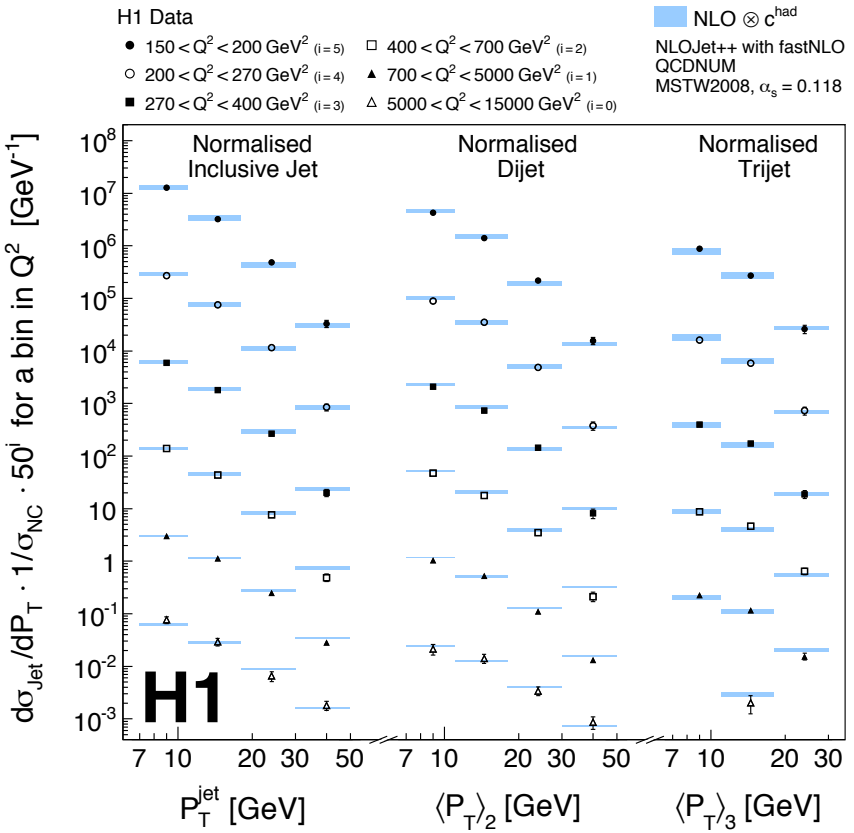
NNLO corrections to jet production at HERA

