The Higgs-Boson Spectrum in the Minimal Supersymmetric Standard Model with *CP* Violation at Higher Orders

Sebastian Paßehr

DESY, Hamburg

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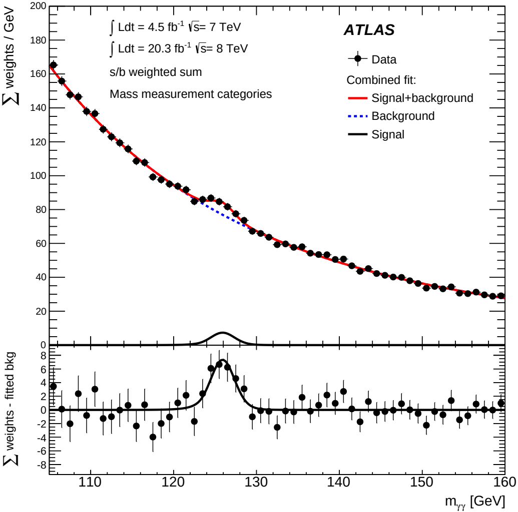
Discovery of a new particle

Higgs-like particle discovered, [ATLAS, arXiv:1207.7214 [hep-ex]], [CMS, arXiv:1207.7235 [hep-ex]],

e. g. latest result of $H \to \gamma\gamma$, [ATLAS, arXiv:1406.3827 [hep-ex]],

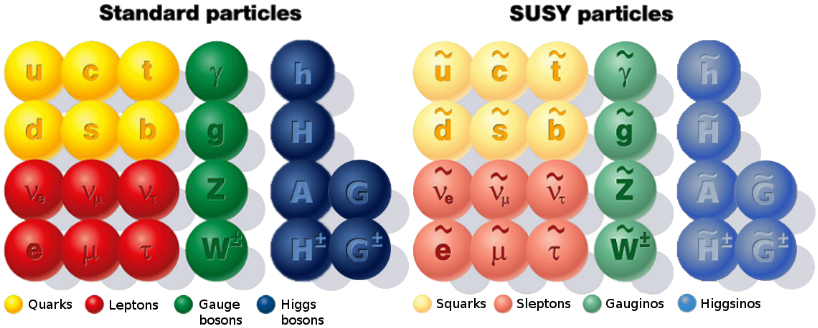


- very good agreement with SM Higgs boson,
- but: SM has many deficiencies,
- test models beyond the Standard Model,
- promising candidate: Minimal Supersymmetric Standard Model (MSSM).

Contents

Particle content of the MSSM

- extension of the Standard Model by Supersymmetry,
- two Higgs doublets.

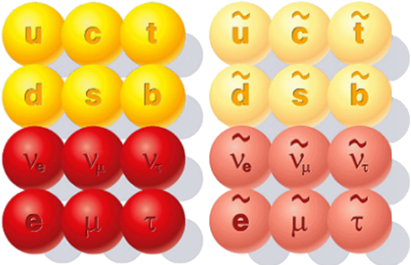


Chiral superfields

for each fermion of the SM:

$$\Phi\left(x,\,\theta,\,\bar{\theta}\right) = \exp\left(-\boldsymbol{i}\,\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\right)\phi\left(x,\,\theta\right)\ ,$$
$$\phi\left(x,\,\theta\right) = A + \sqrt{2}\,\theta\xi + \theta\theta\,F\ ,$$

Weyl spinor for fermion ξ ,
scalar superpartner A ,
auxiliary field F .

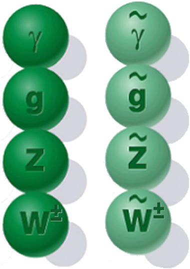


Vector superfields

for each vector boson of the SM (Wess–Zumino gauge):

$$V_{\mathrm{WZ}}\left(x,\,\theta,\,\bar{\theta}\right) = \theta\bar{\theta}\,\sigma^{\mu}A_{\mu} + \theta\theta\,\bar{\theta}\bar{\lambda} + \bar{\theta}\bar{\theta}\,\theta\lambda + \frac{1}{2}\,\theta\theta\,\bar{\theta}\bar{\theta}\,D\ ,$$

vector field for vector boson	A_{μ} ,
fermionic superpartner	λ ,
auxiliary field	D .



Supersymmetric Lagrangian

construct gauge-invariant, renormalizable, supersymmetric Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\mathsf{SUSY}} &= \mathcal{L}_{\mathsf{gauge}} + \mathcal{L}_{\mathcal{W}} + \mathcal{L}_{\mathsf{matter}} \\
&= \left[\int \mathrm{d}^2\theta \left(\frac{1}{4} \left(W^a W_a \right) + \mathcal{W}(\Phi_i) \right) + \mathsf{h.\,c.} \right] \\
&\quad + \int \mathrm{d}^4\theta \, \Phi_i^\dagger e^{2\,g\,V} \Phi_i \ ,
\end{aligned}
$$

with the **chiral** holomorphic superpotential

$$
\mathcal{W}(\Phi_i) = c_i \, \Phi_i + \frac{1}{2} \, m_{ij} \, \Phi_i \, \Phi_j + \frac{1}{6} \, h_{ijk} \, \Phi_i \, \Phi_j \, \Phi_k \ ,
$$

⇒ more than one Higgs field necessary.

Higgs fields in the MSSM

two complex $SU(2)$-Higgs doublets:

$$\mathcal{H}_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}} \left(\phi_1^0 - i \, \chi_1^0\right) \\ -\phi_1^- \end{pmatrix} , \quad \mathcal{H}_2 = e^{i \, \zeta} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}} \left(\phi_2^0 + i \, \chi_2^0\right) \end{pmatrix} ,$$

positive real vacuum expectation values v_1 , v_2 ,
relative phase ζ ,

described by two complex chiral superfield doublets H_1 , H_2 ,
accordingly, fermionic superpartners: higgsinos $\tilde{\mathcal{H}}_1$, $\tilde{\mathcal{H}}_2$.

