Statistics and track reconstruction.

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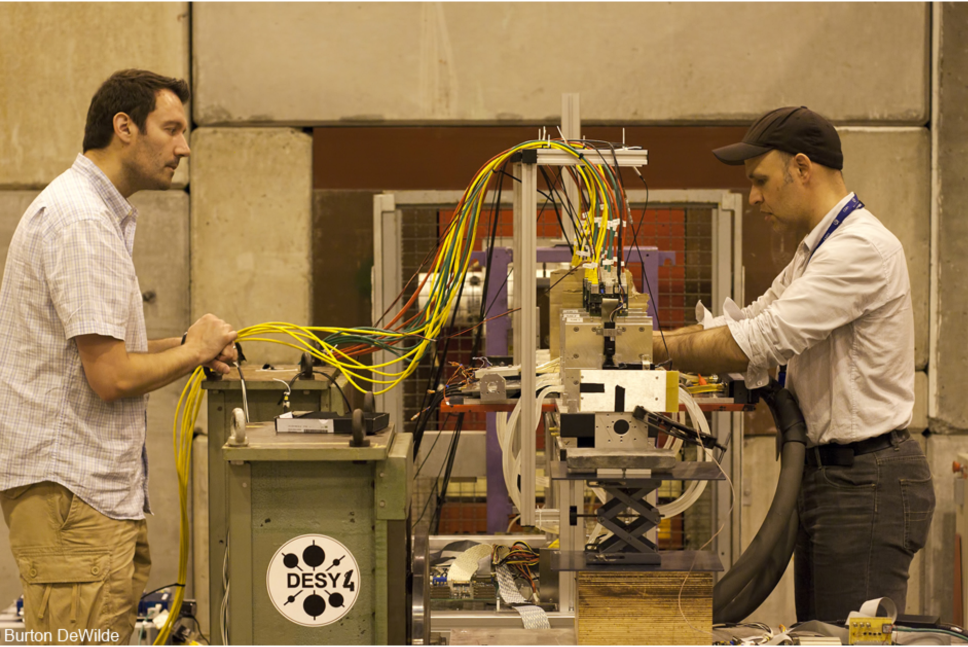

DESY
Burton DeWilde

Test beam reconstruction

Raw data

$\downarrow$

Set of hits

$\downarrow$

Noise filtered set of hits

$\downarrow$

Clusters

$\downarrow$

Measurements

$\downarrow$

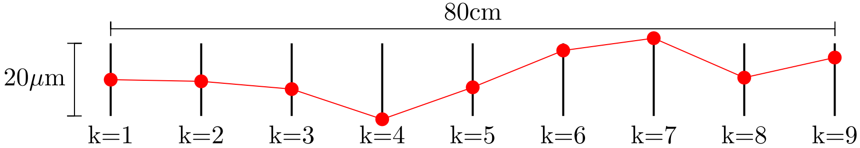
Tracks

Track fitting

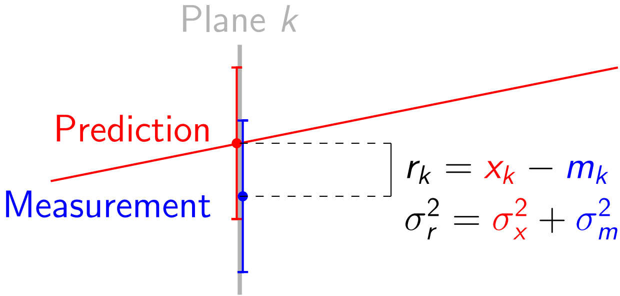
Goal

Get best possible estimate of particle positions and angles in device under test planes

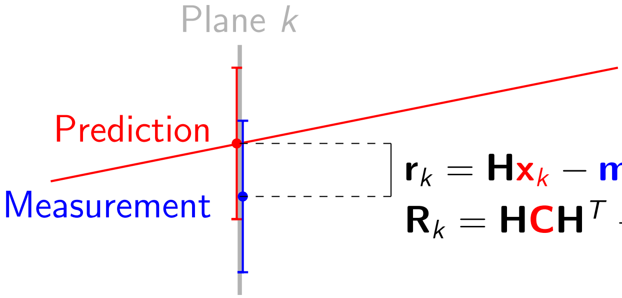
- Least squares estimation of parameters
- Straight lines through air, scatter in planes



Background



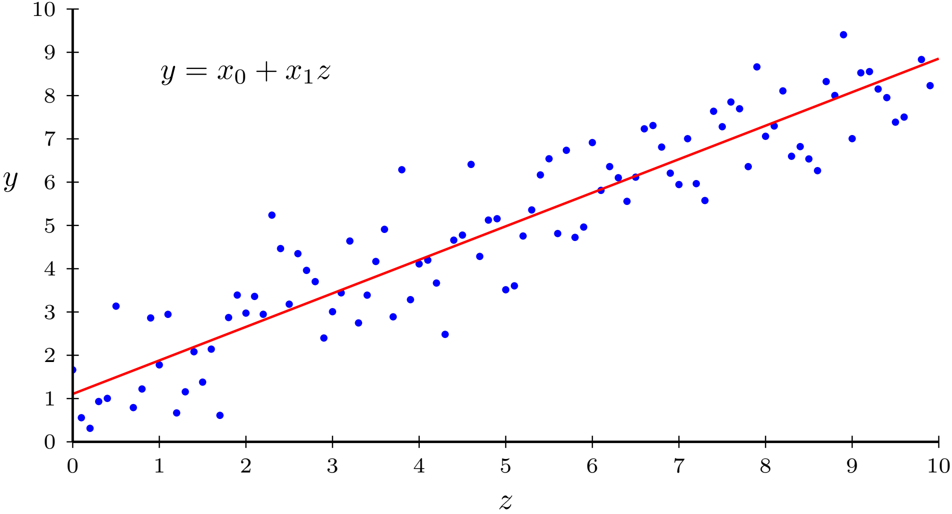
Prediction (x, σ_x^2)
Measurement (m, σ_m^2)
Residuals (r, σ_r^2)



$$\mathbf{r}_k = \mathbf{H}\mathbf{x}_k - \mathbf{m}_k$$
$$\mathbf{R}_k = \mathbf{H}\mathbf{C}\mathbf{H}^T + \mathbf{V}$$

Prediction $(\mathbf{x}, \mathbf{C})$
Measurement $(\mathbf{m}, \mathbf{V})$
Residuals $(\mathbf{r}, \mathbf{R})$

Ordinary least square estimator

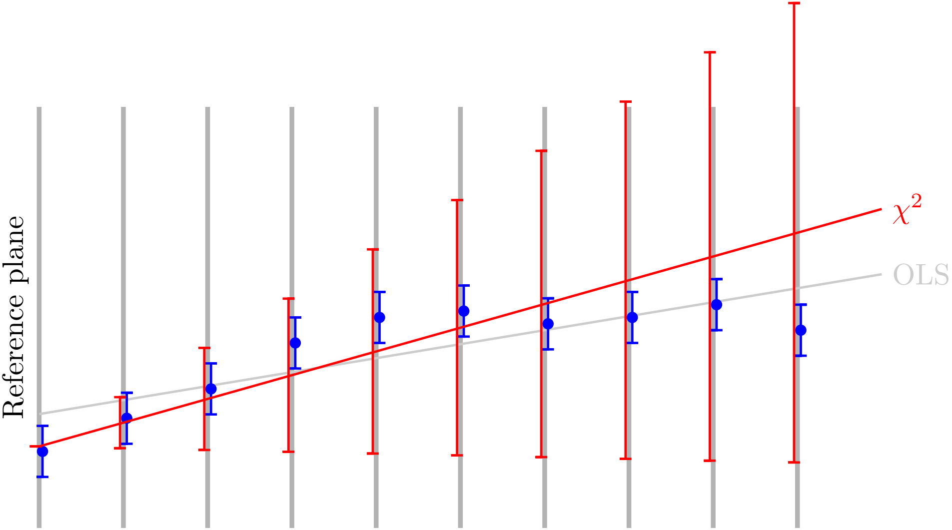


Find the $\boldsymbol{x}$ that minimizes $\sum r^2$

Gauss-Markov assumptions

The ordinary least squares estimator is the optimal linear estimator if

- Measurements are unbiased.
- The uncertainties of all the measurements are the same.
- The measurement errors are uncorrelated.