Thomas Gehrmann, Universität Zürich

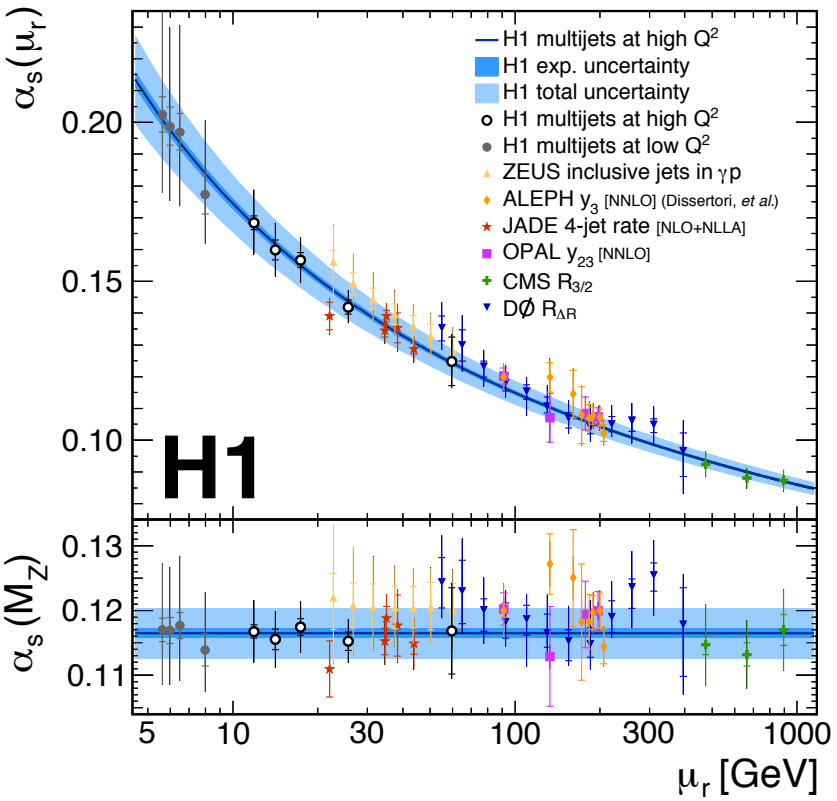
Jet production at HERA

- Large production rates for jet final states
  - Precision measurements
  - Large kinematical coverage
  - Consider inclusive jets, 2 jet, 3 jet
- Allow precision measurements
  - Gluon distribution
  - Strong coupling constant
- Current theory: NLO
  - Studies often theory-limited
  - Require NNLO



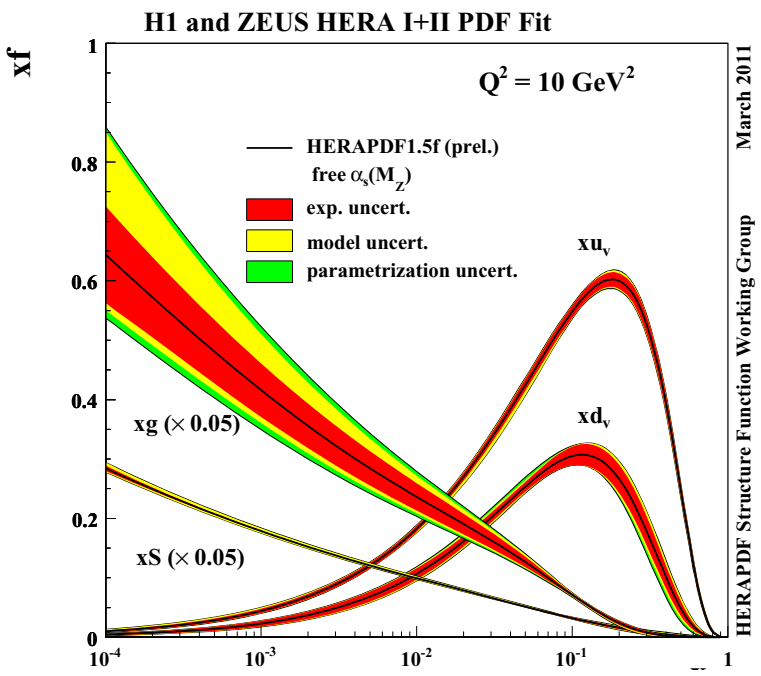
Jet production at HERA

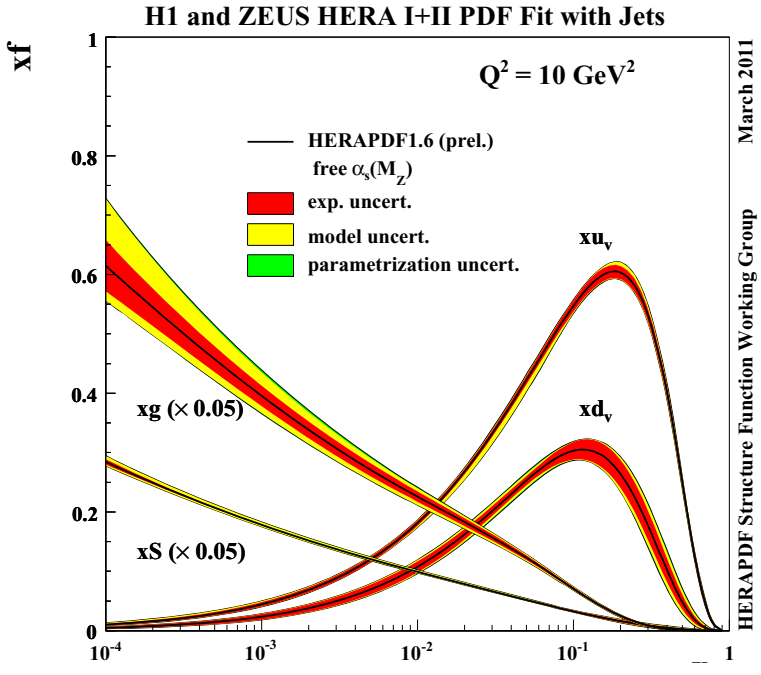
- Extraction of strong coupling constant from jets
  - H1: $\alpha_s(M_Z) = 0.1165 \pm 0.008(\text{exp}) \pm 0.038(\text{pdf,th})$



