Hamburg: Workshop on Astroparticle Physics with multiple messengers.

Outline:

(A) Motivation, phase transition, symmetry breaking, effective action

(B) Gravitational waves.

(C) Electroweak baryogenesis

Motivation: Big bang cosmology describes an expanding and cooling universe. For most parts, the evolution is not very exciting. But certain events leave traces that can be observed today and allow to establish links to particle physics

$T \sim eV$: Ions and electrons combine to form neutral atoms → CMB generated

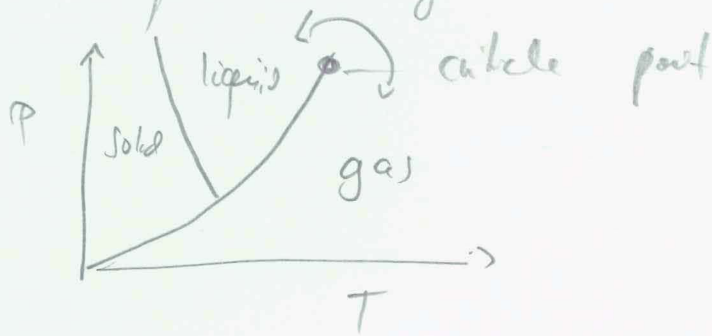
$T \sim MeV$: protons and neutrons combine to form nuclei → BBN predictions of light element abundances

$T \sim 100\,MeV$: QCD phase transition: quarks → hadrons

$T \sim 100\,GeV$: EW phase transition. EW symmetry is broken. Fermions become massive phase transition: Describes the transition of a thermodynamic system from one phase to another (phase = state with certain uniform properties) under change of a thermodynamic variable (e.g. temperature)

E.g. Water:

gas ↔ liquid ↔ solid

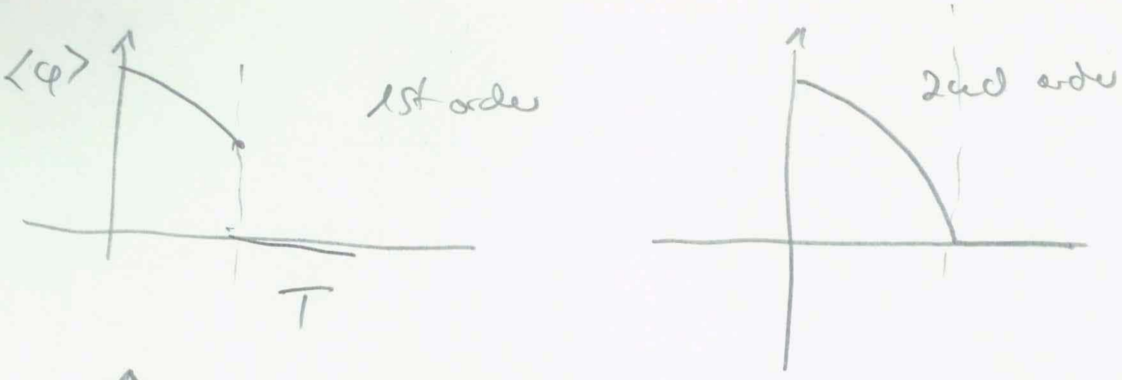


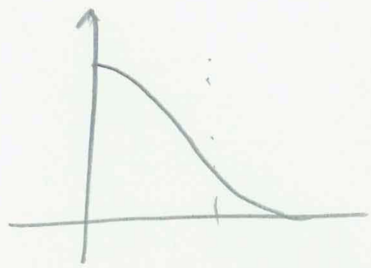
electroweak plasma:

$T_{cr} \sim$ Higgsmass $\sim$ 100 GeV

T	$T > T_{cr}$	$T < T_{cr}$
Higgs vacuum expectation value $\langle\varphi\rangle$	$\langle\varphi\rangle \simeq 0$	$\langle\varphi\rangle \simeq$ 100 GeV
Z/W bosons:	$m_V \simeq 0$ 2 polarizations	$m_V \simeq$ 100 GeV 3 polarizations
Higgs particles	4 polarization	1 polarization.
gauge symmetry	$SU(2)_W \times U(1)_Y$	$U(1)_{EM}$

Different orders of the phase transition:





1st order phase transitions are particularly interesting as they lead to cosmological relics:

(A) Gravitational waves

(B) Electroweak baryogenesis

(C) Magnetic fields

An everyday life example of a 1st order phase transition is boiling water => phase transition proceeds by bubble nucleation.

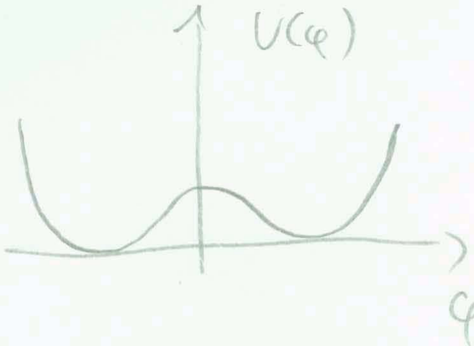
How to decide the order of a phase transition?

Symmetry breaking: An simple example

scalar field: $\mathcal{L}(\varphi, \partial_\mu\varphi) = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)$

$$V = \frac{\lambda}{4}(\varphi^2 - v^2)^2$$

$\varphi \leftrightarrow -\varphi$



- Lagrangian has a $\mathbb{Z}_2$ symmetry
- This symmetry could be spontaneously broken
- What about finite temperature?
- is there a phase transition
- what are it's features?

T=0

The order parameter of this phase transition is the scalar field vev

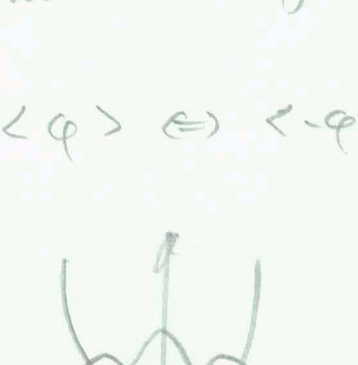
$$\langle\varphi\rangle = \frac{\int D\varphi\,\varphi\, e^{iS}}{\int D\varphi\, e^{iS}}$$

$$S = \int d^4x\, \mathcal{L}(\varphi, \partial_\mu\varphi)$$

Naively: $\varphi \neq 0$ (minimum of potential)

$\varphi = 0$ $\quad \langle\varphi\rangle \Leftrightarrow \langle-\varphi\rangle \quad \mathbb{Z}_2$

QM:



effective action:

naive classical expectation will make the difference

$$e^{-iW(J)} = \int \mathcal{D}\varphi \; e^{iS + i\int d^4x \, J(x)\varphi(x)}$$

J gives a little tilt to the potential

$$\langle\varphi\rangle = -\frac{\partial}{\partial J} W \Big|_{J=0}$$

↳

technically spontaneous symmetry breaking is signaled by the fact that $W(J)$ not analytic!

This is problematic because perturbative methods don't will lead to non-analytic results.

↳ effective action

$$W(J) \simeq \int d^4x \, V(\langle\varphi\rangle) - J\langle\varphi\rangle.$$

$$\langle\varphi\rangle(J) = -\frac{\partial W}{\partial J}(J)$$

+ quantum corrections.

