A new approach to BFKL

H. Kowalski, L.N. Lipatov, D.A. Ross

Motivation: partons in the low x region cannot be free…. (see talk of A. Geiser)

Outline:

Gluon density - analyzed by BFKL equation with running α_s becomes a system of quasi-bound states

(in contrast to the DGLAP evolution)

First approach: BFKL amplitude expressed in terms of discrete eigenfunctions only

Present approach: analysis in terms of a complex BFKL Green function

results are similar to the first approach

with an essential improvement

A new approach to BFKL

H. Kowalski, L.N. Lipatov, D.A. Ross

Application to HERA and LHC data, F_2 and DY processes

Physics motivation

consistent solution of BFKL

Sensitivity to BSM effects

Pomeron-Graviton Correspondence

the talk is based on

The Green Function for BFKL Pomeron and the Transition to DGLAP Evolution.

H. Kowalski, L.N. Lipatov, D.A. Ross, Eur.Phys.J. C74 (2014) 2919

BFKL Evolution as a Communicator Between Small and Large Energy Scales

H. Kowalski, L.N. Lipatov, D.A. Ross, arXiv:1205.6713 and 1109.0432

Using HERA data to determine the infrared behaviour of the BFKL amplitude

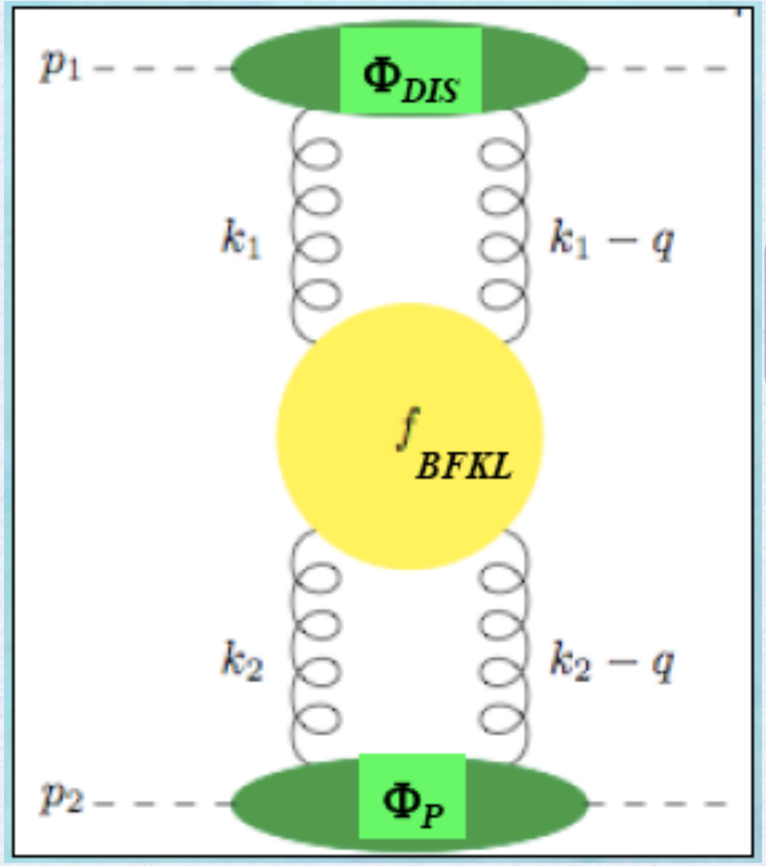
H. Kowalski, L.N. Lipatov, D.A. Ross and G. Watt, EPJC 70: 983, 2010

Evidence for the discrete asymptotically-free BFKL Pomeron from HERA data

J. Ellis, H. Kowalski, D.A. Ross Physics Letters B 668 (2008) 51–56

The dynamics of Gluon Density at low x is determined by **the amplitude for the scattering of a gluon on a gluon**, described by the BFKL equation

$$\frac{\partial}{\partial \ln s}\mathcal{A}(s,\mathbf{k},\mathbf{k}') \;=\; \delta(k^2 - k'^2) + \int dq^2 \mathcal{K}(\mathbf{k},\mathbf{q})\mathcal{A}(s,\mathbf{q},\mathbf{k}')$$



solved by the Green function method, in terms of the eigenfunctions of the kernel

$$\int dk'^2 \mathcal{K}(\mathbf{k},\mathbf{k}') f_\omega(\mathbf{k}') \;=\; \omega f_\omega(\mathbf{k})$$

in LO, with fixed α_s

$$f_\omega(\mathrm{k}) = \exp(i\nu \ln k^2)/k$$

$$\omega \;=\; \alpha_s \chi_0(\nu)$$

Green f. method - preserves the scaling (conformal) invariance of BFKL

⇒ most consistent solution of BFKL

a possible bridge to Pomeron-Graviton?

Properties of the BFKL Kernel

Quasi-locality

$$\mathcal{K}(\mathbf{k},\mathbf{k}') = \frac{1}{kk'}\sum_{n=0}^{\infty} c_n \delta^{(n)}\left(\ln(\mathbf{k}^2/\mathbf{k}'^2)\right)$$

$$c_n = \int_0^\infty dk'^2 \mathcal{K}(\mathbf{k},\mathbf{k}')\frac{k}{k'}\frac{1}{n!}\left(\ln(\mathbf{k}^2/\mathbf{k}'^2)\right)^n$$

Similarity to the Schroedinger equation

$$k\int dk'^2 \mathcal{K}(\mathbf{k},\mathbf{k}') f_\omega(\mathbf{k}') = \sum_{n=0}^{\infty} c_n \left(\frac{d}{d\ln(\mathbf{k}^2)}\right)^n \bar{f}_\omega(\mathbf{k}) = \omega \bar{f}_\omega(\mathbf{k})$$

Characteristic function

$$k\int dk'^2 \mathcal{K}(\mathbf{k},\mathbf{k}') f_\omega(\mathbf{k}') = \chi\left(-i\frac{d}{d\ln k^2}, \alpha_s(k^2)\right) \bar{f}_\omega(k) = \omega \bar{f}_\omega(k)$$

with running α_s, BFKL frequency ν becomes k-dependent, $\nu(k)$

$$\alpha_s(k^2)\chi_0(\nu(\mathbf{k})) + \alpha_s^2(k^2)\chi_1(\nu(\mathbf{k})) \;=\; \omega$$

NLO

ν has to become a function of k because ω is a constant

GS resummation applied

evaluation in diffusion ($\nu \approx 0$) or semiclassical approximation ($\nu > 0$)

For sufficiently large k, there is no longer a real solution for ν.

The transition from real to imaginary $\nu(k)$ singles out a special value of

$k = k_{crit}$, with $\nu(k_{crit}) = 0$.

The solutions below and above this critical momentum k_{crit} have to match. This fixes the phase of ef's.

Near k=k_{crit}, the BFKL eq. becomes the Airy eq. which is solved by the Airy eigenfunctions (to a very good approximation)

$$k\, f_\omega(k) = \bar{f}_\omega(k) = \mathrm{Ai}\left(-(\frac{3}{2}\phi_\omega(k))^{\frac{2}{3}}\right)$$

with

$$\phi_\omega(k) = 2\int_k^{k_{\rm crit}} \frac{d\,k'}{k'} |\nu_\omega(k')|$$

instead of

$f_\omega(\mathrm{k}) = \exp(i\nu \ln k^2)/k$

for k<<k_{crit} the Airy function has the asymptotic behaviour

$$k\, f_\omega(k) \sim \sin\left(\phi_\omega(k) + \frac{\pi}{4}\right)$$

The two fixed phases at k=k_{crit} and at k=k_0 (near Λ_{QCD}) lead to the quantization condition

$$\phi_\omega(k_0) = \left(n - \frac{1}{4}\right)\pi + \eta\,\pi$$

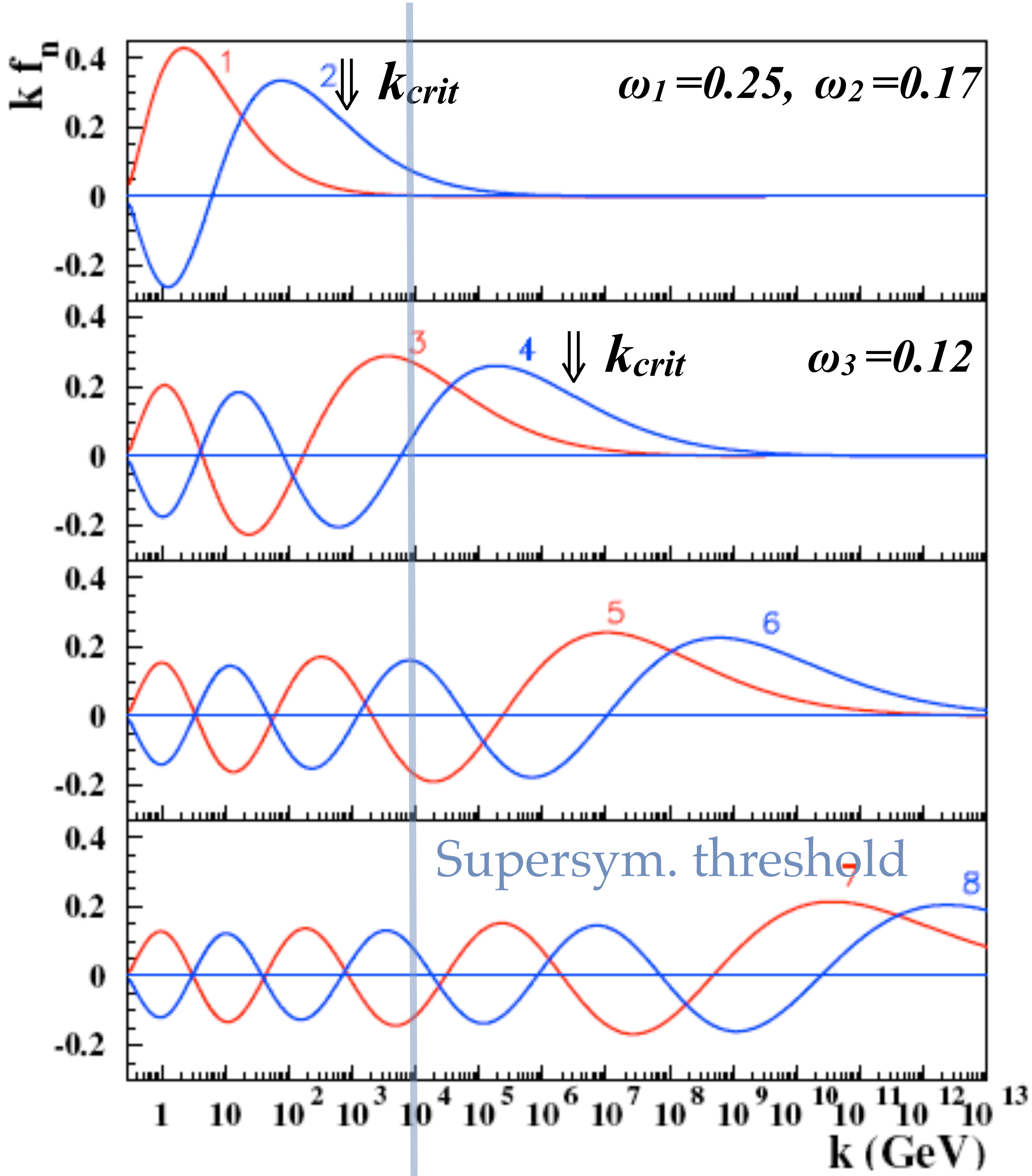
Discrete Pomeron Solution of the BFKL eq