Find the $\boldsymbol{x}$ that minimizes $\chi^2 = \sum (r/\sigma_r)^2 = \sum \mathbf{r}^T \mathbf{R}^{-1} \mathbf{r}$

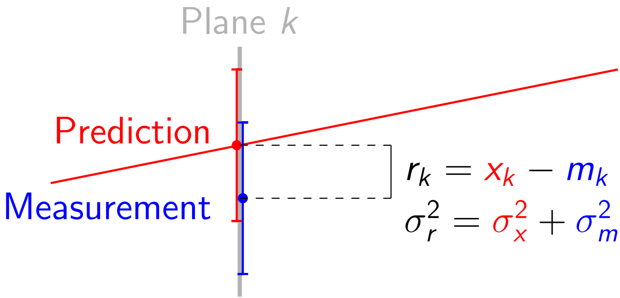
Gauss-Markov assumptions revised

χ^2-minimization is the optimal linear estimator if

- Measurements are unbiased.
- The uncertainties of all residuals are correctly estimated.
- The measurement errors are uncorrelated.

Test statistics

χ^2 minimization

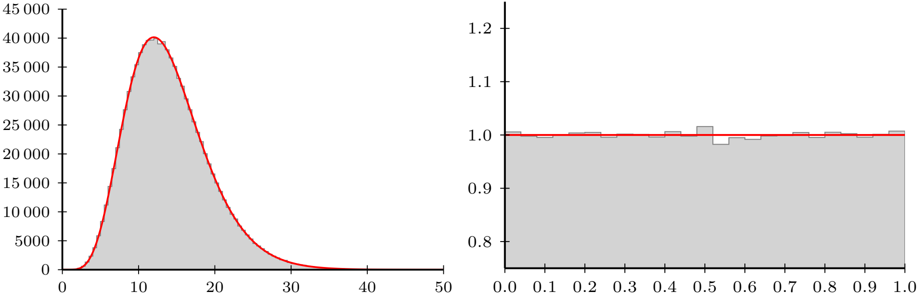


$$r_k/\sigma_r \sim \text{normal distribution}$$

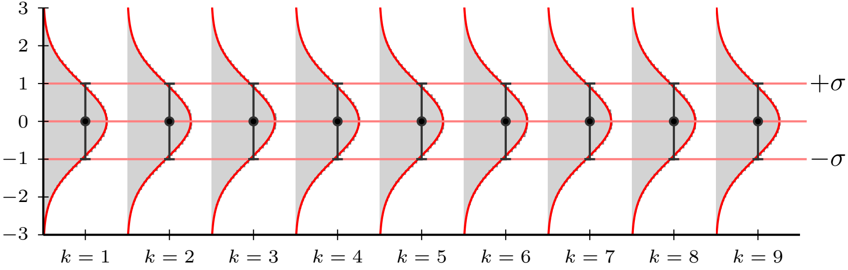
$$\sum^n (r_k/\sigma_r)^2 \sim \chi^2 \text{ with } n \text{ degrees of freedom}$$

$$\sum^n \left(\mathbf{r}_k^T \mathbf{R}_k^{-1} \mathbf{r}_k^T\right)^2 \sim \chi^2 \text{ with } n \times \dim(\mathbf{r}) \text{ degrees of freedom}$$

Test statistics (simulation)

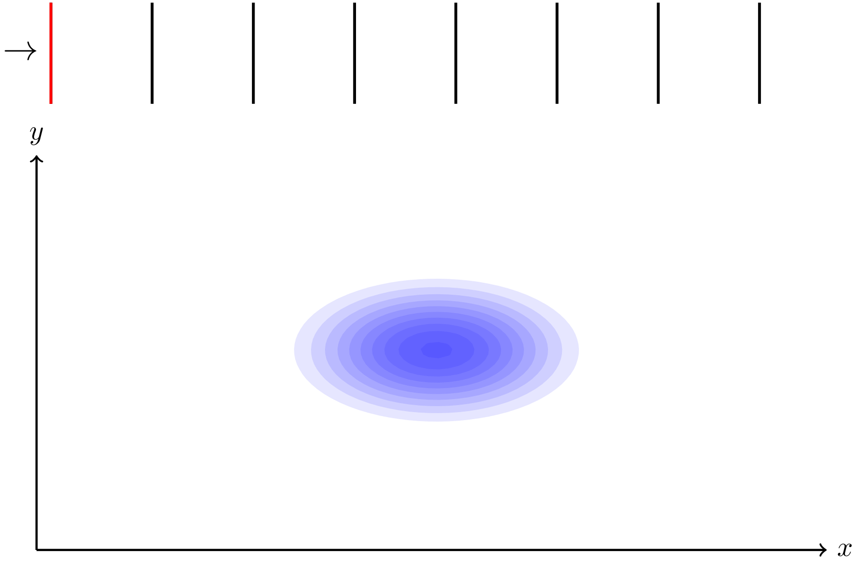


Pull-distributions (simulation)

