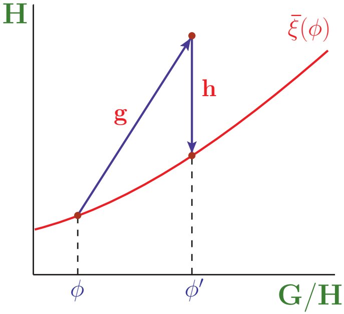
4. High-Energy Dynamics

- CCWZ Formalism
- Heavy Fields
- Low-Energy Constants
- Asymptotic Behaviour
- Signals of Heavy Scales

Energy Scale	Fields	Effective Theory
M_W	t, b, c $s, d, u\ ;\ G^a$	$\mathrm{QCD}^{N_f=6}$
$\lesssim m_c$	$s, d, u\ ;\ G^a$	$\mathrm{QCD}^{N_f=3}$
Λ_χ	V, A, S, P π, K, η	$\mathrm{R}\chi\mathrm{T}$
$\lesssim M_K$	π, K, η	$\chi\mathrm{PT}^{N_f=3}$
$\lesssim M_\pi$	π	$\chi\mathrm{PT}^{N_f=2}$

Coset Space Coordinates: $G \equiv SU(3)_L \otimes SU(3)_R \xrightarrow{SCSB} H \equiv SU(3)_V$



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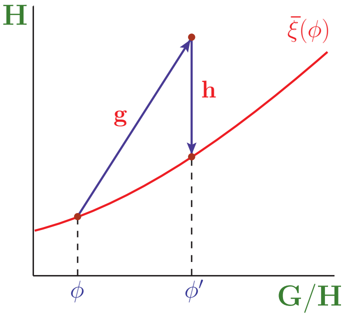

$$\bar{\xi}(\phi) \equiv (\xi_L(\phi), \xi_R(\phi)) \in G$$

$$\xi_L(\phi) \xrightarrow{G} g_L\, \xi_L(\phi)\, h^\dagger(\phi, g)$$

$$\xi_R(\phi) \xrightarrow{G} g_R\, \xi_R(\phi)\, h^\dagger(\phi, g)$$

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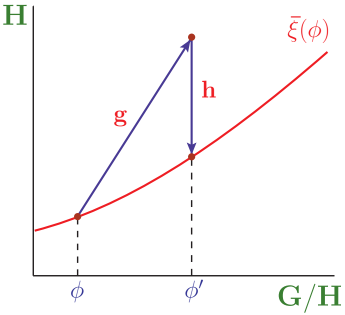

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$$\mathbf{U}(\phi) \equiv \xi_R(\phi)\, \xi_L^\dagger(\phi) \xrightarrow{G} g_R\, \mathbf{U}(\phi)\, g_L^\dagger$$

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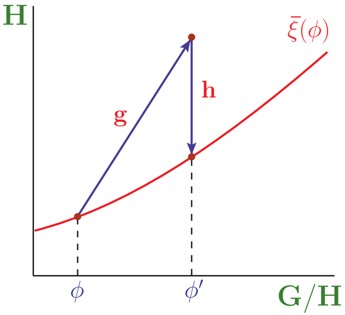

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$$\mathbf{U}(\phi) \equiv \xi_R(\phi)\, \xi_L^\dagger(\phi) \xrightarrow{G} g_R\, \mathbf{U}(\phi)\, g_L^\dagger$$

Canonical choice:

$$\xi_R(\phi) = \xi_L(\phi)^\dagger \equiv \mathbf{u}(\phi) \quad \xrightarrow{G} \quad g_R\, \mathbf{u}(\phi)\, h^\dagger(\phi, g) = h(\phi, g)\, \mathbf{u}(\phi)\, g_L^\dagger$$

$$\mathbf{U}(\phi) = \mathbf{u}(\phi)^2 = \exp\left\{ i \frac{\sqrt{2}}{f} \mathbf{\Phi} \right\}$$

CCWZ Formalism

Callan–Coleman–Wess–Zumino

$$\mathbf{u}(\varphi) \quad \xrightarrow{G} \quad g_R\, \mathbf{u}(\varphi)\, h^\dagger(\phi, g) \;=\; h(\phi, g)\, \mathbf{u}(\phi)\, g_L^\dagger$$

