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Workshop on Astroparticle Physics with Multiple Messengers
September 2014

From Primordial to Extragalactic Magnetic Fields

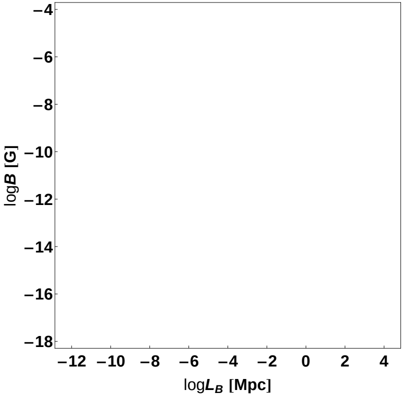
Andrey Saveliev

Universität Hamburg

September 11th, 2014

Contents

EGMF - Standard Constraints [Neronov and Semikoz, 2009]



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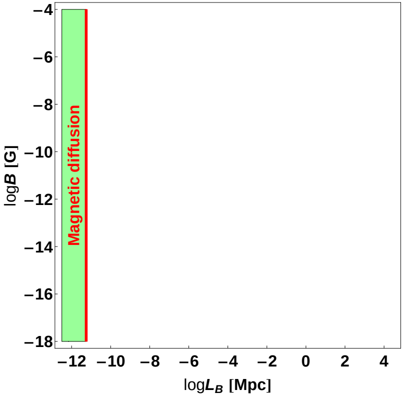

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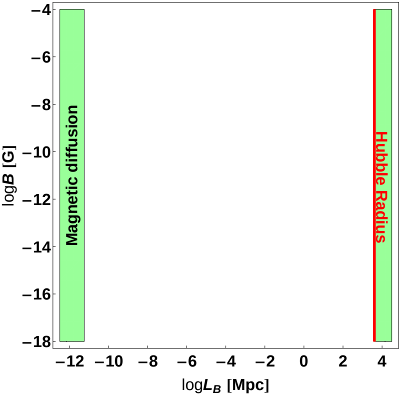

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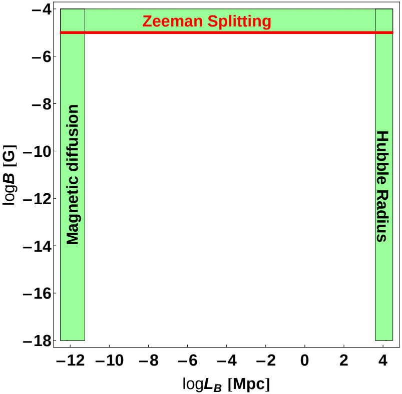

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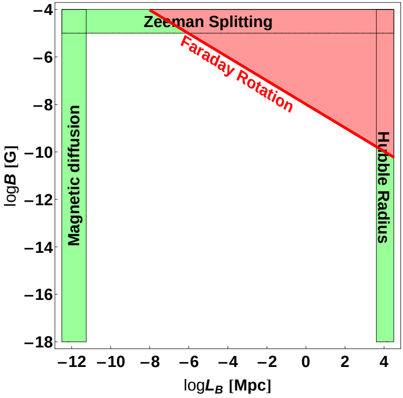

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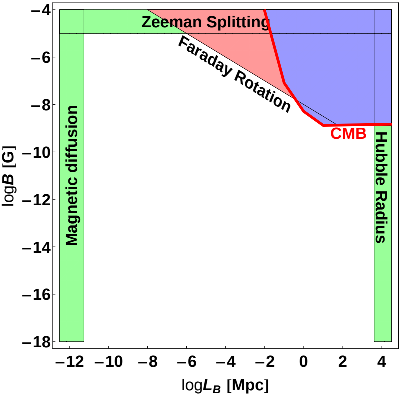

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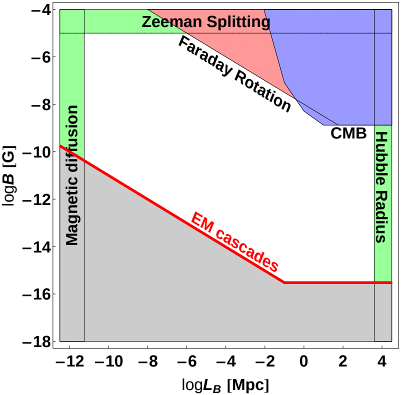

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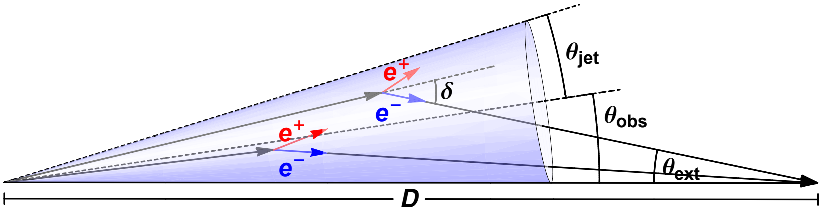
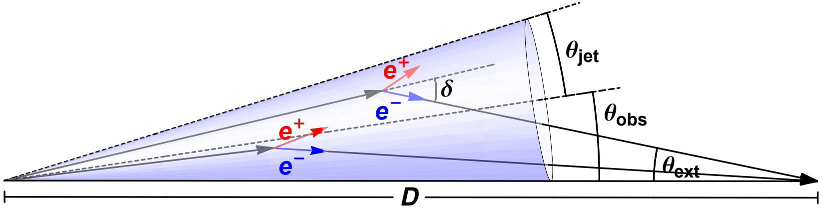
EGMF - Lower Bound on B?



Gamma rays emmitted from a blazar develop an electromagnetic cascade due to interactions with the Extragalactic Background Light (EBL) via Pair Production and Inverse Compton (IC) scattering. The interaction of this cascade with the EGMF results in several observational features.

EGMF - Lower Bound on B?



