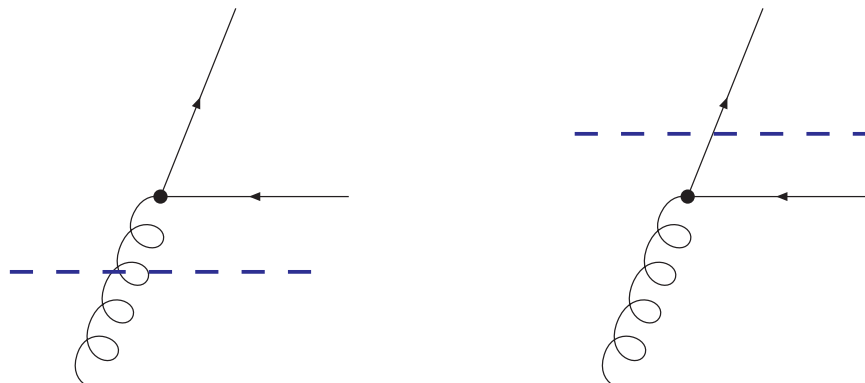
# Treatment of heavy-quarks

## Light quarks

- Neglect “light quark” masses  $m_u, m_d \ll \Lambda_{QCD}$  and  $m_s < \Lambda_{QCD}$  in hard scattering process
  - scale-dependent  $u, d, s, g$  PDFs from mass singularities

## Heavy quarks

- No mass singularities for  $m_c, m_b, m_t \gg \Lambda_{QCD}$ , no (evolving) PDFs
  - $c$  and  $b$  PDFs for  $Q \gg m_c, m_b$  generated perturbatively
  - matching of two distinct theories
    - $n_f$  light flavors + heavy quark of mass  $m$  at low scales
    - $n_f + 1$  light flavors at high scales





# Effective theory for heavy-quarks

## Example: Charm structure function

- $F_2^c$  at HERA (assume no “intrinsic charm”)
  - $Q \not\gg m_c$ : Fixed flavor-number scheme FFNS  
 $u, d, s, g$  partons and massive charm coeff. fcts.
  - $Q \gggg m_c$ : Zero-mass variable flavor-number scheme ZM-VFNS  
terms  $m_c/Q \rightarrow 0$ ,  $n_f = 4$  PDFs (matching),  $m_c = 0$  coeff. fcts.
  - $Q \gg m_c$ : General-mass variable flavor-number scheme GM-VFNS  
terms  $m_c/Q \neq 0$ , but quasi-collinear logs  $\ln(Q/m_c)$  large  
 $n_f = 4$  PDFs, “interpolating” coeff. fcts. (matching prescriptions)

# FFNS

- Perturbative QCD predictions for  $F_2^c$  and  $F_L^c$  (neutral current)
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven '92
  - approximate expressions to NNLO  
Laenen, S.M. '98; Alekhin, S.M. '08; Lo Presti, Kawamura, S.M., Vogt '10
  - asymptotic NNLO terms at large  $Q^2 \gg m^2$  Bierenbaum, Blümlein, Klein '09;  
Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock '14

# VFNS

- Variable flavor number schemes  $\longrightarrow$  matching of two distinct theories  
Aivazis, Collins, Olness, Tung '94; Thorne, Roberts '98;  
Buza, Matiounine, Smith, van Neerven '98  
 $\longrightarrow n_f$  light flavors + heavy quark of mass  $m$  at low scales  
 $\longrightarrow n_f + 1$  light flavors at high scales
- Important aspects of variable flavor number schemes
  - mass factorization to be carried out before resummation  
 $\longrightarrow$  mass factorization involves both heavy and light component of structure function
  - matching conditions required through NNLO  
Chuvakin, Smith, van Neerven '00
- Details of implementation matter in global fits

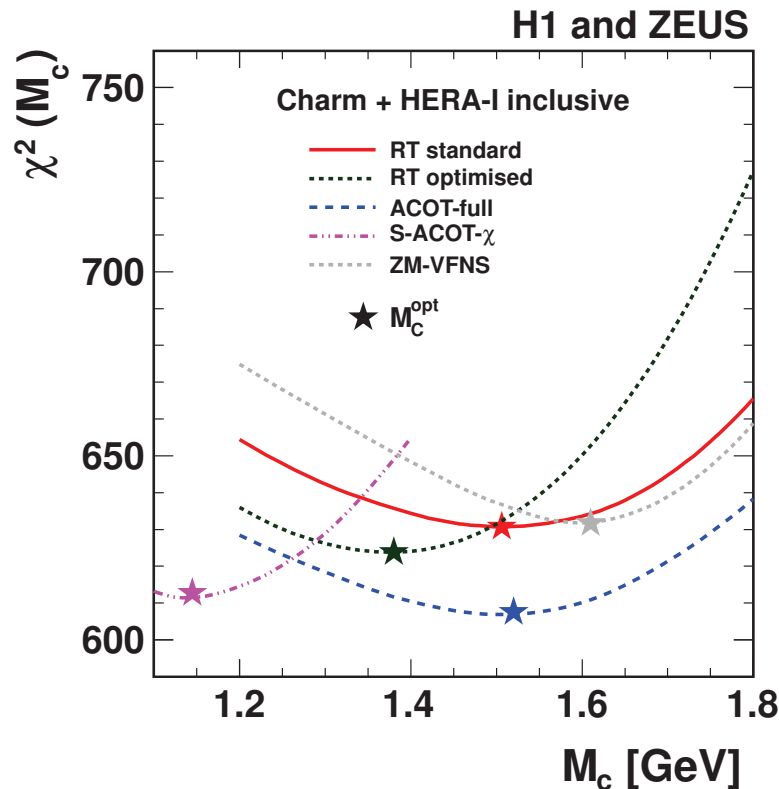
# Heavy quark mass

- Data on  $F_2^c$  at HERA has correlation of  $m_c$ ,  $\alpha_S(M_Z)$ , gluon PDF