$SU(3)_V$ octets: $$\mathbf{X} \quad \xrightarrow{\mathbf{G}} \quad \mathbf{h}(\varphi, \mathbf{g})\; \mathbf{X}\; \mathbf{h}(\varphi, \mathbf{g})^\dagger$$

$$\mathbf{R} \equiv \frac{1}{2}\,\vec{\sigma}\,\vec{R} \quad , \quad \nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R]$$

$$u_\mu \equiv i\, u^\dagger D_\mu U\, u^\dagger = u_\mu^\dagger \quad , \quad h^{\mu\nu} = \nabla^\mu u^\nu + \nabla^\nu u^\mu$$

$$f_\pm^{\mu\nu} = u\, F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu}\, u \quad , \quad \chi_\pm \equiv u^\dagger \chi\, u^\dagger \pm u\, \chi^\dagger u$$

$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu - i\, r_\mu)\, u + u\,(\partial_\mu - i\,\ell_\mu) u^\dagger\right\}$$

CCWZ Formalism

Callan–Coleman–Wess–Zumino

$$\mathbf{u}(\varphi) \quad \stackrel{G}{\longrightarrow} \quad g_R\, \mathbf{u}(\varphi)\, h^\dagger(\phi, g) \;=\; h(\phi, g)\, \mathbf{u}(\phi)\, g_L^\dagger$$

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$$\mathcal{L}_2 = \frac{f^2}{4}\,\langle u^\mu u_\mu + \chi_+\rangle$$

RχT

Resonance Nonet Multiplets: $\mathbf{V}(1^{--})\,,\ \mathbf{A}(1^{++})\,,\ \mathbf{S}(0^{++})\,,\ \mathbf{P}(0^{-+})$

$$\mathcal{L}_2^V = \frac{F_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\, f_+^{\mu\nu}\rangle + \frac{i\,G_V}{\sqrt{2}}\,\langle V_{\mu\nu}\, u^\mu\, u^\nu\rangle$$

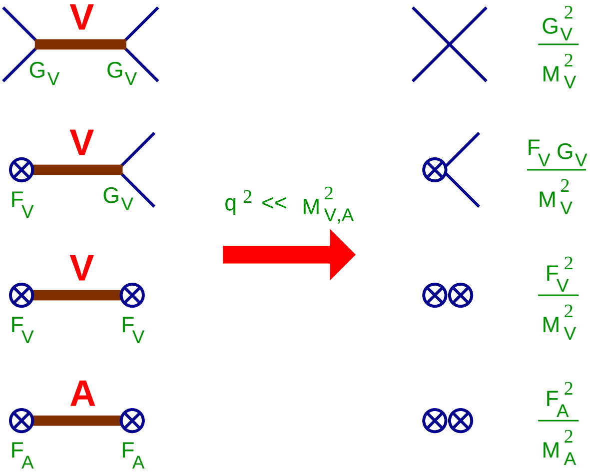
$$\mathcal{L}_2^A = \frac{F_A}{2\sqrt{2}}\,\langle A_{\mu\nu}\, f_-^{\mu\nu}\rangle$$

$$\mathcal{L}_2^S = c_d\,\langle S\, u^\mu\, u^\nu\rangle + c_m\,\langle S\,\chi_+\rangle$$

$$\mathcal{L}_2^P = i\, d_m\,\langle P\,\chi_-\rangle$$

$$u_\mu = i\, u^\dagger\, D_\mu U\, u^\dagger = u_\mu^\dagger \qquad ; \qquad U = u^2$$

$$f_\pm^{\mu\nu} = u\, F_L^{\mu\nu}\, u^\dagger \pm u^\dagger\, F_R^{\mu\nu}\, u \qquad ; \qquad \chi_\pm = u^\dagger\,\chi\, u^\dagger \pm u\,\chi^\dagger\, u$$



$$\left\{ \mathbf{V}\,[M_V, G_V, F_V]\;,\; \mathbf{A}\,[M_A, F_A] \right\} \Longleftrightarrow \left\{ \mathbf{S}\,[M_S, c_d, c_m]\;,\; \mathbf{P}\,[M_P, d_m] \right\}$$