Point-like sources appear extensive [Dolag et al., 2009], [Neronov et al., 2010]

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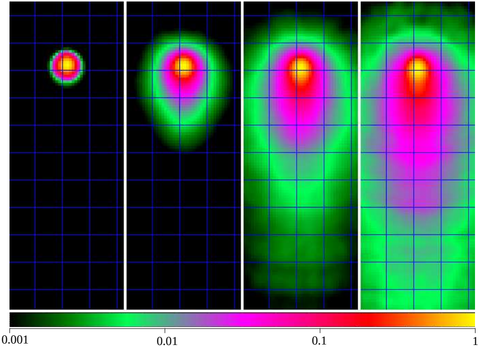

Appearance of a point-like source at $\theta_{obs} = 3$ for magnetic fields $B = 10^{-17}$ G, 10^{-16} G, 10^{-15} G and 10^{-14} G [Neronov et al., 2010]

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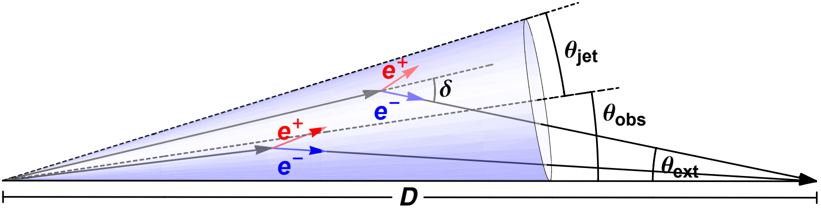
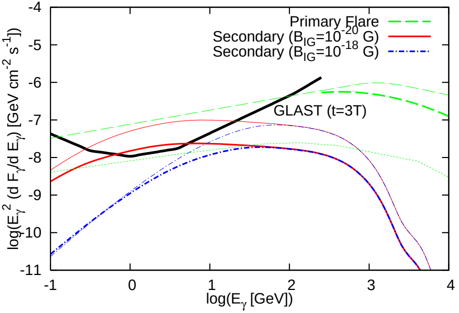


Time-delayed echos of primary gamma rays [Plaga, 1994], [Murase et al., 2008]



Spectrum of the time-delayed spectrum of the 2005 flare of Mrk 501 for different values of the EGMF after 0.5 days (thin) and 1.5 days (thick) [Murase et al., 2008]

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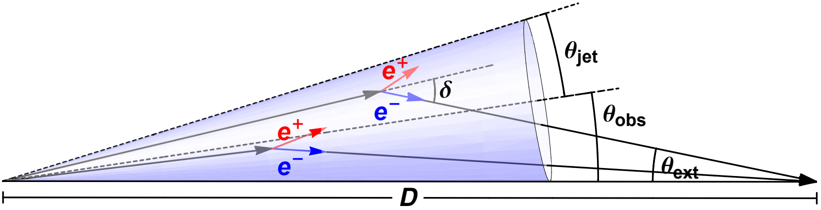

Suppression of observed photon flux in the GeV region
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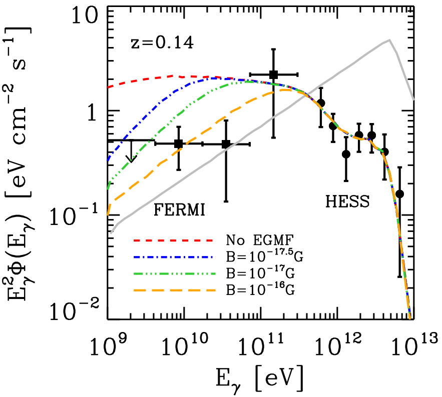

Predicted gamma ray flux of 1ES0229+200 for different magnetic fields with data points of Fermi LAT and HESS [Saveliev et al., 2013a]

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- Basics for the time evolution: Homogeneous and isotropic magnetohydrodynamics in an expanding Universe.

Primordial Magnetic fields - Basic MHD

Magnetohydrodynamics (MHD)

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- Maxwell Equations:
  $\nabla \cdot \mathbf{E} = 4\pi\rho_{\rm ch}, \ \nabla \cdot \mathbf{B} = 0, \ \nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \ \nabla \times \mathbf{B} = 4\pi \mathbf{j}$

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- Navier-Stokes Equations:
  $\rho\left(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\,\mathbf{v}\right) = -\nabla p + \mu \Delta \mathbf{v} + (\lambda + \mu)\nabla\left(\nabla \cdot \mathbf{v}\right) + \mathbf{f}$

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For the magnetic field and the turbulent fluid it follows therefore

$$\partial_t \mathbf{B} = \frac{1}{4\pi\sigma}\Delta\mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})$$
$$\partial_t \mathbf{v} = -(\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{(\nabla\times\mathbf{B})\times\mathbf{B}}{4\pi\rho} + \mathbf{f}_v .$$

Primordial Magnetic fields - Basic MHD

The aspect of interest is the distribution of energies on different scales k, i.e. the magnetic spectral energy density M of the magnetic fields and the kinetic magnetic spectral energy density U

$$\epsilon_B = \frac{1}{8\pi V}\int \mathbf{B}^2(\mathbf{x})\,\mathrm{d}^3x = \int \frac{|\hat{\mathbf{B}}(\mathbf{k})|^2}{8\pi}\mathrm{d}^3k \equiv \rho\int M_k \mathrm{d}k$$

$$\epsilon_K = \frac{\rho}{2V}\int \mathbf{v}^2(\mathbf{x})\,\mathrm{d}^3x = \frac{\rho}{2}\int |\hat{\mathbf{v}}(\mathbf{k})|^2\mathrm{d}^3k \equiv \rho\int U_k \mathrm{d}k$$

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In addition, for magnetic helicity one can define the spectral helicity density $\mathcal{H}$ by

$$h_B = \frac{1}{V}\int \mathbf{A}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x})\,\mathrm{d}^3x = i\int\left(\frac{\mathbf{k}}{k^2}\times\hat{\mathbf{B}}(\mathbf{k})\right)\cdot\hat{\mathbf{B}}(\mathbf{k})^*\mathrm{d}^3k$$

$$\equiv \rho\int \mathcal{H}_k \mathrm{d}k$$

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Switch to Fourier ($\mathbf{q}$-)space: $B(\mathbf{x}) \to \hat{B}(\mathbf{q}), v(\mathbf{x}) \to \hat{v}(\mathbf{q})$

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becomes

$$\partial_t \hat{\mathbf{B}}(\mathbf{q}) = -\frac{1}{4\pi\sigma}q^2\hat{\mathbf{B}}(\mathbf{q}) + \frac{iV^{\frac{1}{2}}}{(2\pi)^{\frac{3}{2}}}\mathbf{q} \times \int \hat{\mathbf{v}}(\mathbf{q}-\mathbf{k}) \times \hat{\mathbf{B}}(\mathbf{k})\,\mathrm{d}^3k$$
$$\partial_t \hat{\mathbf{v}}(\mathbf{q}) = -\frac{iV^{\frac{1}{2}}}{(2\pi)^{\frac{3}{2}}}\int [\hat{\mathbf{v}}(\mathbf{q}-\mathbf{k})\cdot\mathbf{k}]\,\hat{\mathbf{v}}(\mathbf{k})\,\mathrm{d}^3k$$
$$+ \frac{iV^{\frac{1}{2}}}{(2\pi)^{\frac{3}{2}}}\frac{1}{4\pi\rho}\int \left[\mathbf{k} \times \hat{\mathbf{B}}(\mathbf{k})\right] \times \hat{\mathbf{B}}(\mathbf{q}-\mathbf{k})\,\mathrm{d}^3k.$$