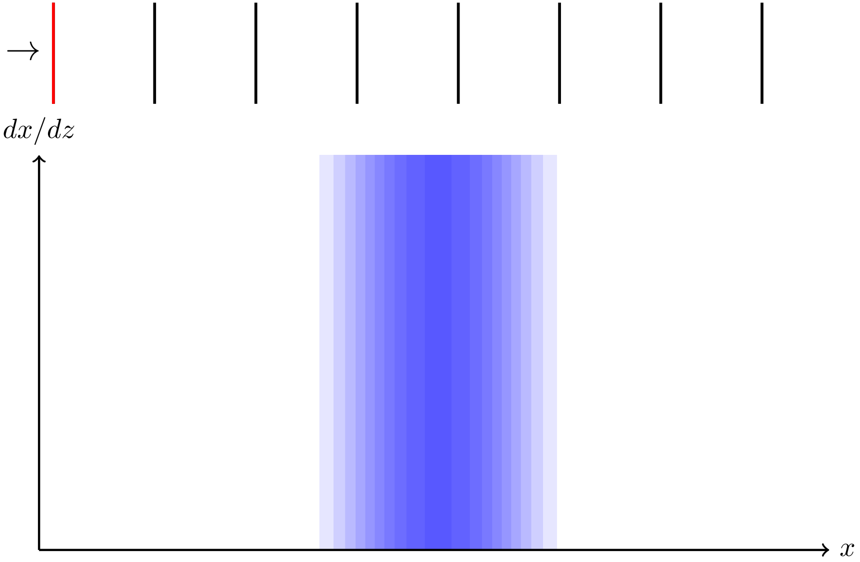


Track fitting with the Kalman filter

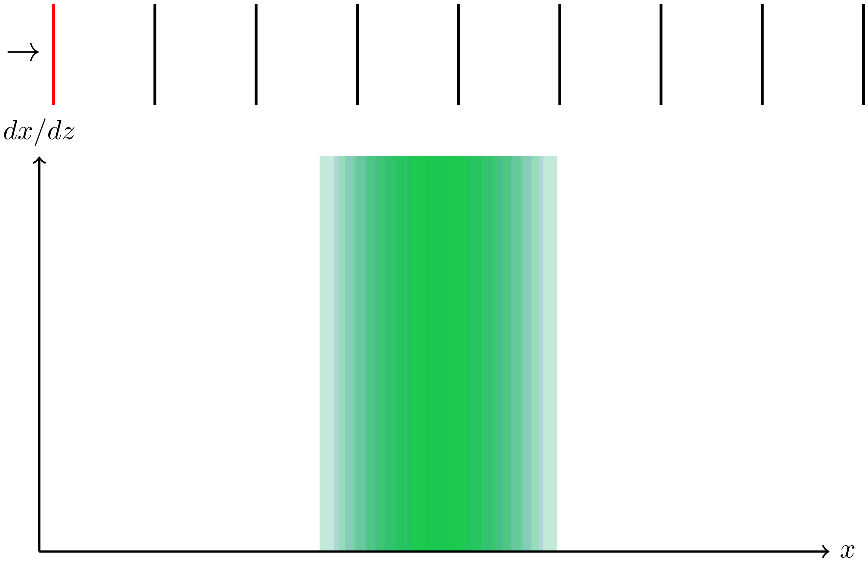
- Recursive formulation of χ^2-minimization.
- Fast compared to global χ^2 minimization.
- Can be used to build other algorithms for:
    - Track finding
    - Outlier rejection
    - Dealing with non-Gaussian energy loss.



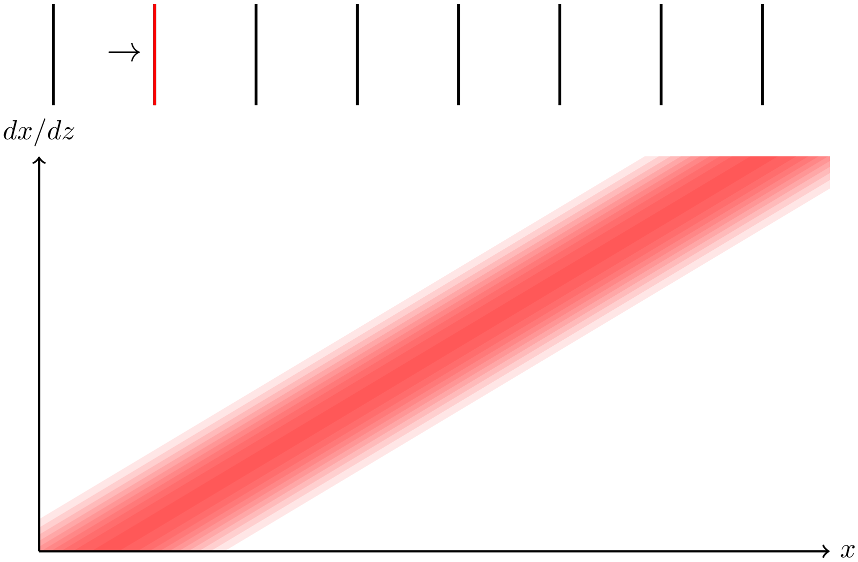
$$\mathcal{N}(\mathbf{x}'; \mathbf{x}, \mathbf{C}) = \frac{1}{2\pi\sqrt{|\mathbf{C}|}} e^{-\frac{1}{2}(\mathbf{x}'-\mathbf{x})^T \mathbf{C}^{-1}(\mathbf{x}'-\mathbf{x})}$$



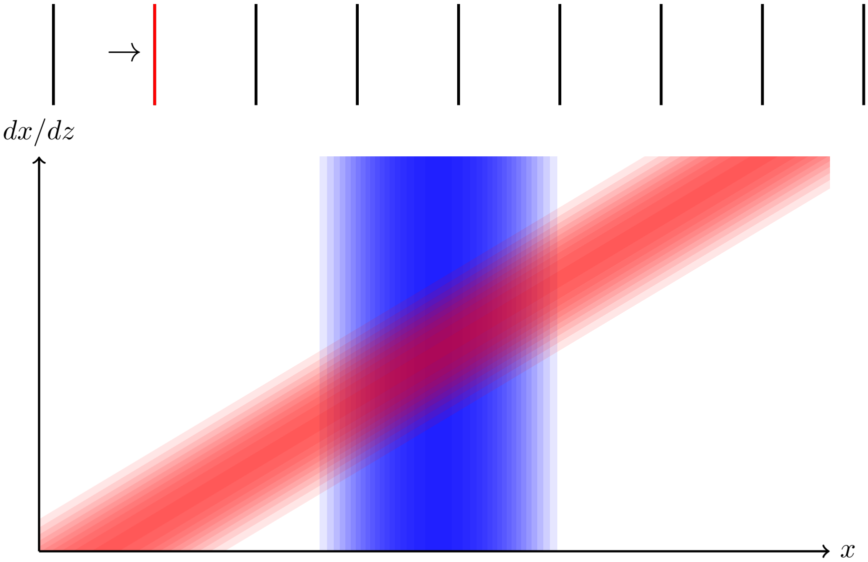
$$\mathcal{N}(\mathbf{x}'; \mathbf{x}, \mathbf{C}) \sim \cancel{\frac{1}{2\pi\sqrt{|\mathbf{C}|}}} e^{-\frac{1}{2}(\mathbf{x}'-\mathbf{x})^T \mathbf{C}^{-1}(\mathbf{x}'-\mathbf{x})}$$