The first eight eigenfunctions determined at $\eta=0$

$k_{crit} \simeq c\ exp(4n)$

$c \simeq \Lambda_{QCD}$

Similarity to WKB solutions of the Schrödinger eq for the potential well

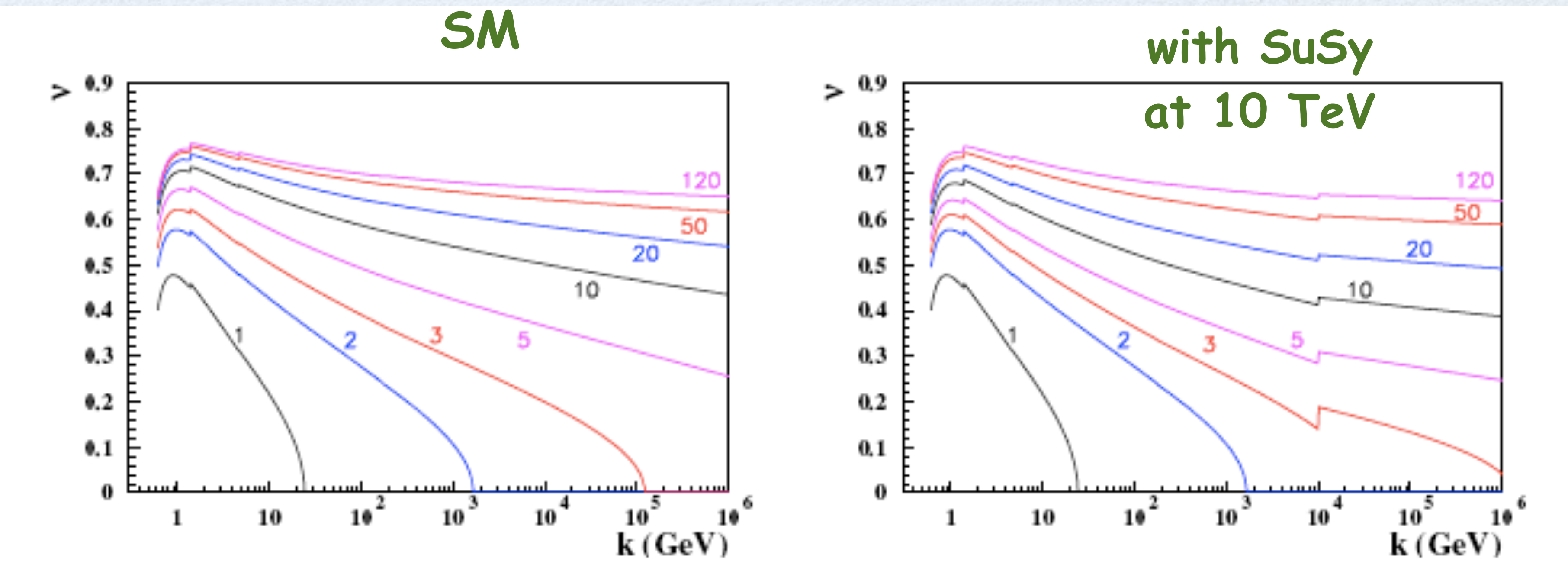


Sensitivity of the frequencies *v(k)* to thresholds

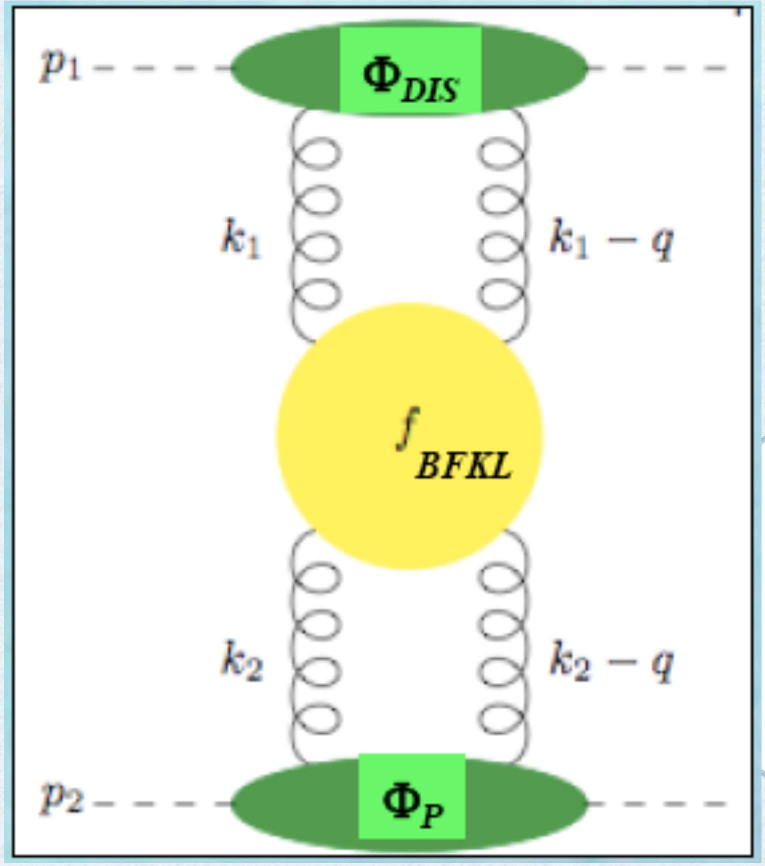
change of β function in α_s (LO)

$$\bar{\beta}_0 = \frac{11}{12} - \frac{n_f}{18}.$$

$$\frac{\beta^{SM}}{\beta^{SUSY}} = \frac{7}{3}$$



Comparison with HERA data



Discreet Pomeron Green function

$$\mathcal{A}(\mathbf{k},\mathbf{k}') = \sum_{m,n} f_m(\mathbf{k}) \mathcal{N}^{-1}_{mn} f_n(\mathbf{k}') \left(\frac{s}{kk'}\right)^{\omega_n}.$$

Integrate with the photon and proton impact factors

$$\mathcal{A}^{(U)}_n \equiv \int_x^1 \frac{d\xi}{\xi} \int \frac{dk}{k} \Phi_{\mathrm{DIS}}(Q^2,k,\xi) \left(\frac{\xi k}{x}\right)^{\omega_n} f_n(\mathbf{k})$$

$$\mathcal{A}^{(D)}_m \equiv \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{1}{k'}\right)^{\omega_m} f_m(\mathbf{k}').$$

$$F_2(x,Q^2) = \sum_{m,n} \mathcal{A}^{(U)}_n \mathcal{N}^{-1}_{nm} \mathcal{A}^{(D)}_m$$

the infrared boundary condition

Proton impact factor

$$\Phi_p(\mathbf{k}) \;=\; A\,k^2 e^{-bk^2}$$

The fit is not sensitive to the particular form of the impact factor.

The support of the proton impact factor is much smaller than the oscillation period of f_n and because the frequencies ν have a limited range

➤ many eigenfunctions have to contribute and η has to be a function of n. Phase condition at $\tilde{k}_0$, (close to Λ_{QCD})

$$\eta \;= \eta_0 \left(\frac{n-1}{n_{\max}-1}\right)^{\kappa}$$

additional parameter k_0 which should be in the perturbative region but close to Λ_{QCD}

$$\phi_n(\tilde{k}_0) \;=\; \phi_n(k_0) - 2\nu_n^0 \ln\left(\frac{k_0}{\tilde{k}_0}\right),$$

The qualities of fits for various numbers of eigenfunctions, Q^2 > 4 GeV^2 (one loop α_s)

$n_{\max}$	χ^2/N_{df}	κ	A	b
1	10811 /125	—	146	30.0
5	350.0 /125	3.78	$3.1 \cdot 10^6$	78.0
20	286.5 /125	0.96	632	15.8
40	193.3 /125	0.84	2315	23.2
60	163.3 /125	0.78	3647	25.6
80	156.5 /125	0.73	3081	24.4
100	149.1 /125	0.69	2414	22.8
120	143.7 /125	0.66	2041	21.8

➤ precise data are crucial for finding the right solution

the differences in the fit qualities would be negligible if the errors where more than 2-times larger

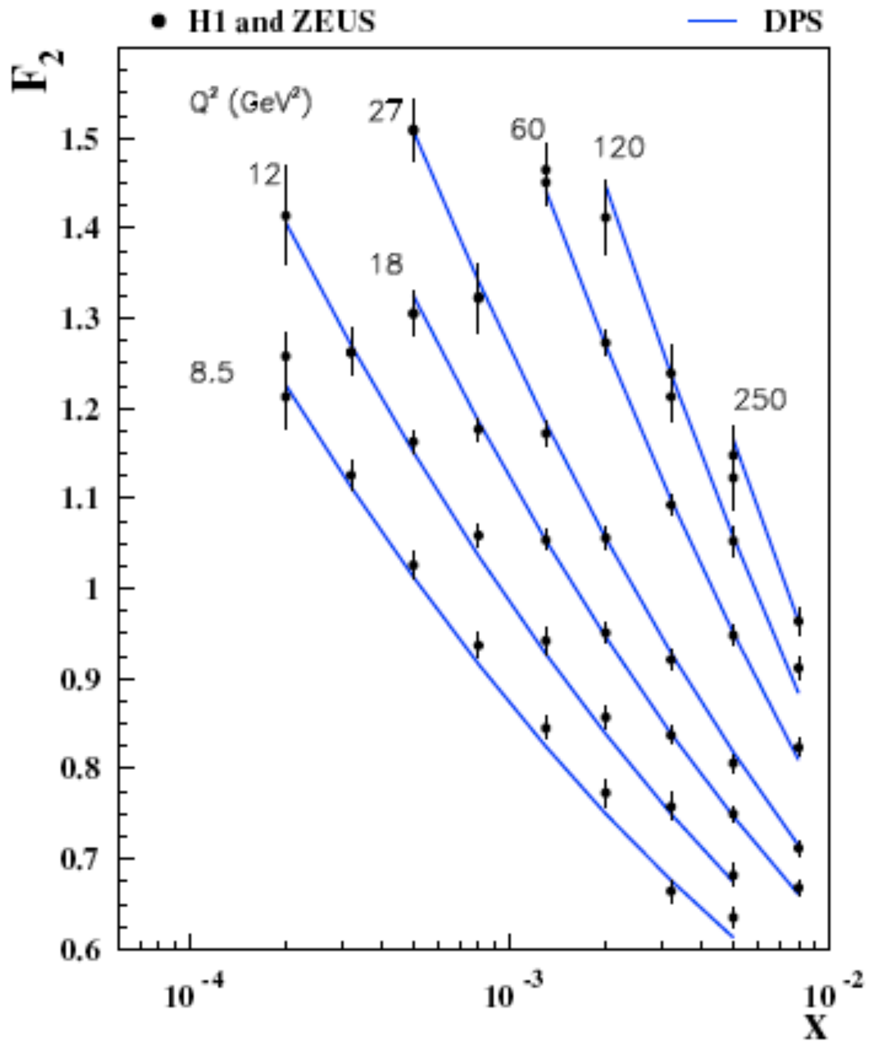
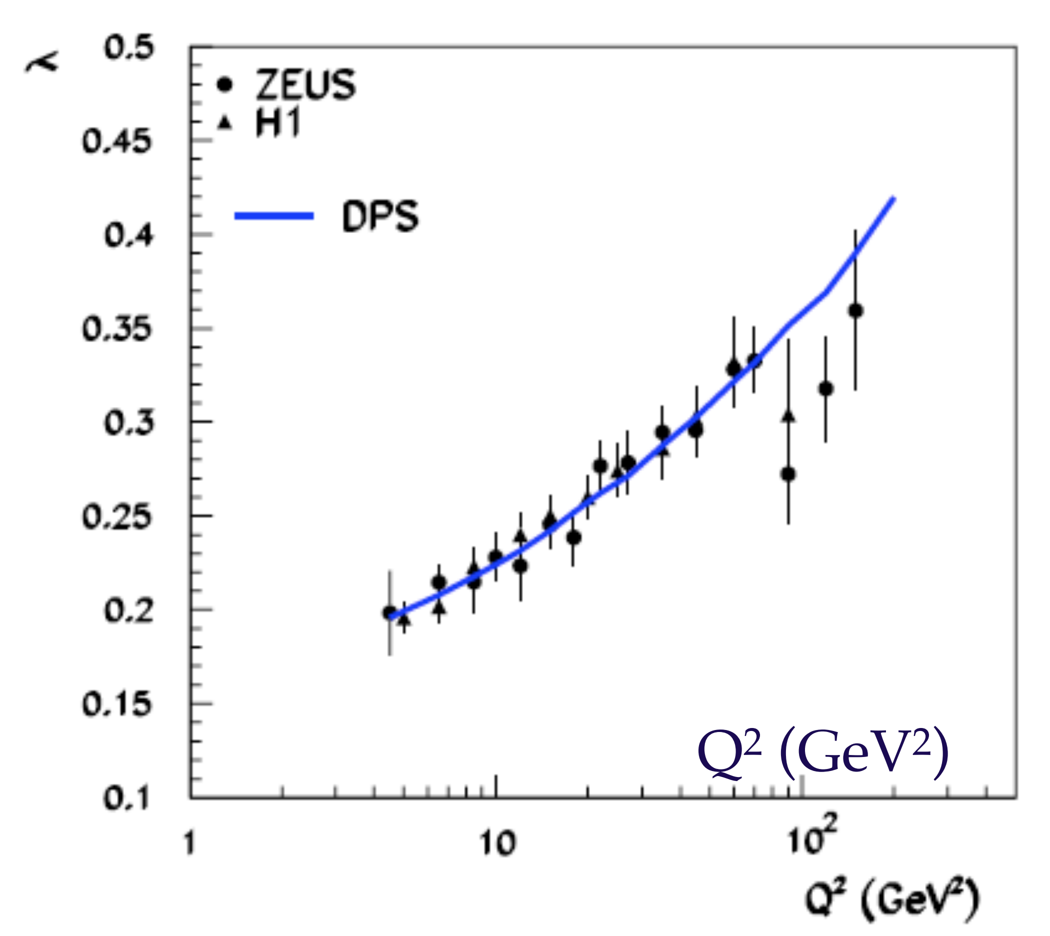


