3. Electroweak Effective Theory

- **Higgs Mechanism**
- **Custodial Symmetry**
- **Equivalence Theorem**
- **Goldstone Electroweak Effective Theory**
- **Fermions, Higgs**
- **Linear Realization**

Energy Scale	Fields	Effective Theory
$\Lambda_{NP} \sim$ TeV	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Underlying Dynamics
Energy Gap		
M_W	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model

Effective Field Theory

$$\mathcal{L}_{\rm eff} \;=\; \mathcal{L}^{(4)} + \sum_{D>4}\sum_{i} \frac{c_i^{(D)}}{\Lambda^{D-4}}\,\mathcal{O}_i^{(D)}$$

- **Most general Lagrangian with the SM gauge symmetries**
- **Light ($m \ll \Lambda_{NP}$) fields only**
- **The SM Lagrangian corresponds to $D = 4$**
- **$c_i^{(D)}$ contain information on the underlying dynamics:**

$$\mathcal{L}_{\rm NP} \doteq g_X\,(\bar{q}_L\gamma^\mu q_L)\,X_\mu \quad \Rightarrow \quad \frac{g_X^2}{M_X^2}\,(\bar{q}_L\gamma^\mu q_L)\,(\bar{q}_L\gamma_\mu q_L)$$

- **Options for H(126):**
  - **$SU(2)_L$ doublet (SM)**
  - **Scalar singlet**
  - **Additional light scalars**

Higgs Mechanism:

Gauge invariance

Massless $W^{\pm}$, Z (spin 1)

$$3 \times 2 \text{ polarizations} = 6$$

Higgs Mechanism: 3 additional degrees of freedom $\theta_i(x)$

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3×2 **polarizations** $=$ **6**

$+$

3 Goldstones $\theta_i(x)$

SSB $\downarrow$

Massive $W^\pm$, Z

3×3 **polarizations** $=$ **9**

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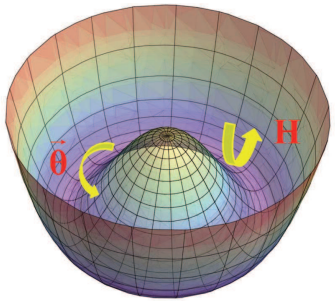
Spontaneous Symmetry Breaking

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$



$\mu^2 < 0$

$$\Phi(x) = \exp\left\{ i \, \frac{\vec{\sigma}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

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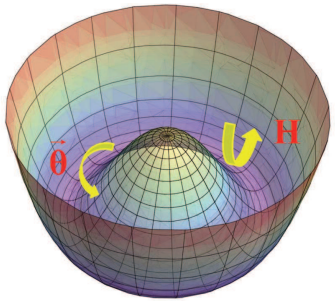

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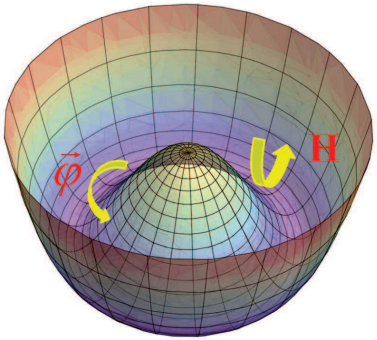
$D_\mu \Phi = (\partial_\mu + \frac{i}{2} g \, \vec{\sigma} \cdot \vec{W}_\mu + \frac{i}{2} g' \, B_\mu) \, \Phi \quad ; \quad v^2 = -\mu^2/\lambda$

$$(D_\mu \Phi)^\dagger D^\mu \Phi \to M_W^2 \, W_\mu^\dagger W^\mu + \frac{M_Z^2}{2} Z_\mu Z^\mu$$

$$\mathbf{M_W = M_Z \cos\theta_W = \frac{1}{2} g \, v}$$



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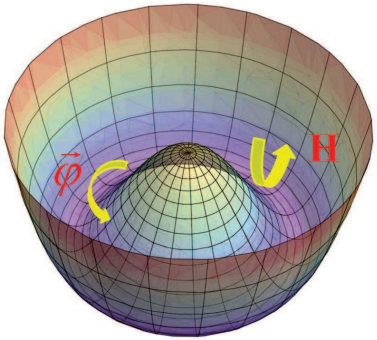

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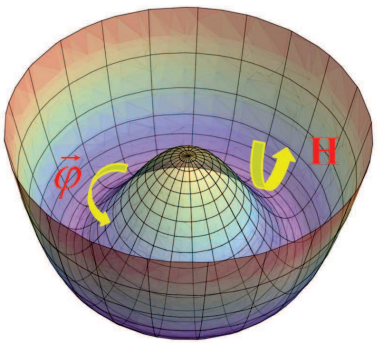

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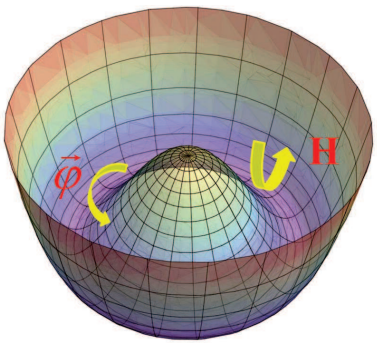

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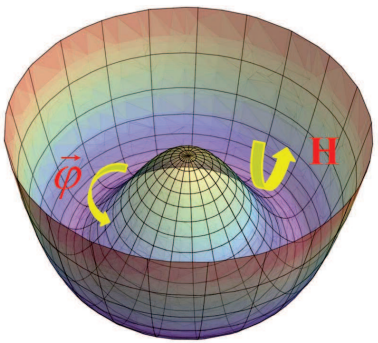

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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \quad , \quad \vec{\pi} \to \vec{\varphi} \to W_L^\pm, Z_L$$

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Derivative Coupling

Goldstones become free at zero momenta

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger\, D^\mu U\right) \xrightarrow{U=1} \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2}\, M_Z^2\, Z_\mu Z^\mu$$

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- **EW Goldstones are responsible for $M_{W,Z}$ (not the Higgs!)**
- **QCD pions also generate small W, Z masses:** $\delta_\pi M_W = \frac{1}{2}\, g\, f_\pi$

Goldstone interactions are determined by the underlying symmetry

$$
\begin{aligned}
\frac{v^2}{4}\,\langle \partial_\mu U^\dagger \partial^\mu U\rangle &= \partial_\mu \varphi^- \partial^\mu \varphi^+ + \frac{1}{2}\,\partial_\mu \varphi^0 \partial^\mu \varphi^0 \\
&+ \frac{1}{6v^2}\left\{\left(\varphi^+ \overleftrightarrow{\partial}_\mu \varphi^-\right)\left(\varphi^+ \overleftrightarrow{\partial}^\mu \varphi^-\right) + 2\left(\varphi^0 \overleftrightarrow{\partial}_\mu \varphi^+\right)\left(\varphi^- \overleftrightarrow{\partial}^\mu \varphi^0\right)\right\} \\
&+ O\left(\varphi^6/v^4\right)
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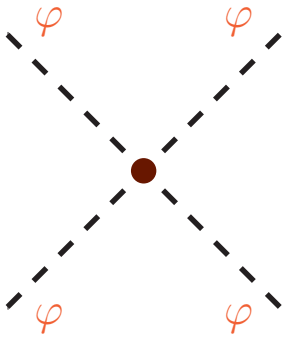