$$\sigma_{c\bar{c}} \sim \alpha_s m_c^2 g(x)$$

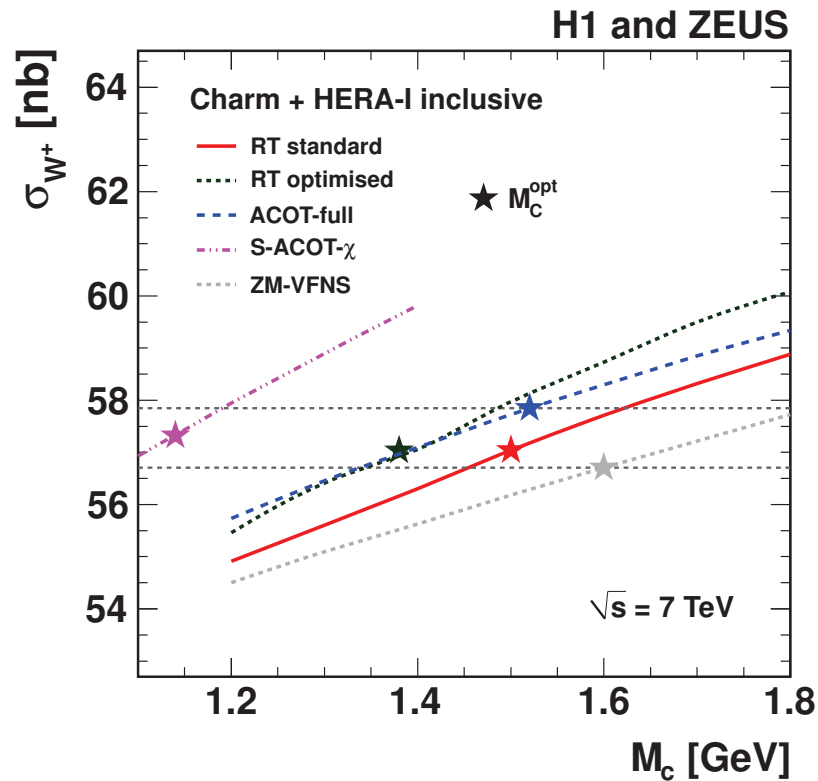
- Comparison of measured data with predictions in various VFNS schemes
  - data shows very good sensitivity to value of  $m_c$
  - fit of value of  $m_c$  strongly dependent on particular choice of VFNS

H1 coll. [arxiv:1211.1182](https://arxiv.org/abs/1211.1182)



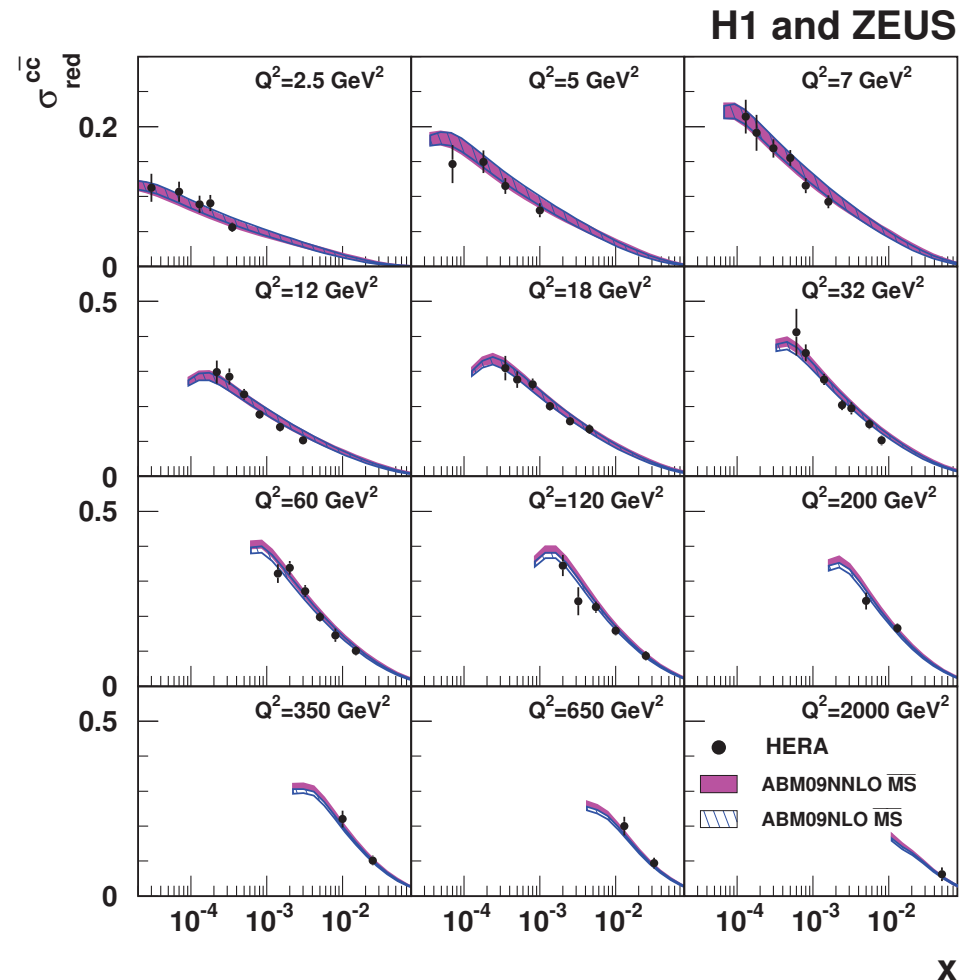
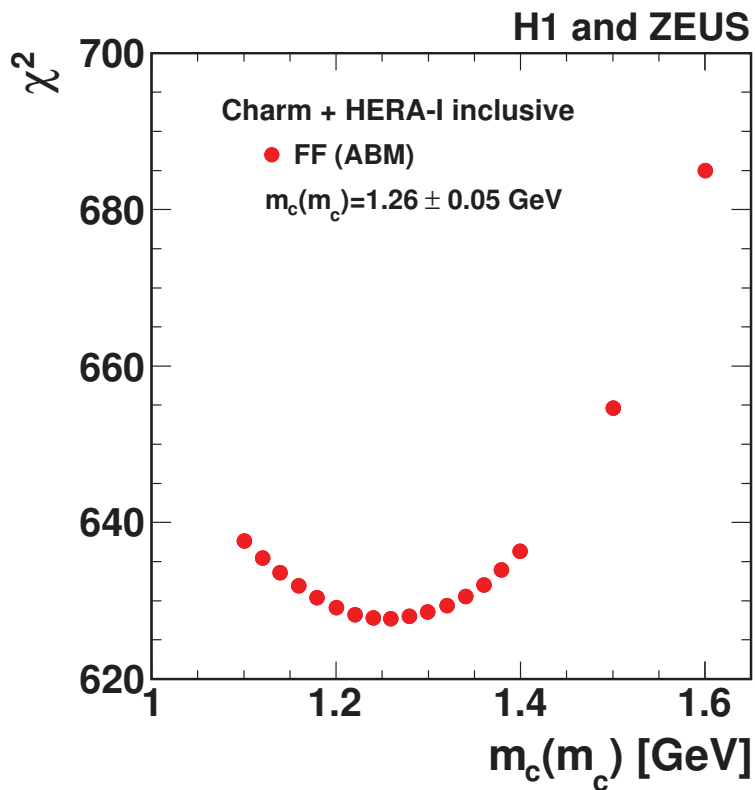
# Heavy quark mass