Figure 7: Comparison of the DPS fit with $M_{SUSY} = 10$ TeV with HERA data.

The rate of rise λ

$F_2 \sim (1/x)^\lambda$



The first successful pure BFKL description of the λ plot.

For many years it was claimed that BFKL analysis was not applicable to HERA data because of the observed substantial variation of λ with Q^2

Fits to F_2, $Q^2 > 8$ GeV^2, $x > 0.01$ N=108, (two loop α_s)

SUSY Scale (TeV)	χ^2	κ	$\tilde{k}_0$ (GeV)	η_0	A	b
3	125.7	0.555	0.288	-0.87	201.2	10.6
6	114.1	0.575	0.279	-0.880	464.8	15.0
10	109.9	0.565	0.275	-0.860	720.1	17.7
15	110.1	0.555	0.279	-0.860	882.2	18.6
30	117.8	0.582	0.278	-0.870	561.6	16.2
50	114.9	0.580	0.279	-0.870	627.4	16.8
90	114.8	0.580	0.279	-0.870	700.2	17.5
∞	122.5	0.600	0.274	-0.800	813.1	17.5

Table 1: Fits for N=1 SUSY at different scales. The bottom row corresponds to the Standard Model. All fits are performed with $n_{max} = 100$.

χ^2/N= 110/108=1.02

Note: we are partially absorbing the SUSY effects into the free parameters of the boundary conditions: e.g best SuSy fit with η_0, κ of SM gives $\chi 2$ ~ 400

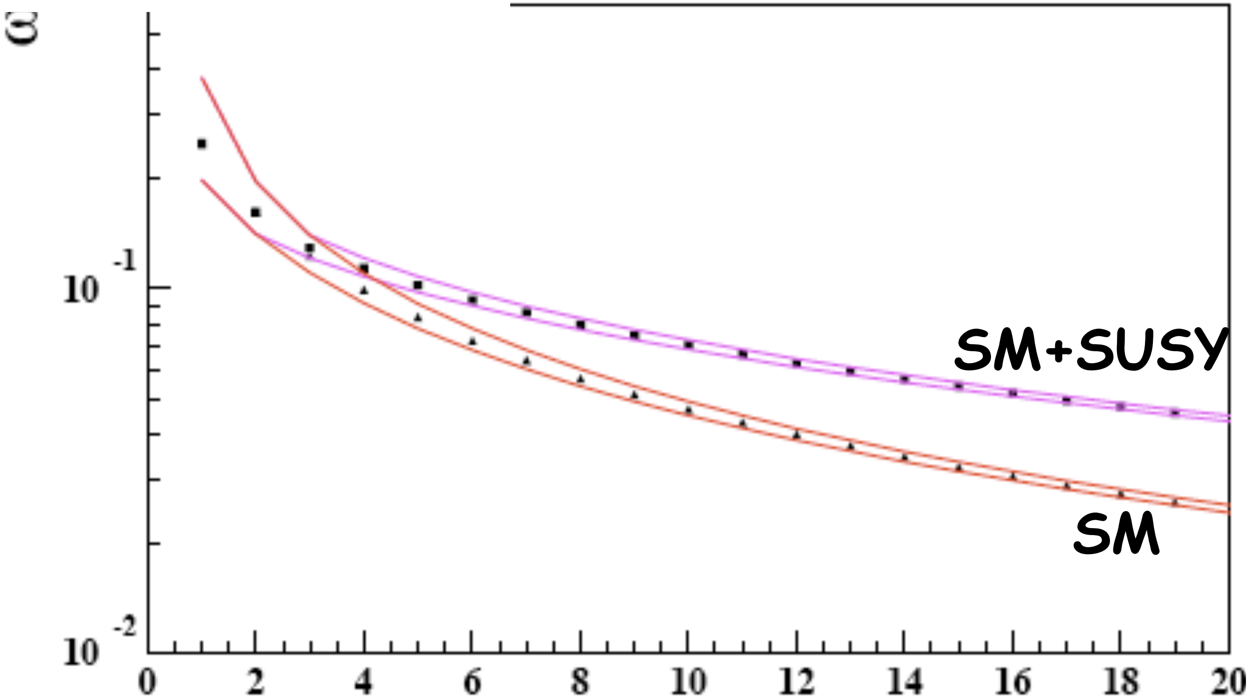
Eigenvalues of the Discrete BFKL-Pomeron

LO evaluation

$$\omega_n = \frac{0.96}{\pi\beta} \cdot \frac{1}{\eta + \mathbf{n} - 1/4}$$

$$\frac{\beta^{SM}}{\beta^{SUSY}} = \frac{7}{3}$$

NLO numerical evaluation



lines indicate the uncertainty of the phase
(η can only vary between 0 and π)

difference between SM and SM+SUSY is substantially larger than the uncertainty of the phase

Discrete BFKL-Pomeron

Very interesting but:

why so many eigenfunctions?

do we have convergence?

suppression of large n contribution only by
the normalization condition for eigenfunctions $\sim 1/\sqrt{n}$

do we have closure ?

paper:

The Green Function for the BFKL Pomeron and the Transition to DGLAP Evolution.

H. Kowalski, L.N. Lipatov, D.A. Ross + ...

Eur.Phys.J. C74 (2014) 2919

closure

Obtain the BFKL Green Function

$$\left(\omega - \hat{\Omega}\left(t, -i\frac{\partial}{\partial t}\right)\right) \mathcal{G}_\omega(t,t') = \delta(t-t'),$$

$t = \ln(k^2/\Lambda^2_{QCD})$

from the generalized Airy operator
(valid in diffusion and semiclassical approximation)

$$\left(\omega - \hat{\Omega}\left(t, -i\frac{\partial}{\partial t}\right)\right) \approx \frac{1}{N_\omega(t)}\left(\dot{z}z - \frac{\partial}{\partial t}\frac{1}{\dot{z}}\frac{\partial}{\partial t}\right)\frac{1}{N_\omega(t)}.$$

$$s_\omega(t) = \int_t^{t_c} dt' \, \nu_\omega(t').$$

$$z(t) = -\left(\frac{3}{2}s_\omega(t)\right)^{\frac{2}{3}}$$

Generalized Airy Green Function

$$\mathcal{G}_\omega(t,t') = \pi N_\omega(t) N_\omega(t') \left(\overline{\mathrm{Bi}}(z(t))\mathrm{Ai}(z(t'))\theta(t'-t) + \mathrm{Ai}(z(t))\overline{\mathrm{Bi}}(z(t'))\theta(t-t')\right)$$

with

$$\overline{\mathrm{Bi}}(z) = \mathrm{Bi}(z) + c(\omega)\mathrm{Ai}(z)$$

$$c(\omega) = \cot\left(\phi(\omega)\right)$$

$$\phi(\omega) = \eta_{np}(\omega, t_0) - \frac{\pi}{4} - s_\omega(t_0)$$

leads to a similar pole term contribution, $\Delta y = \ln(1/x)$

$$\mathcal{G}_\omega^{\mathrm{pole}}(t,t') = \sum_n \pi N_{\omega_n}(t) N_{\omega_n}(t') \frac{A_i\left(z(t)\right) A_i\left(z(t')\right)}{\phi'(\omega_n)(\omega - \omega_n)},$$

$$N_\omega(t) = \frac{(-z(t))^{1/4}}{\sqrt{\frac{1}{2}\left|\Omega'(t,\nu_\omega)\right|}},$$

Unintegrated gluon density

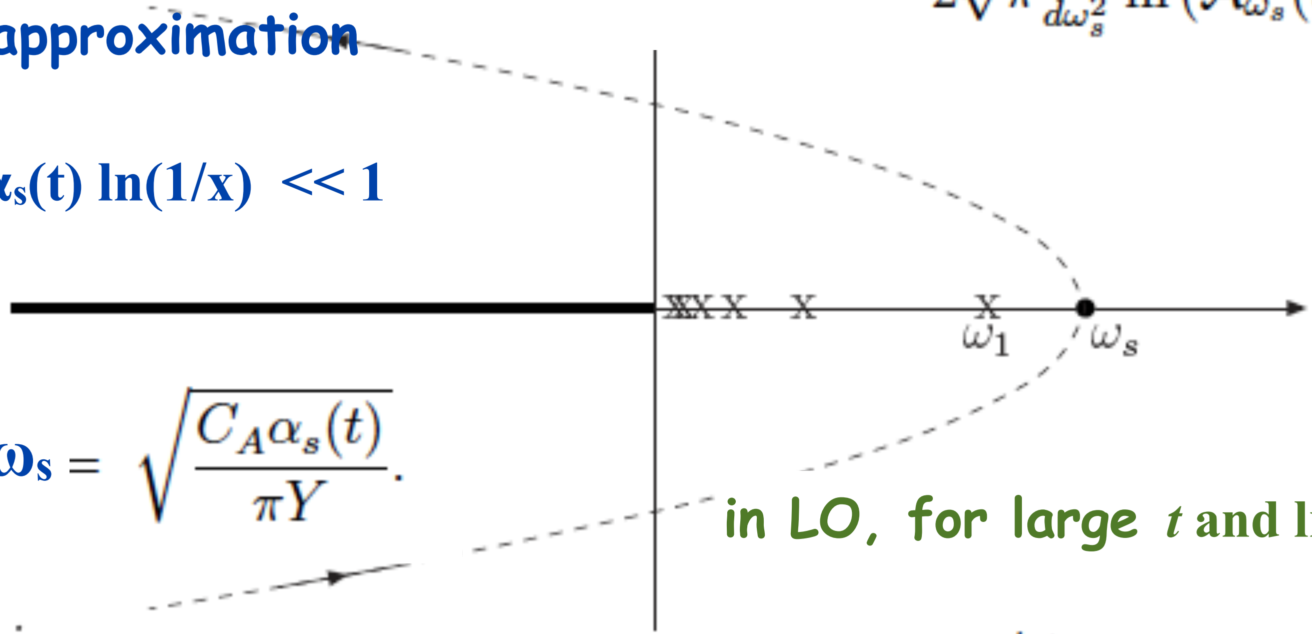
$$\dot{g}(x,t) \;=\; \frac{1}{2\pi i}\int_C d\omega x^{-\omega}\int dt' \mathcal{G}_\omega(t,t')\Phi_P(t'),$$