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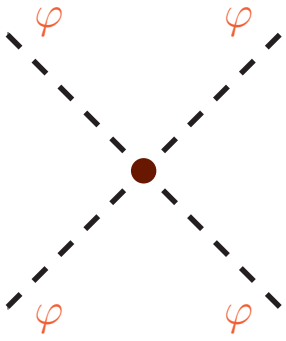

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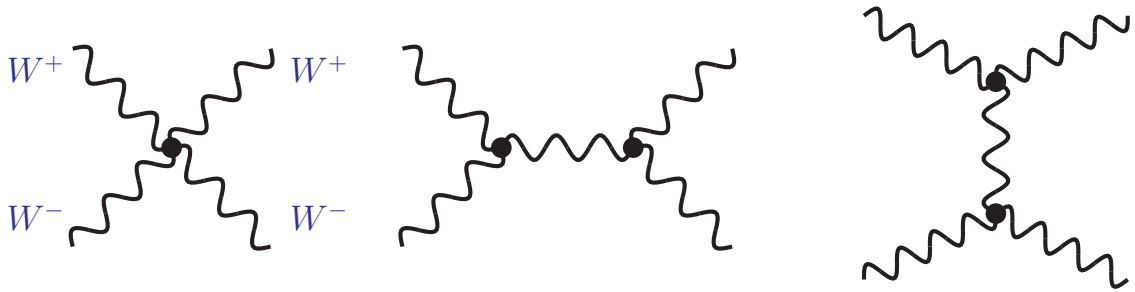
Non-Linear Lagrangian: $2\varphi \to 2\varphi,\ 4\varphi\ \cdots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$

$$= T(\varphi^+ \varphi^- \to \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

The scattering amplitude grows with energy

Goldstone dynamics $\longleftrightarrow$ **derivative interactions**

Tree-level violation of unitarity

Longitudinal Polarizations

$$k^\mu = \left(k^0, 0, 0, |\vec{k}|\right) \quad \Longrightarrow \quad \epsilon_L^\mu(\vec{k}) = \frac{1}{M_W}\left(|\vec{k}|, 0, 0, k^0\right) = \frac{k^\mu}{M_W} + O\left(\frac{M_W}{|\vec{k}|}\right)$$

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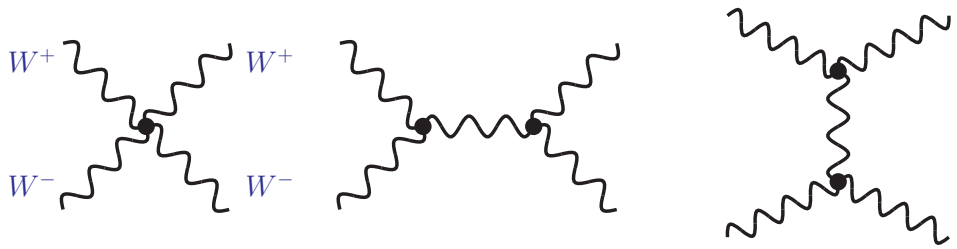

Gauge Cancelation

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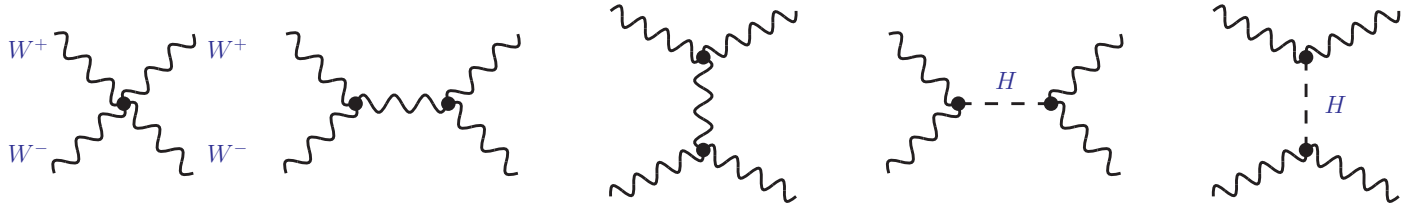
$W_L^+ W_L^- \to W_L^+ W_L^-$:



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Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

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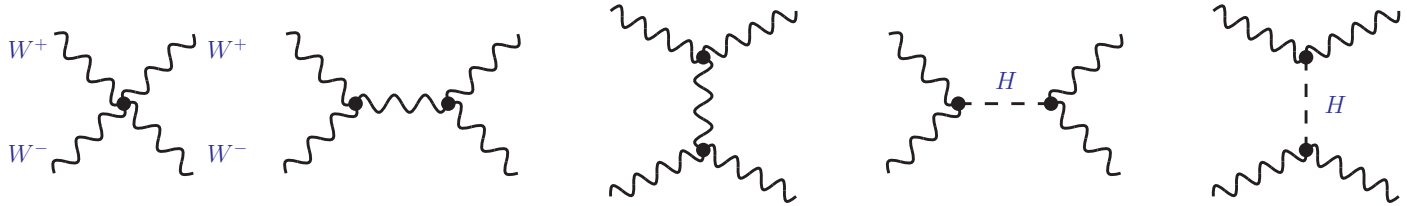

$$T_{\rm SM} = \frac{1}{v^2}\left\{s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

When $s \gg M_H^2$, $\quad T_{\rm SM} \approx -\dfrac{2M_H^2}{v^2}$, $\quad a_0 \equiv \dfrac{1}{32\pi}\displaystyle\int_{-1}^{1} d\cos\theta \; T_{\rm SM} \approx -\dfrac{M_H^2}{8\pi v^2}$

Unitarity:

Lee–Quigg–Thacker

$$|a_0| \leq 1 \quad \Longrightarrow \quad \mathbf{M_H} < \sqrt{\mathbf{8\pi}}\mathbf{v} \underbrace{\sqrt{2/3}}_{\mathbf{W^+W^-,\, ZZ,\, HH}} \approx \mathbf{1\ TeV}$$

What happens in QCD?

- QCD satisfies unitarity (it is a renormalizable theory)
- Pion scattering unitarized by exchanges of resonances (composite objects):
  - P-wave ($J = 1$) unitarized by ρ exchange
  - S-wave ($J = 0$) unitarized by σ exchange
- The σ meson is the QCD equivalent of the SM Higgs
- BUT, the σ is an 'effective' object generated through π rescattering (summation of pion loops)

Does not seem to work this way in the EW case, but . . .