$$\Gamma(J) = \int d^4x - W - J\varphi \simeq \int d^4x - V(\langle\varphi\rangle)$$

effective action

+ quantum corrections

$\Rightarrow$ Γ now is analytic

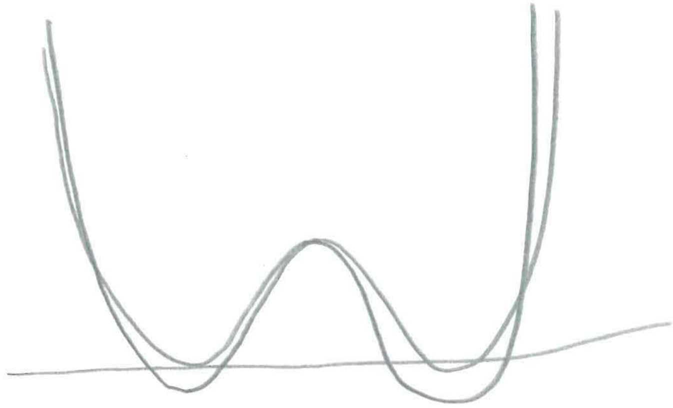
loop corrections:

$$\Gamma = \int d^4x \, V_{eff}(\varphi) \qquad \Delta V^{T=0}$$

$$V_{eff} = V_0(\varphi) \pm \sum_{species} \frac{m_i^4}{64\pi^2} \left(\log \frac{m_i^2}{\mu^2} - c \right)$$

Coleman Weinberg potential

$$+ \quad \Delta V^{T \neq 0}$$



$\Delta V^{T\neq 0}$:

$\boxed{m^2 \ll T^2}$

bosons $\quad -\frac{\pi^2}{90} T^4 + \frac{1}{24} m^2 T^2 - \frac{1}{12\pi} m^3 T$

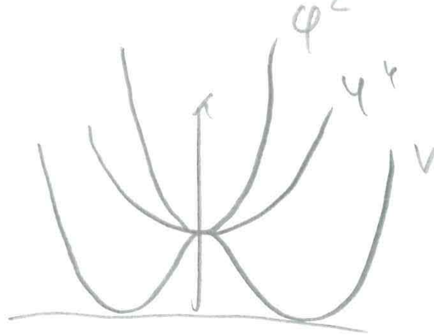
fermions $\quad -\frac{7}{8}\frac{\pi^2}{90} T^4 + \frac{1}{48} m^2 T^2$

1st order phase transition

Symmetry restoration

Stefan Boltzmann limit of a non-interacting gas

Symmetry restoration



Standard model

$$m_W^2 \sim g^2 \varphi^2$$
$$m_Z^2 \sim g'^2 \varphi^2$$
$$m_t^2 \sim \frac{y_t^2}{2} \varphi^2$$

$c \sim 0.17$

$$\Rightarrow \quad V_{eff} \sim \frac{\lambda}{4}(\varphi^2 - v^2)^2 + cT^2\varphi^2$$