Integral over contour C can be evaluated in saddle point approximation

$$\dot{g}(x,t) \;\approx\; \frac{1}{2\sqrt{\pi\frac{d^2}{d\omega_s^2}\ln\left(\mathcal{A}_{\omega_s}(t)\right)}} x^{-\omega_s(t)}\mathcal{A}_{\omega_s}(t)$$

$\alpha_s(t)\ln(1/x) \ll 1$

$$\omega_s = \sqrt{\frac{C_A\alpha_s(t)}{\pi Y}}.$$



$Y = \ln(1/x)$

in LO, for large t and $\ln(1/x)$

$$\mathcal{A}_\omega(t) \overset{t\to\infty}{\sim} e^{-t/2}\,(t)^{\frac{1}{\omega\beta_0}}$$

it agrees with the DLL limit of DGLAP

$\omega = \alpha_s\, \chi(\nu)$

$\alpha_s = 1/\beta_0\, t$

$t = \ln(k^2/\Lambda^2)$

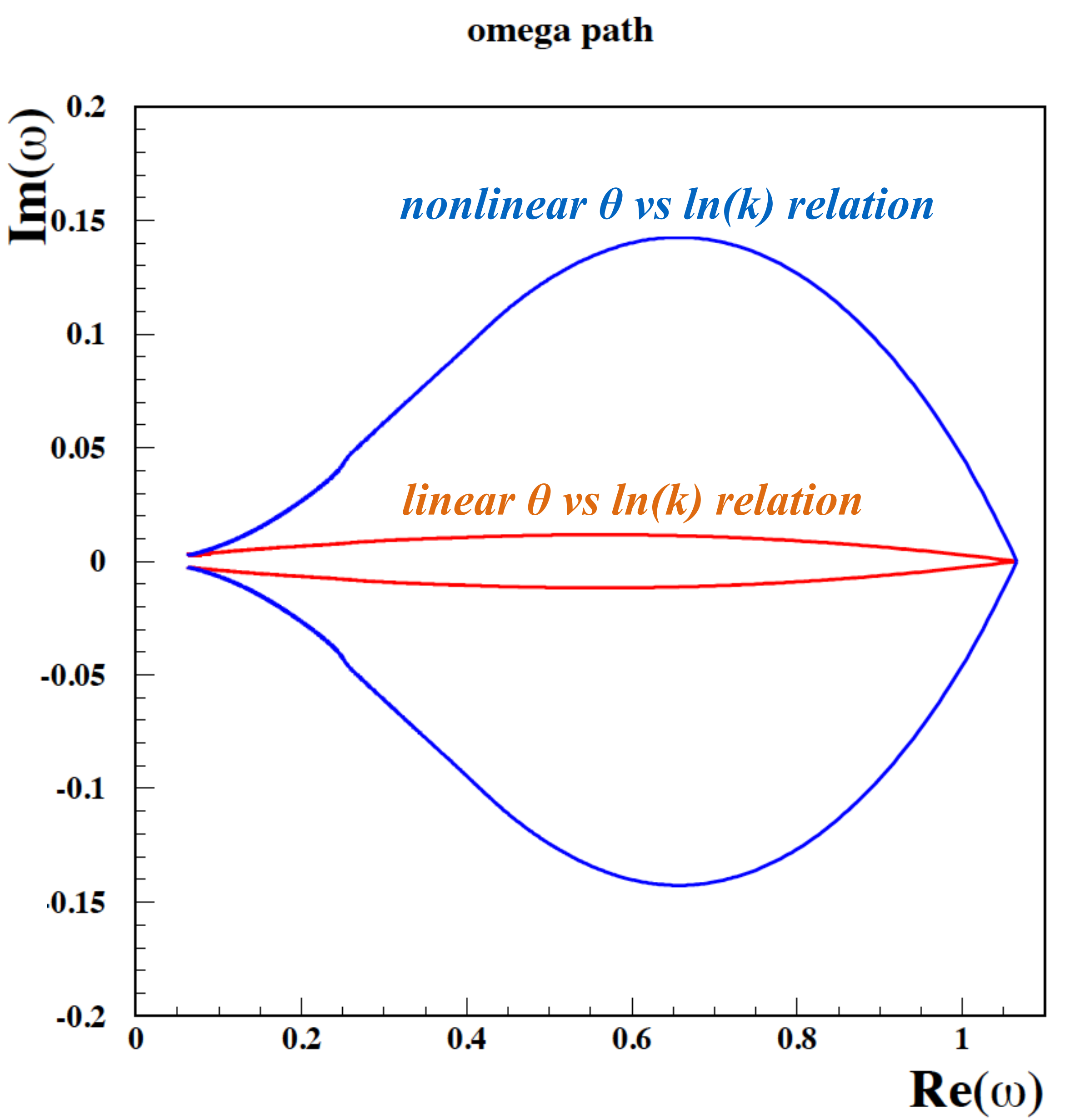
$t_c = \chi(\nu{=}0)/\omega\beta_0$

ω *complex* → α_s *complex*

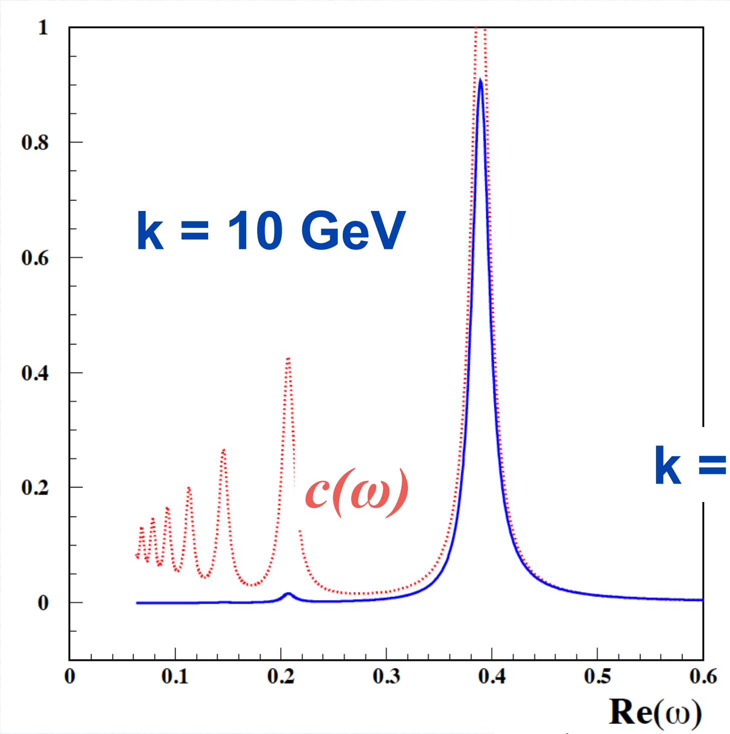
$\alpha_s = 1/(\beta_0 \ln(k^2/\Lambda^2)+i\theta)$

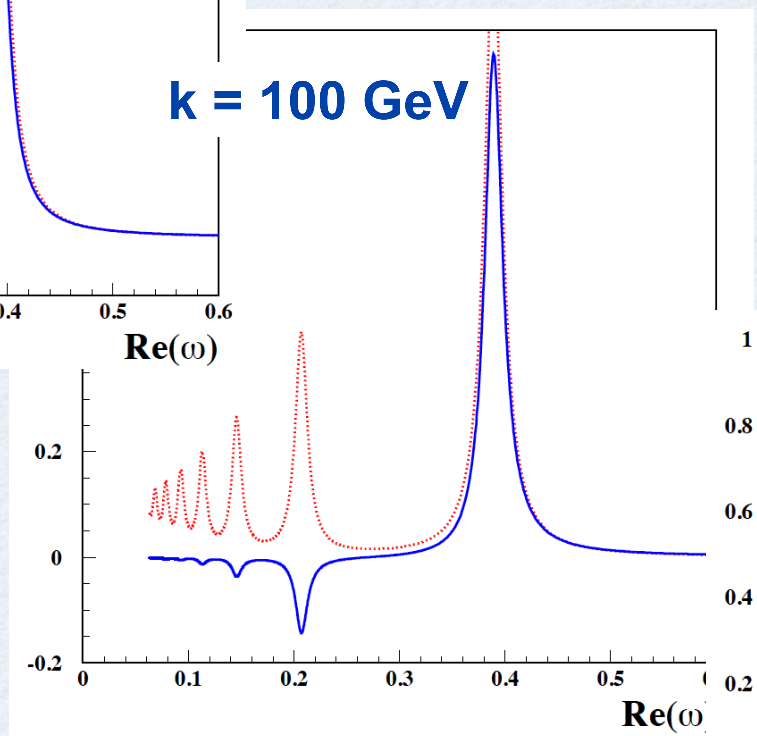
assumption:

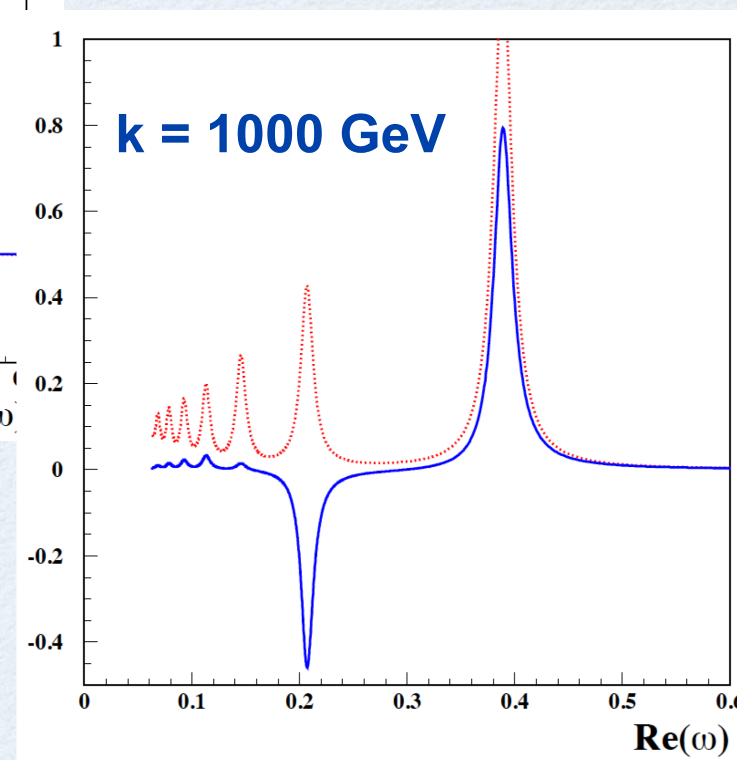
$\theta < \pm\pi$



Integrand of gluon density along the ω path, at x^{-2} *(linear path)*





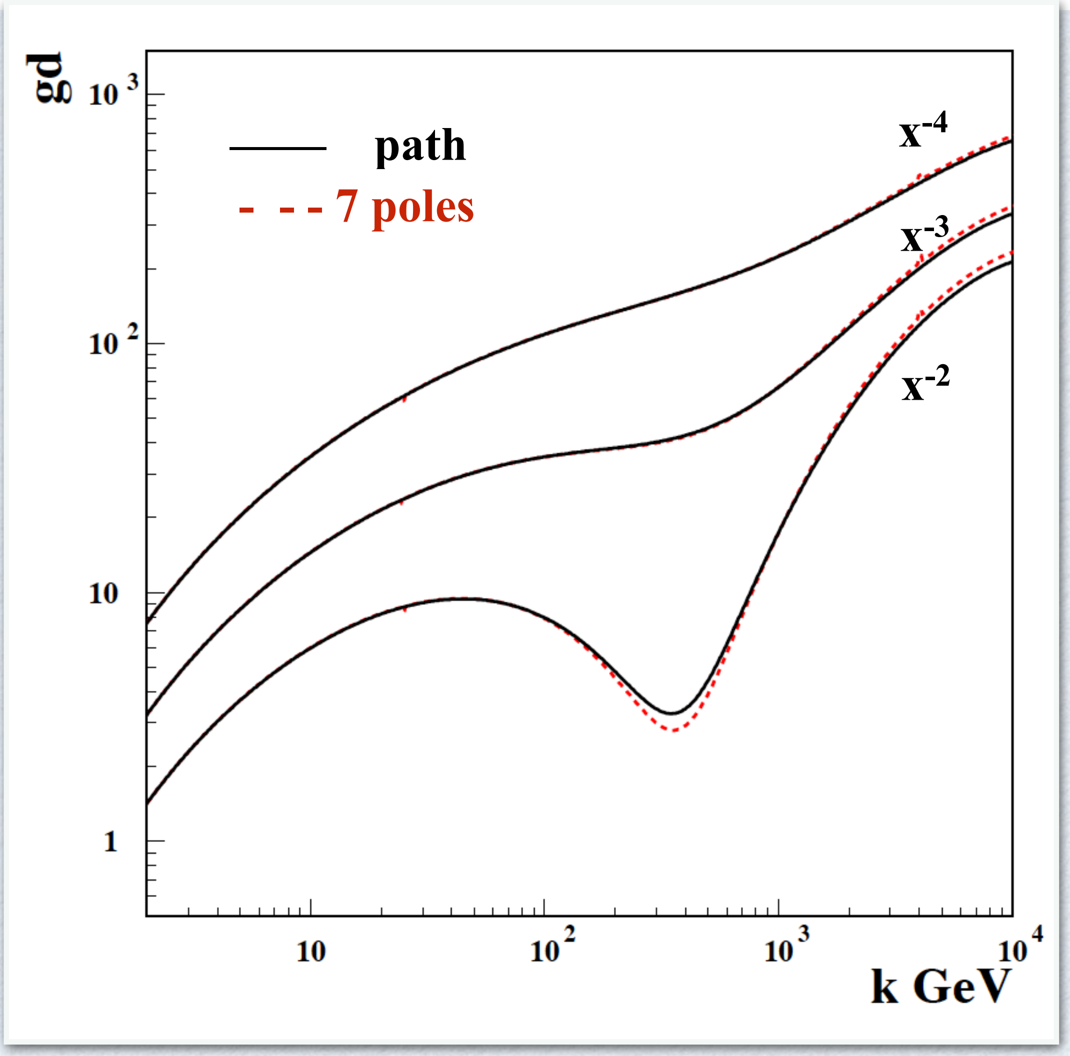


Integrals are convergent for k up to 1000 GeV

results are path independent

is the pole contribution the same as the integral along the ω path?

$$\mathcal{G}^{\text{pole}}_{\omega}(t,t') = \sum_n \pi N_{\omega_n}(t) N_{\omega_n}(t') \frac{\text{Ai}\,(z(t))\,\text{Ai}\,(z(t'))}{\phi'(\omega_n)(\omega-\omega_n)},$$

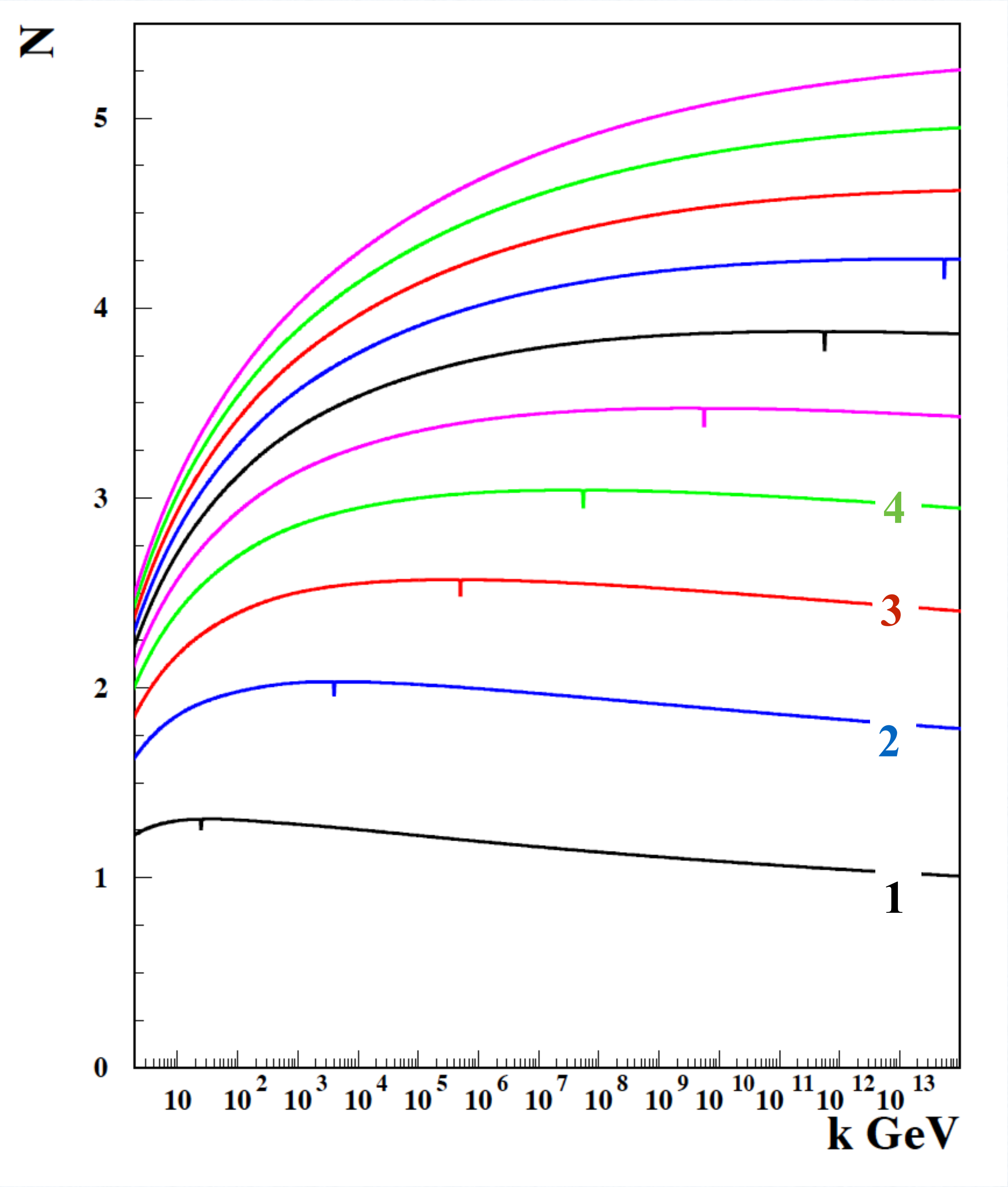


are the eigenfunctions orthonormal?

$$\int_{t=t_0}^{t=\infty} \pi N_i(t) N_j(t)\, Ai(z_i)\, Ai(z_j)/\sqrt{\varphi_i'}\,\sqrt{\varphi_j'}\, dt = \delta_{ij} \;?$$

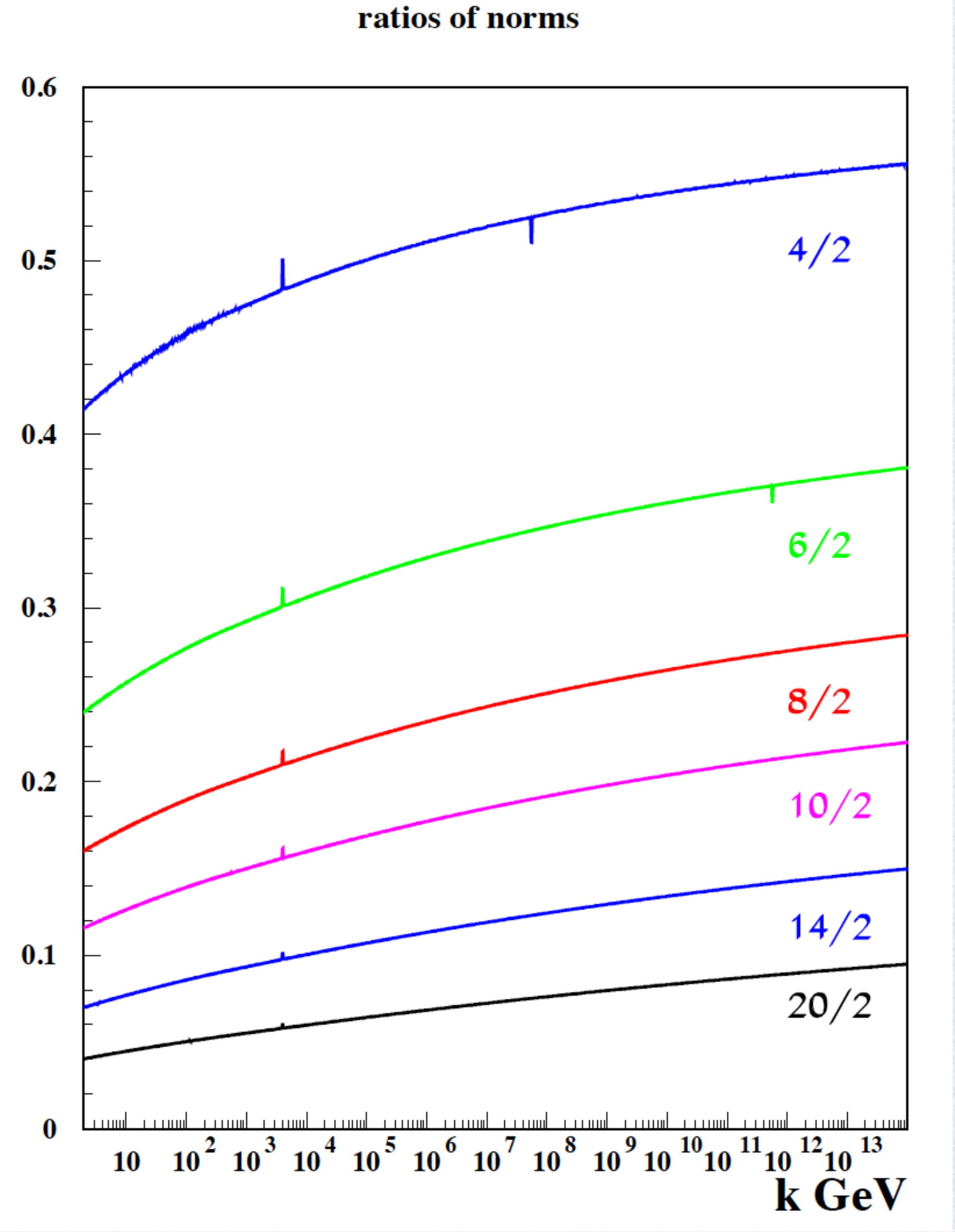
1	1	0.1054E+01	2	1	-0.3009E-01	3	1	0.2040E-01
1	2	-0.3009E-01	2	2	0.1011E+01	3	2	-0.8799E-02
1	3	0.2040E-01	2	3	-0.8799E-02	3	3	0.1004E+01
1	4	-0.1581E-01	2	4	0.7016E-02	3	4	-0.5448E-02
1	5	0.1247E-01	2	5	-0.5243E-02	3	5	0.3562E-02
1	6	-0.1070E-01	2	6	0.4661E-02	3	6	-0.3274E-02
1	7	0.9272E-02	2	7	-0.4059E-02	3	7	0.2806E-02
1	8	-0.8252E-02	2	8	0.3660E-02	3	8	-0.2532E-02
1	9	0.7360E-02	2	9	-0.3256E-02	3	9	0.2224E-02
1	10	-0.6829E-02	2	10	0.3121E-02	3	10	-0.2197E-02

the orthonormality of the computation is a proof that the wave functions disappear below t_0 and that the semiclassical approximation works!!!