- Significant impact on cross section predictions at LHC
  - e.g.,  $W^+$ -production



# Running charm quark mass in $\overline{MS}$ -scheme

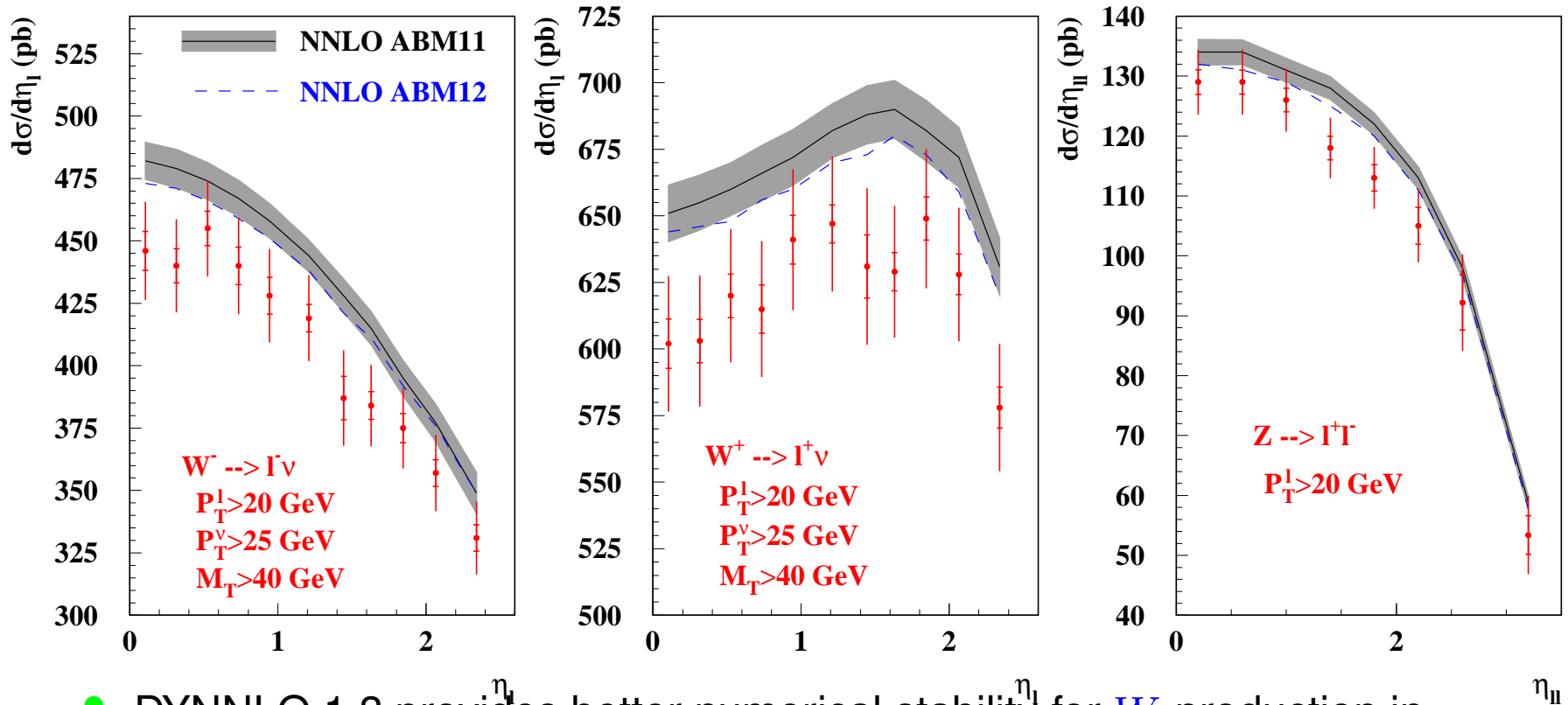
- Determination of  $\overline{MS}$ -mass  $m_c(m_c)$  in DIS [H1 coll. arxiv:1211.1182](#)
- Very good description of data; cf. PDG values:  
 charm:  $m_c(m_c) = 1.27^{+0.07}_{-0.11}$  GeV, bottom:  $m_b(m_b) = 4.20^{+0.17}_{-0.07}$  GeV