Jet production at HERA

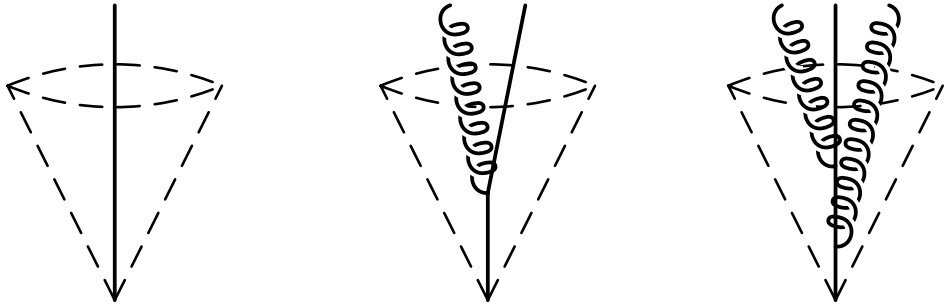
- Impact of jet data on parton distribution fits
  - NLO only, many precision applications now demand NNLO





Jets in perturbative QCD

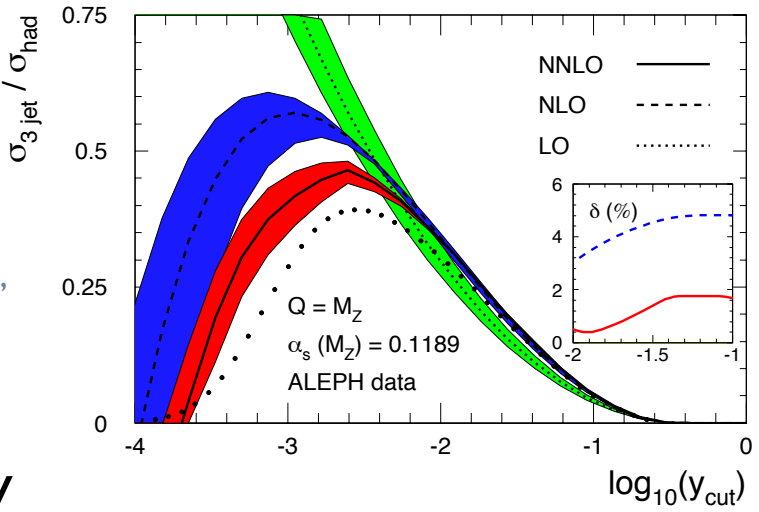
- Partons are combined into jets with the same jet algorithm as in experiment



  - No algorithm dependence at leading order
  - Theoretical description more accurate with increasing order
  - Current status: NNLO starting to become available