$v \sim 246$ GeV

$m_h^2 \sim (125 \text{ GeV})^2 = 2\lambda v^2$

$$\frac{\lambda}{2} v^2 \simeq cT^2 \Rightarrow T = m_h \sqrt{\frac{1}{4c}}$$

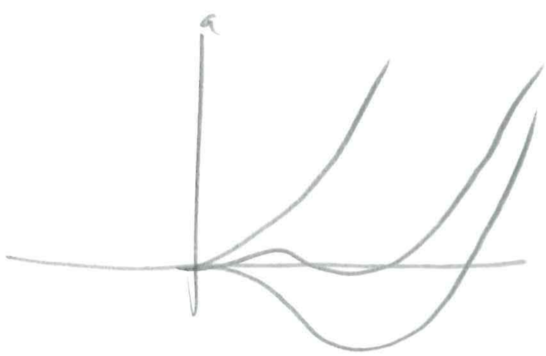
1st order phase transition

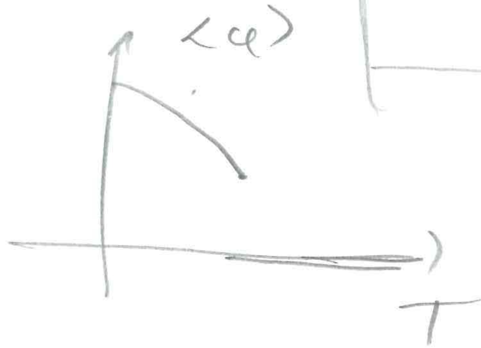
$m^3 \sim \varphi^3$

$E \sim 0.01$

$\Rightarrow m_h < 60$ GeV

$$V \sim \frac{\lambda}{4}(\varphi^2 - v^2)^2 - ET\varphi^3 + cT^2\varphi^2$$



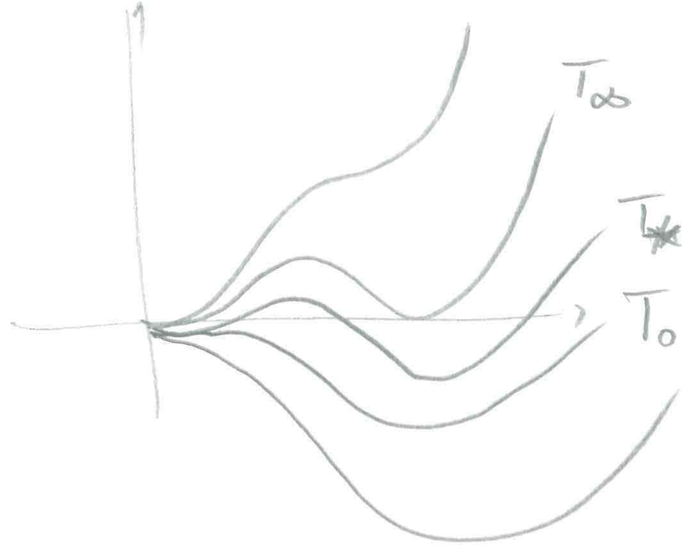


→ system has to tunnel

→ bubble nucleation

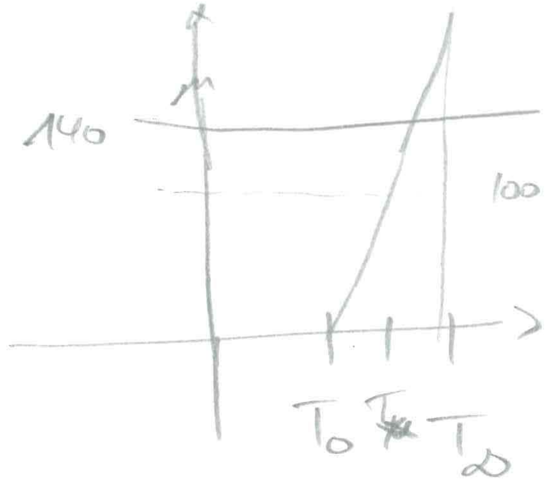
Models

· Bubble size:



euclidean action in the forbidden region

$$\frac{\text{prob}}{\text{vol} \times \text{time}} \simeq T^4 e^{-S_E(T)}$$



It start $\frac{T^4}{H^4} e^{-S_E} \simeq 1$

$$\boxed{\begin{array}{l} H^2 \sim \frac{8\pi}{3} G \rho \sim \frac{1}{3} \frac{a T^4}{M_{Pl}^2} \\ 8\pi G = \frac{1}{M_{Pl}^2} \end{array}}$$

$$S_E = 4 \lg \frac{M_{Pl}}{T_{EW}} \simeq 140$$

$R = v \cdot \Delta t$;

$$S_E(T) = S_E(T_*) + (t - t_*) \partial_t S_E(T_*)$$

$$\frac{1}{\Delta t} = \partial_t S_E(T_*) \propto -H T \frac{\partial}{\partial T} S$$

$T \propto \frac{1}{a} \propto \frac{1}{\sqrt{t}}$

$\frac{\partial T}{\partial t} \sim -HT$

$$\simeq (1-10) H S_E$$

$$\simeq (100 - 1000) H$$

Summary: - The electroweak symmetry is broken in the SM due to the Higgs mechanism