# Quark flavour constraints from LHC data

Fit to LHC Drell-Yan data Alekhin, Bümlin, S.M. '13

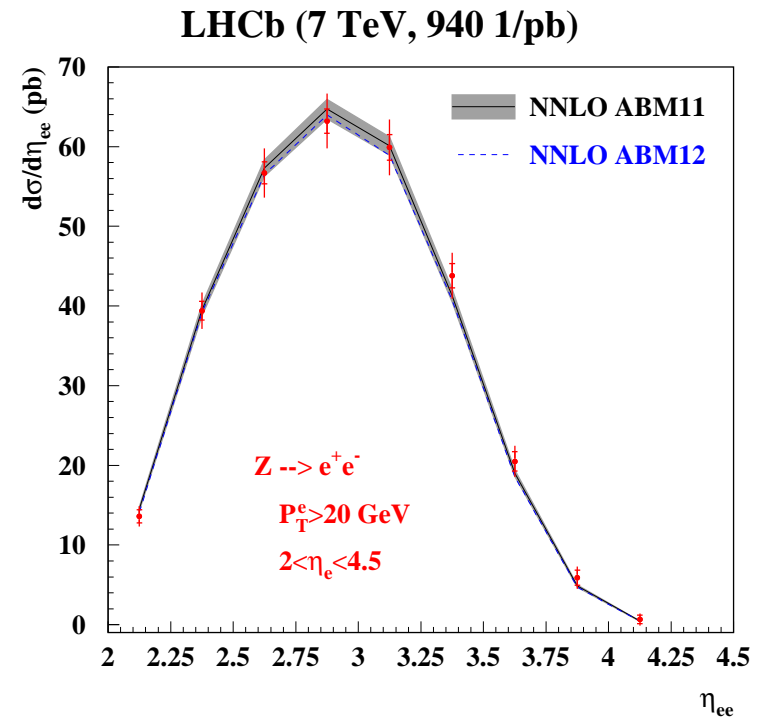
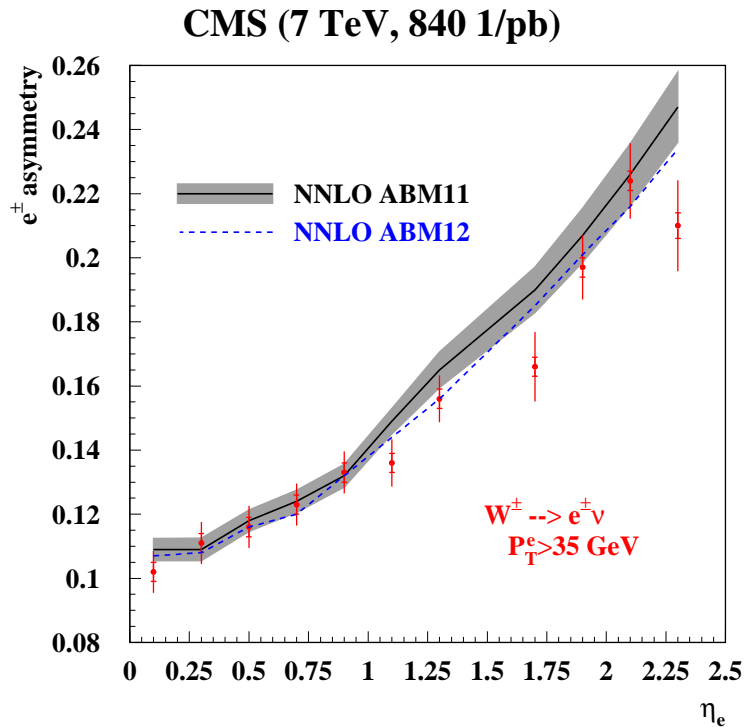
ATLAS (7 TeV, 35 1/pb)



- DYNNLO 1.3 provides better numerical stability for  $W$ -production in central region ( $\sim 200$ h) Catani, Cieri, Ferrera, de Florian, Grazzini '09
- FEWZ 3.1 more convenient/stable for estimation of PDF uncertainties ( $\sim 2d \times 24$  processors) Li, Petriello '12
- Central values computed with DYNNLO and the PDF errors with FEWZ