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\rm EW}^{(4)}\Big|_{\rm CP-even} = \sum_{i=0}^{14} a_i\,\mathcal{O}_i$$

(Appelquist, Longhitano)

$$\mathcal{O}_0 = v^2\,\langle T_L V_\mu\rangle^2$$

$$\mathcal{O}_1 = \langle U\,\hat{B}_{\mu\nu}\,U^\dagger\,\hat{W}^{\mu\nu}\rangle \qquad \mathcal{O}_2 = i\,\langle U\,\hat{B}_{\mu\nu}\,U^\dagger\,[V^\mu, V^\nu]\rangle$$

$$\mathcal{O}_3 = i\,\langle \hat{W}_{\mu\nu}\,[V^\mu, V^\nu]\rangle \qquad \mathcal{O}_4 = \langle V_\mu V_\nu\rangle\,\langle V^\mu V^\nu\rangle$$

$$\mathcal{O}_5 = \langle V_\mu V^\mu\rangle^2 \qquad \mathcal{O}_6 = 4\,\langle V_\mu V_\nu\rangle\,\langle T_L V^\mu\rangle\,\langle T_L V^\nu\rangle$$

$$\mathcal{O}_7 = 4\,\langle V_\mu V^\mu\rangle\,\langle T_L V_\nu\rangle^2 \qquad \mathcal{O}_8 = \langle T_L \hat{W}_{\mu\nu}\rangle^2$$

$$\mathcal{O}_9 = -2\,\langle T_L \hat{W}_{\mu\nu}\rangle\,\langle T_L\,[V^\mu, V^\nu]\rangle \qquad \mathcal{O}_{10} = 16\,\{\langle T_L V_\mu\rangle\,\langle T_L V_\nu\rangle\}^2$$

$$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2\rangle \qquad \mathcal{O}_{12} = 4\,\langle T_L D_\mu D_\nu V^\nu\rangle\,\langle T_L V^\mu\rangle$$

$$\mathcal{O}_{13} = 2\,\langle T_L D_\mu V_\nu\rangle^2 \qquad \mathcal{O}_{14} = -2i\,\varepsilon^{\mu\nu\rho\sigma}\,\langle \hat{W}_{\mu\nu} V_\rho\rangle\,\langle T_L V_\sigma\rangle$$

$$V_\mu \equiv D_\mu U\,U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i\,[\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \to g_L\,(V_\mu, D_\mu V_\nu, T_L)\,g_L^\dagger$$

Symmetry breaking: $\quad T_L \equiv U\,\frac{\sigma_3}{2}\,U^\dagger \quad , \quad \hat{B}_{\mu\nu} \equiv -g'\,\frac{\sigma_3}{2}\,B_{\mu\nu}$

Low-Energy Effective Theory ➡ Power Counting

- **Momentum expansion:** $\Lambda \sim 4\pi v\,,\ M_X$

$$T = \sum_n T_n \left(\frac{p}{\Lambda}\right)^n$$

- $U \sim O(p^0)$, $D_\mu U,\ \hat{W}_\mu,\ \hat{B}_\mu \sim O(p^1)$, $\hat{W}_{\mu\nu},\ \hat{B}_{\mu\nu} \sim O(p^2)$

- **A general connected diagram with N_d vertices of $O(p^d)$ and L Goldstone loops has a power dimension:**

$$D = 2L + 2 + \sum_d N_d\,(d-2)$$

➡ **Finite number of divergences / counterterms**

NLO Predictions

- $\mathcal{L}_{\rm EW}^{(2)}$ **at one loop:** **Unitarity**

Non-local (logarithmic) dependences unambiguously predicted

- $\mathcal{L}_{\rm EW}^{(4)}$ **at tree level:** **Local (polynomic) amplitude**

Short-distance information encoded in the a_i couplings

Loop divergences reabsorbed through renormalized a_i

$$a_i \;=\; a_i^r(\mu) + \frac{\gamma_i}{16\pi^2}\left[\frac{2\,\mu^{D-4}}{4-D} + \log\left(4\pi\right) - \gamma_E\right]$$

$\hat{a}_i \equiv a_i/(16\pi)^2$ for different limits of the SM

	$M_H \to \infty$	$M_{t',b'} \to \infty$	$M_t \to \infty$
$\hat{a}_0$	$-\frac{3}{4}g'^2\left[\log(M_H/\mu) - \frac{5}{12}\right]$	0	$\frac{3}{2}\frac{M_t^2}{v^2}$
$\hat{a}_1$	$-\frac{1}{6}\log(M_H/\mu) + \frac{5}{72}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{1}{4}$
$\hat{a}_2$	$-\frac{1}{12}\log(M_H/\mu) + \frac{17}{144}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{3}{4}$
$\hat{a}_3$	$\frac{1}{12}\log(M_H/\mu) - \frac{17}{144}$	$\frac{1}{2}$	$\frac{3}{8}$
$\hat{a}_4$	$\frac{1}{6}\log(M_H/\mu) - \frac{17}{72}$	$\frac{1}{4}$	$\log(M_t/\mu) - \frac{5}{6}$
$\hat{a}_5$	$\frac{2\pi^2 v^2}{M_H^2} + \frac{1}{12}\log(M_H/\mu) - \frac{79}{72} + \frac{9\pi}{16\sqrt{3}}$	$-\frac{1}{8}$	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_6$	0	0	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_7$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_8$	0	0	$\log(M_t/\mu) - \frac{7}{12}$
$\hat{a}_9$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_{10}$	0	0	$-\frac{1}{64}$
$\hat{a}_{11}$	—	$-\frac{1}{2}$	$-\frac{1}{2}$
$\hat{a}_{12}$	—	0	$-\frac{1}{8}$
$\hat{a}_{13}$	—	0	$-\frac{1}{4}$
$\hat{a}_{14}$	0	0	$\frac{3}{8}$

Equations of Motion ➡ Redundant Operators

Equivalent to field redefinitions which only affect higher-order terms

For massless fermions: $\partial_\mu \langle T_L V^\mu \rangle = 0 \quad , \quad D_\mu V^\mu = 0$

$$\Rightarrow \begin{cases} \mathcal{O}_{11} = \mathcal{O}_{12} = 0 \\ \mathcal{O}_{13} = -\frac{g'^2}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{O}_1 - \mathcal{O}_4 + \mathcal{O}_5 - \mathcal{O}_6 + \mathcal{O}_7 + \mathcal{O}_8 \end{cases}$$

Equations of Motion ➡ Redundant Operators

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$$\Rightarrow \left\{ \begin{array}{c} \mathcal{O}_{11} = \mathcal{O}_{12} = 0 \\ \mathcal{O}_{13} = -\frac{g'^2}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{O}_1 - \mathcal{O}_4 + \mathcal{O}_5 - \mathcal{O}_6 + \mathcal{O}_7 + \mathcal{O}_8 \end{array} \right.$$

Heavy top: $\mathcal{O}_{11} \doteq \frac{g^4}{8 M_W^4} m_t^2 \left\{ (\bar{t}\gamma_5 t)^2 - 4 \sum_{i,j} (\bar{d}_{iL} t_R)(\bar{t}_R d_{jL}) V_{tj} V_{ti}^* \right\}$

Unitary Gauge: $\mathbf{U = 1}$

All invariants reduce to polynomials of gauge fields

- **Bilinear terms:** $\mathcal{O}_0$, $\mathcal{O}_1$, $\mathcal{O}_8$, $\mathcal{O}_{11}$, $\mathcal{O}_{12}$, $\mathcal{O}_{13}$