$O(N_C)$:

$$2\,L_1 \;=\; L_2 \;=\; \sum_i \frac{G_{V_i}^2}{4\,M_{V_i}^2} \qquad ; \qquad L_3 \;=\; \sum_i \left\{ -\frac{3\,G_{V_i}^2}{4\,M_{V_i}^2} + \frac{c_{d_i}^2}{2\,M_{S_i}^2} \right\}$$

$$L_5 \;=\; \sum_i \frac{c_{d_i}\,c_{m_i}}{M_{S_i}^2} \qquad ; \qquad L_8 \;=\; \sum_i \left\{ \frac{c_{m_i}^2}{2\,M_{S_i}^2} - \frac{d_{m_i}^2}{2\,M_{P_i}^2} \right\}$$

$$L_9 \;=\; \sum_i \frac{F_{V_i}\,G_{V_i}}{2\,M_{V_i}^2} \qquad ; \qquad L_{10} \;=\; \frac{1}{4} \sum_i \left\{ \frac{F_{A_i}^2}{M_{A_i}^2} - \frac{F_{V_i}^2}{M_{V_i}^2} \right\}$$

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$$2\,L_1 - L_2 \,=\, L_4 \,=\, L_6 \,=\, 0 \qquad ; \qquad L_7 \,=\, -\frac{\tilde{d}_m^{\,2}}{2\,M_{\eta_1}^2}$$

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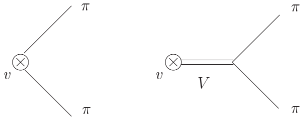
BUT $\qquad M_{\eta_1}^2 \;\sim\; O\left(\frac{1}{N_C},\, \mathcal{M}\right)$

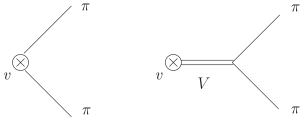
Vector Form Factor



$\langle \pi | \mathbf{v}_\mu | \pi \rangle$ **:**

$$F_V(t) \,=\, 1 + \sum_i \frac{F_{V_i}\, G_{V_i}}{f^2}\, \frac{t}{M_{V_i}^2 - t}$$

Vector Form Factor



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Short-distance QCD constraint: $\lim_{t\to\infty} F_V(t) = 0$

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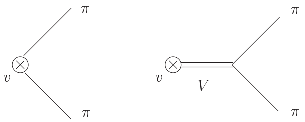

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$\Longrightarrow$ $$\sum_i F_{V_i}\, G_{V_i} \; = \; f^2$$

Weinberg Sum Rules

Chiral Symmetry:

$$\Pi_{LR}^{\mu\nu}(q) \equiv \int d^4x \, \mathrm{e}^{iqx} \, \langle 0| T(J_L^\mu(x) \, J_R^\nu(0)^\dagger)|0\rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \, \Pi_{LR}(q^2) = 0$$

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OPE: $\Pi_{LR}^{\mu\nu}(q) \neq 0$ only through **order parameters of EWSB** (operators invariant under **H** but not under **G**)

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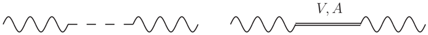
→

$$\frac{1}{\pi} \int_0^\infty \mathrm{d}s \, \left[\mathrm{Im}\Pi_{VV}(s) - \mathrm{Im}\Pi_{AA}(s) \right] \, = \, f^2 \qquad (1^{\mathrm{st}} \text{ WSR})$$

$$\frac{1}{\pi} \int_0^\infty \mathrm{d}s \, s \, \left[\mathrm{Im}\Pi_{VV}(s) - \mathrm{Im}\Pi_{AA}(s) \right] \, = \, 0 \qquad (2^{\mathrm{nd}} \text{ WSR})$$

WSRs @ LO



$$\Pi_{LR}(s) \;=\; \frac{f^2}{s} + \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 - s} - \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 - s}$$

- **1st WSR:** $\sum_i F_{V_i}^2 - \sum_i F_{A_i}^2 \;=\; f^2$
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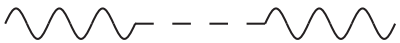

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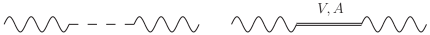
SRA ➡ $F_V^2 = v^2 \frac{M_A^2}{M_A^2 - M_V^2} \quad , \quad F_A^2 = v^2 \frac{M_V^2}{M_A^2 - M_V^2} \quad , \quad M_A > M_V$

WSRs @ LO



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1st WSR likely valid also in gauge theories with non-trivial UV fixed points