Higgs Lagrangian

supersymmetric Lagrangian for the Higgs superfields:

$$\mathcal{L}_{\text{Higgs}} = \left(\int \mathrm{d}^2\theta \, \mathcal{W}_{\text{MSSM}} + \text{h. c.} \right) + \int \mathrm{d}^4\theta \sum_{j=1}^{2} H_j^{\dagger} \exp\left(g_Y \, Y \, V_Y + g_{\text{w}} \, \tau_a \, V_{\text{w}}^a\right) H_j \ ,$$

with the superpotential

$$\mathcal{W}_{\text{MSSM}} = \mu \, H_1 \overset{SU}{\odot} H_2 - h_{u,\,ij} \, Q_i \overset{SU}{\odot} H_2 \, U_j^C - h_{e,\,ij} \, H_1 \overset{SU}{\odot} L_i \, E_j^C - h_{d,\,ij} \, H_1 \overset{SU}{\odot} Q_i \, D_j^C \ .$$

bilinear mass parameter μ , Yukawa couplings $h_{f,\,ij}$.

$$\Phi_1 \overset{SU}{\odot} \Phi_2 = \epsilon_{\alpha\beta} \, \Phi_1^{\alpha} \, \Phi_2^{\beta} \ , \quad \epsilon_{12} = -1 \ .$$

Supersymmetry breaking in the MSSM

soft supersymmetry-breaking terms involving only Higgs fields:

$$\begin{aligned}\mathcal{L}_{\text{breaking, Higgs}} = &-\tilde{m}_1^2\,\mathcal{H}_1^\dagger\,\mathcal{H}_1 - \tilde{m}_2^2\,\mathcal{H}_2^\dagger\,\mathcal{H}_2 \\ &-\left(b_{\mathcal{H}_1\mathcal{H}_2}\,\mu\,\mathcal{H}_1 \overset{SU}{\odot} \mathcal{H}_2 + \text{h.c.}\right),\end{aligned}$$

bilinear mass terms $\tilde{m}_1^2\ ,\ \tilde{m}_2^2\ ,\ b_{\mathcal{H}_1\mathcal{H}_2}\,\mu\ .$

1. Particle content of the MSSM
2. Higgs fields in the MSSM
3. **Higgs potential at the tree-level**
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Higgs potential at the tree-level

non-kinetic part of the Lagrangian involving only Higgs fields:

$$\begin{aligned} V_H &= V_H^{\text{SUSY}} + V_H^{\text{breaking}} \ , \\ V_H^{\text{SUSY}} &= \frac{1}{8}\left(g_Y^2 + g_{\text{w}}^2\right)\left(\mathcal{H}_2^\dagger \mathcal{H}_2 - \mathcal{H}_1^\dagger \mathcal{H}_1\right)^2 \\ &\quad + \frac{1}{2} g_{\text{w}}^2 \left(\mathcal{H}_1^\dagger \mathcal{H}_2\right)\left(\mathcal{H}_2^\dagger \mathcal{H}_1\right) + |\mu|^2 \left(\mathcal{H}_1^\dagger \mathcal{H}_1 + \mathcal{H}_2^\dagger \mathcal{H}_2\right) \ , \\ V_H^{\text{breaking}} &= \tilde{m}_1^2\, \mathcal{H}_1^\dagger \mathcal{H}_1 + \tilde{m}_2^2\, \mathcal{H}_2^\dagger \mathcal{H}_2 + \left(b_{\mathcal{H}_1 \mathcal{H}_2}\, \mu\, \mathcal{H}_1 \overset{SU}{\odot} \mathcal{H}_2 + \text{h. c.}\right) \ , \end{aligned}$$

common abbreviations:

$$\begin{aligned} m_1^2 &\equiv \tilde{m}_1^2 + |\mu|^2 \ , \\ m_2^2 &\equiv \tilde{m}_2^2 + |\mu|^2 \ , \\ m_{12}^2 &\equiv b_{\mathcal{H}_1 \mathcal{H}_2}\, \mu = |m_{12}^2| e^{i\,\zeta'} \ . \end{aligned}$$

Components of the Higgs potential at the tree-level

insert the components of the Higgs fields:

$$\begin{aligned} V_H = {} & \text{constant} \\ & - T_{\phi_1}\,\phi_1 - T_{\phi_2}\,\phi_2 - T_{\chi_1}\,\chi_1 - T_{\chi_2}\,\chi_2 \\ & + \frac{1}{2}\left(\phi_1,\,\phi_2,\,\chi_1,\,\chi_2\right)\mathbf{M}_{\phi_1\phi_2\chi_1\chi_2}\begin{pmatrix}\phi_1\\ \phi_2\\ \chi_1\\ \chi_2\end{pmatrix} \\ & + \left(\phi_1^-,\,\phi_2^-\right)\mathbf{M}_{\phi_1^\pm\phi_2^\pm}\begin{pmatrix}\phi_1^+\\ \phi_2^+\end{pmatrix} \\ & + \text{triple} + \text{quartic}\ . \end{aligned}$$

tadpole coefficients $T_{\phi_1}\ ,\ T_{\phi_2}\ ,\ T_{\chi_1}\ ,\ T_{\chi_2}\ ,$

mass matrices $\mathbf{M}_{\phi_1\phi_2\chi_1\chi_2}\ ,\ \mathbf{M}_{\phi_1^\pm\phi_2^\pm}\ .$

Tadpole coefficients and mass matrices at the tree-level

explicit expressions for the tadpole coefficients

$$
\begin{aligned}
T_{\phi_1} &= -\sqrt{2}\left[m_1^2\, v_1 - |m_{12}^2|\, v_2\, c_{\zeta+\zeta'} + \tfrac{1}{4}\left(g_Y^2 + g_{\mathrm{w}}^2\right)\left(v_1^2 - v_2^2\right) v_1\right] \ , \\
T_{\phi_2} &= -\sqrt{2}\left[m_2^2\, v_2 - |m_{12}^2|\, v_1\, c_{\zeta+\zeta'} - \tfrac{1}{4}\left(g_Y^2 + g_{\mathrm{w}}^2\right)\left(v_1^2 - v_2^2\right) v_2\right] \ , \\
T_{\chi_1} &= \sqrt{2}\,|m_{12}^2|\, v_2\, s_{\zeta+\zeta'} \ , \\
T_{\chi_2} &= -\frac{v_1}{v_2}\, T_{\chi_1} = -\sqrt{2}\,|m_{12}^2|\, v_1\, s_{\zeta+\zeta'} \ ,
\end{aligned}
$$

mass matrices expressed similarly,

set of eight independent parameters:

$$
m_1^2 \ , \ m_2^2 \ , \ |m_{12}^2| \ , \ \zeta + \zeta' \ , \ v_1 \ , \ v_2 \ , \ g_Y^2 \ , \ g_{\mathrm{w}}^2 \ .
$$