Primordial Magnetic Fields - Correlation Function

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In Fourier space this means that the most general Ansatz is [von Kármán and Howarth, 1938, Junklewitz and Enßlin, 2011]

$$\langle \hat{B}_l(\mathbf{k})\hat{B}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm} - \frac{k_l k_m}{k^2})M(k) - \frac{i}{8\pi}\epsilon_{lmj}k_j\mathcal{H}(k)]$$

$$\langle \hat{v}_l(\mathbf{k})\hat{v}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm} - \frac{k_l k_m}{k^2})U(k) - \frac{i\rho}{2k^2}\epsilon_{lmj}k_j\mathcal{H}^{\mathrm{K}}(k)]$$

Master Equations for the Time Evolution of M, U and $\mathcal{H}$

$$\left\langle \partial_t M_q \right\rangle = \int_0^\infty \left(\Delta t \left\{ -\frac{2}{3} q^2 \left\langle M_q \right\rangle \langle U_k \rangle - \frac{4}{3} q^2 \left\langle M_q \right\rangle \langle M_k \rangle + \frac{1}{3} \frac{1}{(4\pi)^2} q^2 k^2 \left\langle \mathcal{H}_q \right\rangle \langle \mathcal{H}_k \rangle \right.\right.$$
$$\left.\left. + \int_0^\pi \left[\frac{1}{2} \frac{q^4}{k_1^4} \left(q^2 + k^2 - qk \cos\theta \right) \sin^3\theta \, \langle M_k \rangle \left\langle U_{k_1} \right\rangle \right] \mathrm{d}\theta \right\} \right) \mathrm{d}k$$

$$\left\langle \partial_t U_q \right\rangle = \int_0^\infty \left(\Delta t \left\{ -\frac{2}{3} q^2 \langle M_k \rangle \left\langle U_q \right\rangle - \frac{2}{3} q^2 \left\langle U_q \right\rangle \langle U_k \rangle \right.\right.$$
$$+ \int_0^\pi \left[\frac{1}{4} \frac{q^3 k}{k_1^4} \left(qk \sin^2\theta + 2k_1^2 \cos\theta \right) \sin\theta \, \langle M_k \rangle \left\langle M_{k_1} \right\rangle + \frac{1}{4} \frac{q^4 k}{k_1^4} (3k - q\cos\theta) \sin^3\theta \, \langle U_k \rangle \left\langle U_{k_1} \right\rangle \right.$$
$$\left.\left.\left. + \frac{1}{(16\pi)^2} \frac{q^3 k^2}{k_1^2} \left(-2q - q\sin^2\theta + 2k\cos\theta \right) \sin\theta \, \langle \mathcal{H}_k \rangle \left\langle \mathcal{H}_{k_1} \right\rangle \right] \mathrm{d}\theta \right\} \right) \mathrm{d}k$$

$$\langle \partial_t \mathcal{H}_q \rangle = \int_0^\infty \left\{ \Delta t \left[\frac{4}{3} k^2 \langle M_q \rangle \langle \mathcal{H}_k \rangle - \frac{4}{3} q^2 \langle M_k \rangle \langle \mathcal{H}_q \rangle \right.\right.$$
$$\left.\left. - \frac{2}{3} q^2 \langle U_k \rangle \langle \mathcal{H}_q \rangle + \int_0^\pi \left(\frac{1}{2} \frac{q^4 k^2}{k_1^4} \sin^3\theta \left\langle U_{k_1} \right\rangle \langle \mathcal{H}_k \rangle \right) \mathrm{d}\theta \right] \right\} \mathrm{d}k$$

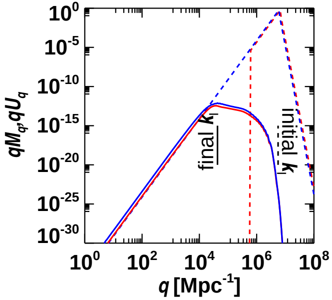
Definitions:
$\mathbf{k_1} \equiv \mathbf{q} - \mathbf{k}$
$\mathbf{q} \cdot \mathbf{k} \equiv qk \cos\theta$

Energy/helicity conservation: $\partial_t \epsilon_{\mathrm{tot}} = \rho \int \left(\partial_t M_q + \partial_t U_q \right) \mathrm{d}q = 0$ and $\partial_t h_{\mathrm{B}} = \rho \int \partial_t \mathcal{H}_q \mathrm{d}q = 0$

Results on the Time Evolution of Primordial Magnetic Fields without Helicity

[Saveliev et al., 2012]

- Starting either with an initial power-law ...



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- ... or a concentration of the spectral energies on a single scale the qualitative result is similar: a tendence to equipartition and both $M_q \propto q^4 \propto L^{-4}$ (i.e. $B \propto q^{\frac{5}{2}} \propto L^{-\frac{5}{2}}$) and $U_q \propto q^4$ at large scales.

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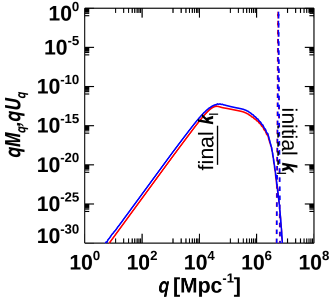

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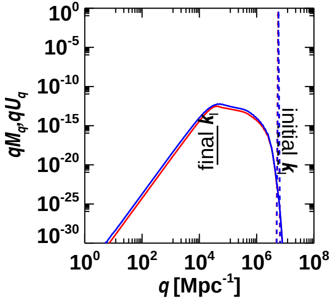
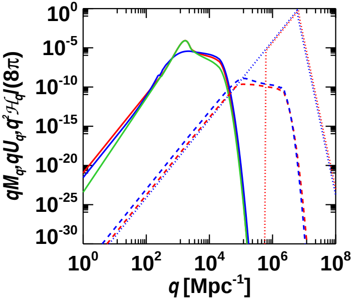


- A rough estimate for B (for the QCD phase transition) is given by $B(200\,\mathrm{pc}) \lesssim 5 \times 10^{-12}\,\mathrm{G}$

Results on the Time Evolution of Primordial Magnetic Fields with Helicity

- Including magnetic helicity for the same initial conditions results in an Inverse Cascade, a fast transport of big amounts of magnetic energy to large scales. This is due to helicity conservation.