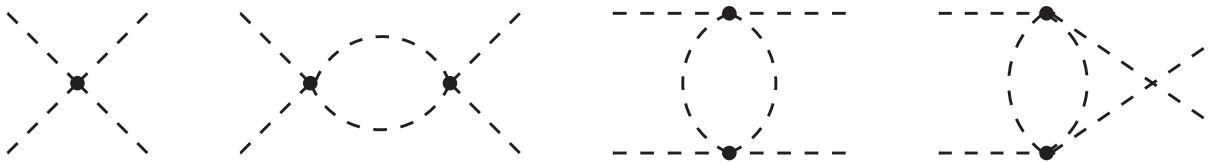
  ➡ **Oblique corrections** $(\Delta r,\ \Delta\rho,\ \Delta k \Longleftrightarrow S,\ T,\ U)$

- **Trilinear terms:** $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_9$, $\mathcal{O}_{14}$

- **Quartic terms:** $\mathcal{O}_4$, $\mathcal{O}_5$, $\mathcal{O}_6$, $\mathcal{O}_7$, $\mathcal{O}_{10}$

- $\mathcal{O}_{11} \sim m_t^2\,(\bar\psi\psi)(\bar\psi\psi)$**:** $Z\bar b b$, B^0–$\bar B^0$, $\varepsilon_K \ldots$

$\varphi^a \varphi^b \to \varphi^c \varphi^d$:



$$\mathcal{A}(\varphi^a \varphi^b \to \varphi^c \varphi^d) \;=\; A(s,t,u)\,\delta_{ab}\,\delta_{cd} + A(t,s,u)\,\delta_{ac}\,\delta_{bd} + A(u,t,s)\,\delta_{ad}\,\delta_{bc}$$

$$\boxed{\begin{aligned} A(s,t,u) \;=\; & \frac{s}{v^2} + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\,a_5^r(\mu)\,s^2\right] \\ & + \frac{1}{16\pi^2 v^2}\left\{\frac{5}{9}\,s^2 + \frac{13}{18}\,(t^2+u^2) + \frac{1}{12}\,(s^2-3t^2-u^2)\,\log\left(\frac{-t}{\mu^2}\right)\right. \\ & \left. + \frac{1}{12}\,(s^2-t^2-3u^2)\,\log\left(\frac{-u}{\mu^2}\right) - \frac{1}{2}\,s^2\,\log\left(\frac{-s}{\mu^2}\right)\right\} \end{aligned}}$$

$$a_i \;=\; a_i^r(\mu) + \frac{\gamma_i}{16\pi^2}\left[\frac{2\,\mu^{D-4}}{4-D} + \log(4\pi) - \gamma_E\right] \quad , \quad \gamma_4 \;=\; -\frac{1}{12} \quad , \quad \gamma_5 \;=\; -\frac{1}{24}$$

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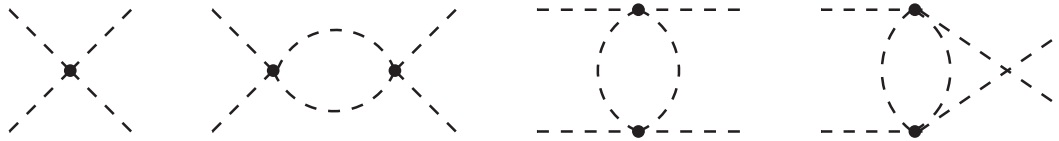


+ Higgs (tree + 1-loop) contributions

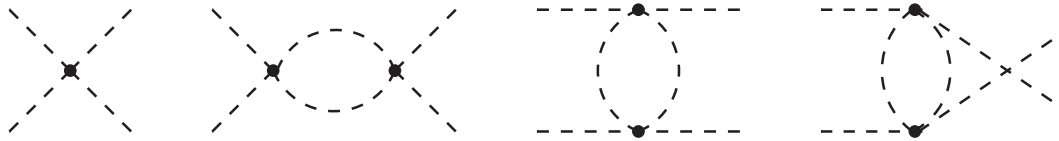
$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2\, a\, \frac{H}{v} + b\, \frac{H^2}{v^2} \right]$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned} A(s,t,u) \;=\; & \frac{s}{v^2}\,(1-a^2) + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\, a_5^r(\mu)\, s^2 \right] \\ & + \frac{1}{16\pi^2 v^2}\left\{ \frac{1}{9}\,(14\,a^4 - 10\,a^2 - 18\,a^2 b + 9\,b^2 + 5)\, s^2 + \frac{13}{18}\,(1-a^2)^2\,(t^2+u^2) \right. \\ & \quad - \frac{1}{2}\,(2\,a^4 - 2\,a^2 - 2\,a^2 b + b^2 + 1)\, s^2 \log\left(\frac{-s}{\mu^2}\right) \\ & \quad \left. + \frac{1}{12}\,(1-a^2)^2 \left[(s^2 - 3t^2 - u^2) \log\left(\frac{-t}{\mu^2}\right) + (s^2 - t^2 - 3u^2) \log\left(\frac{-u}{\mu^2}\right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\tfrac{1}{12}\,(1-a^2)^2 \qquad , \qquad \gamma_5 = -\tfrac{1}{48}\,(2 + 5\,a^4 - 4\,a^2 - 6\,a^2 b + 3\,b^2)$$

$$\varphi^a \varphi^b \to \varphi^c \varphi^d$$



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$$\begin{aligned} A(s,t,u) \;=\; & \frac{s}{v^2}\,(1-a^2) + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\, a_5^r(\mu)\, s^2 \right] \\ & + \frac{1}{16\pi^2 v^2}\left\{ \frac{1}{9}\,(14\,a^4 - 10\,a^2 - 18\,a^2 b + 9\,b^2 + 5)\, s^2 + \frac{13}{18}\,(1-a^2)^2\,(t^2+u^2) \right. \\ & - \frac{1}{2}\,(2\,a^4 - 2\,a^2 - 2\,a^2 b + b^2 + 1)\, s^2 \log\left(\frac{-s}{\mu^2}\right) \\ & \left. + \frac{1}{12}\,(1-a^2)^2 \left[(s^2 - 3t^2 - u^2) \log\left(\frac{-t}{\mu^2}\right) + (s^2 - t^2 - 3u^2) \log\left(\frac{-u}{\mu^2}\right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\tfrac{1}{12}\,(1-a^2)^2 \quad , \quad \gamma_5 = -\tfrac{1}{48}\,(2 + 5\,a^4 - 4\,a^2 - 6\,a^2 b + 3\,b^2)$$

SM: $a = b = 1$, $a_4 = a_5 = 0$ $\Rightarrow$ $A(s,t,u) \sim \mathcal{O}(M_H^2/v^2)$

Yukawa Couplings

$$\boxed{\mathcal{L}_Y \;=\; -v\,\left\{\bar{Q}_L\,U(\varphi)\left[\hat{\mathbf{Y}}_{\mathbf{u}}\,\mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}}\,\mathcal{P}_-\right]Q_R \;+\; \bar{L}_L\,U(\varphi)\,\hat{\mathbf{Y}}_{\boldsymbol{\ell}}\,\mathcal{P}_+\,L_R \;+\; \mathrm{h.c.}\right\}}$$

$$Q \;=\; \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L \;=\; \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \;\to\; g_L\,U(\varphi)\,g_R^\dagger \quad , \quad Q_L \;\to\; g_L\,Q_L \quad , \quad Q_R \;\to\; g_R\,Q_R \quad , \quad \mathcal{P}_\pm \;\to\; g_R\,\mathcal{P}_\pm\,g_R^\dagger$$