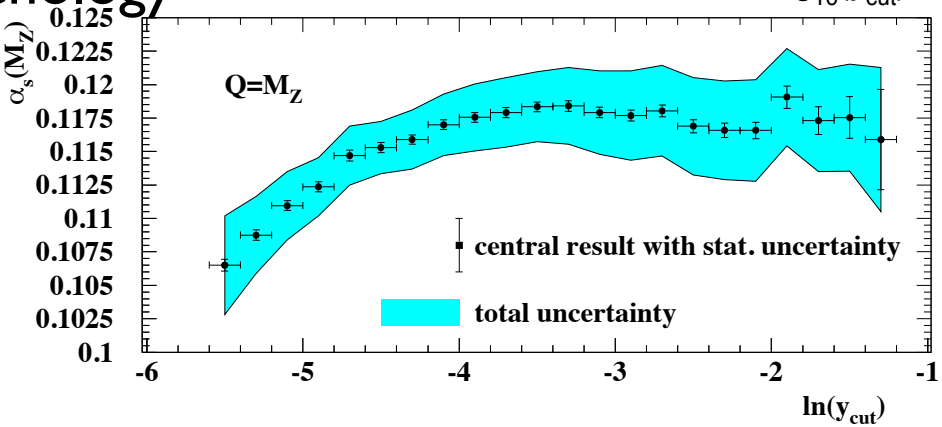
NNLO corrections to $e^+e^- \to 3$ jets

- Calculation of NNLO corrections to $e^+e^- \to 3$ jets and related event shapes (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)



- Revisit LEP precision phenomenology (with G. Dissertori, G. Luisoni, H. Stenzel)

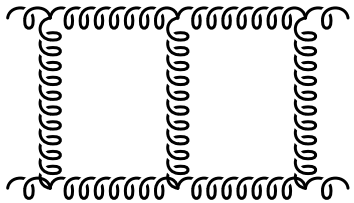
- Strong coupling from R_{3jet}



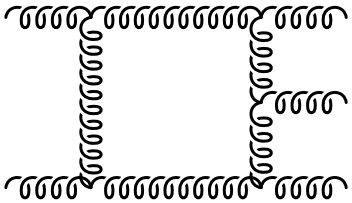
$$\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\text{exp}) \pm 0.0015\,(\text{theo})$$

Ingredients to jet production at NNLO

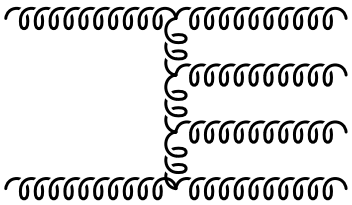
- Two-loop matrix elements
  - Explicit infrared poles from loop integrals



- One-loop matrix elements
  - Explicit infrared poles from loop integral
  - Implicit infrared poles from real radiation



- Tree-level matrix elements
  - Implicit infrared poles from real radiation



Real radiation at NNLO: factorization

- Single unresolved radiation at one loop
  - One-loop correction to collinear splitting factors (Z. Bern, V. Del Duca, W. Kilgore, C. Schmidt)
  - One-loop correction to soft eikonal factor (S. Catani, M. Grazzini)
- Double unresolved radiation factors at tree level (J. Campbell, E.W.N. Glover; S. Catani, M. Grazzini)
  - Double soft
  - Soft/Collinear
  - Triple collinear
  - Double single collinear
- Require method to extract singular contributions

NNLO: methods & results

- Sector decomposition
  (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)
  - $pp \to H$, $pp \to V$, including decays
    (C. Anastasiou, K. Melnikov, F. Petriello)
- Sector-improved subtraction schemes
  (M. Czakon; R. Boughezal, K. Melinkov, F. Petriello)
  - $pp \to t\bar{t}$ (M. Czakon, P. Fiedler, A. Mitov)
  - $pp \to H+j$ (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
- q_T-subtraction (S. Catani, M. Grazzini)
  - $pp \to H$, $pp \to V$, $pp \to \gamma\,\gamma$, $pp \to VH$, pp $pp \to VV$
    (S. Catani, M. Grazzini, et al.)
- Antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - $e^+e^- \to 3j$ (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)
  - $pp \to 2j$ (A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
  - $pp \to H+j$ (X. Chen, E.W.N. Glover, M. Jaquier, TG)