Minimization of the Higgs potential at the lowest order

minimum of the Higgs potential described by v_1 , v_2 , i. e.

$$V_H^{\text{min}} = V_H\big|_{\phi_1 = 0, \phi_2 = 0, \chi_1 = 0, \chi_2 = 0} \;,$$

necessary conditions:

$$\left.\frac{\partial V_H}{\partial \phi_1}\right|_{\phi_1 = 0, \phi_2 = 0, \chi_1 = 0, \chi_2 = 0} = 0 \;,$$
$$\left.\frac{\partial V_H}{\partial \phi_2}\right|_{\phi_1 = 0, \phi_2 = 0, \chi_1 = 0, \chi_2 = 0} = 0 \;,$$
$$\left.\frac{\partial V_H}{\partial \chi_1}\right|_{\phi_1 = 0, \phi_2 = 0, \chi_1 = 0, \chi_2 = 0} = 0 \;,$$
$$\left.\frac{\partial V_H}{\partial \chi_2}\right|_{\phi_1 = 0, \phi_2 = 0, \chi_1 = 0, \chi_2 = 0} = 0 \;.$$

Minimization conditions at the lowest order

necessary conditions yield:

$$\begin{aligned} m_1^2 &= |m_{12}^2| \, \frac{v_2}{v_1} \, c_{\zeta+\zeta'} - \frac{1}{4} \left(g_Y^2 + g_{\mathrm{w}}^2\right) \left(v_1^2 - v_2^2\right) \ , \\ m_2^2 &= |m_{12}^2| \, \frac{v_1}{v_2} \, c_{\zeta+\zeta'} + \frac{1}{4} \left(g_Y^2 + g_{\mathrm{w}}^2\right) \left(v_1^2 - v_2^2\right) \ , \\ s_{\zeta+\zeta'} &= 0 \ , \end{aligned}$$

eliminate m_1^2 and m_2^2 ,
apply relation $\zeta = -\zeta'$,

moreover, with the help of a Peccei–Quinn transformation:

$$\zeta' = 0 \quad \Rightarrow \quad \zeta = 0 \ .$$

Tadpole coefficients and mass matrices at the lowest order

tadpole coefficients:

$$\begin{aligned} T^{(0)}_{\phi_1} &= 0 \ , \\ T^{(0)}_{\phi_2} &= 0 \ , \\ T^{(0)}_{\chi_1} &= 0 \ , \\ T^{(0)}_{\chi_2} &= 0 \ , \end{aligned}$$

neutral mass matrix:

$$\mathbf{M}^{(0)}_{\phi_1\phi_2\chi_1\chi_2} = \begin{pmatrix} \mathbf{M}^{(0)}_{\phi_1\phi_2} & \mathbf{M}^{(0)}_{\phi\chi} \\ \mathbf{M}^{(0)\,\dagger}_{\phi\chi} & \mathbf{M}^{(0)}_{\chi_1\chi_2} \end{pmatrix} \ ,$$

Lowest-order relations: substitutions

apply following substitutions:

$$\tan\beta \equiv \frac{v_2}{v_1} \ ,$$
$$M_W^2 \equiv \frac{1}{2}\, g_\text{w}^2 \left(v_1^2 + v_2^2\right) \ ,$$
$$M_Z^2 \equiv \frac{1}{2}\left(g_Y^2 + g_\text{w}^2\right)\left(v_1^2 + v_2^2\right) \ ,$$

short reminder:

$\phi_1 \ , \ \phi_2$ CP even,

$\chi_1 \ , \ \chi_2$ CP odd.

Lowest-order relations: masses I

$$\mathbf{M}_{\phi\chi}^{(0)} = \mathbf{0} \ ,$$

$\Rightarrow$ **no *CP*-mixing at the lowest order**,

$$\mathbf{M}_{\chi_1\chi_2}^{(0)} = \begin{pmatrix} |m_{12}^2| \tan\beta & -|m_{12}^2| \\ -|m_{12}^2| & |m_{12}^2| \cot\beta \end{pmatrix}$$

real and symmetric,

masses of *CP* odd bosons:

$$m_A^2 \equiv \frac{2\,|m_{12}^2|}{\sin(2\beta)} \ ,$$
$$m_G^2 \equiv 0 \ .$$

Lowest-order relations: masses II

$$\mathbf{M}^{(0)}_{\phi_1\phi_2} = \begin{pmatrix} \frac{1}{2}\left(g_Y^2 + g_\mathrm{w}^2\right) v_1^2 + |m_{12}^2|\,\frac{v_2}{v_1} & -\frac{1}{2}\left(g_Y^2 + g_\mathrm{w}^2\right) v_1\, v_2 - |m_{12}^2| \\ -\frac{1}{2}\left(g_Y^2 + g_\mathrm{w}^2\right) v_1\, v_2 - |m_{12}^2| & \frac{1}{2}\left(g_Y^2 + g_\mathrm{w}^2\right) v_2^2 + |m_{12}^2|\,\frac{v_1}{v_2} \end{pmatrix},$$

real and symmetric,

masses of *CP* even bosons:

$$m_{h/H}^2 \equiv \frac{1}{2}\left[m_A^2 + M_Z^2 \mp \sqrt{\left(m_A^2 + M_Z^2\right)^2 - \left(2\, m_A\, M_Z\, \cos\left(2\beta\right)\right)^2}\right] \;,$$

upper bound on the lightest Higgs mass:

$$m_h^2 \leq M_Z^2\, \cos^2(2\beta)\;.$$

Lowest-order relations: masses III

$$\mathbf{M}^{(0)}_{\phi_1^\pm \phi_2^\pm} = \begin{pmatrix} \frac{1}{2}\, g_\mathrm{w}^2\, v_2^2 + |m_{12}^2|\, \frac{v_2}{v_1} & -\frac{1}{2}\, g_\mathrm{w}^2\, v_1\, v_2 - |m_{12}^2| \\ -\frac{1}{2}\, g_\mathrm{w}^2\, v_1\, v_2 - |m_{12}^2| & \frac{1}{2}\, g_\mathrm{w}^2\, v_1^2 + |m_{12}^2|\, \frac{v_1}{v_2} \end{pmatrix},$$

real and symmetric,

masses of charged bosons:

$$m_{H^\pm}^2 = m_A^2 + M_W^2 \ ,$$
$$m_{G^\pm}^2 = 0 \ .$$

Lowest-order relations: parameters

- all masses at the lowest order determined by two parameters of the MSSM:

$$m_A \text{ and } \tan\beta\ ,$$

- tadpole coefficients (linear terms) are zero,

  mass matrices (bilinear terms) are real,

  triple and quartic couplings determined by real g_Y^2 , g_w^2 , v_1 and v_2 ,

  $\Rightarrow$ **Higgs sector is *CP*-conserving at the lowest order**.