- The symmetry is restored for $T > v$

- the breaking can happen via 1st order, 2nd order, crossover depending on the Higgs sector

- E.g. bosons strongly coupled to the Higgs increase the strength (stops, more Higgses / singlets)

SM ⊖
MSSM stops ⊕
THD
singlet + SM ⊕

cosmological probes link to particle physics via the implications of the EWPT

Gravitational Waves:

$$\frac{dE}{d\Omega} = 2G \int d\omega\, \omega^2 A^{ij} A_{ij}$$

$A_{ij} = \Lambda_{ij,ab} T^{ab}$

Λ transverse traceless

$\mathrm{Tr}\, A = 0$

$k_i A_{ij} = k_j A_{ij} = 0$

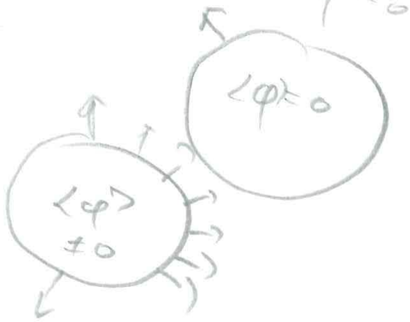
The Einstein equations allow for wave-solutions => gravitational waves.

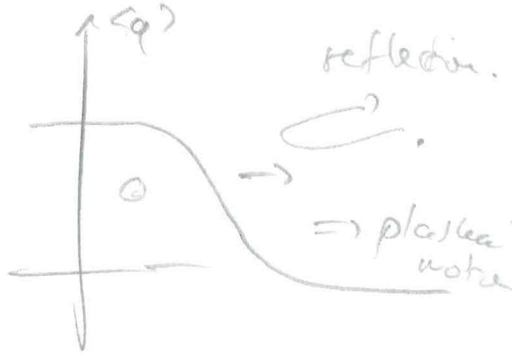
The source for GWs is a special projection of the energy-momentum tensor.

I.e. in principle almost everything produces tiny GWs. However, certain configurations do not contribute (e.g. spherical symmetry)

In the far field approximation, the Quadrupole (<-> dipole) of the energy-momentum tensor is the leading source.

The phase transition produces GWs if of first order:





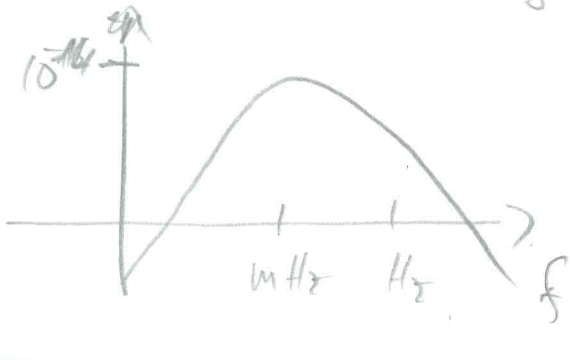
spherical symmetry

back on the envelope: The frequency of the GWs is related to the size of the bubbles at the end of the phase transition.

$$f \sim H \sim \frac{T_{ew}^2}{M_{Pl}} \; ; \quad f_{today} = \frac{T_{today}}{T_{ew}} f \simeq mHz$$

energy density $\Omega \sim \, ?$ $\qquad \Omega_{today} \simeq \Omega \cdot 10^{-5}$

$\frac{\Delta}{\rho}$; efficiencies $\qquad \simeq 10^{-14}$



elisa: $\Omega \sim 10^{-11}$ $\quad f \sim mHz$

BBO: $\Omega \sim 10^{-16}$ $\quad f \sim 100\,mHz$

Electroweak baryogenesis:

Baryogenesis: explain $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 10^{-10}$