NNLO Infrared Subtraction

Structure of NNLO cross section

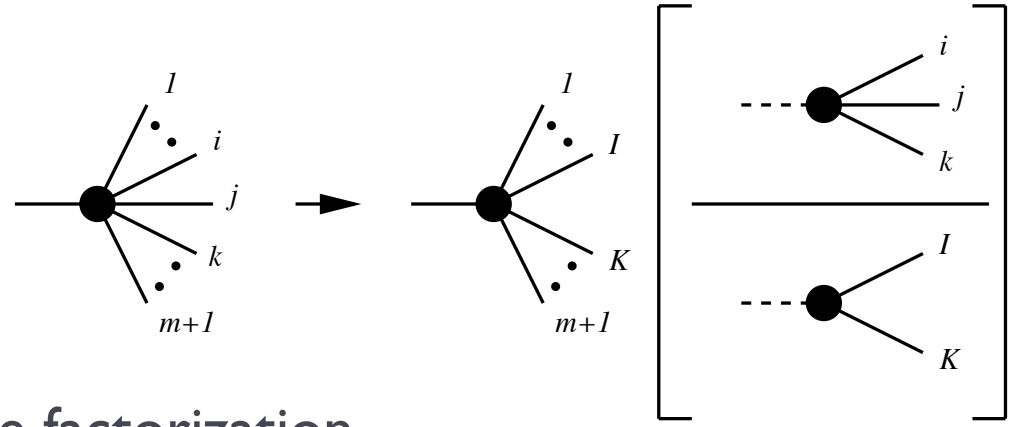
$$\begin{aligned}
\mathrm{d}\sigma_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{MF,1}_{NNLO} \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{MF,2}_{NNLO}
\end{aligned}$$

- Real and virtual contributions: $\mathrm{d}\sigma^{R}_{NNLO}, \mathrm{d}\sigma^{V,1}_{NNLO}, \mathrm{d}\sigma^{V,2}_{NNLO}$
- Subtraction term for double real radiation: $\mathrm{d}\sigma^{S}_{NNLO}$
- Subtraction term for one-loop single real radiation: $\mathrm{d}\sigma^{VS,1}_{NNLO}$
- Mass factorization terms: $\mathrm{d}\sigma^{MF,1}_{NNLO}, \mathrm{d}\sigma^{MF,2}_{NNLO}$

- Each line finite and free of poles
  → numerical implementation

Antenna subtraction

- Subtraction terms constructed from antenna functions
  - Antenna function contains all emission between two partons



  - Phase space factorization

$$d\Phi_{m+1}(p_1,\ldots,p_{m+1};q) = d\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_K,\ldots,p_{m+1};q)\cdot d\Phi_{X_{ijk}}(p_i,p_j,p_k;\tilde{p}_I+\tilde{p}_K)$$

  - Integrated subtraction term

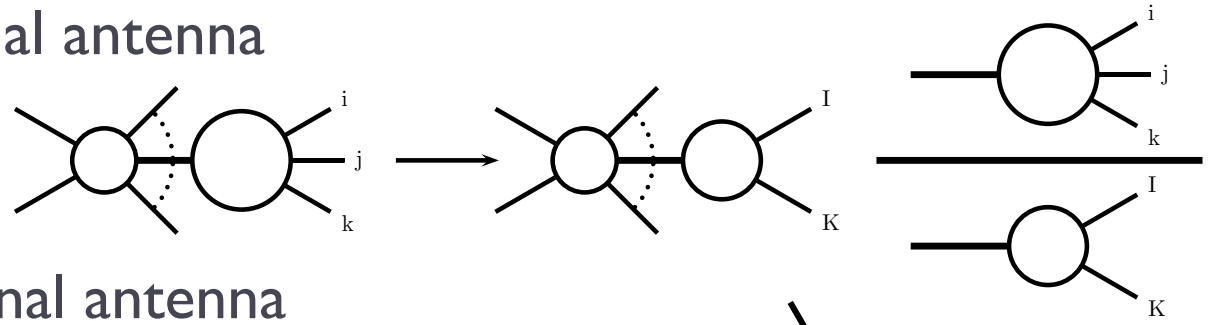
$$\mathcal{X}_{ijk} = \int d\Phi_{X_{ijk}} X_{ijk}$$