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Lowest-order relations: mass-eigenstate basis

five massive, physical Higgs bosons h , H , A , $H^\pm$,
three massless, unphysical Goldstone bosons G , $G^\pm$
(only acquire masses by gauge-fixing),

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathbf{D}_\alpha \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} , \quad \begin{pmatrix} A \\ G \end{pmatrix} = \mathbf{D}_{\beta_n} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} , \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \mathbf{D}_{\beta_c} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} ,$$

mixing matrix

$$\mathbf{D}_x = \begin{pmatrix} -\sin(x) & \cos(x) \\ \cos(x) & \sin(x) \end{pmatrix} \ .$$

Higgs potential in mass-eigenstate basis at the tree-level

parametrization of the Higgs potential in mass-eigenstate basis

$$\begin{aligned} V_H = {} & \text{constant} \\ & - T_h\, h - T_H\, H - T_A\, A - T_G\, G \\ & + \frac{1}{2} \left(h,\, H,\, A,\, G \right) \mathbf{M}_{hHAG} \begin{pmatrix} h \\ H \\ A \\ G \end{pmatrix} \\ & + \left(H^-,\, G^- \right) \mathbf{M}_{H^\pm G^\pm} \begin{pmatrix} H^+ \\ G^+ \end{pmatrix} \\ & + \text{triple} + \text{quartic} \ . \end{aligned}$$

tadpole coefficients $T_h\ ,\ T_H\ ,\ T_A\ ,\ T_G\ ,$

mass matrices $\mathbf{M}_{hHAG}\ ,\ \mathbf{M}_{H^\pm G^\pm}\ .$

Tree level versus lowest order

tree level:

- **non-zero** tadpole coefficients $T_h\ ,\ T_H\ ,\ T_A\ ,\ T_G\ ,$
- **full** (4×4) matrix $\mathbf{M}_{hHAG} = \begin{pmatrix} m_h^2 & m_{hH}^2 & m_{hA}^2 & m_{hG}^2 \\ m_{hH}^2 & m_H^2 & m_{HA}^2 & m_{HG}^2 \\ m_{hA}^2 & m_{HA}^2 & m_A^2 & m_{AG}^2 \\ m_{hG}^2 & m_{HG}^2 & m_{AG}^2 & m_G^2 \end{pmatrix}\ ,$
- **full** (2×2) matrix $\mathbf{M}_{H^\pm G^\pm} = \begin{pmatrix} m_{H^\pm}^2 & m_{H^- G^+}^2 \\ m_{G^- H^+}^2 & m_{G^\pm}^2 \end{pmatrix}\ ,$

lowest order:

- **zero** tadpole coefficients $T_h^{(0)}\ ,\ T_H^{(0)}\ ,\ T_A^{(0)}\ ,\ T_G^{(0)}\ ,$
- **diagonal** (4×4) matrix $\mathbf{M}_{hHAG}^{(0)} = \mathrm{diag}\left(m_h^2,\ m_H^2,\ m_A^2,\ m_G^2\right)\ ,$
- **diagonal** (2×2) matrix $\mathbf{M}_{H^\pm G^\pm}^{(0)} = \mathrm{diag}\left(m_{H^\pm}^2,\ m_{G^\pm}^2\right)\ .$

Lowest-order relations: mixing angles

evaluate tree-level mass matrices in mass-eigenstate basis:

$$m^2_{AG} = -m^2_A \, t_{\beta-\beta_n} - \tfrac{e}{2\, s_{\rm w}\, M_W\, c_{\beta-\beta_n}} \left(T_H \, s_{\alpha-\beta_n} - T_h \, c_{\alpha-\beta_n} \right) \;,$$

$$m^2_{H^- G^+} = -m^2_{H^\pm} \, t_{\beta-\beta_c} - \tfrac{e}{2\, s_{\rm w}\, M_W} \left(T_H \, \tfrac{s_{\alpha-\beta_c}}{c_{\beta-\beta_c}} + T_h \, \tfrac{c_{\alpha-\beta_c}}{c_{\beta-\beta_c}} + i \, T_A \, \tfrac{1}{c_{\beta-\beta_n}} \right) \;,$$

$$s_x \equiv \sin(x) \;,\; c_x \equiv \cos(x) \;,\; t_x \equiv \tan(x) \;,$$

apply lowest-order relations:

$$\begin{aligned} 0 &= -m^2_A \, \tan(\beta - \beta_n) & \Rightarrow \quad & \beta_n = \beta \;, \\ 0 &= -m^2_{H^\pm} \, \tan(\beta - \beta_c) & \Rightarrow \quad & \beta_c = \beta \;, \end{aligned}$$

similar relation for *CP* even Higgs-boson mass matrix:

$$\tan(2\alpha) = \frac{m^2_A + M^2_Z}{m^2_A - M^2_Z} \, \tan(2\beta) \;.$$

Lowest-order relations: summary

- two neutral CP even Higgs bosons, h: 🔵, H: 🟣,
- one neutral CP odd Higgs boson, A: 🟢,
- two charged Higgs bosons, $H^{\pm}$: 🔴,
- CP conservation,
- two independent parameters to describe masses and mixing, common choice: $\tan\beta$ and m_A ,
- important relation $m_{H^{\pm}}^2 = m_A^2 + M_W^2$.