# ABM PDFs with LHC data

Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. '13

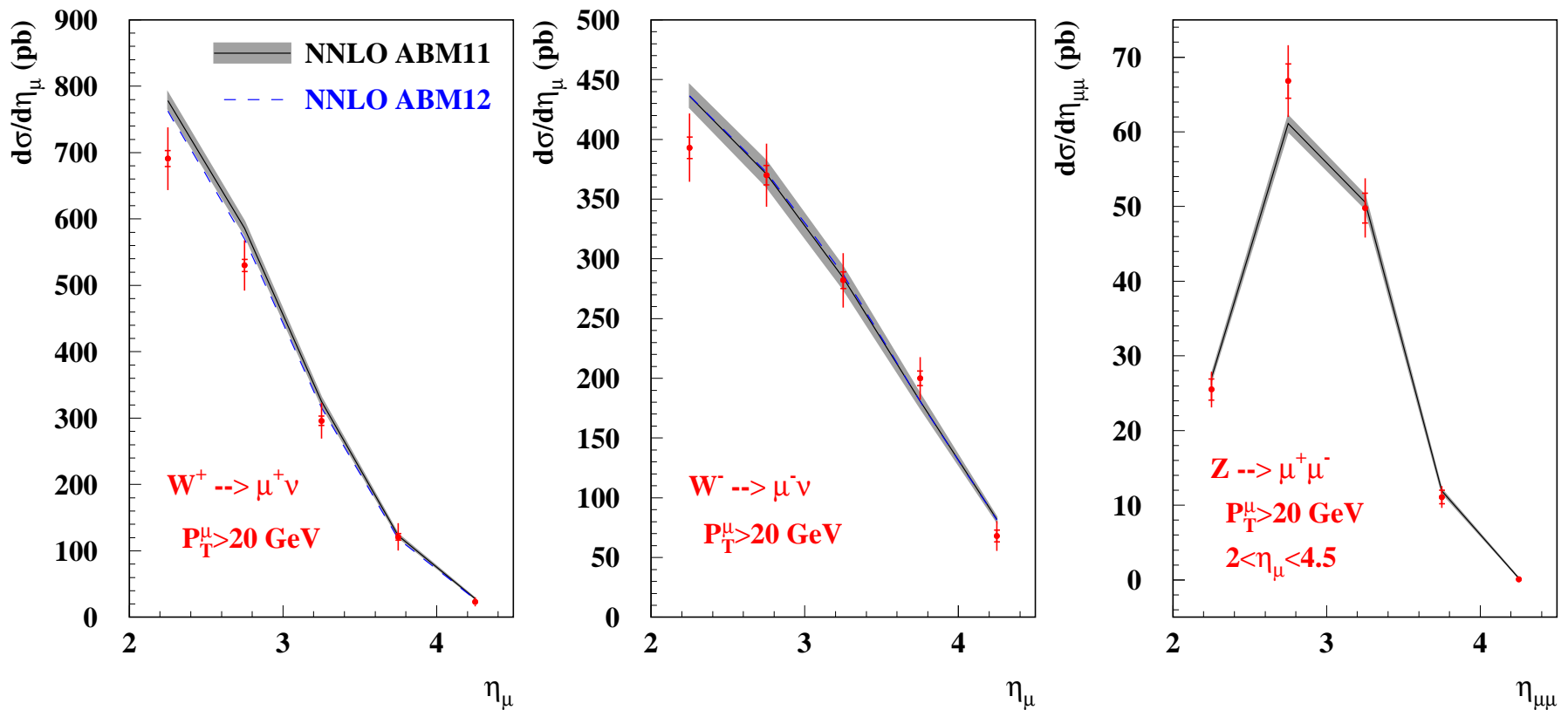


- Good overall agreement with data of CMS '10 and LHCb '12, '13

# ABM PDFs with LHC data

Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. '13

LHCb (7 TeV, 37 1/pb)



- Good overall agreement with data of CMS '10 and LHCb '12, '13



# Benchmarking of ABM PDFs

Experiment	ATLAS '11	CMS '12	LHCb '12	LHCb '12
Final states	$W^+ \rightarrow l^+ \nu$ $W^- \rightarrow l^- \nu$ $Z \rightarrow l^+ l^-$	$W^+ \rightarrow e^+ \nu$ $W^- \rightarrow e^- \nu$	$W^+ \rightarrow \mu^+ \nu$ $W^- \rightarrow \mu^- \nu$	$Z \rightarrow e^+ e^-$
Luminosity (1/pb)	35	840	37	940
$NDP$	30	11	10	9
$\chi^2$ (ABM11)	35.7(7.7)	10.6(4.7)	13.1(4.5)	11.3(4.2)
$\chi^2$ (ABM12)	35.6	9.3	14.4	13.4