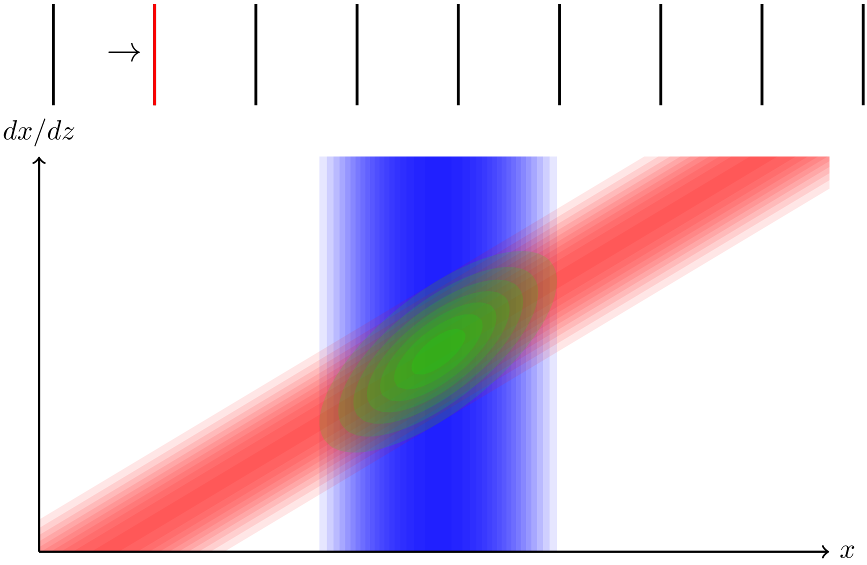
Prediction, measurement, update



Prediction, measurement, update



Prediction, measurement, update



Prediction, measurement, update

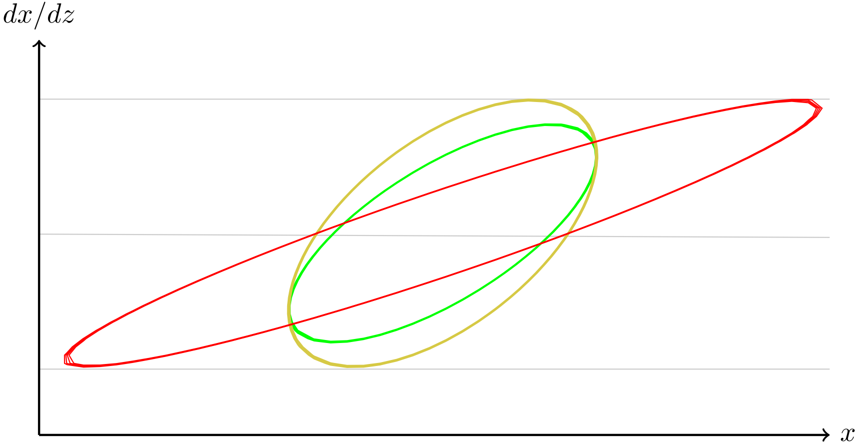
Front side of plane

$\mathbf{x}_k$, $\mathbf{C}_k$

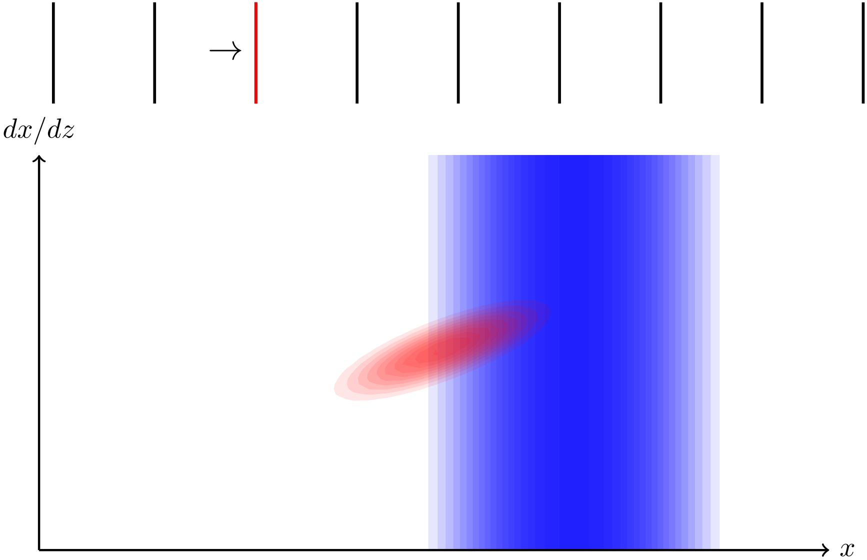
Back side of plane

$$\mathbf{x}'_k = \mathbf{x}_k,\ \mathbf{C}'_k = \mathbf{C}_k + \mathbf{Q}_k$$

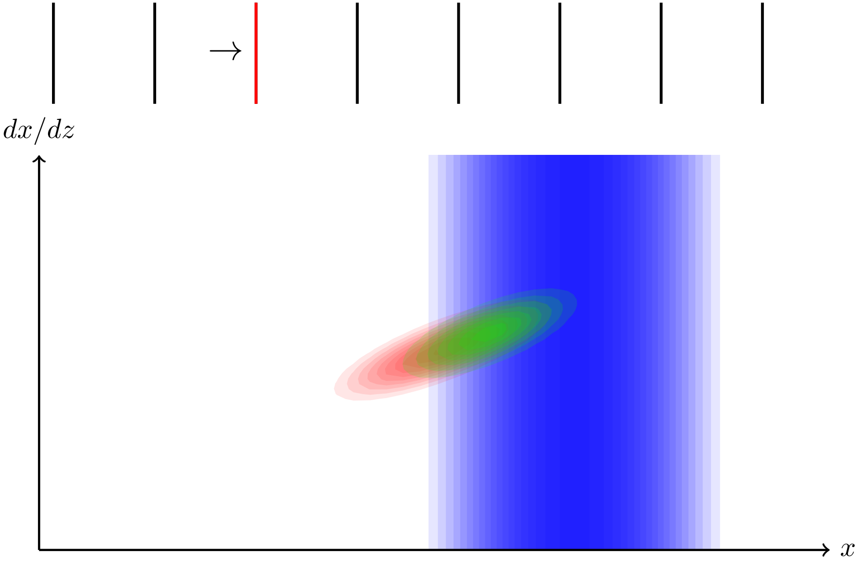
Front side of next plane



$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}'_k,\ \mathbf{C}_{k+1} = \mathbf{F}\mathbf{C}'_k\mathbf{F}^T$$

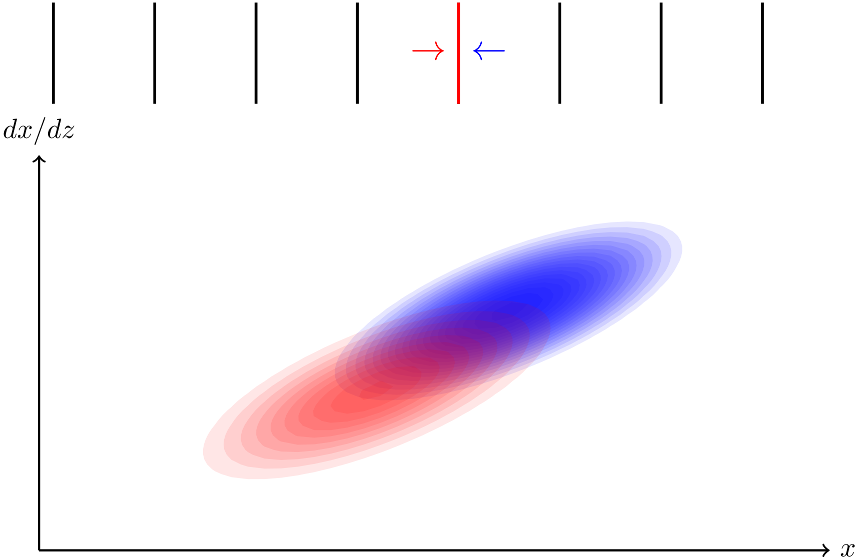


Prediction, measurement, update

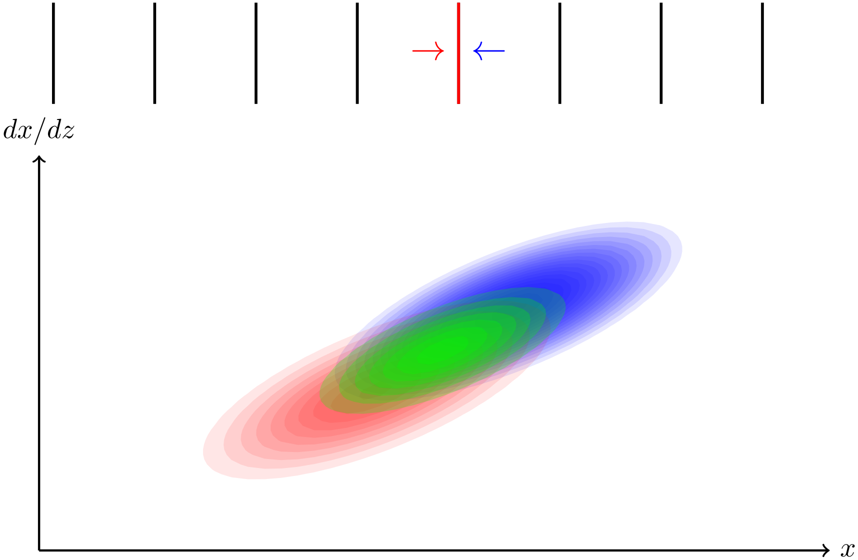


$$\mathbf{x}_{k|k} = \mathbf{C}_{k|k}\left(\mathbf{C}_{k|k-1}^{-1}\mathbf{x}_{k|k-1} + \mathbf{H}^{T}\mathbf{V}_{k}^{-1}\mathbf{m}_{k}\right)$$

$$\mathbf{C}_{k|k} = \left(\mathbf{C}_{k|k-1}^{-1} + \mathbf{H}^{T}\mathbf{V}_{k}^{-1}\mathbf{H}\right)^{-1}$$