2nd WSR questionable (not valid) in walking (conformal) TC scenarios

Appelquist–Sannino, Orgogozo–Rychkov

Short-Distance Constraints

Vector Form Factor $\langle\pi|v_\mu|\pi\rangle$:

$$F_V(t) \;=\; 1 \;+\; \sum_i \frac{F_{V_i}\, G_{V_i}}{f^2}\;\frac{t}{M_{V_i}^2 - t}$$

$$\lim_{t\to\infty} F_V(t) = 0 \quad \longrightarrow \quad \boxed{\sum_i \; F_{V_i}\, G_{V_i} \;=\; f^2}$$

Axial Form Factor $\langle\gamma|a_\mu|\pi\rangle$:

$$G_A(t) = \sum_i \left\{ \frac{2\, F_{V_i}\, G_{V_i} - F_{V_i}^2}{M_{V_i}^2} + \frac{F_{A_i}^2}{M_{A_i}^2 - t} \right\}$$

$$\lim_{t\to\infty} G_A(t) = 0 \quad \longrightarrow \quad \boxed{\sum_i \; \left(2\, F_{V_i}\, G_{V_i} - F_{V_i}^2\right) / M_{V_i}^2 \;=\; 0}$$

Weinberg Sum Rules:

$$\Pi_{LR}(t) \;=\; -\frac{f^2}{t} \;+\; \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 + t} \;-\; \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 + t}$$

$$\lim_{t\to\infty} \; t\;\Pi_{LR}(t) = 0$$

$$\lim_{t\to\infty} \; t^2\,\Pi_{LR}(t) = 0$$

$$\longrightarrow \quad \boxed{\begin{array}{c} \sum_i \; \left(F_{V_i}^2 - F_{A_i}^2\right) = f^2 \\ \sum_i \; \left(M_{V_i}^2\, F_{V_i}^2 - M_{A_i}^2\, F_{A_i}^2\right) = 0 \end{array}}$$

Scalar FF: $$F^S_{K\pi}(s) = 1 + \sum_i \frac{4c_{m_i}}{f^2}\left[c_{d_i} + (c_{m_i} - c_{d_i})\,\frac{M_K^2 + M_\pi^2}{M_{S_i}^2}\right]\frac{s}{M_{S_i}^2 - s}$$

$$\lim_{s\to\infty} F^S_{K\pi}(s) = 0 \quad\longrightarrow\quad \boxed{4\sum_i c_{d_i} c_{m_i} = f^2 \quad ; \quad \sum_i \frac{c_{m_i}}{M_{S_i}^2}\,(c_{m_i} - c_{d_i}) = 0}$$

S − P Sum Rules: $$\Pi_{SS-PP}(t) = 16 B_0^2 \left\{\sum_i \frac{c_{m_i}^2}{M_{S_i}^2 + t} - \sum_i \frac{d_{m_i}^2}{M_{P_i}^2 + t} - \frac{f^2}{8\,t}\right\}$$

$$\lim_{t\to\infty} t\;\Pi_{SS-PP}(t) = 0 \quad\longrightarrow\quad \boxed{8\sum_i \left(c_{m_i}^2 - d_{m_i}^2\right) = f^2}$$

Pseudoscalar Nonet:

$$\mathcal{L}_2 \;\doteq\; \frac{f^2}{4}\,\langle \chi\, U^\dagger + U\,\chi^\dagger\rangle \;\sim\; -i\,\frac{f}{\sqrt{24}}\,\eta_1\,\langle\chi_-\rangle \quad\longrightarrow\quad \boxed{\tilde{d}_m = -\frac{f}{\sqrt{24}}}$$

1–Resonance Approximation:

Ecker, Gasser, Leutwyler, Pich, de Rafael

$$F_V = 2\,G_V = \sqrt{2}\,F_A = \sqrt{2}\,f \quad ; \quad M_A = \sqrt{2}\,M_V \quad ; \quad d_m = \frac{1}{2\sqrt{2}}\,f$$

$$c_m = c_d = \frac{1}{2}\,f$$

Jamin, Oller, Pich

$$M_P \approx \sqrt{2}\,M_S$$

$\Rightarrow$

$$2\,L_1 = L_2 = \frac{1}{4}\,L_9 = -\frac{1}{3}\,L_{10} = \frac{f^2}{8\,M_V^2}$$

$$L_3 = -\frac{3\,f^2}{8\,M_V^2} + \frac{f^2}{8\,M_S^2} \quad ; \quad L_5 = \frac{f^2}{4\,M_S^2}$$

$$L_8 = \frac{f^2}{8\,M_S^2} - \frac{f^2}{16\,M_P^2} \quad ; \quad L_7 = -\frac{f^2}{48\,M_{\eta_1}^2}$$