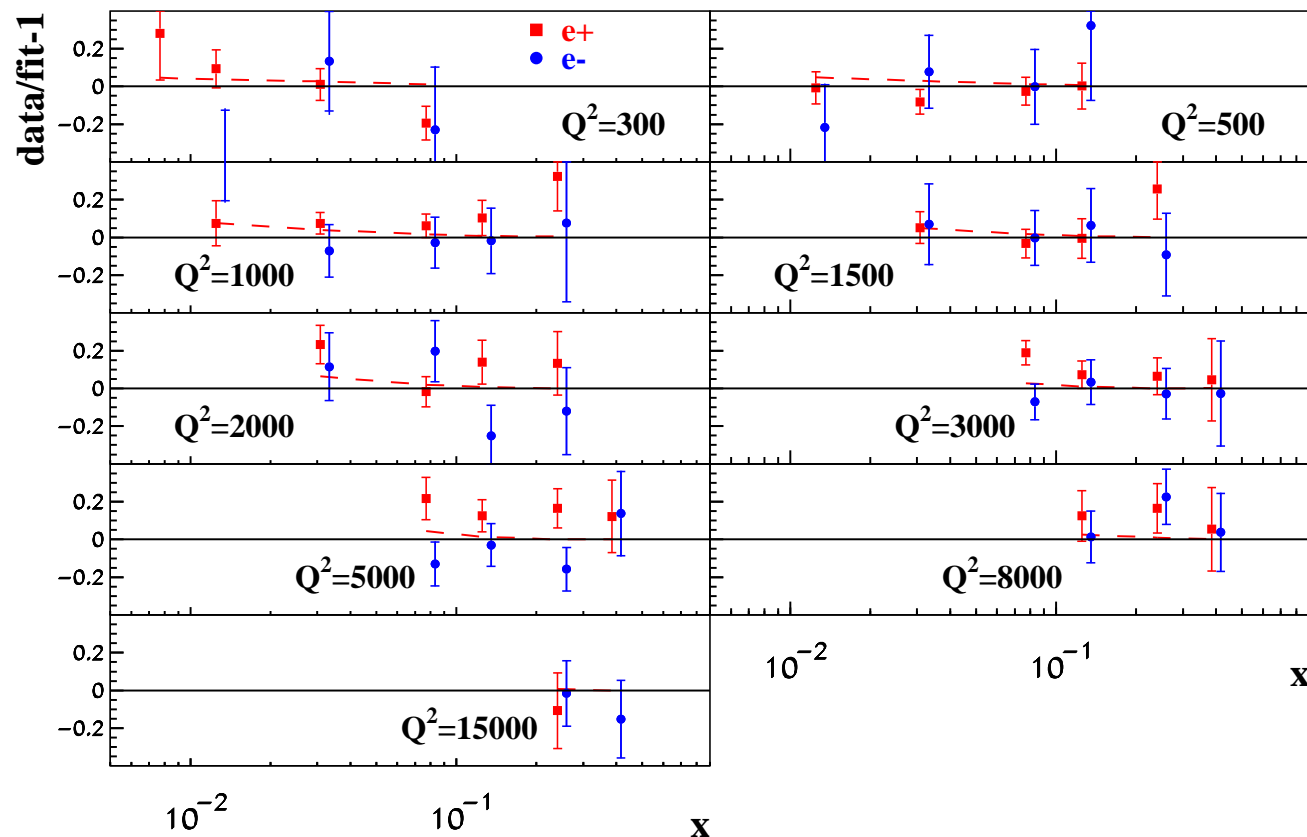
- value of  $\chi^2$  for Drell-Yan data at the LHC with NNLO ABM11 PDFs (+  $1\sigma$  standard deviation of  $\chi^2$  equal to  $\sqrt{2NDP}$ )
- Caution: If benchmarking, make sure to get it right
  - Wrong  $\chi^2$  values for PDF comparison to ABM11 (NLO theory with MCFM and K-factors, no PDF errors, shifted  $\alpha_s$ ) reported in [arXiv:1211.5142](https://arxiv.org/abs/1211.5142) (Ball, Carrazza, Del Debbio, Forte, Gao, Hartland, Huston, Nadolsky, Rojo, Stump, Thorne, Yuan)

# Strange sea determination

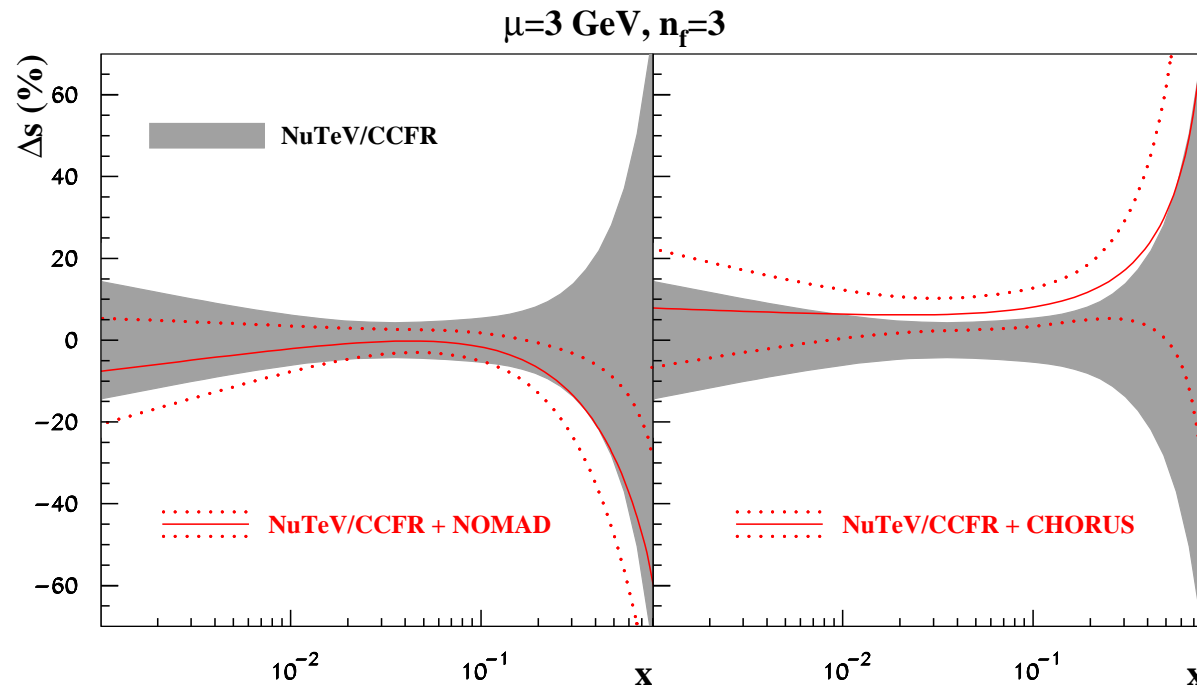
## Charged current DIS

Alekhin, Bümlein, Caminada, Lipka, Lohwasser, S.M. Petti, Placakyte '14

- CC DIS inclusive data (HERA), CC DIS di-muon production data (NOMAD) and CC DIS charmed-hadron production data (CHORUS)
- Theory description with exact NLO QCD corrections and asymptotic NNLO terms at large  $Q^2 \gg m^2$  Buza van Neerven '97