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Symmetry Breaking: $\mathcal{P}_\pm \;=\; \frac{1}{2}\left(\mathrm{I}_2 \pm \sigma_3\right)$

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Symmetry Breaking: $\mathcal{P}_\pm \;=\; \frac{1}{2}\left(\mathrm{I}_2 \pm \sigma_3\right)$

Flavour Structure: $\hat{\mathbf{Y}}_{\mathbf{u},\mathbf{d},\ell}$ **3×3 matrices in flavour space**

NLO Operators

Buchalla–Catá

$$\mathcal{O}_{\psi V1} = i\,\bar{Q}_L\gamma^\mu Q_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V2} = i\,\bar{Q}_L\gamma^\mu T_L Q_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V3} = i\,\bar{Q}_L\gamma^\mu \tilde{P}_{12} Q_L\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V4} = i\,\bar{u}_R\gamma^\mu u_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V5} = i\,\bar{d}_R\gamma^\mu d_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V6} = i\,\bar{u}_R\gamma^\mu d_R\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V7} = i\,\bar{L}_L\gamma^\mu L_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V8} = i\,\bar{L}_L\gamma^\mu T_L L_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V9} = \bar{L}_L\gamma^\mu \tilde{P}_{12} L_L\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V10} = \bar{\ell}_R\gamma^\mu \ell_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}^\dagger_{\psi V3} \qquad \mathcal{O}^\dagger_{\psi V6}$$

$$\mathcal{O}_{\psi S1,2} = \bar{Q}_L\tilde{P}_\pm U Q_R\,\langle D_\mu U^\dagger D^\mu U\rangle \qquad \mathcal{O}_{\psi S3,4} = \bar{Q}_L\tilde{P}_\pm U Q_R\,\langle V_\mu T_L\rangle^2$$

$$\mathcal{O}_{\psi S5} = \bar{Q}_L\tilde{P}_{12} U Q_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\mu T_L\rangle \qquad \mathcal{O}_{\psi S6} = \bar{Q}_L\tilde{P}_{12} Q_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V^\mu T_L\rangle$$

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$$\mathcal{O}_{\psi T3,4} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_\pm U Q_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V_\nu \tilde{P}_{21}\rangle \qquad \mathcal{O}_{\psi T5} = \bar{L}_L\sigma^{\mu\nu}\tilde{P}_{12} U L_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\nu T_L\rangle$$

$$\mathcal{O}_{\psi T6} = \bar{L}_L\sigma^{\mu\nu}\tilde{P}_- U L_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V_\nu \tilde{P}_{21}\rangle$$

$$V_\mu = D_\mu U\,U^\dagger \quad , \quad T_L = U\,\frac{\sigma_3}{2}\,U^\dagger \quad , \quad \tilde{P}_{12} = U\,\frac{\sigma_{1+i2}}{2}\,U^\dagger \quad , \quad \tilde{P}_{21} = U\,\frac{\sigma_{1-i2}}{2}\,U^\dagger \quad , \quad \tilde{P}_\pm = U\,P_\pm\,U^\dagger$$

NLO Operators (cont.)

Buchalla–Catá

$$\mathcal{O}_{LL6} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{Q}_L\gamma_\mu T_L Q_L \qquad \mathcal{O}_{LL7} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{Q}_L\gamma_\mu Q_L \qquad \mathcal{O}_{LL8} = \bar{q}_{L\alpha}\gamma^\mu T_L Q_{L\beta}\ \bar{Q}_{L\beta}\gamma_\mu T_L Q_{L\alpha}$$

$$\mathcal{O}_{LL10} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LL11} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{L}_L\gamma_\mu L_L \qquad \mathcal{O}_{LL9} = \bar{Q}_{L\alpha}\gamma^\mu T_L Q_{L\beta}\ \bar{Q}_{L\beta}\gamma_\mu Q_{L\alpha}$$

$$\mathcal{O}_{LL12} = \bar{Q}_L\gamma^\mu Q_L\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LL13} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu T_L Q_L \qquad \mathcal{O}_{LL14} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu Q_L$$

$$\mathcal{O}_{LL15} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LL16} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu L_L$$

$$\mathcal{O}_{LR10} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{u}_R\gamma_\mu u_R \qquad \mathcal{O}_{LR12} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{d}_R\gamma_\mu d_R \qquad \mathcal{O}_{LR11} = \bar{Q}_L\gamma^\mu t^a T_L Q_L\ \bar{u}_R\gamma_\mu t^a u_R$$

$$\mathcal{O}_{LR14} = \bar{u}_R\gamma^\mu u_R\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LR15} = \bar{d}_R\gamma^\mu d_R\ \bar{L}_L\gamma_\mu T_L L_L \qquad \mathcal{O}_{LR13} = \bar{Q}_L\gamma^\mu t^a T_L Q_L\ \bar{d}_R\gamma_\mu t^a d_R$$

$$\mathcal{O}_{LR16} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{\ell}_R\gamma_\mu \ell_R \qquad \mathcal{O}_{LR17} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{\ell}_R\gamma_\mu \ell_R \qquad \mathcal{O}_{LR18} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{\ell}_R\gamma_\mu d_R$$

$$\mathcal{O}_{ST5} = \bar{Q}_L\tilde{P}_+ U Q_R\ \bar{Q}_L\tilde{P}_- U Q_R \qquad \mathcal{O}_{ST6} = \bar{Q}_L\tilde{P}_{21} U Q_R\ \bar{Q}_L\tilde{P}_{12} U Q_R \qquad \mathcal{O}_{ST7} = \bar{Q}_L t^a\tilde{P}_+ U Q_R\ \bar{Q}_L t^a\tilde{P}_- U Q_R$$

$$\mathcal{O}_{ST9} = \bar{Q}_L\tilde{P}_+ U Q_R\ \bar{L}_L\tilde{P}_- U L_R \qquad \mathcal{O}_{ST10} = \bar{Q}_L\tilde{P}_{21} U Q_R\ \bar{L}_L\tilde{P}_{12} U L_R \qquad \mathcal{O}_{ST8} = \bar{Q}_L t^a\tilde{P}_{21} U Q_R\ \bar{Q}_L t^a\tilde{P}_{12} U Q_R$$

$$\mathcal{O}_{ST11} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_+ U Q_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_- U L_R \qquad \mathcal{O}_{ST12} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_{21} U Q_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_{12} U L_R$$

$$\mathcal{O}_{FY4} = \bar{Q}_L t^a\tilde{P}_- U Q_R\ \bar{Q}_L t^a\tilde{P}_- U Q_R \qquad \mathcal{O}_{FY8} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_- U Q_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_- U L_R$$

$$\mathcal{O}_{FY1} = \bar{Q}_L\tilde{P}_+ U Q_R\ \bar{Q}_L\tilde{P}_+ U Q_R \qquad \mathcal{O}_{FY3} = \bar{Q}_L\tilde{P}_- U Q_R\ \bar{Q}_L\tilde{P}_- U Q_R \qquad \mathcal{O}_{FY2} = \bar{Q}_L t^a\tilde{P}_+ U Q_R\ \bar{Q}_L t^a\tilde{P}_+ U Q_R$$