N
5
4
3
2
1
0
4
3
2
1
10
10 2
10 3
10 4
10 5
10 6
10 7
10 8
10 9
10 10
10 11
10 12
10 13
k GeV

why is the pole contribution so quickly convergent?



$N(t)N(t')/\varphi'$ factor is droping like $1/n\sqrt{n}$ at low t

$x^{-\omega}$ factor enhances the first poles

Conclusions

The Discrete-Pomeron solution of BFKL provides a very good description of HERA data

The new complex Green function evaluation of BFKL provides a similar set of eigenfunctions than the previous one but which are faster converging. We need now O(10) poles instead of O(100) as previously.

Self-consistency of semiclassical approximation

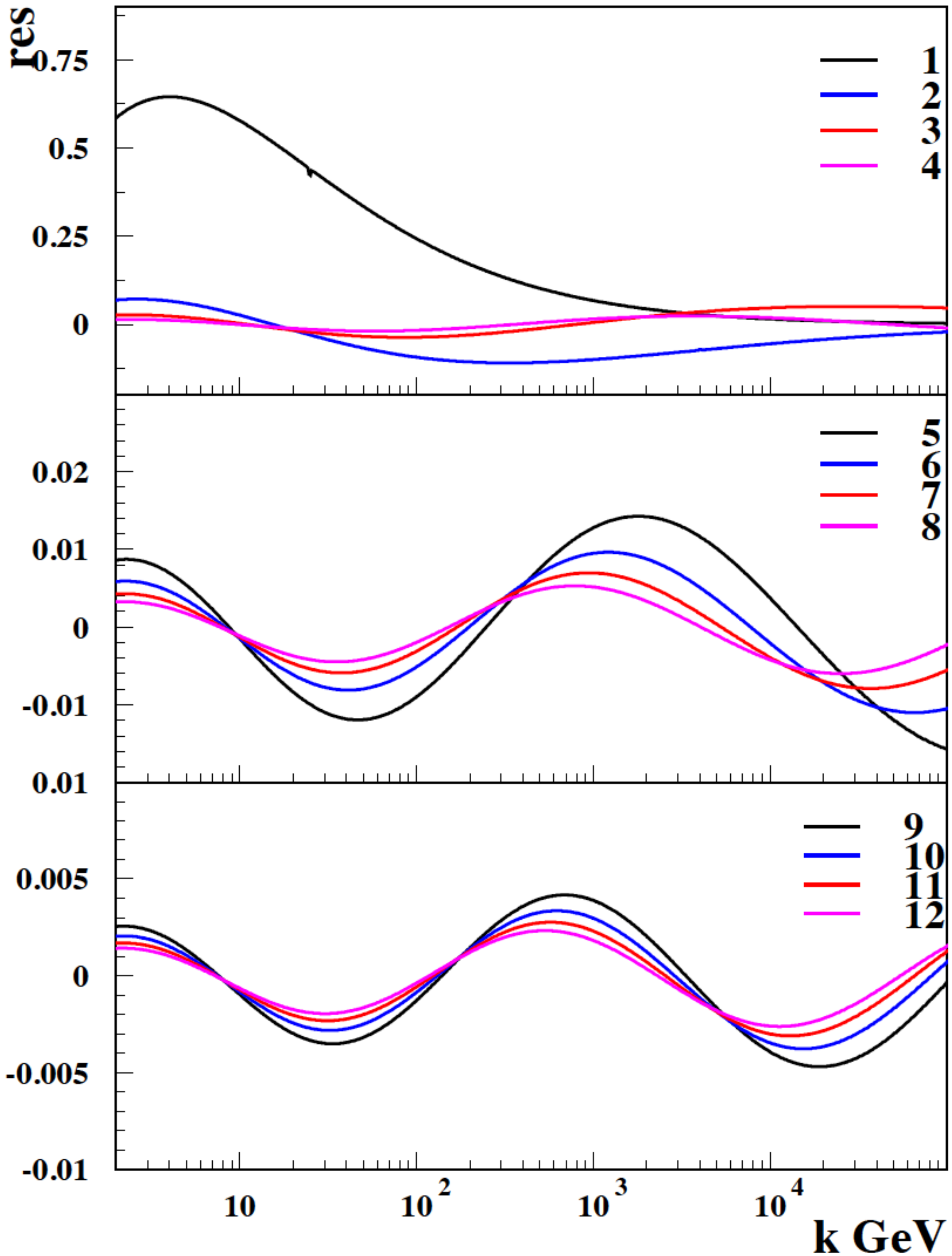
The investigation of properties of the new solution is close to the end

After it is finished we will apply it to HERA (F2) and LHC (DY) data (in the NLO version).

Similar sensitivity to BSM effects as previously?

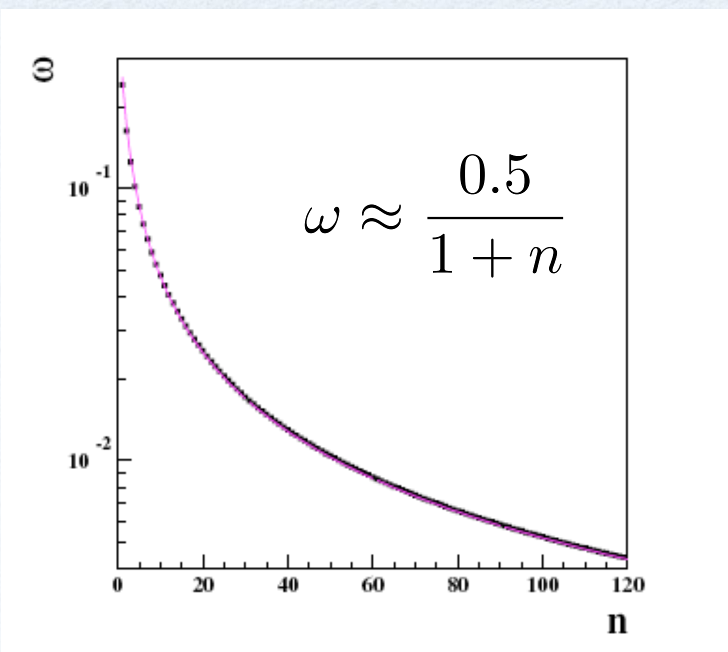
Back up slides

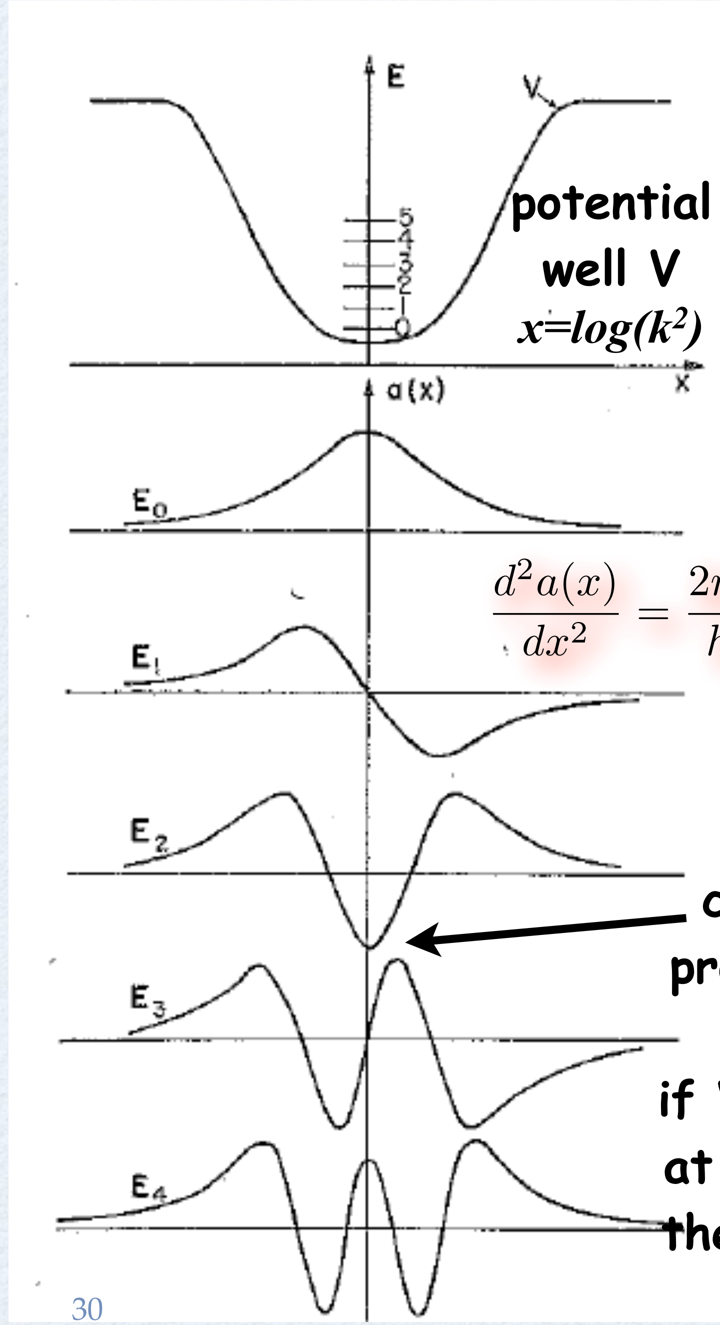
residue functions x^{-2}



Similarity with the Schroedinder eq. for the potential well Feynman Lecture III

BFKL eq is similar to S. eq for the potential well with the dynamically increasing width