[Saveliev et al., 2013b]



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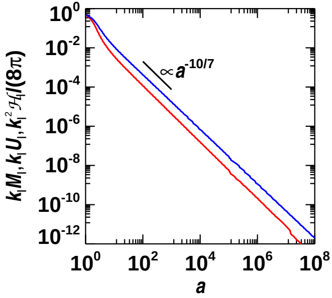

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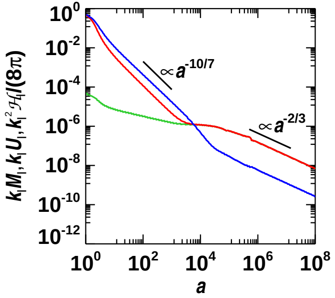

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- One possible scenario for the generation of EGMF is the time evolution of Primordial magnetic Fields during which energy, among other things, can be transported from smaller to larger scales
- The expicit computation of the backreaction of the magnetic field on the medium gives the result of a power law behavior with $M_q \propto q^4 \propto L^{-4}$ (i.e. $B \propto q^{\frac{5}{2}} \propto L^{-\frac{5}{2}}$) and $U_q \propto q^4 \propto L^{-4}$ and equipartition at large scales.

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- One possible scenario for the generation of EGMF is the time evolution of Primordial magnetic Fields during which energy, among other things, can be transported from smaller to larger scales
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- Helicity enhances this effect by creating an inverse cascade which results in much higher magnetic fields today compared to the non-helical case

Current Projects I - Master Equations Continued

with G. Sigl (U Hamburg) and K. Jedamzik (U Montpellier)

Further analysis of the Master Equations (Super-equipartition, other applications) and consideration of kinetic helicity:

$$\langle \hat{B}_l(\mathbf{k})\hat{B}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm}-\frac{k_l k_m}{k^2})M(k)-\frac{i}{8\pi}\epsilon_{lmj}k_j\mathcal{H}(k)]$$

$$\langle \hat{v}_l(\mathbf{k})\hat{v}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm}-\frac{k_l k_m}{k^2})U(k)-\frac{i\rho}{2k^2}\epsilon_{lmj}k_j\mathcal{H}^{\mathrm{K}}(k)]$$

This will give insights to the evolution and influence of vorticity.

Current Projects II - Numerical Simulations

with B. Chetverushkin (Keldysh Institute, Moscow) and N. D'Ascenzo (DESY Hamburg)

The starting point is the Boltzmann Equation:

$$\partial_t f(\mathbf{x}, \mathbf{v}, t) + \mathbf{v} \cdot \nabla f(\mathbf{x}, \mathbf{v}, t) = C(f)$$

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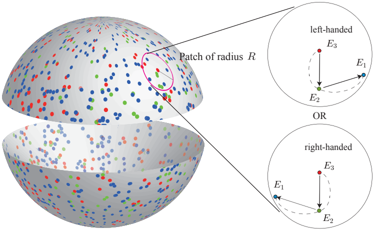
- This approach allows to use efficient explicit numerical schemes suitable for large parallel computing systems
- Various applications, in particular in astrophysics

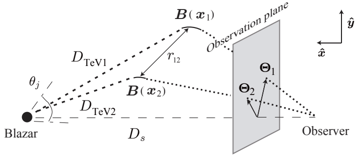
Current Projects III - CP Violation/Helicity Determination

with T. Vachaspati (Arizona State University) and R. Alves Batista (U Hamburg)



[Tashiro et al., 2013]

- The propagation of electromagnetic cascades can be used to statistically analyze the properties of the magnetic helicity and therefore CP violation



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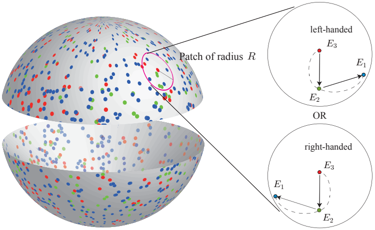

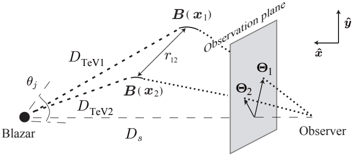

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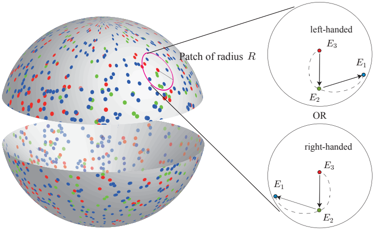

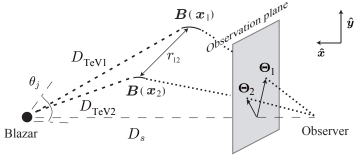

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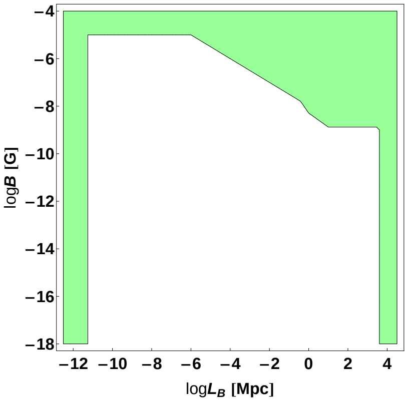
- The propagation of electromagnetic cascades can be used to statistically analyze the properties of the magnetic helicity and therefore CP violation
- This can be done for single sources as well as for the gamma ray sky
- Development of 3D simlations of cascades in magnetic fields

Additional EGMF Constraints from Primordial Fields



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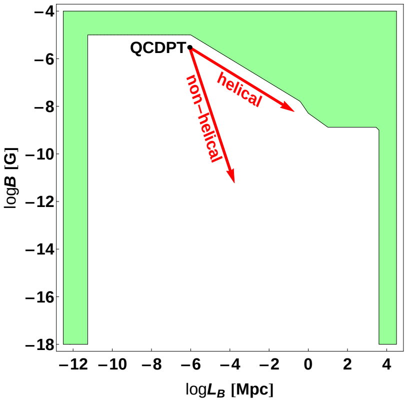
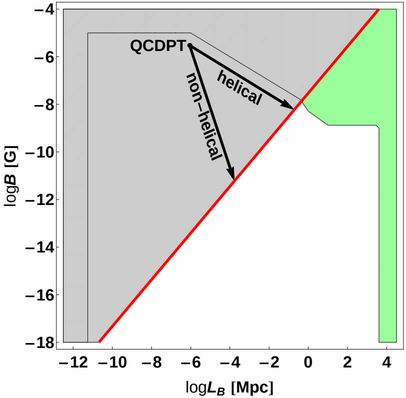


- Taking the resulting time evolution for helical and non-helical fields...