$$\mathcal{O}_{FY5} = \bar{Q}_L\tilde{P}_- U Q_R\ \bar{Q}_R U^\dagger\tilde{P}_+ Q_L \qquad \mathcal{O}_{FY7} = \bar{Q}_L\tilde{P}_- U Q_R\ \bar{L}_L\tilde{P}_- U L_R \qquad \mathcal{O}_{FY6} = \bar{Q}_L t^a\tilde{P}_- U Q_R\ \bar{Q}_R t^a U^\dagger\tilde{P}_+ Q_L$$

$$\mathcal{O}_{FY9} = \bar{L}_L\tilde{P}_- U L_R\ \bar{Q}_R U^\dagger\tilde{P}_+ Q_R \qquad \mathcal{O}_{FY10} = \bar{L}_L\tilde{P}_- U L_R\ \bar{L}_L\tilde{P}_- U L_R \qquad \mathcal{O}_{FY11} = \bar{L}_L\tilde{P}_- U Q_R\ \bar{Q}_R U^\dagger\tilde{P}_+ L_L$$

Including a Light (singlet) Higgs in the EWET

Let us assume that h(126) is an $SU(2)_{L+R}$ scalar singlet

All Higgsless operators can be multiplied by an arbitrary function of **h**:

$$\mathcal{O}_X \quad \Longrightarrow \quad \tilde{\mathcal{O}}_X \equiv \mathcal{F}_X(h)\,\mathcal{O}_X$$

$$\mathcal{F}_X(h) = \sum_{n=0} c_X^{(n)} \left(\frac{h}{v}\right)^n$$

In addition, the LO Lagrangian should include the **scalar potential**:

$$V(h) = v^4 \sum_{n=2} c_V^{(n)} \left(\frac{h}{v}\right)^n$$

Plus operators with derivatives $(\partial_\mu h)$: $F_X \equiv F_X(h)$

$$\mathcal{O}_{D7} = -\langle V_\mu V^\mu \rangle \frac{\partial_\nu h\, \partial^\nu h}{v^2} F_{D7} \qquad \mathcal{O}_{D8} = -\langle V_\mu V_\nu \rangle \frac{\partial^\mu h\, \partial^\nu h}{v^2} F_{D8} \qquad \mathcal{O}_{D11} = \frac{(\partial_\mu h\, \partial^\mu h)^2}{v^4} F_{D11}$$

$$\mathcal{O}_{D6} = -\langle T_L V_\mu V_\nu \rangle \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{D6} \qquad \mathcal{O}_{D9} = -\langle T_L V_\mu \rangle \langle T_L V^\mu \rangle \frac{\partial_\nu h\, \partial^\nu h}{v^2} F_{D9}$$

$$\mathcal{O}_{D10} = -\langle T_L V_\mu \rangle \langle T_L V_\nu \rangle \frac{\partial^\mu h\, \partial^\nu h}{v^2} F_{D10} \qquad \mathcal{O}_{\psi S18} = \bar{L}_L \tilde{P}_- U L_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S18}$$

$$\mathcal{O}_{\psi S10} = -i\, \bar{Q}_L \tilde{P}_+ U Q_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S10} \qquad \mathcal{O}_{\psi S11} = -i\, \bar{Q}_L \tilde{P}_- U Q_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S11}$$

$$\mathcal{O}_{\psi S12} = -i\, \bar{Q}_L \tilde{P}_{12} U Q_R \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S12} \qquad \mathcal{O}_{\psi S13} = -i\, \bar{Q}_L \tilde{P}_{21} U Q_R \langle \tilde{P}_{12} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S13}$$

$$\mathcal{O}_{\psi S14} = \bar{Q}_L \tilde{P}_+ U Q_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S14} \qquad \mathcal{O}_{\psi S15} = \bar{Q}_L \tilde{P}_- U Q_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S15}$$

$$\mathcal{O}_{\psi S16} = -i\, \bar{L}_L \tilde{P}_- U L_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S16} \qquad \mathcal{O}_{\psi S17} = -i\, \bar{L}_L \tilde{P}_{12} U L_R \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S17}$$

$$\mathcal{O}_{\psi T7} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_+ U Q_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T7} \qquad \mathcal{O}_{\psi T8} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_- U Q_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T8}$$

$$\mathcal{O}_{\psi T9} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{21} U Q_R \langle \tilde{P}_{12} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T9} \qquad \mathcal{O}_{\psi T10} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{12} U Q_R \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T10}$$

$$\mathcal{O}_{\psi T11} = -i\, \bar{L}_L \sigma_{\mu\nu} \tilde{P}_- U L_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T11} \qquad \mathcal{O}_{\psi T12} = -i\, \bar{L}_L \sigma_{\mu\nu} \tilde{P}_{12} U L_R \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T12}$$

Buchalla–Catà–Krause

Linear Realization: $SU(2)_L \otimes U(1)_Y$

Assumes that H(126) and $\vec{\varphi}$ combine into an $SU(2)_L$ doublet:

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \tfrac{1}{2}(v+H)\, U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The SM Lagrangian is the low-energy effective theory with $D = 4$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}}\, \mathcal{O}_i^{(D)}$$

- **1 operator with D = 5:** $\mathcal{O}^{(5)} = \bar{L}_L \tilde{\Phi} \tilde{\Phi}^T L_L^c$ **(violates L by 2 units)**
  Weinberg
- **59 independent $\mathcal{O}_i^{(6)}$ preserving B and L** **(for 1 generation)**
  Buchmuller–Wyler, Grzadkowski–Iskrzynski–Misiak–Rosiek
- **5 independent $\mathcal{O}_i^{(6)}$ violating B and L** **(for 1 generation)**
  Weinberg, Wilczek–Zee, Abbott–Wise,

D = 6 Operators (other than 4-fermion ones)