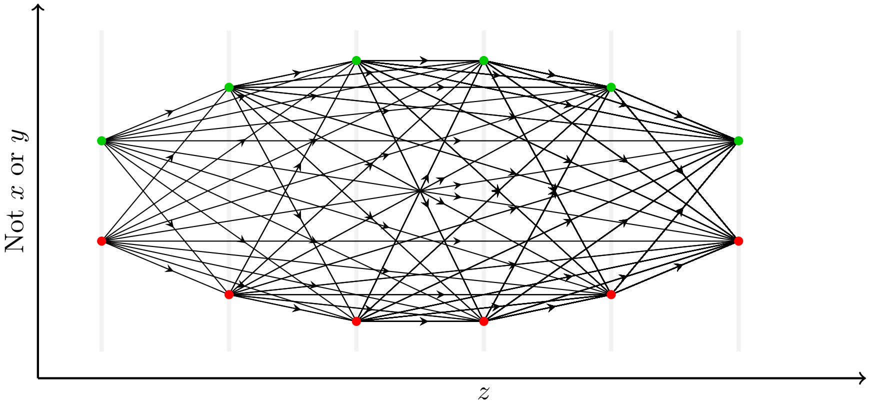
Prediction, prediction, update



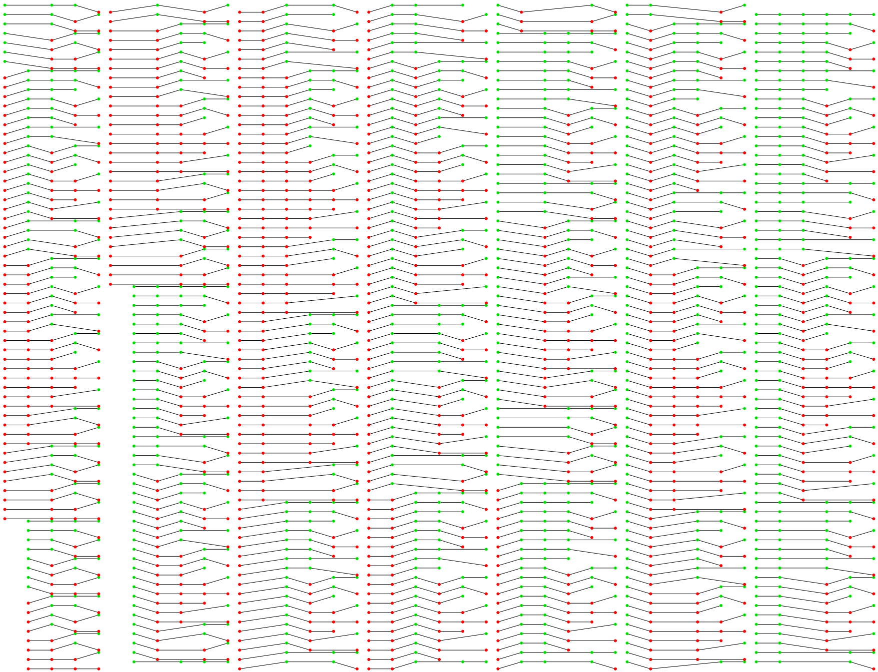
$$\mathbf{x}_{k|k} = \mathbf{C}_{k|k}\left(\mathbf{C}^{-1}_{k|k-1}\mathbf{x}_{k|k-1} + \mathbf{C}^{-1}_{k|k+1}\mathbf{x}_{k|k+1}\right)$$

$$\mathbf{C}_{k|k} = \left(\mathbf{C}^{-1}_{k|k-1} + \mathbf{C}^{-1}_{k|k+1}\right)^{-1}$$

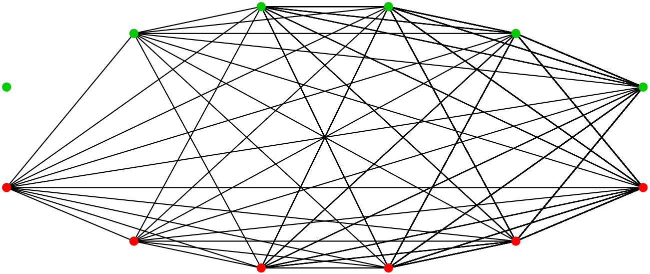
Track finding



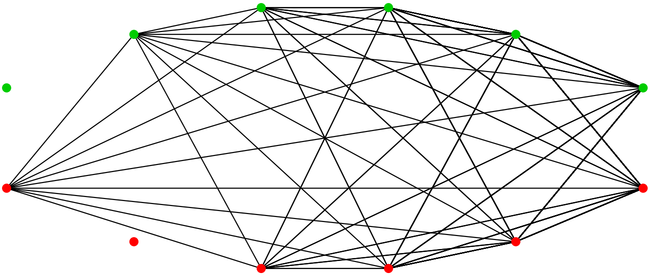
Not x or y
z



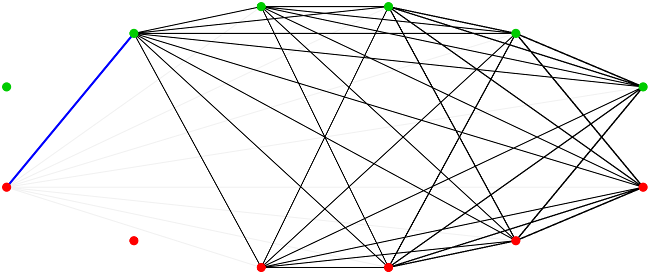
A node in the first plane is chosen as a starting point



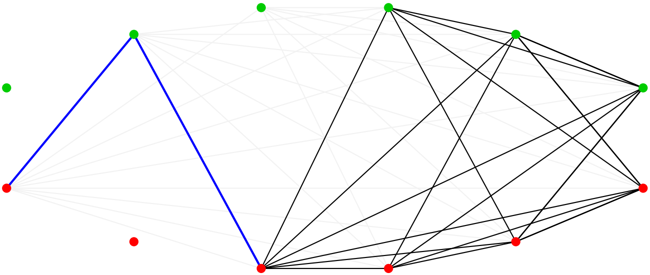
Discarding an edge excludes sub-graph



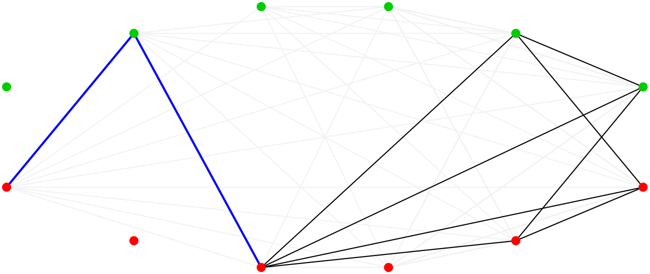
Initial edges are followed based on angle cuts



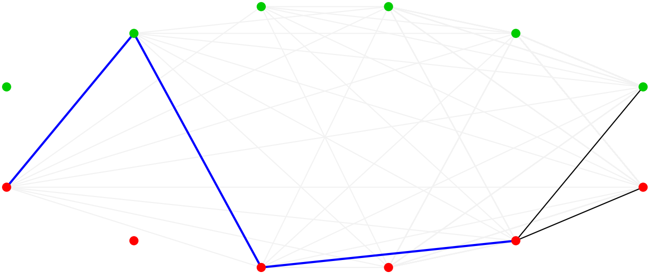
The following edges are followed based on χ^2 cuts