Two-point vertex function at higher orders

Higgs masses given by poles of propagator matrix

$$\mathbf{\Delta}_{hHAG}\left(p^2\right) = i\,\left[p^2\mathbf{1} - \mathbf{M}^{(k)}_{hHAG}\left(p^2\right)\right]^{-1} ,$$

matrix of renormalized two-point vertex functions:

$$\widehat{\mathbf{\Gamma}}^{(k)}_{hHAG}\left(p^2\right) = -\left[\mathbf{\Delta}_{hHAG}\left(p^2\right)\right]^{-1},$$

masses determined by

$$\det\left[\widehat{\mathbf{\Gamma}}^{(k)}_{hHAG}\left(p^2\right)\right]_{p^2\,=\,x_i^2} = 0\ , \quad m^2_{h_i} = \mathfrak{Re}\left[x_i^2\right]\ , \quad i \in \{1,\,2,\,3\}\ ,$$

(fourth solution belongs to Goldstone boson, equal to zero).

Mass matrix at higher orders

- lowest order: $\mathbf{M}^{(k)}_{hHAG}(p^2)\Big|_{k\,=\,0} = \mathbf{M}^{(0)}_{hHAG}$, diagonal,

- higher order: $\mathbf{M}^{(k)}_{hHAG}(p^2)\Big|_{k\,\geq\,1} = \mathbf{M}^{(0)}_{hHAG} - \sum_{j\,=\,1}^{k} \widehat{\mathbf{\Sigma}}^{(j)}_{hHAG}(p^2)$,

  shift by renormalized self-energies

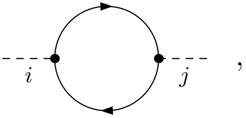
$$\widehat{\mathbf{\Sigma}}^{(j)}_{hHAG}(p^2) = \begin{pmatrix} \hat{\Sigma}^{(j)}_{h}(p^2) & \hat{\Sigma}^{(j)}_{hH}(p^2) & \hat{\Sigma}^{(j)}_{hA}(p^2) & \hat{\Sigma}^{(j)}_{hG}(p^2) \\ \hat{\Sigma}^{(j)}_{hH}(p^2) & \hat{\Sigma}^{(j)}_{H}(p^2) & \hat{\Sigma}^{(j)}_{HA}(p^2) & \hat{\Sigma}^{(j)}_{HG}(p^2) \\ \hat{\Sigma}^{(j)}_{hA}(p^2) & \hat{\Sigma}^{(j)}_{HA}(p^2) & \hat{\Sigma}^{(j)}_{A}(p^2) & \hat{\Sigma}^{(j)}_{AG}(p^2) \\ \hat{\Sigma}^{(j)}_{hG}(p^2) & \hat{\Sigma}^{(j)}_{HG}(p^2) & \hat{\Sigma}^{(j)}_{AG}(p^2) & \hat{\Sigma}^{(j)}_{G}(p^2) \end{pmatrix} .$$

Example at the one-loop order

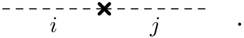
renormalized self-energies composed of
self-energies and counterterms:

$$\hat{\Sigma}^{(1)}_{ij}\left(p^2\right) = \Sigma^{(1)}_{ij}\left(p^2\right) + \left(p^2 - m^2_{ij}\right)\delta Z_{ij} - \delta m^2_{ij} \ ,$$

self-energy:



counterterm:



Renormalized self-energy of the lightest Higgs h

tree-level mass:

$$m_h^2 = M_Z^2\, s_{\alpha+\beta}^2 + m_A^2\, \frac{c_{\alpha-\beta}^2}{c_{\beta-\beta_n}^2} \\ + \frac{e\, s_{\alpha-\beta_n}}{2\, s_\mathrm{w}\, M_W\, c_{\beta-\beta_n}^2} \left[T_H\, c_{\alpha-\beta}\, s_{\alpha-\beta_n} + T_h\, \tfrac{1}{2} \left(c_{2\alpha-\beta-\beta_n} + 3\, c_{\beta-\beta_n} \right) \right],$$

independent parameters:

M_Z , M_W , e , $\tan\beta$, m_A (or $m_{H^\pm}$) , T_h , T_H , T_A ,

one-loop self-energy: $\Sigma_h^{(1)}(p^2)$, calculate Feynman diagrams,

one-loop counterterm: δm_h^2 , apply renormalization transformation to m_h^2 .

Counterterm for the self-energy of the lightest Higgs h

renormalization transformations:

$$\begin{aligned} M_Z &\to M_Z + \delta M_Z \ , & T_h &\to T_h + \delta T_h \ , \\ M_W &\to M_W + \delta M_W \ , & T_H &\to T_H + \delta T_H \ , \\ e &\to e + \delta e \ , & T_A &\to T_A + \delta T_A \ , \\ m_A^2 &\to m_A^2 + \delta m_A^2 \ , & \tan\beta &\to \tan\beta + \delta\tan\beta \ , \end{aligned}$$

mixing angles α , β_n , β_c **not** renormalized,

apply to tree-level mass m_h^2 and **utilize lowest-order relations**

$$\begin{aligned} \delta m_h^2 = {} & \delta M_Z^2\, s_{\alpha+\beta}^2 + \delta m_A^2\, c_{\alpha-\beta}^2 \\ & + \delta\tan\beta\, c_\beta^2 \left(M_Z^2\, s_{2(\alpha+\beta)} + m_A^2\, s_{2(\alpha-\beta)} \right) \\ & + \frac{e\, s_{\alpha-\beta}}{2\, s_{\mathrm{w}}\, M_W} \left[\delta T_H\, c_{\alpha-\beta}\, s_{\alpha-\beta} + \delta T_h \left(1 + c_{\alpha-\beta}^2 \right) \right] . \end{aligned}$$

CP mixing at higher orders

off-diagonal entries of $\widehat{\boldsymbol{\Sigma}}^{(j)}_{hHAG}(p^2)$:

- $\hat{\Sigma}^{(j)}_{hH}(p^2) \neq 0$: *CP* even bosons h, H mix,
- $\hat{\Sigma}^{(j)}_{hA}(p^2) \neq 0$: *CP* even boson h and *CP* odd boson A mix,
- $\hat{\Sigma}^{(j)}_{HA}(p^2) \neq 0$: *CP* even boson H and *CP* odd boson A mix,

$\Rightarrow$ in general no *CP* eigenstates at higher orders,

more precisely:

$$\hat{\Sigma}^{(j)}_{hA}\left(p^2\right) \propto \mathfrak{Im}[\ldots]\ ,\ \hat{\Sigma}^{(j)}_{HA}\left(p^2\right) \propto \mathfrak{Im}[\ldots]\ ,$$

i. e. ***CP* mixing** introduced by **complex parameters** from other sectors of the MSSM (e. g. μ, A_t, $\ldots$).