Additional EGMF Constraints from Primordial Fields



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- ...further constraints are possible.

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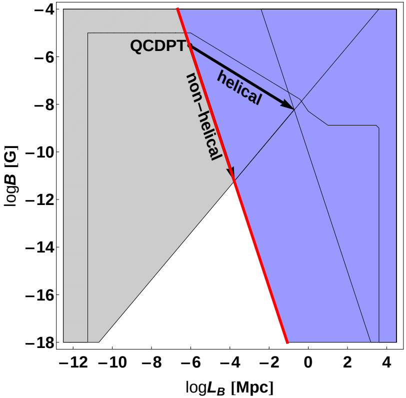


- Taking the resulting time evolution for helical and non-helical fields...
- ...further constraints are possible.
- Considering the power-law slope for the spectral energies, causality dictates further limits.

Constraints on EGMF - Resistive Decay

Diffusion equation for magnetic fields: $\partial_t \mathbf{B} = \frac{1}{4\pi\sigma}\Delta\mathbf{B}$

- Estimate for the diffusion time $\tau_{\rm diff}$: $\frac{B}{\tau_{\rm diff}} \simeq \frac{1}{4\pi\sigma}\frac{B}{L^2}$
- Therefore: $L = \left(\frac{\tau_{\rm diff}}{4\pi\sigma}\right)^{\frac{1}{2}} \propto \tau_{\rm diff}^{\frac{1}{2}}$, i.e. B on smaller scales decays faster than on larger scales
- With $\tau_{\rm diff} \simeq H^{-1}$ and $\sigma \simeq 10^{11}\,\mathrm{s}$ it is $L_B \gtrsim (4\pi\sigma H)^{-\frac{1}{2}} \simeq 6 \times 10^{-12}\,\mathrm{Mpc}$

Constraints on EGMF - Faraday Rotation

The rotation of the polarization plane of radiation rotates by an angle β depending on the wavelength λ according to

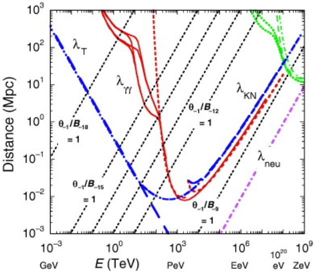
$$\beta = \lambda^2 \mathrm{RM} \propto \lambda^2 \int_0^{l(z_{\mathrm{emit}})} n_e B_{\parallel}(z)(1+z)^{-2} \mathrm{d}l(z)$$

- The difference $\Delta\beta$ of the rotation angles between wavelengths λ and $\lambda + \Delta\lambda$ is given by $\mathrm{RM} = \Delta\beta / \Delta\lambda^2$
- After transversing $N = D/\lambda_B$ coherence lengths with the Rotation Measure RM_{λ_B} each, the average total rotation measure is $\Delta\beta$ is

$$\overline{\frac{\Delta\beta}{\Delta\lambda_B^2}} = \frac{N \mathrm{RM}_{\lambda_B}}{N^{\frac{1}{2}}} = N^{\frac{1}{2}} \mathrm{RM}_{\lambda_B}$$

$$= \left(\frac{D}{\lambda_B}\right)^{\frac{1}{2}} \mathrm{RM}_{\lambda_B} \propto \left(\frac{D}{\lambda_B}\right)^{\frac{1}{2}} \lambda_B B_{\parallel} \propto \lambda_B^{\frac{1}{2}} B_{\parallel}$$

Propagation Paths



[Dermer et al., 2009]

Due to their importance the following lengths are shown:

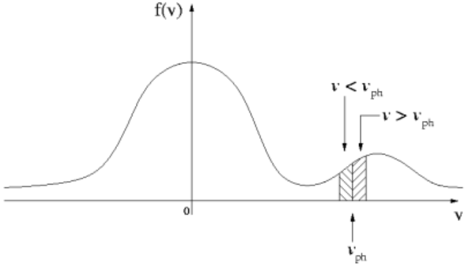
- λ_{T}: Electrons in the Thomson limit
- λ_{KN}: Electrons in the Klein-Nishina limit
- $\lambda_{\gamma\gamma}$: Photons due to Pair Production

Two-Stream-Like Electrostatic Instabilities

One dimension:

- $v \gtrsim v_{\rm ph} \rightarrow$ particles decelerate, wave gains energy
- $v \lesssim v_{\rm ph} \rightarrow$ particles accelerate, wave loses energy

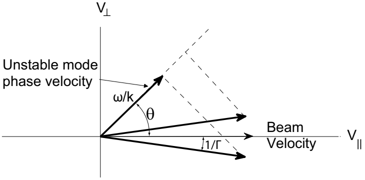
- Result: Wave gains more energy than it loses $\rightarrow$ instability grows

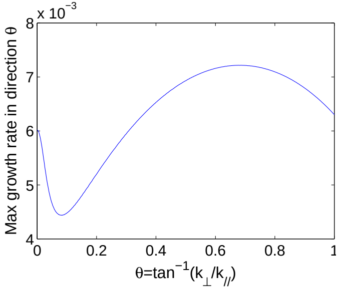


Two-Stream-Like Electrostatic Instabilities

Three dimensions [Nakar et al., 2011]:

- Similar to 1D case, but transversal spread and propagation oblique to wave mode increases number of configurations
- Therefore two maximum growth cases: Parallel (in analogy to 1D) and oblique
- Resonance condition: $\mathfrak{Re}(\omega) \simeq ck_{\parallel} - ck_{\perp}/\Gamma$

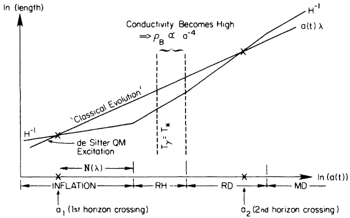




Magnetic Fields from Inflation

- Adding terms to the Lagrangian to couple the electromagnetic fields to Curvature (e.g. $RA_\mu A^\mu$, $RF_{\mu\nu}F^{\mu\nu}$) or to the Inflaton field ($f^2(\phi)F_{\mu\nu}F^{\mu\nu}$) [Turner and Widrow, 1988]
- Similar to the quantum fluctuation component of ϕ, electromagnetic superhorizon modes are frozen in during inflation, giving non-negligible magnetic fields

  - Predictions and constraints are difficult due to strong dependence on model of Inflation and coupling