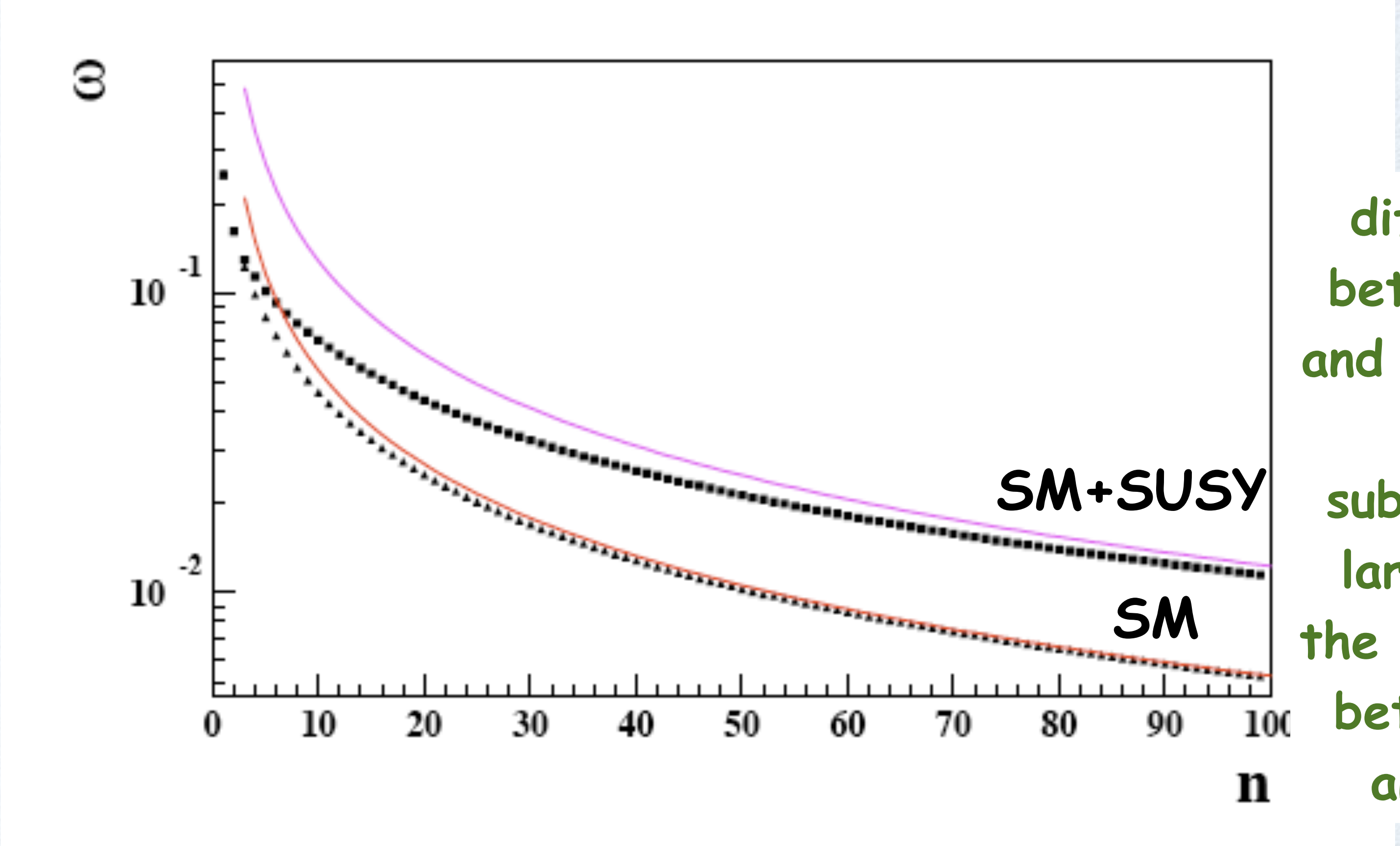


analogy worked out with J.Bartels

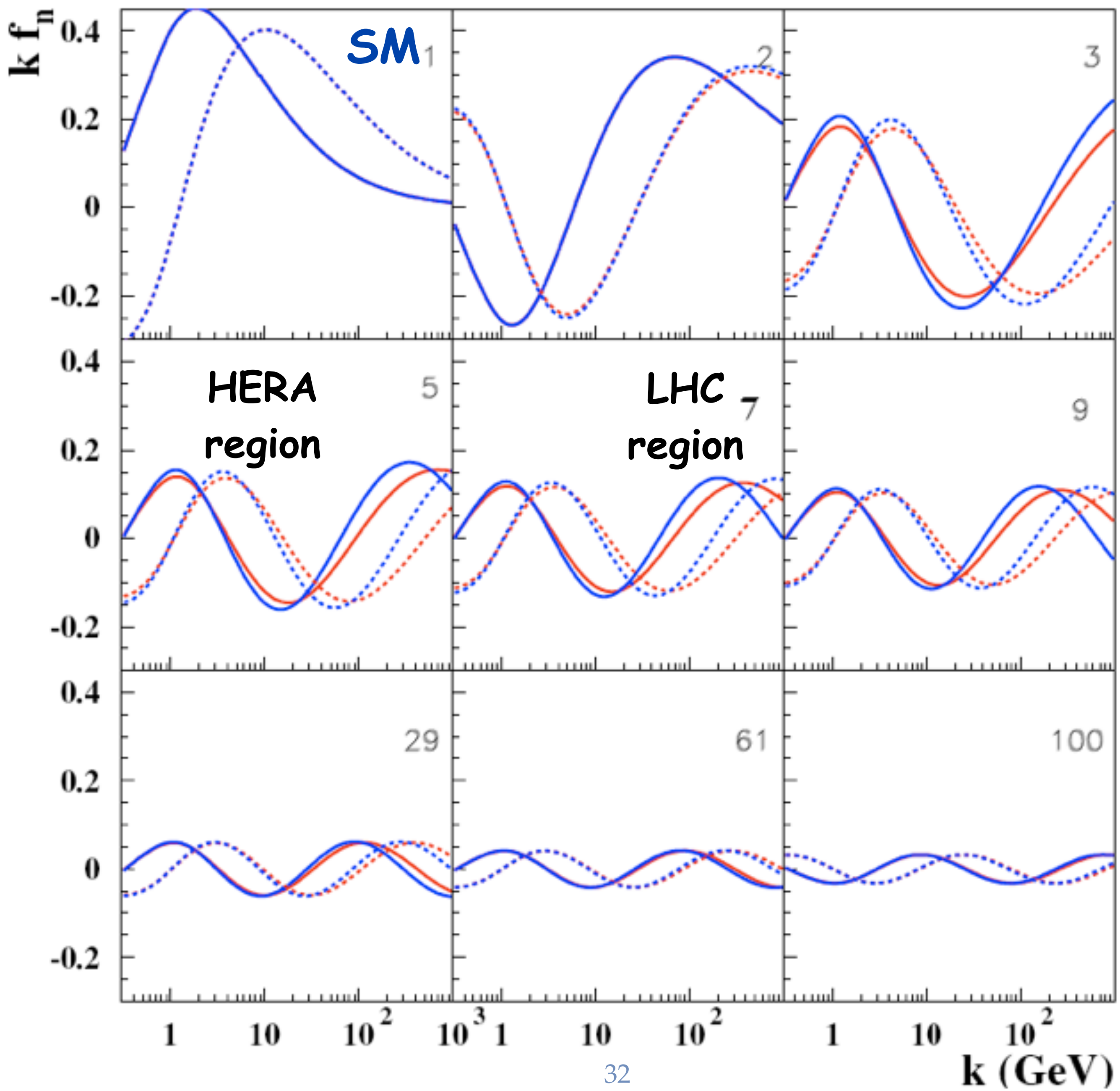
curvature is proportional to (V-E)
if V is modified at some x then the whole wf is changed

Eigenvalues of the Discrete BFKL-Pomeron

Comparison of the LO analytical (lines) and the NLO numerical evaluation (symbols)

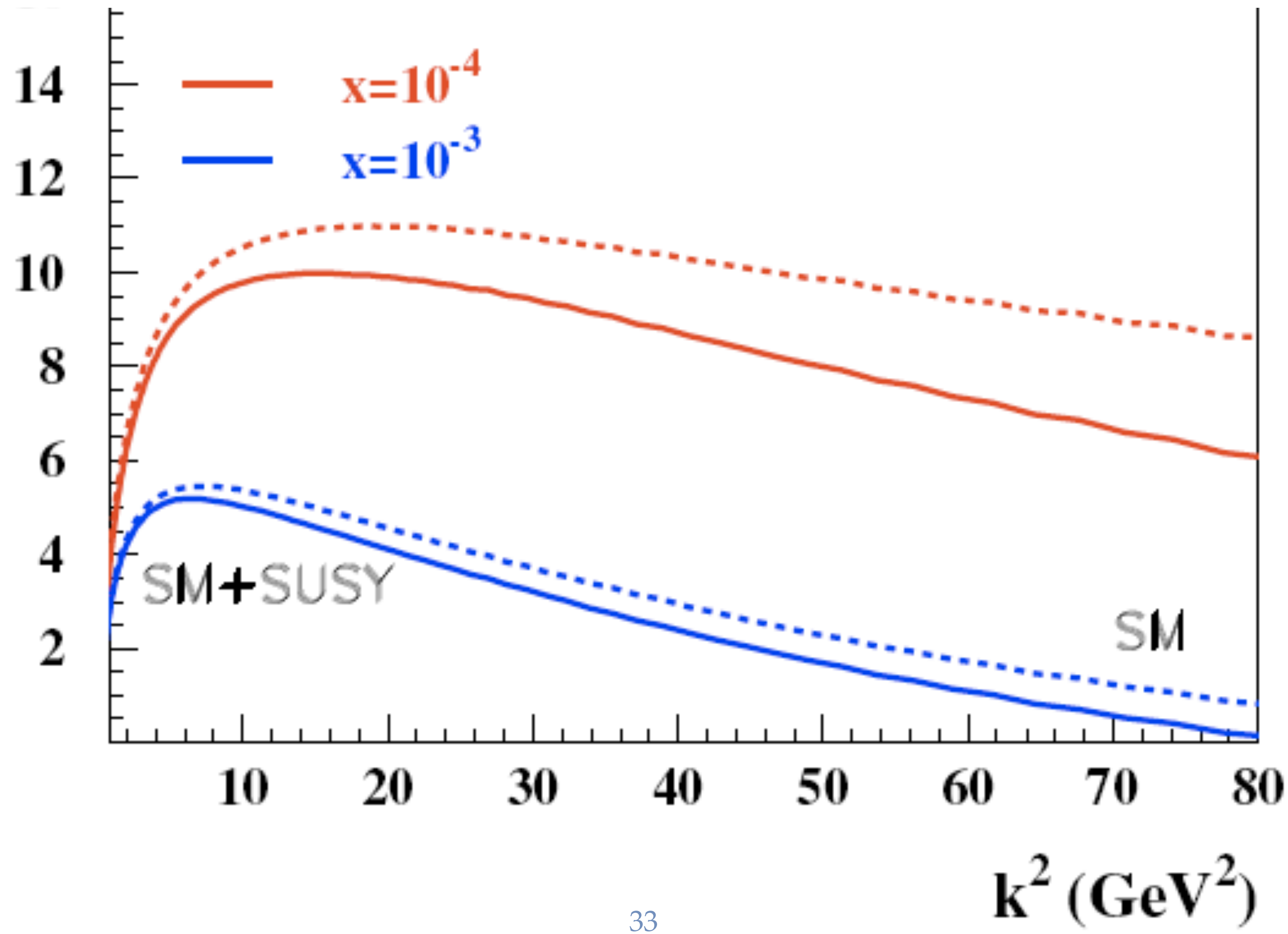


difference between SM and SM+SUSY is substantially larger than the difference between LO and NLO


k f_n
SM
1
2
3
HERA
region
5
LHC
region
7
9
29
61
100
0.4
0.2
0
-0.2
1
10
10^2
10^3
k (GeV)