L_i'S from Resonance Exchange

i	$10^3 \cdot L_i^r(M_\rho)$	V	A	S	η_1	Total	Total[b]
1	0.7 ± 0.3	0.6	0	0	0	0.6	0.9
2	1.3 ± 0.3	1.2	0	0	0	1.2	1.8
3	-3.5 ± 1.1	-3.6	0	0.6	0	-3.0	-4.3
4	-0.3 ± 0.5	0	0	0	0	0.0	0.0
5	1.4 ± 0.5	0	0	1.4[a]	0	1.4	2.1
6	-0.2 ± 0.3	0	0	0	0	0.0	0.0
7	-0.4 ± 0.2	0	0	0	-0.3	-0.3	-0.3
8	0.9 ± 0.3	0	0	0.9[a]	0	0.9	0.8
9	6.9 ± 0.7	6.9[a]	0	0	0	6.9	7.2
10	-5.5 ± 0.7	-10.0	4.0	0	0	-6.0	-5.4

[a] Input   [b] Short-Distance Constraints

EW Resonance Effective Theory

- **Towers of heavy states are usually present in strongly-coupled models of EWSB: Technicolour, Walking TC...**
- **The low-energy constants (LECs) of the Goldstone Lagrangian contain information on the heavier states. The lightest states not included in the Lagrangian dominate**

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1. Build $\mathcal{L}_{\mathrm{eff}}(\varphi_i, R_k)$ with the lightest R_k coupled to the φ_i
2. Require a good UV behaviour ➡ Low # of derivatives
3. Match the two effective Lagrangians ➡ LECs

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- **Towers of heavy states are usually present in strongly-coupled models of EWSB: Technicolour, Walking TC...**

- **The low-energy constants (LECs) of the Goldstone Lagrangian contain information on the heavier states. The lightest states not included in the Lagrangian dominate**

1. Build $\mathcal{L}_{\rm eff}(\varphi_i, R_k)$ with the lightest R_k coupled to the φ_i
2. Require a good UV behaviour ➡ Low # of derivatives
3. Match the two effective Lagrangians ➡ LECs

This program works in QCD: RχT (Ecker–Gasser–Leutwyler–Pich–de Rafael)

Good dynamical understanding at large N_C

LO Resonance EW Lagrangian:

Pich–Rosell–Sanz-Cillero

$$\mathcal{L}_{\rm eff} \;=\; \mathcal{L}_{\varphi} \;+\; \sum_{R} \mathcal{L}_R \;+\; \sum_{R,R'} \mathcal{L}_{RR'} \;+\; \cdots$$

Triplets: $\mathbf{V}(1^{--})$, $\mathbf{A}(1^{++})$; **Singlet:** $\mathbf{S_1}(0^{++})$

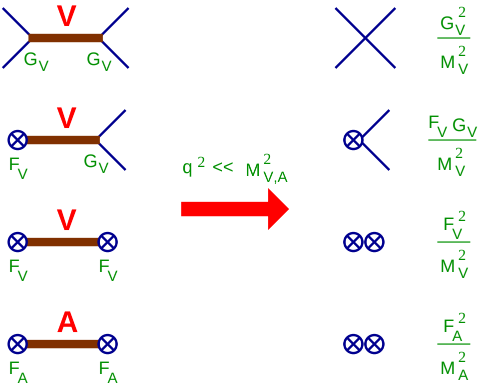
$$\mathcal{L}_{S_1} + \mathcal{L}_A + \mathcal{L}_V \;=\; \frac{v}{2}\,\kappa_W\; S_1\,\langle u^\mu u_\mu\rangle \;+\; \frac{F_A}{2\sqrt{2}}\,\langle A_{\mu\nu} f_-^{\mu\nu}\rangle$$
$$+\; \frac{F_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\, f_+^{\mu\nu}\rangle \;+\; \frac{i\,G_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\;[u^\mu u^\nu]\rangle$$

$$\mathcal{L}_{S_1 A} \;=\; \sqrt{2}\;\lambda_1^{SA}\;\partial_\mu S_1\;\langle A^{\mu\nu} u_\nu\rangle$$