Sakharov conditions: $\cancel{B}$

$\cancel{C}$

$\cancel{CP}$

non-equilibrium $(n = n(T, m)$

$m = \bar{m})$

n_L, n_B

$n_L \xrightarrow{C} \bar{n}_L$

$n_L \xrightarrow{CP} \bar{n}_R$

$n_L \xrightarrow{B} \bar{n}_L$

Standard model:

sphaleron process

$B-L$ conserving

$B+L$ violating

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particle reflection: wall frame:

$\bar{E} = E$ $\qquad E^2 - p_z^2 = m^2$

$\bar{p}_z \neq p_z$ $\qquad \bar{E} - \bar{p}_z = \bar{m}^2$

$\bar{m} \neq m$

$\left.\bar{p}_z^2 = \hat{p}_z^2 - 4m^2\right\}$ $p_z^2 < 4m^2 \Rightarrow$ reflection fermions:

Dirac eq:

$$(i\cancel{\partial}-m)\psi=0$$

$$(\cancel{p}-m)\psi=0$$

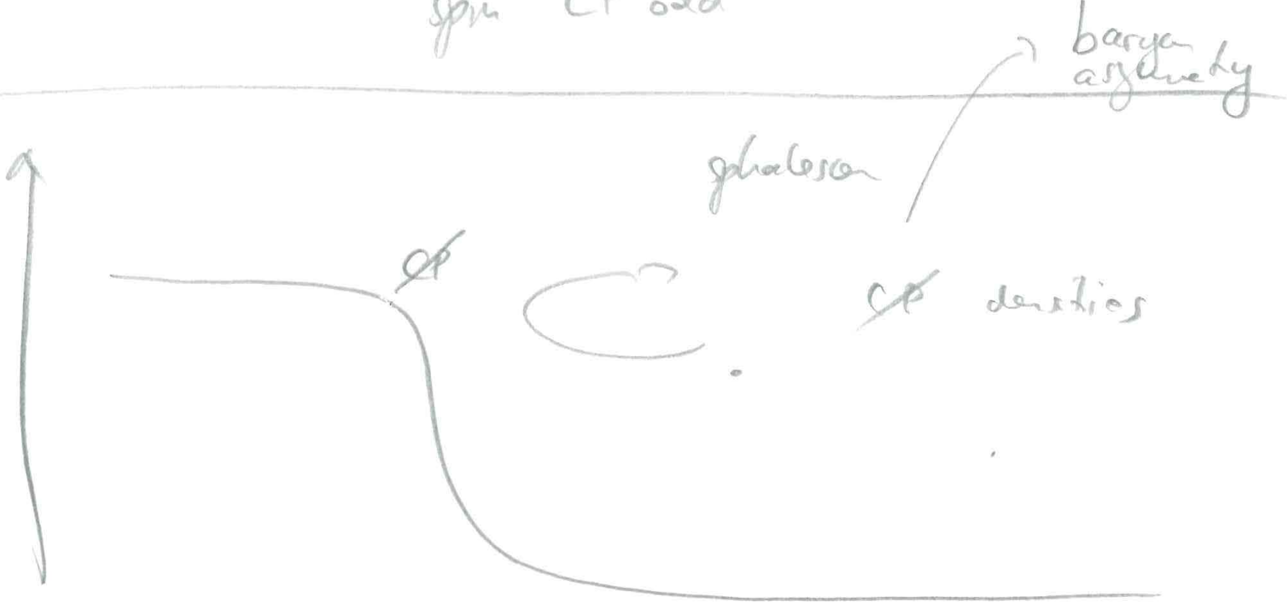
$$\hookrightarrow \boxed{p^2-m^2=0}$$

m complex and space-dependent

$$m(z)=|m|(z)\,e^{i\theta(z)}$$

$$\hookrightarrow \boxed{p^2-m^2\pm s\frac{m^2\theta'}{E}=0}$$

spin CP odd



Models that don't work: SM

MSSM

that work: THD

SM + singlet + low cutoff

smoking gun EDMs