Input parameter m_A or $m_{H^\pm}$

so far: m_A chosen as an input parameter,
convenient if only h and H mix,

on-shell renormalization of A possible, i. e.

$$\hat{\Sigma}_A^{(j)}(m_A^2) = 0\,,$$

however: input of m_A makes no sense for complex parameters,
because higher-order corrections to m_A also induced by
off-diagonal self-energies $\hat{\Sigma}_{hA}^{(j)}(p^2)\,,\ \hat{\Sigma}_{HA}^{(j)}(p^2)\,,$

instead: choose $m_{H^\pm}$ as an input
utilizing the relation $m_{H^\pm}^2 = m_A^2 + M_W^2\,,$

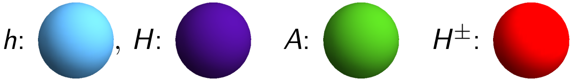
on-shell renormalization of $m_{H^\pm}$ possible, i. e.

$$\hat{\Sigma}_{H^\pm}^{(j)}(m_{H^\pm}^2) = 0\,.$$

Mixed particles at higher orders

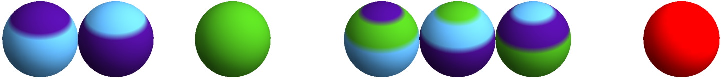
- lowest order mass eigenstates:

CP even	*CP* odd	charged



- higher orders:

real MSSM		complex MSSM	both
CP even	*CP* odd	*CP* mixed	charged



input parameter:
m_A or $m_{H^{\pm}}$

input parameter:
$m_{H^{\pm}}$

Field-renormalization constants I

one field-renormalization constant for each Higgs doublet:

$$\mathcal{H}_1 \to \mathcal{H}_1\sqrt{Z_{\mathcal{H}_1}}\ , \quad Z_{\mathcal{H}_1} = 1 + \delta Z_{\mathcal{H}_1}\ ,$$
$$\mathcal{H}_2 \to \mathcal{H}_2\sqrt{Z_{\mathcal{H}_2}}\ , \quad Z_{\mathcal{H}_2} = 1 + \delta Z_{\mathcal{H}_2}\ .$$

determined by $\overline{\text{DR}}$-conditions:

$$\delta Z_{\mathcal{H}_1} = -\mathfrak{Re}\left[\frac{\partial\Sigma^{(1)}_{\phi_1}(p^2)}{\partial p^2}\right]_{\text{div}} = -\mathfrak{Re}\left[\frac{\partial\Sigma^{(1)}_{H}\Big|_{\alpha\,=\,0}(p^2)}{\partial p^2}\right]_{\text{div}}\ ,$$
$$\delta Z_{\mathcal{H}_2} = -\mathfrak{Re}\left[\frac{\partial\Sigma^{(1)}_{\phi_2}(p^2)}{\partial p^2}\right]_{\text{div}} = -\mathfrak{Re}\left[\frac{\partial\Sigma^{(1)}_{h}\Big|_{\alpha\,=\,0}(p^2)}{\partial p^2}\right]_{\text{div}}\ .$$

Field-renormalization constants II

commonly, field-renormalization constants in mass-eigenstate basis:

$$\begin{pmatrix} h \\ H \end{pmatrix} \to \mathbf{D}_\alpha \begin{pmatrix} \sqrt{Z_{\mathcal{H}_1}} & 0 \\ 0 & \sqrt{Z_{\mathcal{H}_2}} \end{pmatrix} \mathbf{D}_\alpha^{-1} \begin{pmatrix} h \\ H \end{pmatrix}$$

$$\equiv \left[\mathbf{1} + \frac{1}{2} \begin{pmatrix} \delta Z_{hh} & \delta Z_{hH} \\ \delta Z_{Hh} & \delta Z_{HH} \end{pmatrix} \right] \begin{pmatrix} h \\ H \end{pmatrix},$$

$$\delta Z_{hh} = \left(s_\alpha^2 \delta Z_{\mathcal{H}_1} + c_\alpha^2 \delta Z_{\mathcal{H}_2} \right) \ ,$$

$$\delta Z_{HH} = \left(c_\alpha^2 \delta Z_{\mathcal{H}_1} + s_\alpha^2 \delta Z_{\mathcal{H}_2} \right) \ ,$$

$$\delta Z_{hH} = \delta^{(1)} Z_{Hh} = c_\alpha s_\alpha \left(\delta Z_{\mathcal{H}_2} - \delta Z_{\mathcal{H}_1} \right) \ ,$$

analogously for neutral *CP* odd and charged Higgs fields,
no *CP* mixing by δZ_{ij} at all orders.

Tadpole-renormalization constants I

Higgs potential at the one-loop order:

$$V_H^{(1)} = -T_h^{(1)}\,h - T_H^{(1)}\,H - T_A^{(1)}\,A - T_G^{(1)}\,G + \ldots \ ,$$

tadpole coefficients:

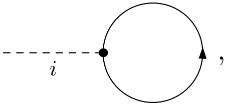
$$T_i^{(1)} = T_i^{(0)} + \hat{\tau}_i^{(1)} \ ,$$

renormalized tadpoles:

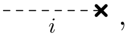
$$\hat{\tau}_i^{(1)} = \tau_i^{(1)} + \delta T_i^{(1)} \ .$$

Tadpole-renormalization constants II

tadpole diagrams:



tadpole counterterms



renormalization condition:
minimum of Higgs potential not shifted, i. e.

$$\hat{\tau}_i^{(1)} = 0 \quad \Rightarrow \quad \delta T_i^{(1)} = -\tau_i^{(1)} \ .$$

Renormalization of m_A and $m_{H^\pm}$

input parameter:	m_A	$m_{H^\pm}$
on-shell particle:	A	$H^\pm$
renormalization condition:	$\mathfrak{Re}\left[\hat{\Sigma}_A^{(1)}(m_A^2)\right] = 0$	$\mathfrak{Re}\left[\hat{\Sigma}_{H^\pm}^{(1)}(m_{H^\pm}^2)\right] = 0$
renormalization constant:	$\delta m_A^2 = \mathfrak{Re}\left[\Sigma_A^{(1)}(m_A^2)\right]$	$\delta m_{H^\pm}^2 = \mathfrak{Re}\left[\Sigma_{H^\pm}^{(1)}(m_{H^\pm}^2)\right]$
relations:	$m_{H^\pm}^2 = m_A^2 + M_W^2$ $\delta m_{H^\pm}^2 = \delta m_A^2 + \delta M_W^2$	