Grzadkowski–Iskrzynski–Misiak–Rosiek

X^3		Φ^6 and $\Phi^4 D^2$		$\psi^2\Phi^3$	
$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_\Phi$	$(\Phi^\dagger\Phi)^3$	$\mathcal{O}_{e\Phi}$	$(\Phi^\dagger\Phi)\,(\bar l_p e_r \Phi)$
$\mathcal{O}_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{\Phi\Box}$	$(\Phi^\dagger\Phi)\,\Box\,(\Phi^\dagger\Phi)$	$\mathcal{O}_{u\Phi}$	$(\Phi^\dagger\Phi)\,(\bar q_p u_r \widetilde\Phi)$
$\mathcal{O}_W$	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{\Phi D}$	$\left(\Phi^\dagger D^\mu\Phi\right)^\star \left(\Phi^\dagger D_\mu\Phi\right)$	$\mathcal{O}_{d\Phi}$	$(\Phi^\dagger\Phi)\,(\bar q_p d_r \Phi)$
$\mathcal{O}_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\Phi^2$		$\psi^2 X\Phi$		$\psi^2\Phi^2 D$	
$\mathcal{O}_{\Phi G}$	$\Phi^\dagger\Phi\; G^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar l_p \sigma^{\mu\nu} e_r)\,\tau^I\Phi\, W^I_{\mu\nu}$	$\mathcal{O}_{\Phi l}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)\,(\bar l_p \gamma^\mu l_r)$
$\mathcal{O}_{\Phi\widetilde G}$	$\Phi^\dagger\Phi\; \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar l_p \sigma^{\mu\nu} e_r)\,\Phi\, B_{\mu\nu}$	$\mathcal{O}_{\Phi l}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)\,(\bar l_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{\Phi W}$	$\Phi^\dagger\Phi\; W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\,\widetilde\Phi\, G^A_{\mu\nu}$	$\mathcal{O}_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)\,(\bar e_p \gamma^\mu e_r)$
$\mathcal{O}_{\Phi\widetilde W}$	$\Phi^\dagger\Phi\; \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar q_p \sigma^{\mu\nu} u_r)\,\tau^I\widetilde\Phi\, W^I_{\mu\nu}$	$\mathcal{O}_{\Phi q}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)\,(\bar q_p \gamma^\mu q_r)$
$\mathcal{O}_{\Phi B}$	$\Phi^\dagger\Phi\; B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar q_p \sigma^{\mu\nu} u_r)\,\widetilde\Phi\, B_{\mu\nu}$	$\mathcal{O}_{\Phi q}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)\,(\bar q_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{\Phi\widetilde B}$	$\Phi^\dagger\Phi\; \widetilde B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\,\Phi\, G^A_{\mu\nu}$	$\mathcal{O}_{\Phi u}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)\,(\bar u_p \gamma^\mu u_r)$
$\mathcal{O}_{\Phi WB}$	$\Phi^\dagger\tau^I\Phi\; W^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar q_p \sigma^{\mu\nu} d_r)\,\tau^I\Phi\, W^I_{\mu\nu}$	$\mathcal{O}_{\Phi d}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)\,(\bar d_p \gamma^\mu d_r)$
$\mathcal{O}_{\Phi\widetilde WB}$	$\Phi^\dagger\tau^I\Phi\; \widetilde W^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar q_p \sigma^{\mu\nu} d_r)\,\Phi\, B_{\mu\nu}$	$\mathcal{O}_{\Phi ud}$	$i(\widetilde\Phi^\dagger D_\mu\Phi)\,(\bar u_p \gamma^\mu d_r)$

$q = q_L$, $l = l_L$, $u = u_R$, $d = d_R$, $e = e_R$, $p, r =$ generation indices

D = 6 Four-Fermion Operators

Grzadkowski–Iskrzynski–Misiak–Rosiek

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$\mathcal{O}_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

$q = q_L$, $l = l_L$, $u = u_R$, $d = d_R$, $e = e_R$, p, r, s, t = generation indices

Backup Slides

Goldstone Electroweak Effective Theory

$$\boxed{\mathcal{L}_{\rm EW}^{(2)} \;=\; -\frac{1}{2g^2}\,\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle \;-\; \frac{1}{2g'^2}\,\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\,\langle D^\mu U^\dagger D_\mu U\rangle}$$

$$U(\varphi) \;=\; \exp\left\{\frac{i\sqrt{2}}{v}\,\Phi\right\} \qquad , \qquad \Phi \;\equiv\; \frac{1}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{\varphi} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\,\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\,\varphi^0 \end{pmatrix}$$

$$D^\mu U \;=\; \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \qquad , \qquad D^\mu U^\dagger \;=\; \partial^\mu U^\dagger + i\,U^\dagger\,\hat{W}^\mu - i\,\hat{B}^\mu\,U^\dagger \qquad , \qquad \langle A\rangle \;\equiv\; {\rm Tr}(A)$$

$$\hat{W}^{\mu\nu} \;=\; \partial^\mu\,\hat{W}^\nu - \partial^\nu\,\hat{W}^\mu - i\,[\hat{W}^\mu,\hat{W}^\nu] \qquad , \qquad \hat{B}^{\mu\nu} \;=\; \partial^\mu\,\hat{B}^\nu - \partial^\nu\,\hat{B}^\mu - i\,[\hat{B}^\mu,\hat{B}^\nu]$$

Goldstone Electroweak Effective Theory

$$\mathcal{L}_{\rm EW}^{(2)} \;=\; -\frac{1}{2g^2}\,\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle \;-\; \frac{1}{2g'^2}\,\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\,\langle D^\mu U^\dagger D_\mu U\rangle$$

$$U(\varphi) \;=\; \exp\left\{\frac{i\sqrt{2}}{v}\,\Phi\right\} \qquad , \qquad \Phi \;\equiv\; \frac{1}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{\varphi} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\,\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\,\varphi^0 \end{pmatrix}$$

$$D^\mu U \;=\; \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \qquad , \qquad D^\mu U^\dagger \;=\; \partial^\mu U^\dagger + i\,U^\dagger\hat{W}^\mu - i\,\hat{B}^\mu U^\dagger \qquad , \qquad \langle A\rangle \;\equiv\; {\rm Tr}(A)$$

$$\hat{W}^{\mu\nu} \;=\; \partial^\mu\hat{W}^\nu - \partial^\nu\hat{W}^\mu - i\,[\hat{W}^\mu,\hat{W}^\nu] \qquad , \qquad \hat{B}^{\mu\nu} \;=\; \partial^\mu\hat{B}^\nu - \partial^\nu\hat{B}^\mu - i\,[\hat{B}^\mu,\hat{B}^\nu]$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $U(\varphi) \;\to\; g_L\,U(\varphi)\,g_R^\dagger$

$$\hat{W}^\mu \to g_L\,\hat{W}^\mu g_L^\dagger + i\,g_L\,\partial^\mu g_L^\dagger \qquad , \qquad \hat{B}^\mu \to g_R\,\hat{B}^\mu g_R^\dagger + i\,g_R\,\partial^\mu g_R^\dagger$$

Goldstone Electroweak Effective Theory

$$\mathcal{L}^{(2)}_{\rm EW} \;=\; -\frac{1}{2g^2}\,\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle \;-\; \frac{1}{2g'^2}\,\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\,\langle D^\mu U^\dagger D_\mu U\rangle$$

$$U(\varphi) \;=\; \exp\left\{\frac{i\sqrt{2}}{v}\,\Phi\right\} \qquad , \qquad \Phi \;\equiv\; \frac{1}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{\varphi} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\,\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\,\varphi^0 \end{pmatrix}$$

$$D^\mu U \;=\; \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \qquad , \qquad D^\mu U^\dagger \;=\; \partial^\mu U^\dagger + i\,U^\dagger\hat{W}^\mu - i\,\hat{B}^\mu U^\dagger \qquad , \qquad \langle A\rangle \;\equiv\; {\rm Tr}(A)$$