Antenna functions

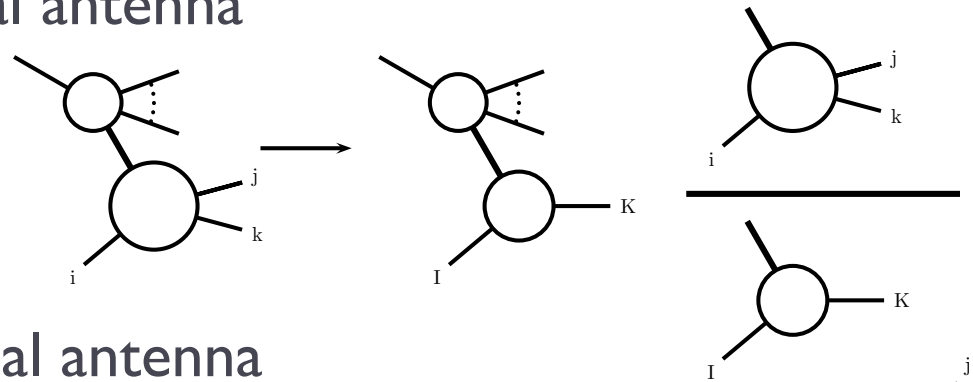
- Colour-ordered pair of hard partons (radiators)
  - Hard quark-antiquark pair
  - Hard quark-gluon pair
  - Hard gluon-gluon pair
- NLO (D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover)
  - Three-parton antenna: one unresolved parton
- NNLO (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - Four-parton antenna: two unresolved partons
  - Three-parton antenna at one loop
  - Products of NLO antenna functions
  - Soft antenna function

Antenna subtraction: incoming hadrons

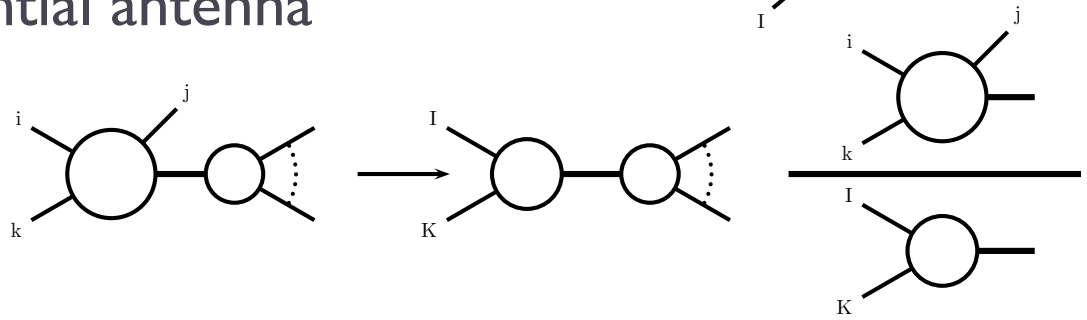
- **Three antenna types** (A. Daleo, D. Maitre, TG)
  - Final-final antenna



  - Initial-final antenna



  - Initial-intial antenna



Integrated NNLO antenna functions

- Analytical integration over unresolved part of phase space only
  - phase space integrals reduced to masters (C. Anastasiou, K. Melnikov)
  - Final-final: $q \to k_1 + k_2 + k_3(+k_4)$, one scale: q^2
    - $1 \to 4$ tree level
    - $1 \to 3$ one loop
  - Initial-final: $q + p_1 \to k_1 + k_2(+k_3)$, two scales: q^2, x
    (A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG)
    - $2 \to 3$ tree level
    - $2 \to 2$ one loop
  - Initial-initial: $p_1 + p_2 \to q + k_1(+k_2)$, three scales: q^2, x_1, x_2
    (R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann, TG)
    - $2 \to 3$ tree level
    - $2 \to 2$ one loop