Renormalization of gauge sector

gauge sector renormalized as in the Standard Model,

gauge-boson masses on-shell

$$0 = \mathfrak{Re}\left[\hat{\Sigma}_{W}^{(1)}\left(p^2\right)\right]_{p^2 = M_W^2} ,$$

$$\delta M_W^2 = \mathfrak{Re}\left[\Sigma_W^{(1)}\left(M_W^2\right)\right] ,$$

$$0 = \mathfrak{Re}\left[\hat{\Sigma}_{Z}^{(1)}\left(p^2\right)\right]_{p^2 = M_Z^2} ,$$

$$\delta M_Z^2 = \mathfrak{Re}\left[\Sigma_Z^{(1)}\left(M_Z^2\right)\right] ,$$

δe not required for renormalized Higgs potential at one-loop.

Renormalization of $\tan\beta$

definition:

$$\tan\beta = \frac{v_2}{v_1} \ ,$$

$v_1\,,\ v_2$ part of Higgs doublets and parameter, hence

$$v_i \to \sqrt{Z_{\mathcal{H}_i}}\,(v_i + \delta v_i) \ ,$$

presently best option: $\overline{\text{DR}}$-definition of $\tan\beta$,

utilize result for δv_i:

$$\left.\frac{\delta v_1}{v_1}\right|_{\text{div}} = \left.\frac{\delta v_2}{v_2}\right|_{\text{div}} \ ,$$

$$\delta\tan\beta = \tfrac{1}{2}\,\tan\beta\,(\delta Z_{\mathcal{H}_2} - \delta Z_{\mathcal{H}_1}) \ .$$

Computational parts

definitions above leave following tasks:

- evaluation of all neutral Higgs-boson self-energies,

  if complex parameters:
  evaluation of charged Higgs-boson self-energy,
- evaluation of derivatives $\left.\frac{\partial \Sigma^{(1)}_{\phi_1}(p^2)}{\partial p^2}\right|_{\text{div}}$, $\left.\frac{\partial \Sigma^{(1)}_{\phi_2}(p^2)}{\partial p^2}\right|_{\text{div}}$,
- evaluation of Higgs-boson tadpoles,
- evaluation of W- and Z-boson self-energies,

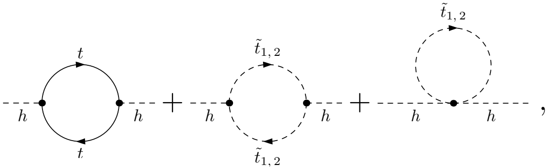
⇒ divergences of each renormalized self-energy cancel,
evaluate zeroes of determinant of two-point vertex function.

1. Particle content of the MSSM
2. Higgs fields in the MSSM
3. Higgs potential at the tree-level
4. Mass-eigenstate basis at the tree-level
5. Higher-Order Corrections
6. Renormalization of the Higgs sector
7. **Present status**
8. Order α_t^2 Corrections
9. Numerical Results
10. Outlook

One-loop corrections

- full one-loop result including dependence on p^2 known in the MSSM with complex parameters,
- main contributions: t and $\tilde{t}$ loops, order α_t , but proportional to m_t^4:

$\Sigma_h\left(p^2\right) =$



- additional parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, A_t , μ ,
  complex case: ϕ_{A_t} , ϕ_μ , mixing of h , H , A ,
- mass contribution to m_h up to 50% of tree-level result,
- higher-order corrections necessary.

Higher-order corrections

most important parts:
leading corrections to m_t-enhanced one-loop contributions:

2-loop

complex MSSM

- corrections of $\mathcal{O}(\alpha_t\alpha_s)$ in complex MSSM,
  [Heinemeyer, Hollik, Rzehak, Weiglein, arXiv:hep-ph/0705.0746, 2007],
- corrections of $\mathcal{O}(\alpha_t^2)$ in complex MSSM,
  [Hollik, SP, arXiv:1401.8275 [hep-ph], arXiv:1409.1687 [hep-ph], 2014],

real MSSM

- corrections of $\mathcal{O}(\alpha_t^2) + \ldots$ in real MSSM
  in effective potential approach,
  [Brignole, Degrassi, Slavich, Zwirner, arXiv:hep-ph/0112177, 2002],
- corrections of $\mathcal{O}(\alpha_t\alpha_s)$ in real MSSM,
  momentum dependent parts,
  [Borowka, Hahn, Heinemeyer, Heinrich, Hollik, arXiv:1404.7074 [hep-ph], 2014],

3-loop

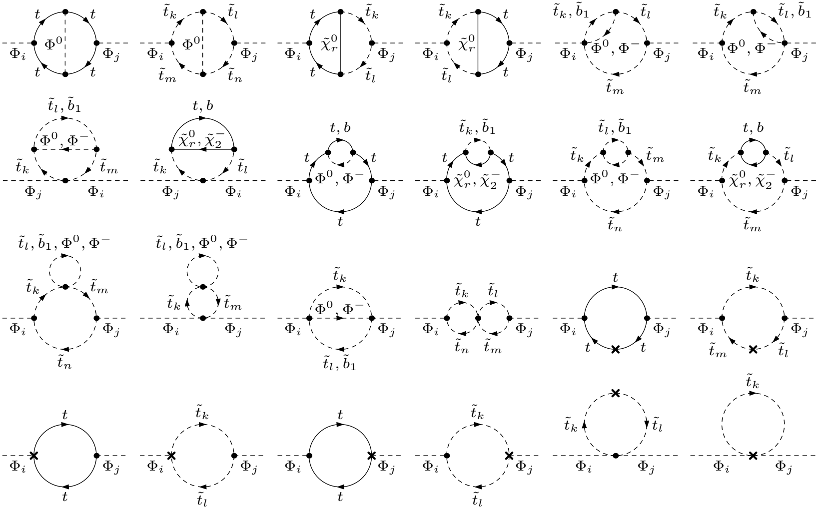
- corrections of $\mathcal{O}(\alpha_t\alpha_s^2)$ in real MSSM,
  only for the lightest Higgs,
  [Harlander, Kant, Mihaila, Steinhauser, arXiv:1005.5709 [hep-ph], 2010].