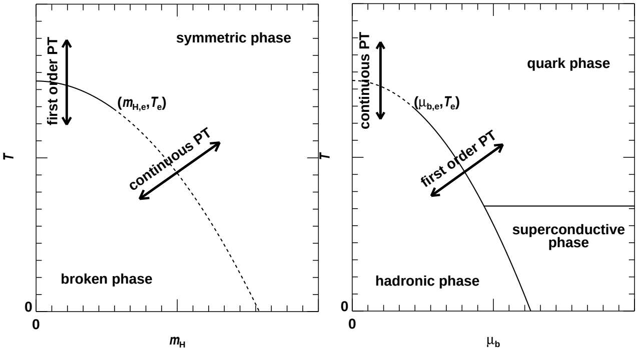


Magnetic Fields from Inflation

Example: $\mathcal{L}'_{\mathrm{EM}} = f^2(\phi) F_{\mu\nu} F^{\mu\nu}$, $f^2(\phi) \propto a^{-\alpha}$

- $B = B(k, \alpha, H)$ (k is the inverse scale of interest and H the Hubble Parameter at Inflation).
- Anisotropies and the claim of negligible backreaction onto Inflation by the electromagnetic fields give a constraint of the form $\alpha \gtrsim \alpha_0(H, k)$
- Recent measurements of the BICEP2 collaboration [Ade et al., 2014] give $H \simeq 1.1 \times 10^{23}\,\mathrm{eV}$ and therefore $B \lesssim 8.1 \times 10^{-35}\,\mathrm{G}$ [Ferreira et al., 2014]

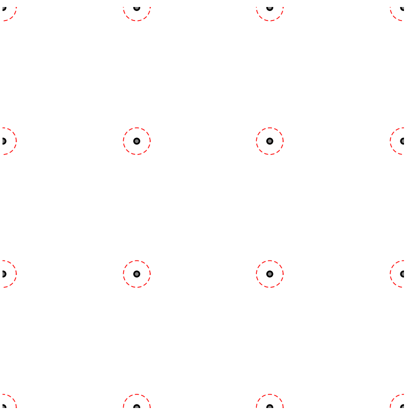
Cosmological Phase Transitions



The phase diagrams of the EWPT (left) and QCDPT (right) suggest a continuous phase transition, however, due to BSM approaches, also a first order is possible.

Magnetic Fields from Cosmological Phase Transitions

- At a first order phase transition seeds of the high temperature phase (gray) nucleates at some specific average length scale and starts to grow. The resulting latent heat is released in form of shock fronts (red, dashed).
- Magnetic fields might emerge due to phase velocity differences in the two phases at the bubble walls [Sigl et al., 1997]



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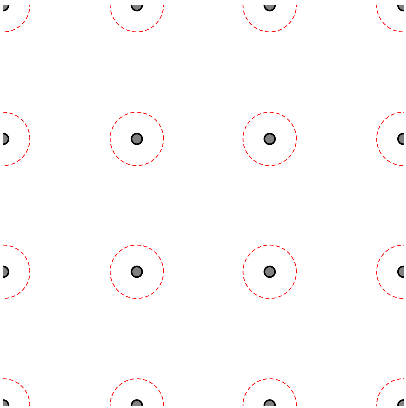

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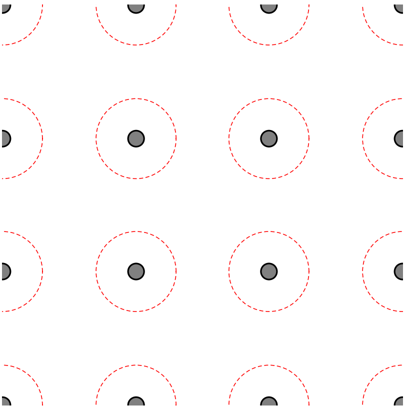

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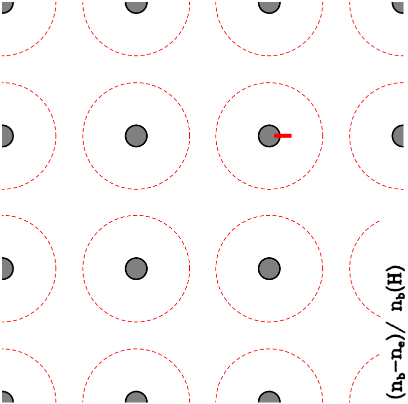
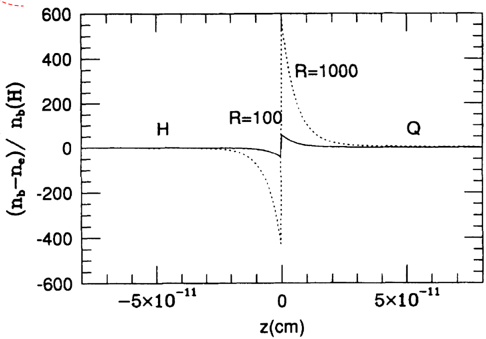


- QCDTP: Charge separation due to the the bubbles "pushing" the quarks ahead of them produces surface currents which result in magnetic fields [Cheng and Olinto, 1994]



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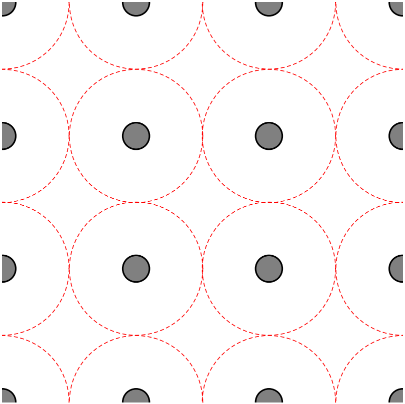
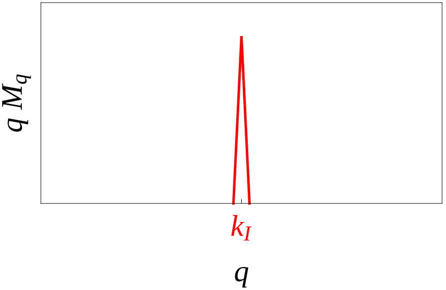


- Collisions of the shock fronts produce turbulence which gives rise to magnetic fields via a Biermann-Battery-like mechanism [Quashnock et al., 1989] [Baym et al., 1996]