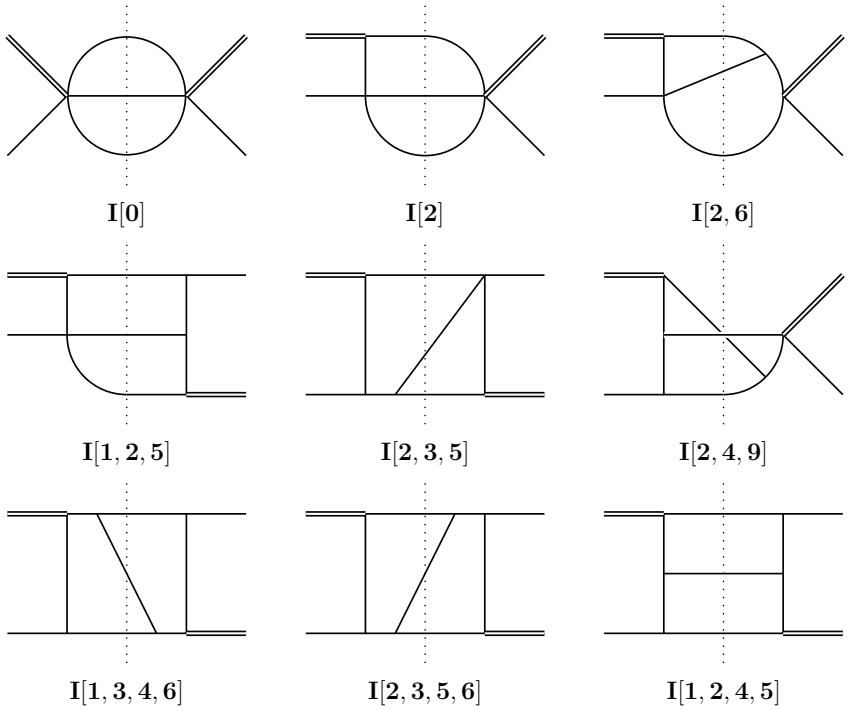
Integration of antenna functions

- Initial-final antenna functions (A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG)
  - resemble NNLO coefficient functions in DIS

- Validated
  - Coefficient functions F_2, F_L
    (W. van Neerven. E. Zijlstra)

$$\frac{1}{C_F N_F}(\mathcal{F}_{2,q}^{(2,\mathrm{PS})} - \frac{3-2\epsilon}{2-2\epsilon}\mathcal{F}_{L,q}^{(2,\mathrm{PS})}) = \bar{\mathcal{B}}_{q,q'\bar{q}'q}^{0}$$

  - Higgs-induced DIS
    (G. Soar, S. Moch, J. Vermaseren, A. Vogt)



NNLO corrections to ep → 2j

- Status of the calculation (antenna subtraction)
  (J. Currie, J. Niehues, TG)
  - Matrix elements from MCFM (tree, 1-loop) and EERAD3 (2-loop)
  - Double real radiation
    - Subtraction terms constructed and 90% implemented and tested
    - Azimuthal correlations from gluon splitting
  - Single real radiation at one loop
    - Subtraction terms constructed, implementation started
    - Interplay of antenna functions and mass factorization
  - Two-loop contributions
    - Need to add integrated subtraction terms from above
    - Should yield analytic cancellation of all infrared poles
- All implemented in parton-level event generator

Summary

- Precision physics with jets demands NNLO corrections

- Current status of ep → 2j
  - All matrix elements known
  - Implementation of all parton-level processes at NNLO
    (J. Currie, J. Niehues, TG)
  - Double real radiation: nearly complete, others in progress

- Will provide flexible NNLO parton-level program
  - Ensure future availability of data for re-analysis
  - Decrease theory error on partons and strong coupling constant