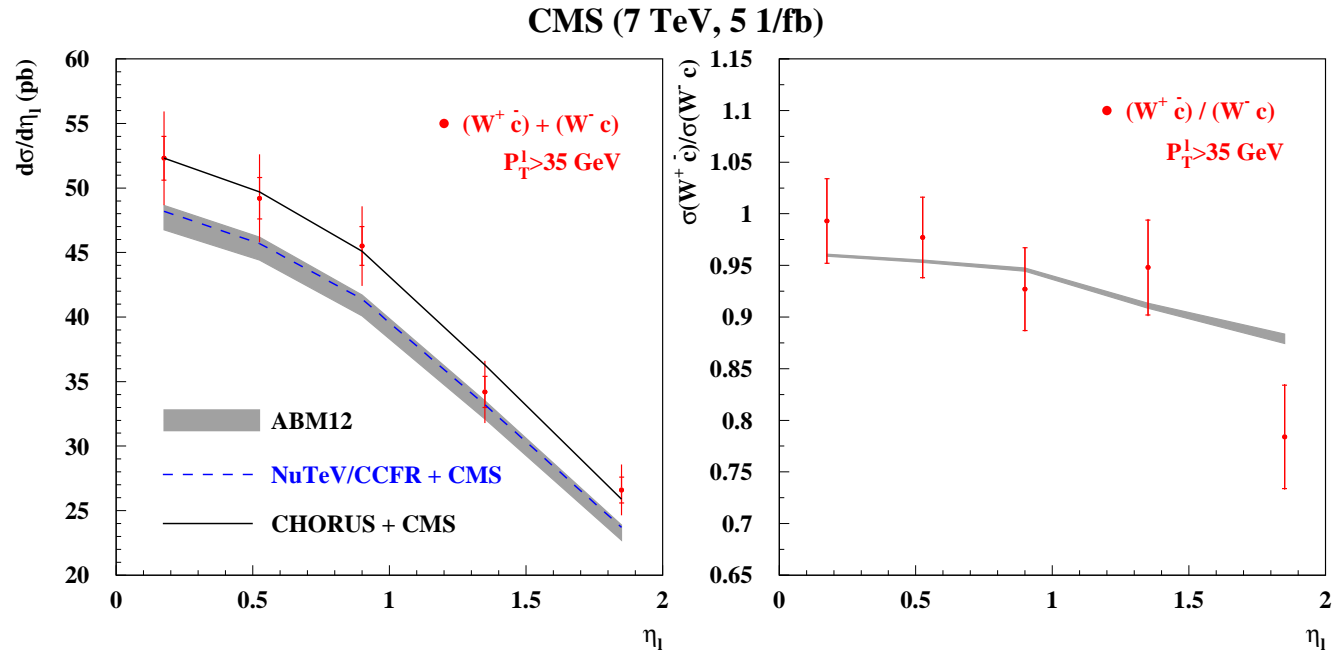
# Strange sea from new fixed target data



- Nomad data on ratio of di-muon sample to incl. CC DIS with statistics of 15000 events (much more than CCFR and NuTeV samples)
  - systematics, nuclear corrections, etc. cancel in ratio
  - pull down strange quarks at  $x > 0.1$ ; sizable reduction of uncertainty
  - $m_c(m_c) = 1.23 \pm 0.03(\text{exp.})\text{GeV}$
- Chorus data pull strangeness up
  - statistical significance the effect is poor