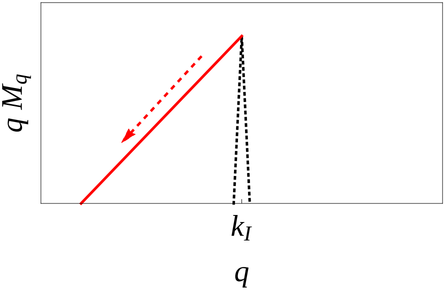
Cosmological Implications of EGMF

- BBN: Strong magnetic fields increase the neutron decay rate which results in smaller relic abundances of Helium [Matese and O'Connell, 1969]; Increase of the expansion rate and thus change of the n/p-ratio freeze-out temperature [Matese and O'Connell, 1970]
- Homogeneous magnetic field: The energy-momentum tensor becomes anisotropic, hence leading to an anisotropic Expansion of the Universe [Barrow et al., 1997]
- Deposition of dissipated magnetic field energy in the heat bath of the CMB (Sunyaev-Zel'dovich Effect) [Jedamzik et al., 1998],[Jedamzik et al., 2000]
- Due to magnetic pressure inhomogeneous magnetic fields slow down the growth rate of density perturbations which, backreacting, at the same time slows down the decay of the magnetic fields [Barrow et al., 2007]
- Production of B-modes in CMB anisotropies, thus mimicking primordial gravitational waves [Bonvin et al., 2014]

Time Evolution without Helicity - Predictions



- In the beginning the energy is concentrated on the integral scale k_I



- In the beginning the energy is concentrated on the integral scale k_I

- Due to the interaction with the IGM some of the energy is transported to larger scales (smaller q), such that $M_q \propto q^{\alpha-1}$ with $\alpha = 3$ [Hogan, 1983] or $\alpha = 5$ [Durrer and Caprini, 2003]

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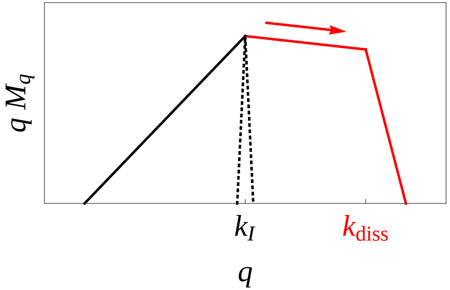

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- Energy cascades down to small scales (large q) and then dissipates beyond k_{diss}; slope as $\alpha = -5/3$ [Kolmogorov, 1941] or $-3/2$ [Iroshnikov, 1964] [Kraichnan, 1965]

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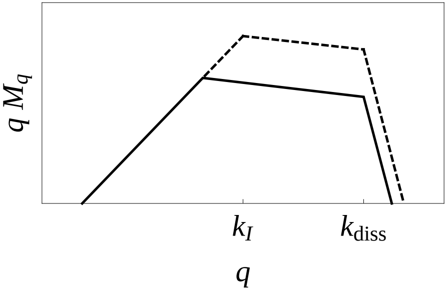

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- Due to this energy transport k_I decreases (selective decay)

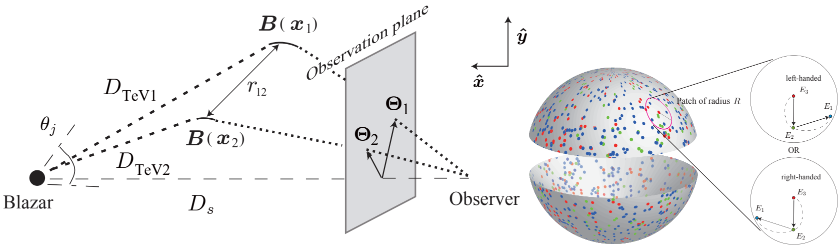
Origin of Primordial Magnetic Helicity

It has been shown that [Tashiro and Vachaspati, 2013]

$$G(E_1, E_2) = \left\langle (\Theta_1 \times \Theta_2) \cdot \frac{\mathbf{x}}{|\mathbf{x}|} \right\rangle \propto \frac{1}{2}\mathcal{H}(r_{12}) r_{12}$$

for a known blazar position; otherwise (with $E_3 > E_2 > E_1$)

$$G(E_1, E_2, E_3) = \left\langle [(\Theta_1 - \Theta_3) \times (\Theta_2 - \Theta_3)] \cdot \frac{\mathbf{x_3}}{|\mathbf{x_3}|} \right\rangle \propto \frac{1}{2}\mathcal{H}(r_{12}) r_{12}$$



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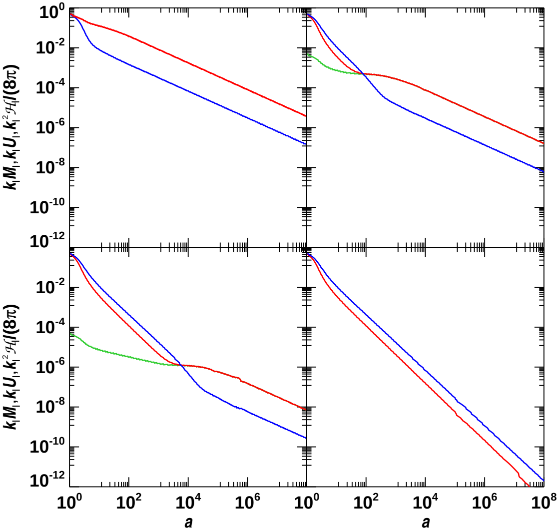
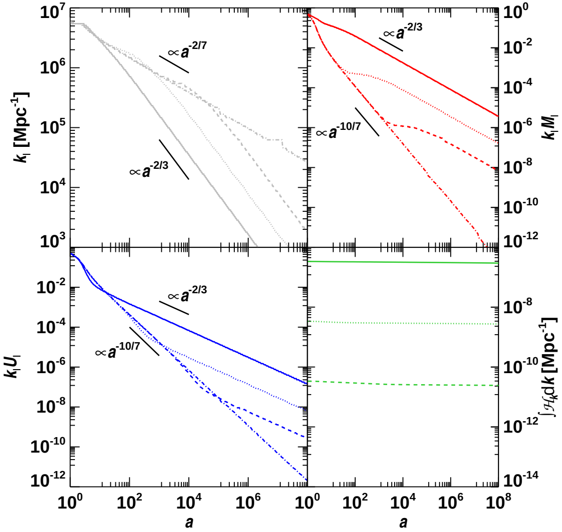
- The time evolution is now governed by the claim of helicity conservation, i.e. $k_\mathrm{I}\mathcal{H}_\mathrm{I} \simeq 8\pi M_\mathrm{I} \simeq const$
- The relaxation time τ_I on the integral scale L_I is given by $\tau_\mathrm{I} = \frac{L_\mathrm{I}}{v^\mathrm{eff}_\mathrm{A,I}} \propto \frac{L_\mathrm{I}}{(k_\mathrm{I} M_\mathrm{I})^{\frac{1}{2}}} \propto \frac{L_\mathrm{I}}{(k_\mathrm{I})^{\frac{1}{2}}} \propto L_\mathrm{I}^{\frac{3}{2}} \propto k_\mathrm{I}^{-\frac{3}{2}} \Rightarrow k_\mathrm{I} \propto t^{-\frac{2}{3}}$
  $\Rightarrow \mathcal{H}_\mathrm{I} \propto \frac{M_\mathrm{I}}{k_\mathrm{I}} \propto t^{\frac{2}{3}}$