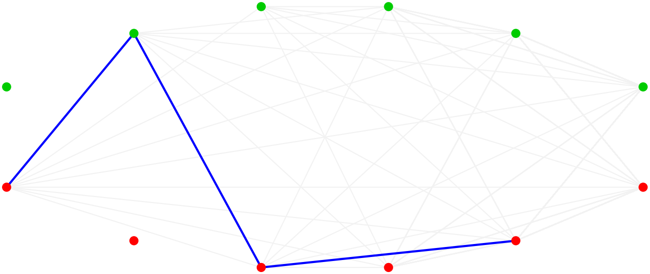
Follow short edges depth first



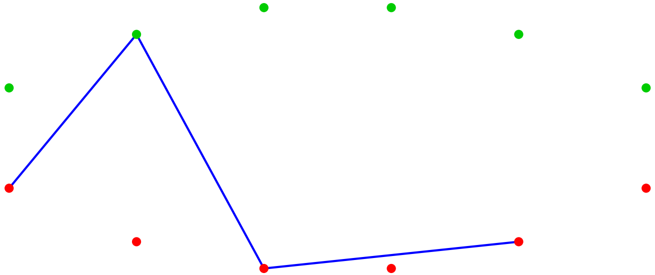
Follow short edges depth first



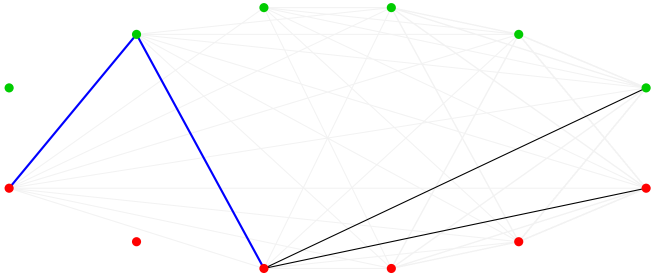
When the path can no longer grow, it is rejected or accepted



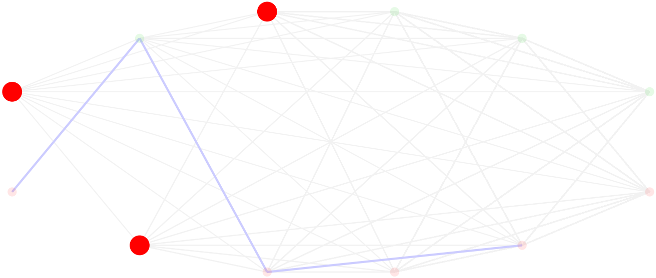
If accepted, the track is stored for analysis



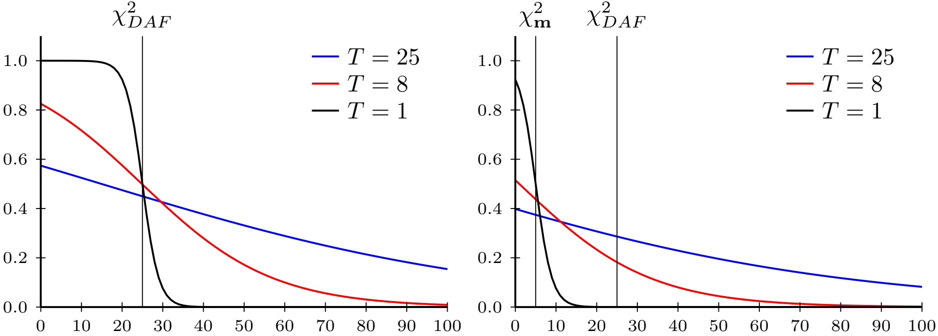
If rejected, edges skipping more planes are considered



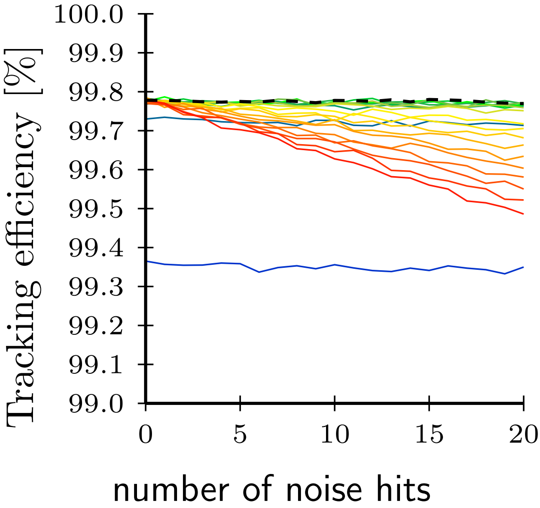
Searches are started in all nodes that can lead to a track



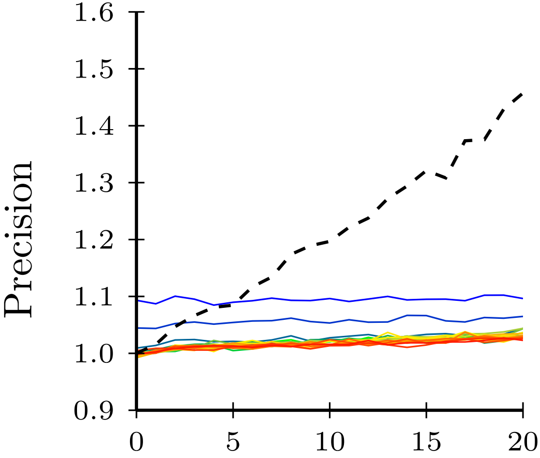
Deterministic annealing filter



Toy simulation

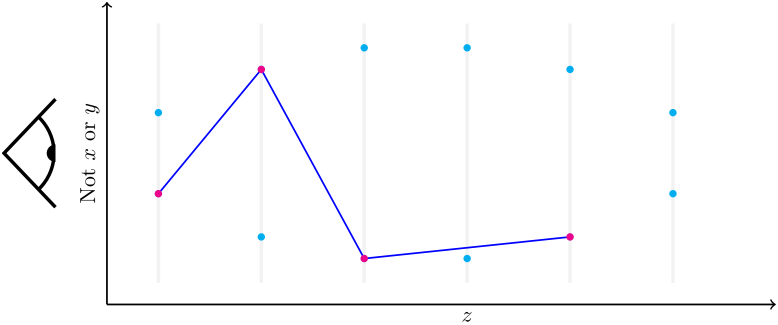


Toy simulation

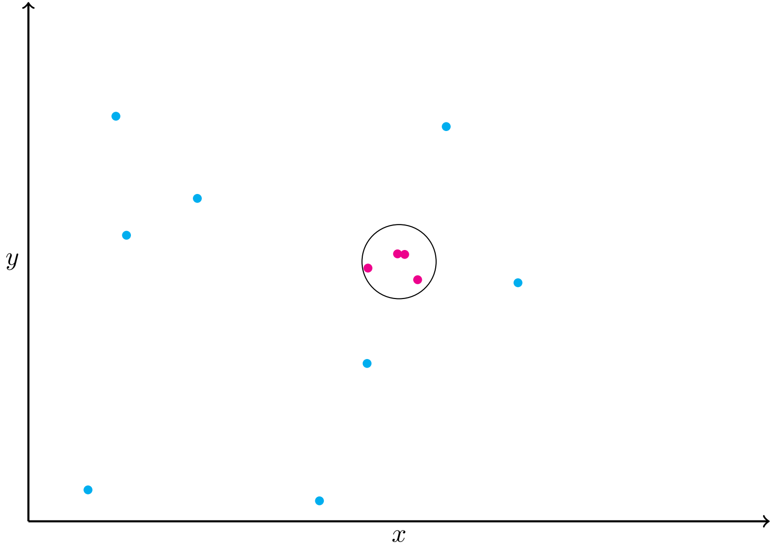


number of noise hits

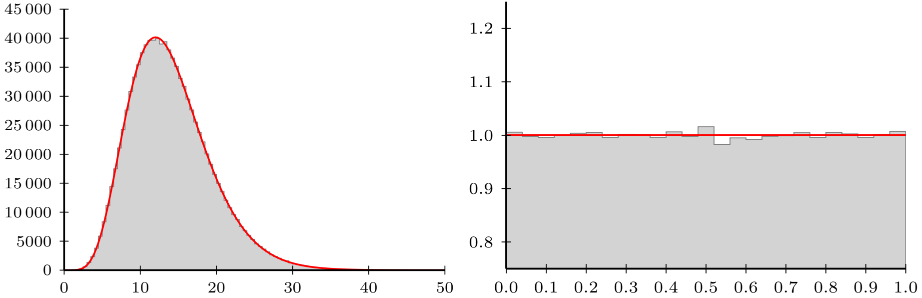
The cluster track finder takes advantage of the collimated beam.