# $W$ +charm production at LHC

- Cross check with LHC data for  $W$ +charm production

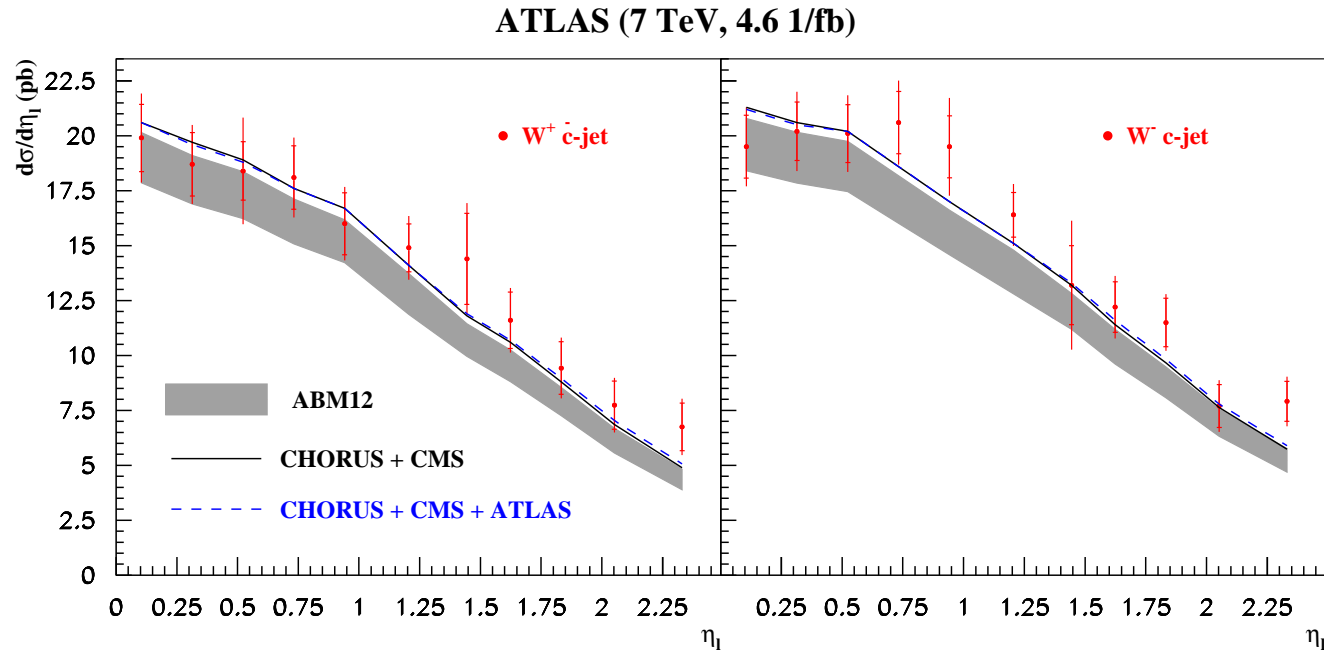


## CMS

- CMS data above NuTeV/CCFR by  $1\sigma$
- Charge asymmetry in a good agreement with charge-symmetric strange sea

# $W$ +charm production at LHC

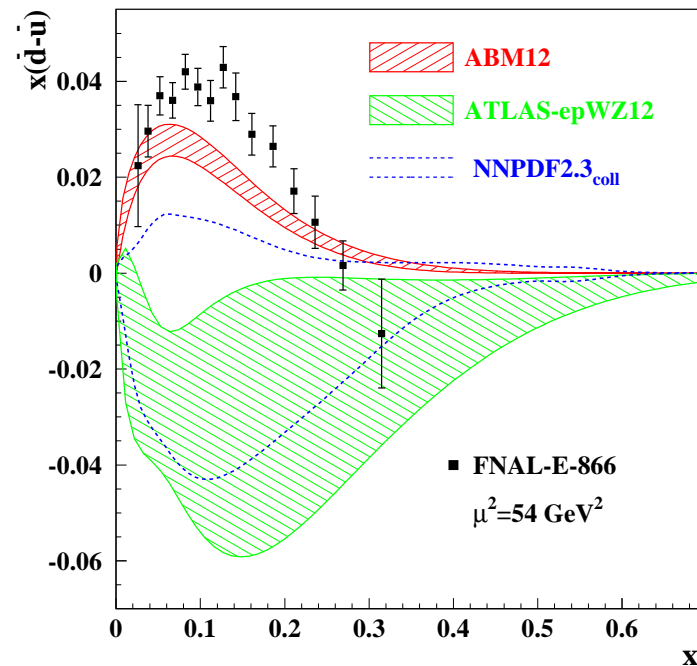
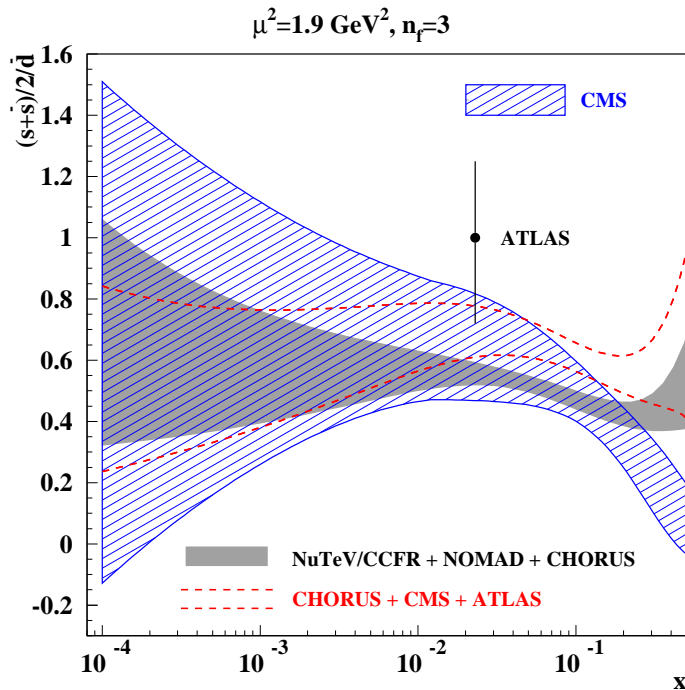
- Cross check with LHC data for  $W$ +charm production



## ATLAS

- ATLAS data in good agreement with NuTeV/CCFR
- Highest bin in  $\eta_l$  deviates

# Comparison with earlier determinations



- ABM update (NuTeV/CCFR+NOMAD+CHORUS) in good agreement with CMS results
- ATLAS strange-sea is enhanced, but correlated with  $d$ -quark sea suppression (disagreement with the FNAL-E-866 data)
- Upper margin of ABM analysis (CHORUS+CMS+ATLAS) is lower than ATLAS

# Summary

## *Parton distributions at the LHC*

- Precision determinations of non-perturbative parameters is essential
  - parton content of proton (PDFs)
  - coupling constants  $\alpha_s(M_Z)$
  - masses  $m_c, m_b, m_t, M_W, m_H, \dots$

## *Precision PDFs*

- Precision measurements require careful definition of observable
  - experimental precision makes NNLO in QCD mandatory
  - global analyses sensitive to theory description of cross section and to analysis procedures  $\longrightarrow$  main source of discrepancy
- Confronting LHC data requires continuous benchmarking