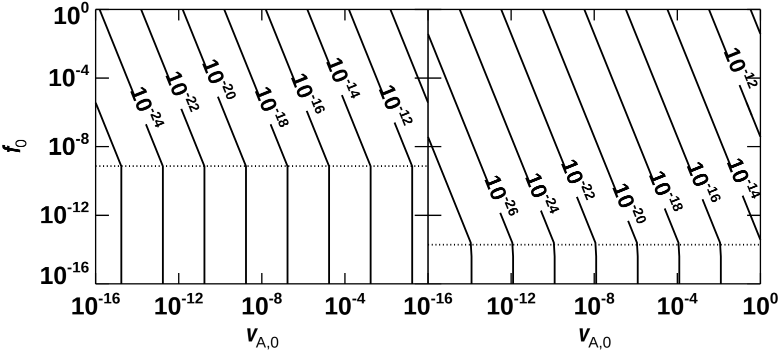
Dependence of the Time Evolution on Helicity



Dependence of the Time Evolution on Helicity



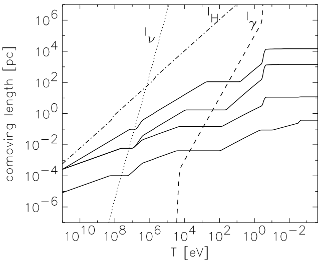
Present-Day EGMF from PMF

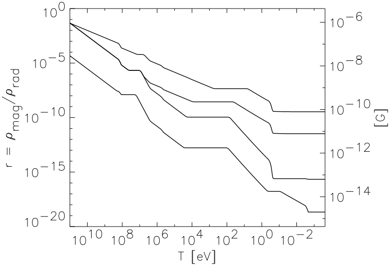


Present-Day Extragalactic Magnetic Fields depending on the initial Alfvén Velocity $v_{\mathrm{A},0}$ and the initial magnetic helicity in terms of $f_0 = \mathcal{H}_0/\mathcal{H}_{0,\max}$ for magnetogenesis at QCDPT (left) and EWPT (right).

Evolution of Primordial Magnetic Fields including Viscosity

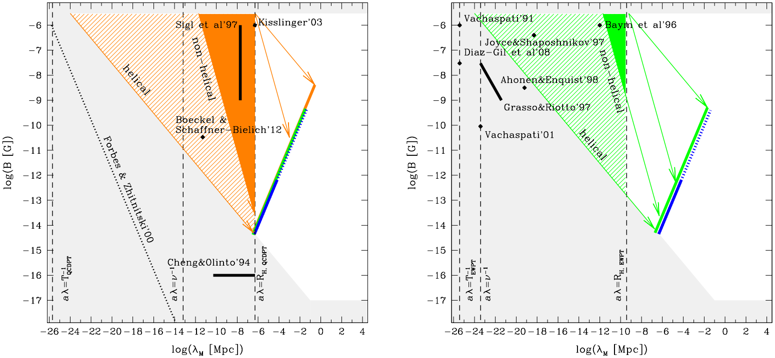
Taking into account viscosity in the Early Universe the actual time evolution of the integral scale $L_{\rm I}$ (*left*) and $M_{\rm I}$ (*right*) has the following form [Banerjee and Jedamzik, 2004]:



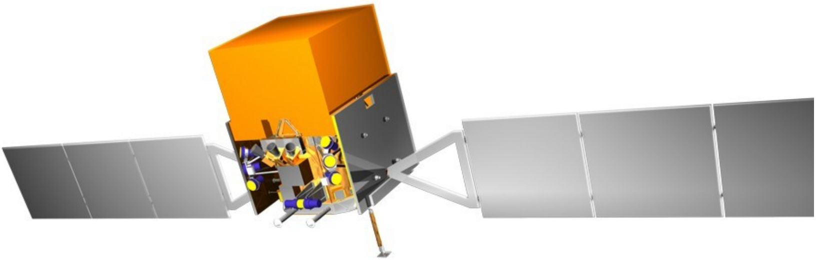


Predictions from different Models of Magnetogenesis at QCDPT and EWPT

Over the years there has been a large variety of different models how Primordial Magnetic Fields were generated during QCDPT (*left*) and EWPT (*right*) [Durrer and Neronov, 2013]:



Gamma Ray Instruments - Fermi LAT



- Located on the Fermi Gamma-Ray Space Telescope (FGST), formerly Gamma-ray Large Area Space Telescope (GLAST) launched in 2008

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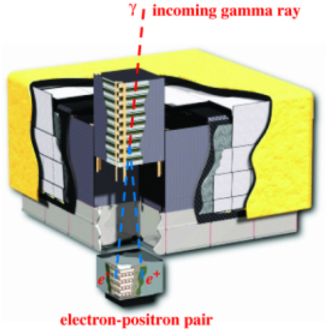
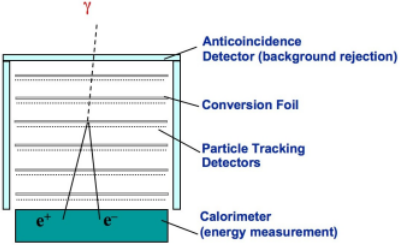
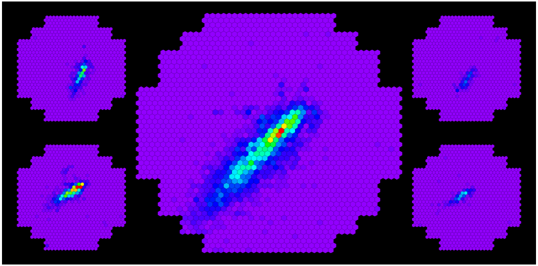




- Located on the Fermi Gamma-Ray Space Telescope (FGST), formerly Gamma-ray Large Area Space Telescope (GLAST) launched in 2008
- The Large Area Telescope (LAT) is a highly sensitive gamma-ray detector for energies of 30 MeV to 300 GeV

- H.E.S.S. (High Energy Stereoscopic System) is an Imaging Atmospheric Cherenkov Telescopes (IACT) located and Namibia and taking data since 2002; upgrade 2012
- Detection of Very High Energy (100 GeV to TeV) gamma-rays by Cherenkov light from cascades initiated by them in the atmosphere

Gamma Ray Instruments - H.E.S.S. Observatory



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