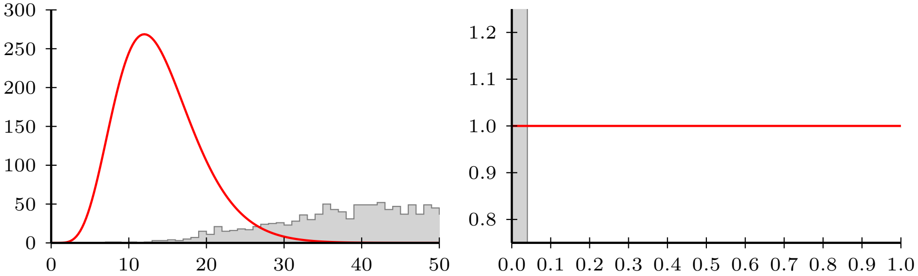
The cluster track finder takes advantage of the collimated beam.



P-values, simulated data



P-values, real data

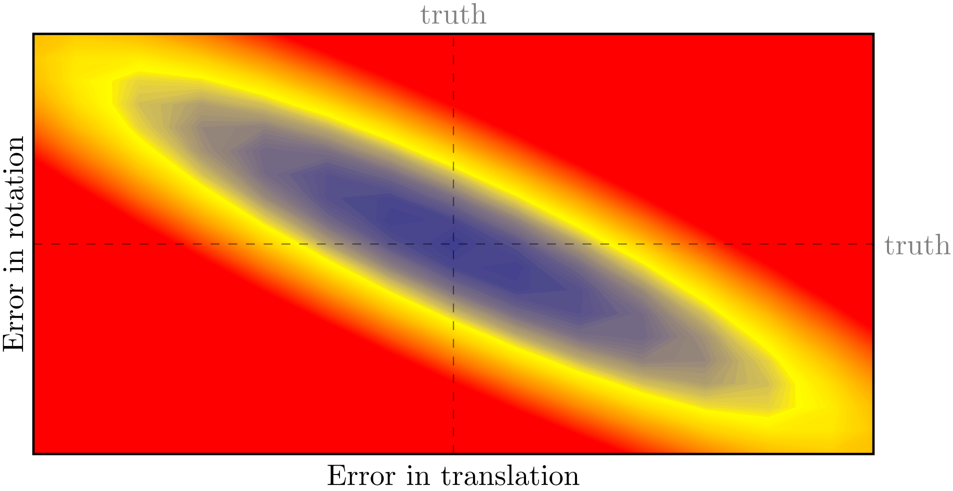


Detector alignment

- Obtaining a geometry description of the detector system
- Measurements during mounting not enough.
- The sum of independent random numbers following χ^2-distributions is also a χ^2 distribution

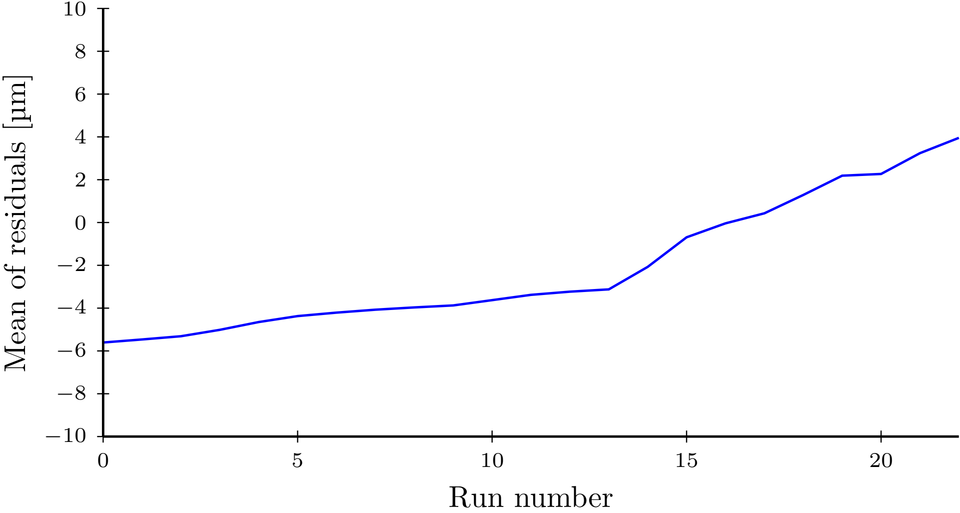
$$\chi^2(n_{doftot} = n_{track} \times n_{dof}) = \sum^{n_{track}} \chi^2(n_{dof})$$

Track sample χ^2 as a function of alignment parameters

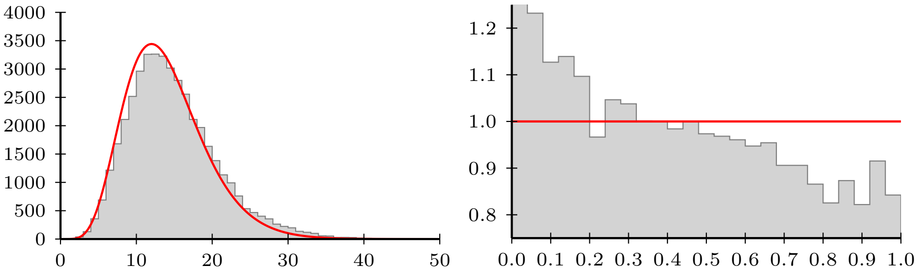


Alignment: Find alignment parameters that minimize track sample χ^2.

Mechanical stability



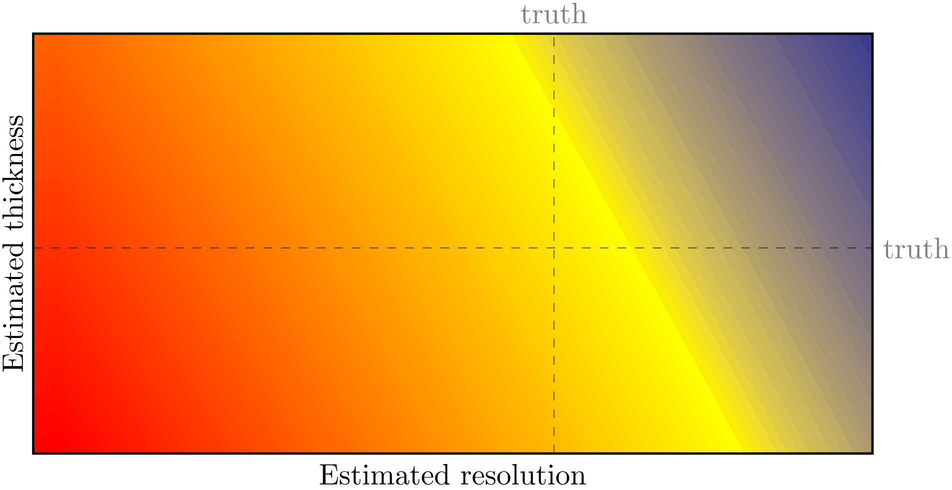
P-values, real data, after alignment



Material distribution and resolutions

- Affect uncertainties of measurements, predictions and updates
- Not known at time of data taking
- Estimation from data is new!