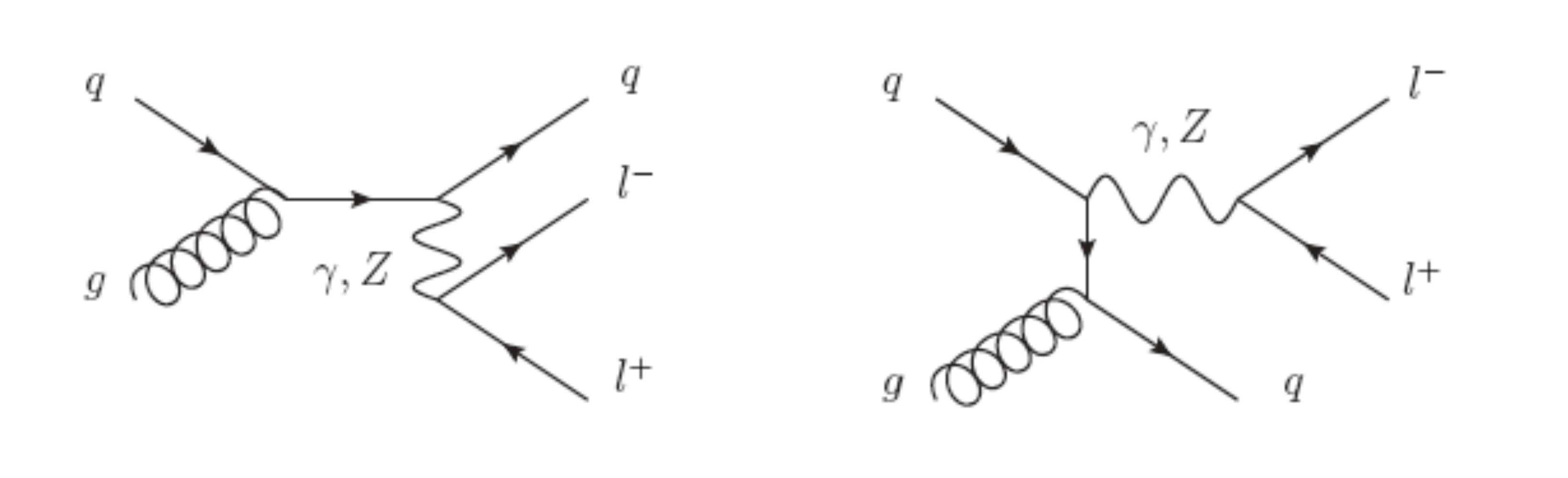
Evolution of the gluon density in DPS

$$x\dot{g}(x,k^2) = k^2 \int d\omega \int \frac{dk'^2}{k'^2}\left(\frac{kx}{k'}\right)^{-\omega} \hat{\mathcal{G}}_\omega(k,k')\Phi_p(k'),$$



Drell-Yan processes at LHC

Dominant process at LHC

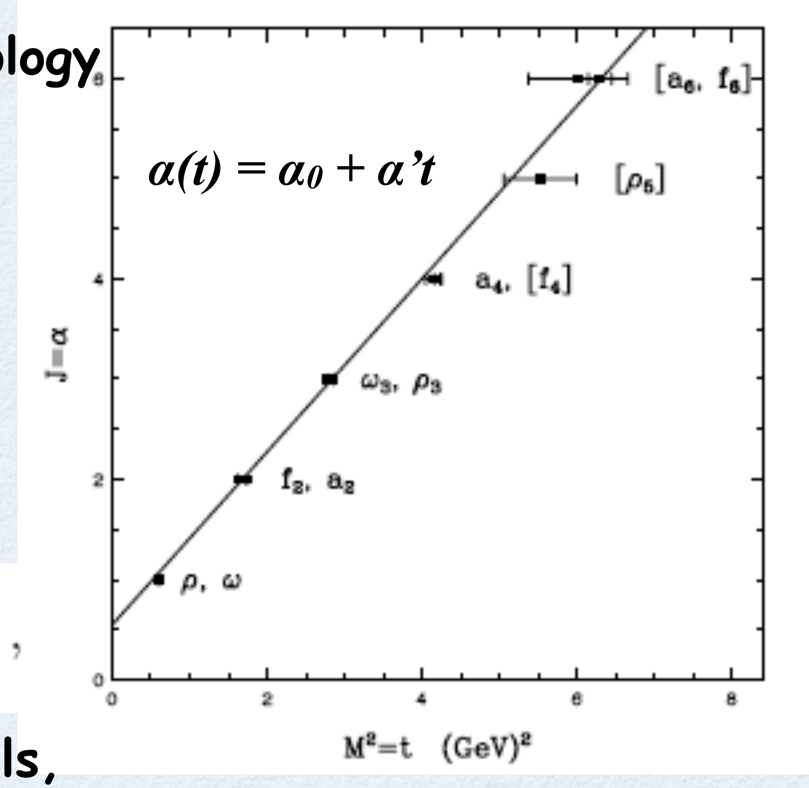


Additional requirement: add valence quarks contribution, i.e; gluon and sea-quark contribution like in DPS and valence quarks like in DGLAP

necessary requirement: obtain DGLAP from DP-BFKL

Pomeron - Graviton Correspondence

String theory emerged out of phenomenology of hadron-hadron scattering - Dolan-Horn-Schmid duality



$$\sum_r \frac{g_r^2(t)}{s-(M_r-i\Gamma_r)^2} \simeq \beta(t)(-\alpha' s)^{\alpha(t)}$$

- Veneziano amplitude

$$A_{\pi^+\pi^-\to\pi^+\pi^-}(s,t) = g_o^2 \frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[1-\alpha_\rho(s)-\alpha_\rho(t)]},$$

- generalization to dual resonance models,

Veneziano amplitude for the pomeron trajectory has a pole for $s=t=0$ *with* $J=2$

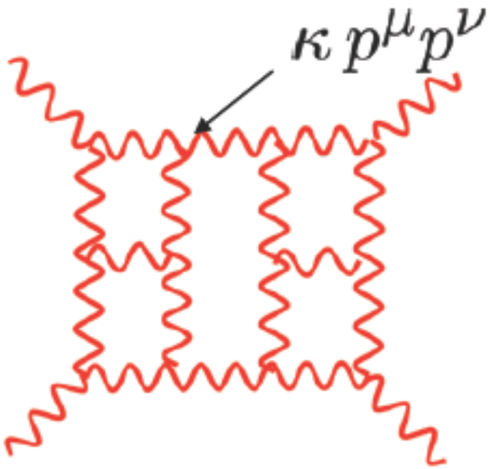
- starting point for a theory of quantum gravity

Maldacena Conjecture: (N=4 SUSY QCD) = (CFT in $ADS_5 \times S^5$)

from a talk by ZVI BERN

Is a UV finite theory of gravity possible?

$\kappa = \sqrt{32\pi G_N}$ ⟵ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{dp_i^D}{(2\pi)^D} \frac{(\kappa\, p_j^\mu p_j^\nu)\cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g\, p_j^\nu)\cdots}{\text{propagators}}$$

Extra powers of loop momenta in numerator means integrals are badly behaved in the UV

Focus on N=8 supergravity and N=4 SUSY YM
High degree of symmetry => technical simplicity
new methods developed:

Modern Unitarity, symbology, BDS ...

focus on order by order finiteness - now up to 6 loops
Infinite loop calculation could be possible in the Multi-Regge limit

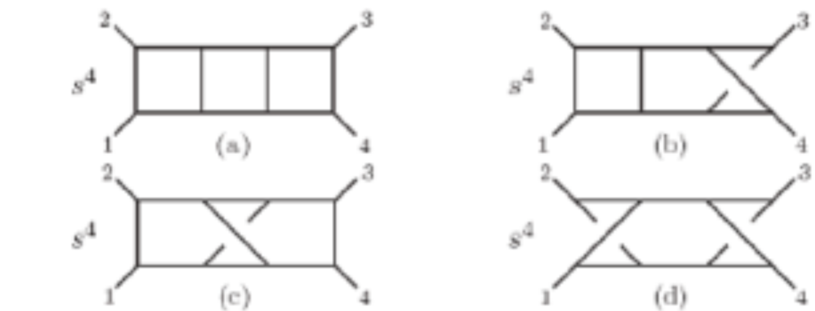
Complete Three Loop Result

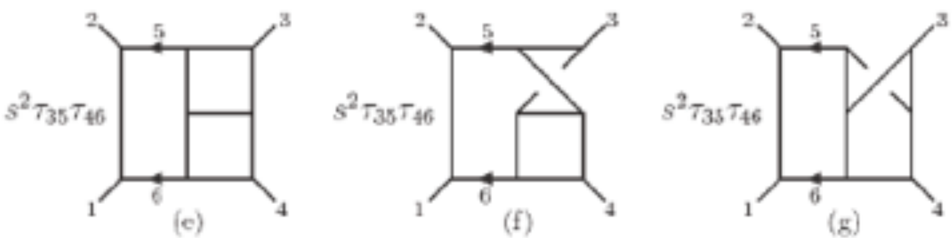
ZB, Carrasco, Dixon, Johansson, Kosower, Roiban; hep-th/0702112
ZB, Carrasco, Dixon, Johansson, Roiban arXiv:0808.4112 [hep-th]

Obtained via maximal cut method:

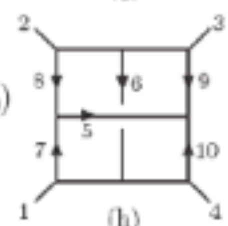
from a talk by ZVI BERN

$$\tau_{ij} = 2k_i \cdot k_j$$

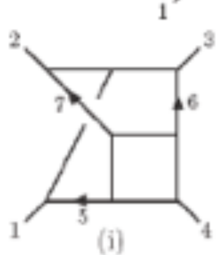




$$\begin{aligned}&(s(\tau_{26}+\tau_{36})+t(\tau_{15}+\tau_{25})+st)^2\\&+(s^2(\tau_{26}+\tau_{36})-t^2(\tau_{15}+\tau_{25}))(\tau_{17}+\tau_{28}+\tau_{39}+\tau_{4,10})\\&+s^2(\tau_{17}\tau_{28}+\tau_{39}\tau_{4,10})+t^2(\tau_{28}\tau_{39}+\tau_{17}\tau_{4,10})\\&+u^2(\tau_{17}\tau_{39}+\tau_{28}\tau_{4,10})\end{aligned}$$

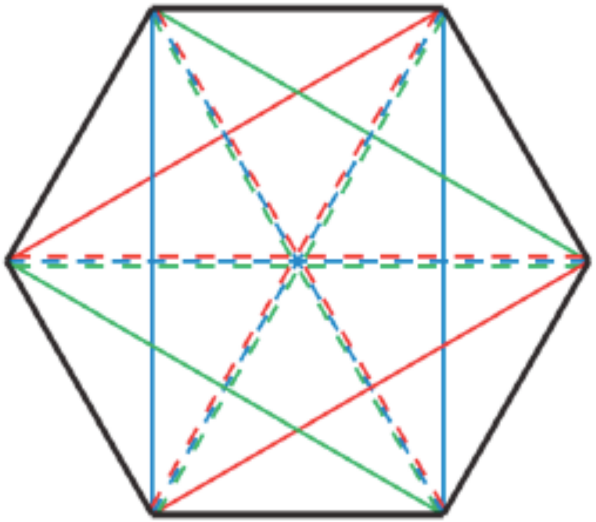


$$\begin{aligned}&(s\tau_{45}-t\tau_{46})^2-\tau_{27}(s^2\tau_{45}+t^2\tau_{46})\\&-\tau_{15}(s^2\tau_{47}+u^2\tau_{46})-\tau_{36}(t^2\tau_{47}+u^2\tau_{45})\\&+l_5^2s^2t+l_6^2st^2-\tfrac{1}{3}l_7^2stu\end{aligned}$$



Three-loop is not only ultraviolet finite it is "superfinite"—cancellations beyond those needed for finiteness!

Scattering
in Planar N=4 Super-Yang-Mills Theory
and the Multi-Regge-Limit



Lance Dixon (SLAC)

ICHEP Melbourne, Australia July 5, 2012

from the Summary:

• Multi-Regge limit of 6-gluon amplitude may well be first case solved to all orders