Antisymmetric $V_{\mu\nu}$ and $A_{\mu\nu}$ fields **(better UV properties):**

$$\mathcal{L}_{\rm Kin} = -\frac{1}{2}\sum_{R=V,A}\langle\nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{1}{2}\,M_R^2\,R_{\mu\nu}R^{\mu\nu}\rangle + \frac{1}{2}\,\partial^\mu S_1\,\partial_\mu S_1 - \frac{1}{2}\,M_{S_1}^2\,S_1^2$$

Resonance Exchange



$$a_1 = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2} \quad , \quad a_2 = a_3 = \frac{F_V\, G_V}{4M_V^2} \quad , \quad a_4 = -a_5 = \frac{G_V^2}{4M_V^2}$$

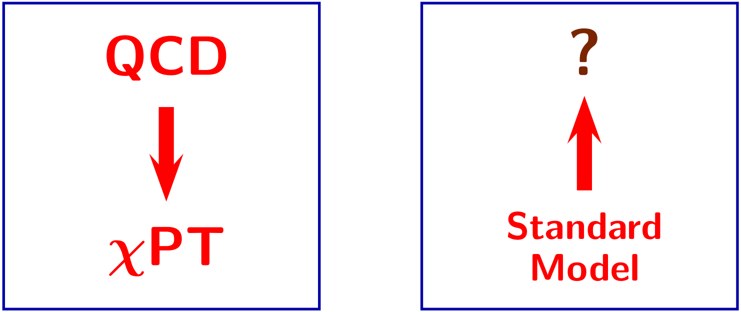
OUTLOOK

- **Effective Field Theory:** powerful low-energy tool
- **Mass Gap:** $E, m_{light} \ll \Lambda_{NP}$
- **Assumption:** relevant symmetries (breakings) & light fields
- Most general $\mathcal{L}_{eff}(\phi_{light})$ allowed by symmetry
- Short-distance dynamics encoded in LECs
- LECs constrained phenomenologically
- **Goal:** get hints on the underlying fundamental dynamics

$\longrightarrow$ **New Physics**

Learning from QCD experience. **EW problem more difficult**

Fundamental Underlying Theory unknown

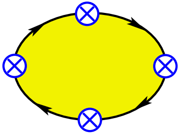


Additional dynamical input (fresh ideas!) needed

Backup Slides

Large–N_C Counting Rules

$g_s \sim 1/\sqrt{N_C}$; $\alpha_s \sim 1/N_C$; $\langle T\,(J_1 \cdots J_n)\rangle \;\sim\; N_C$

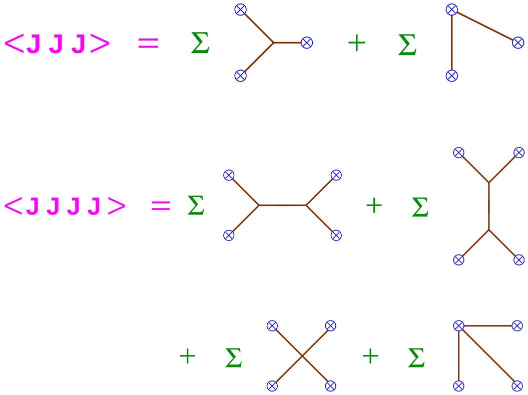


- Dominance of planar gluonic exchanges
- Non-planar diagrams suppressed by $1/N_C^2$
- Internal quark loops suppressed by $1/N_C$

Colour Confinement ➡ $J\,|0\rangle \;\sim\; |1\ \text{Meson}\rangle$

$$\langle J(k)\,J(-k)\rangle \;=\; \sum_n \frac{f_n^2}{k^2 - M_n^2}$$

- **Infinite** number of mesons $(\sim \ln k^2)$
- $f_n = \langle 0|J|n\rangle \sim \sqrt{N_C}$; $M_n \sim \mathcal{O}(1)$
- Mesons are **free**, **stable** and **non-interacting**



Crossing + Unitarity ➡ **Tree Approximation to some Local Effective Meson Lagrangian**