Average χ^2 as a function of thickness and resolution



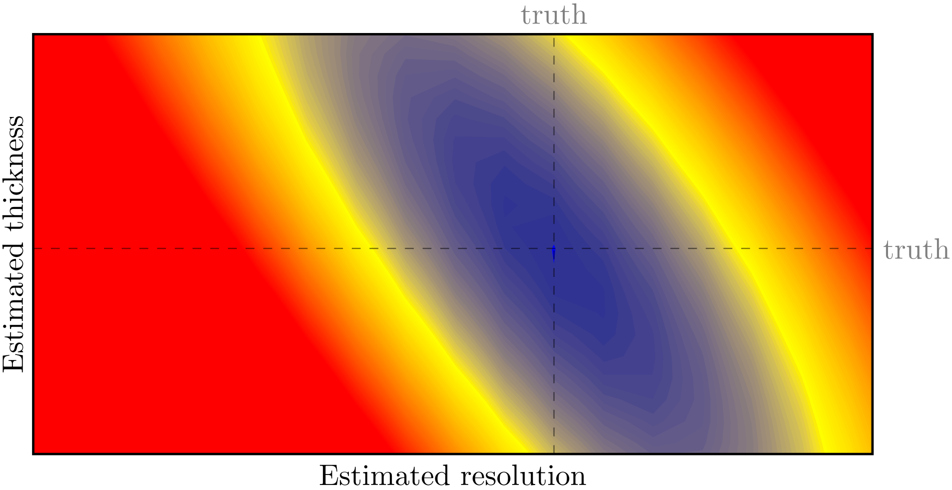
$$\sum \left(\left\langle \frac{d_i}{\sigma_{d_i}} \right\rangle^2 + \left[\left\langle \left(\frac{d_i}{\sigma_{df_i}} \right)^2 \right\rangle - 1 \right] \right)^2$$

$d_0, d_1, \ldots d_n$ are all residuals and state parameter differences

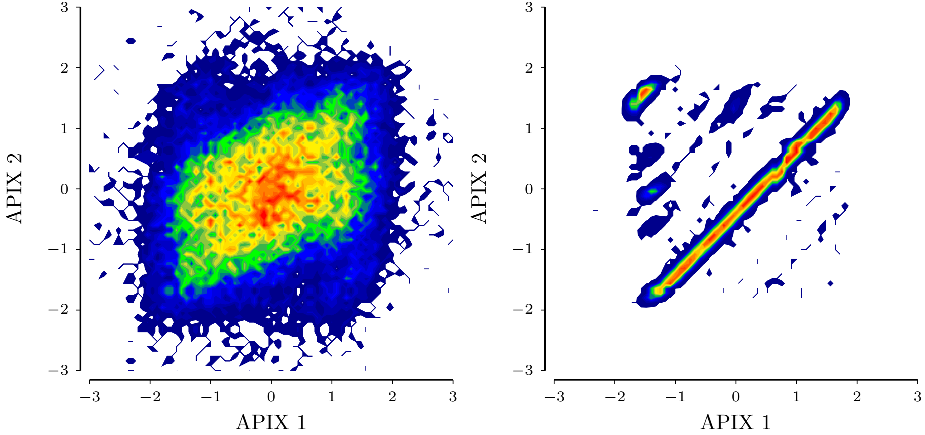
The mean, $\left\langle \frac{d_i}{\sigma_{d_i}} \right\rangle$ should be 0 from alignment

The variance, $\left\langle \left(\frac{d_i}{\sigma_{df_i}} \right)^2 \right\rangle$ should be 1 if uncertainties are correct

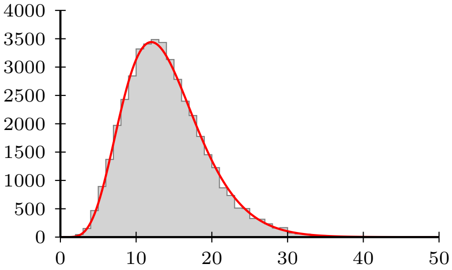
SDR3 as a function of thickness and resolution

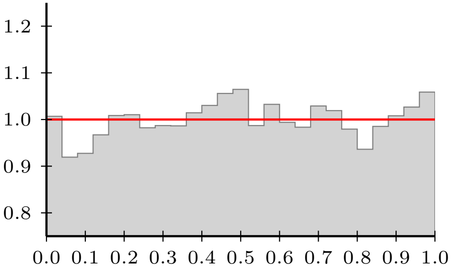


Third Gauss-Markov assumption

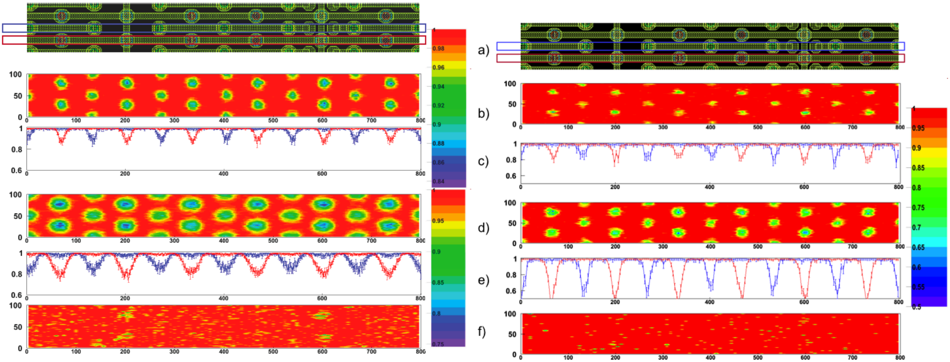


P-values, real data, material and resolution estimated

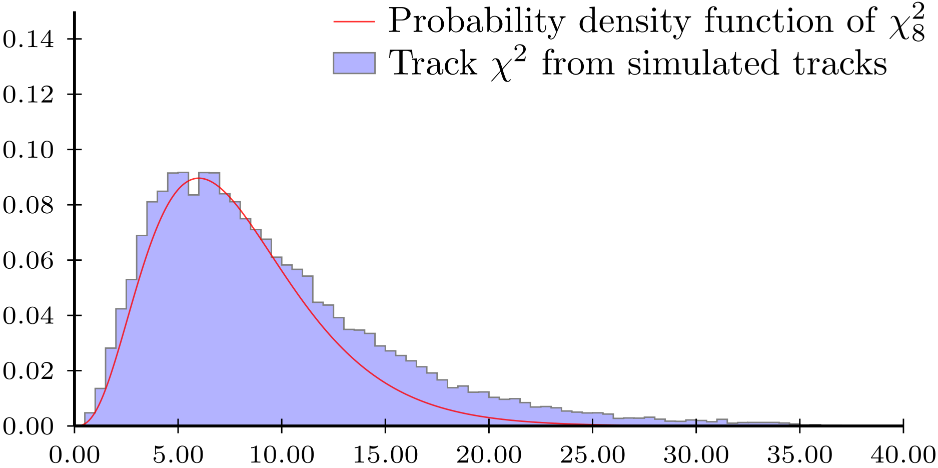




Detection efficiency



Electron beams



Summary

More detail:

Straight line track reconstruction for the ATLAS IBL testbeam with the EUDET telescope. Gjersdal, Røhne, Strandlie. ATL-INDET-PUB-2014-003.

Optimizing track reconstruction by simultaneous estimation of material and resolutions. Gjersdal, Frühwirth, Nadler, Strandlie. JINST, 8, 2013.

Test beam track reconstruction and analysis of ATLAS 3D pixel detectors. Håvard Gjersdal. PhD thesis, UiO, 2014.

Code:

https://github.com/hgjersdal/eigen-track-fitter

Thanks for listening!