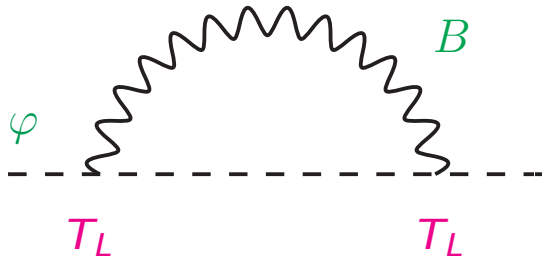
$$\hat{W}^{\mu\nu} \;=\; \partial^\mu\hat{W}^\nu - \partial^\nu\hat{W}^\mu - i\,[\hat{W}^\mu,\hat{W}^\nu] \qquad , \qquad \hat{B}^{\mu\nu} \;=\; \partial^\mu\hat{B}^\nu - \partial^\nu\hat{B}^\mu - i\,[\hat{B}^\mu,\hat{B}^\nu]$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ Symmetry: $\quad U(\varphi) \;\to\; g_L\,U(\varphi)\,g_R^\dagger$

$$\hat{W}^\mu \to g_L\,\hat{W}^\mu g_L^\dagger + i\,g_L\,\partial^\mu g_L^\dagger \qquad , \qquad \hat{B}^\mu \to g_R\,\hat{B}^\mu g_R^\dagger + i\,g_R\,\partial^\mu g_R^\dagger$$

SM Symmetry Breaking: $\quad \hat{W}^\mu \;=\; -\frac{g}{2}\,\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu \;=\; -\frac{g'}{2}\,\sigma_3\,B^\mu$

Custodial Symmetry Breaking: $\hat{B}_\mu \equiv -g' \frac{\sigma_3}{2} B_\mu$



$\Rightarrow \quad \langle V_\mu T_L V^\mu T_L \rangle$

$$V_\mu \equiv D_\mu U U^\dagger = i \frac{\sqrt{2}}{v} D_\mu \Phi + \cdots \quad , \quad T_L \equiv U \frac{\sigma_3}{2} U^\dagger \quad , \quad T_L T_L = \frac{1}{4} I_2$$

$$\langle V_\mu T_L V^\mu T_L \rangle = \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle - \frac{1}{2} \langle V_\mu V^\mu \rangle \langle T_L T_L \rangle$$

$$= \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle - \frac{1}{4} \langle V_\mu V^\mu \rangle$$

$$\Rightarrow \quad \mathcal{O}_0 = v^2 \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle$$

Decoupling:

Appelquist–Carazzone

The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles

SM: $\mathbf{M_W = M_Z \cos\theta_W = \frac{1}{2}\, g\, v}$, $\mathbf{M_H = \sqrt{2\lambda}\, v}$, $\mathbf{M_f = \frac{1}{\sqrt{2}}\, y_f\, v}$

- **Decoupling occurs when $\mathbf{v \to \infty}$, keeping the couplings fixed**

- **There is no decoupling if some $\mathbf{M_i \to \infty}$, keeping $\mathbf{v = 246}$ GeV**

  $\mathbf{(g,\ \lambda,\ y_f \to \infty)}$

$\varphi^a \varphi^b \to \varphi^c \varphi^d$:

- **Isospin:**

$$
\begin{aligned}
A_0(s,t,u) &= 3\,A(s,t,u) + A(t,s,u) + A(u,t,s) \\
A_1(s,t,u) &= A(t,s,u) - A(u,t,s) \\
A_2(s,t,u) &= A(t,s,u) + A(u,t,s)
\end{aligned}
$$

- **Partial Waves:**

$$
\begin{aligned}
A_{IJ}(s) &= \frac{1}{64\pi} \int_{-1}^{+1} d\cos\theta \; P_J(\cos\theta)\; A_I(s,t,u) \\
\sigma(s) &= \frac{64\pi}{s} \sum_{I,J} (2I+1)\,(2J+1)\,|A_{IJ}|^2
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
A_{00}(s) &= \frac{s}{16\pi v^2} \left\{ 1 + \frac{s}{16\pi^2 v^2} \left[\frac{101}{36} + \frac{64\pi^2}{3} (7\,a_4^r + 11\,a_5^r) - \frac{25}{18} \log\left(\frac{s}{\mu^2}\right) + i\,\pi \right] + \cdots \right\} \\
A_{11}(s) &= \frac{s}{96\pi v^2} \left\{ 1 + \frac{s}{16\pi^2 v^2} \left[\frac{1}{9} + 64\pi^2 (a_4^r - 2\,a_5^r) + i\,\frac{\pi}{6} \right] + \cdots \right\} \\
A_{20}(s) &= \frac{-s}{32\pi v^2} \left\{ 1 + \frac{s}{16\pi^2 v^2} \left[-\frac{91}{36} - \frac{256\pi^2}{3} (2\,a_4^r + a_5^r) + \frac{10}{9} \log\left(\frac{s}{\mu^2}\right) - i\,\frac{\pi}{2} \right] + \cdots \right\}
\end{aligned}
}
$$

Power Counting

- **Momentum expansion:** $\Lambda \sim 4\pi v\,,\, M_X$

$$T = \sum_n T_n \left(\frac{p}{\Lambda}\right)^n$$

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- **Loop Expansion:** **A generic L-loop diagram $\mathcal{D}$ scales as**

$$\mathcal{D} \sim \frac{(yv)^\nu (gv)^{m+2r+2x+u+z}}{v^{F_L+F_R-2-2\omega}} \frac{p^d}{\Lambda^{2L}} \bar{\psi}_L^{F_L^1} \psi_L^{F_L^2} \bar{\psi}_R^{F_R^1} \psi_R^{F_R^2} \left(\frac{X_{\mu\nu}}{v}\right)^V \left(\frac{\varphi}{v}\right)^B \left(\frac{h}{v}\right)^H$$

$$d \equiv 2L + 2 - \tfrac{1}{2}(F_L + F_R) - V - \nu - m - 2r - 2x - u - z - 2\omega$$

Buchalla–Catà–Krause

\# external fields: $F_L = F_L^1 + F_L^2$, $F_R = F_R^1 + F_R^2$, B , H , V (X_μ = gauge boson)

\# vertices: $m \equiv \sum_l m_l\ (X_\mu \varphi^l)$, $r \equiv \sum_s r_s\ (X_\mu^2 \varphi^s)$, $u\ (X_\mu^3)$, $x\ (X_\mu^4)$, $\omega \equiv \sum_q \omega_q\ (h^q)$

$\nu \equiv \sum_k \nu_k\ (\bar{\psi}\psi\varphi^k) + \sum_{t,b} \tau_{tb}\ (\bar{\psi}\psi\varphi^t h^b)$, $z \equiv z_L + z_R\ (\bar{\psi}_\lambda \psi_\lambda X_\mu\,,\ \lambda = L, R)$

Too many operators/couplings → **Further input needed**

$$\mathcal{F}_X(h) \;=\; \sum_{n=0} c_X^{(n)} \left(\frac{h}{v}\right)^n \;=\; \sum_{n=0} \tilde{c}_X^{(n)} \left(\frac{g_h\, h}{\Lambda_{\rm NP}}\right)^n$$

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- **Weak coupling:** $g_h \ll 1$
- **Strong coupling:** $g_h \sim 4\pi = \Lambda_{\rm NP}/f$ $\Rightarrow$ $\mathcal{F}_X(h/f)$
- **v $\ll$ f** $\Rightarrow$ $\xi \equiv \dfrac{v^2}{f^2}$, $c_X^{(n)} = \tilde{c}_X^{(n)}\, \xi^{n/2}$