Leading order α_t^2 corrections

- complex MSSM, *CP*-mixing,
- self-energies for neutral and charged Higgs bosons computed,
- analytical calculation in Feynman-diagrammatic approach,
- approximations to yield dominant parts:
  - gauge-less limit: $g_Y = 0\,,\ g_{\mathrm{w}} = 0$ (just as $\mathcal{O}(\alpha_t\alpha_s)$),
  - also $g_s = 0\,,$
  - external momentum equal to zero,
  - bottom mass equal to zero,
- total enhancement of $m_t^6\,,$
- Higgs-boson masses evaluated from

$$\widehat{\mathbf{\Gamma}}^{(2)}_{hHAG}\left(p^2\right) = \boldsymbol{i}\left[p^2\mathbf{1} - \mathbf{M}^{(0)}_{hHAG} + \widehat{\mathbf{\Sigma}}^{(1)}_{hHAG}\left(p^2\right)\right. \\ \left. + \widehat{\mathbf{\Sigma}}^{(2),\,\alpha_t\alpha_s}_{hHAG}(0) + \widehat{\mathbf{\Sigma}}^{(2),\,\alpha_t^2}_{hHAG}(0)\right].$$

Feynman diagrams for neutral Higgs bosons



a cross denotes a one-loop counterterm insertion,

$\Phi_i = h\ ,\ H\ ,\ A\ ,\ \Phi^0 = h\ ,\ H\ ,\ A\ ,\ G\ ,\ \Phi^- = H^-\ ,\ G^-\ .$

Renormalization scheme

required renormalization constants:

1-loop, 2-loop

- on-shell:
  δt_h , δt_H , δt_A ,
  either $\delta m^2_{H^\pm}$ or δm^2_A ,

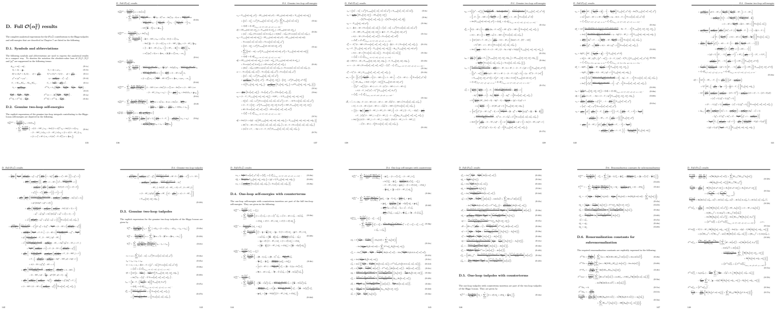
1-loop

- on-shell:
  $\delta M_W/M_W$, $\delta M_Z/M_Z$,
- $\overline{\text{DR}}$:
  $\delta Z_{\mathcal{H}_1}$, $\delta Z_{\mathcal{H}_2}$, $\delta \tan\beta$,
- on-shell or $\overline{\text{DR}}$:
  δm_t , $\delta m_{\tilde{t}_1}$, $\delta m_{\tilde{t}_2}$, $\delta\mu$, δA_t .

Procedure of calculation

- creation of Feynman-diagrams and amplitudes with help of `FeynArts`, [Hahn, arXiv:hep-ph/0012260, 2001],
- applying approximations,
- reducing one-loop diagrams to master integrals with help of `FormCalc`, [Hahn, arXiv:hep-ph/0901.1528, 2009],
- reducing two-loop diagrams to master integrals with help of `TwoCalc`, [Weiglein, Scharf, Böhm, arXiv:hep-ph/9310358, 1993],
- creating counterterms from the Higgs potential,
- applying renormalization scheme,
- evaluating renormalization constants.

Analytical result



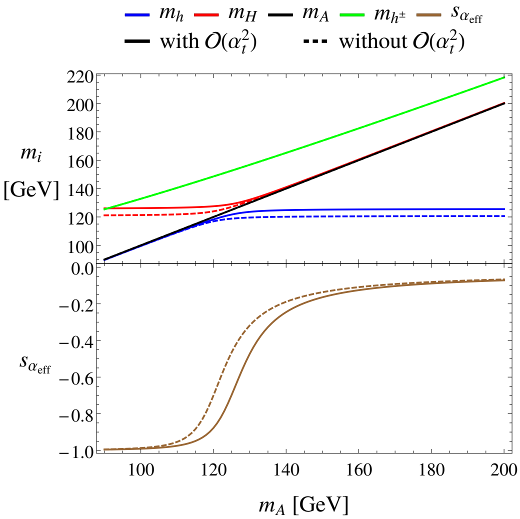
all complex parameters combined into real quantities,
$p^2 = 0 \quad \Rightarrow \quad$ no imaginary parts from loop integrals,
result real.

Numerical results

Default input values of the MSSM and SM parameters.

MSSM input	SM input
$M_2 = 200$ GeV,	$m_t = 173.2$ GeV,
$M_1 = \left(5 s_\mathrm{w}^2\right)/\left(3 c_\mathrm{w}^2\right) M_2$,	$m_b = 4.2$ GeV,
$m_{\tilde{\ell}_1} = m_{\tilde{e}_\mathrm{R}} = 2000$ GeV,	$m_\tau = 1.77703$ GeV,
$m_{\tilde{q}_1} = m_{\tilde{u}_\mathrm{R}} = m_{\tilde{d}_\mathrm{R}} = 2000$ GeV,	$M_W = 80.385$ GeV,
$A_u = A_d = A_e = 0$ GeV,	$M_Z = 91.1876$ GeV,
$m_{\tilde{\ell}_2} = m_{\tilde{\mu}_\mathrm{R}} = 2000$ GeV,	$G_\mathrm{F} = 1.16639 \cdot 10^{-5}$,
$m_{\tilde{q}_2} = m_{\tilde{c}_\mathrm{R}} = m_{\tilde{s}_\mathrm{R}} = 2000$ GeV,	$\alpha_s = 0.118$.
$A_c = A_s = A_\mu = 0$ GeV,	

Higgs masses in the real MSSM with input m_A



$t_\beta = 30$

$\mu = 200$ GeV

$m_{\tilde{q}_3} = m_{\tilde{t}_R} = m_{\tilde{b}_R} = 1000$ GeV

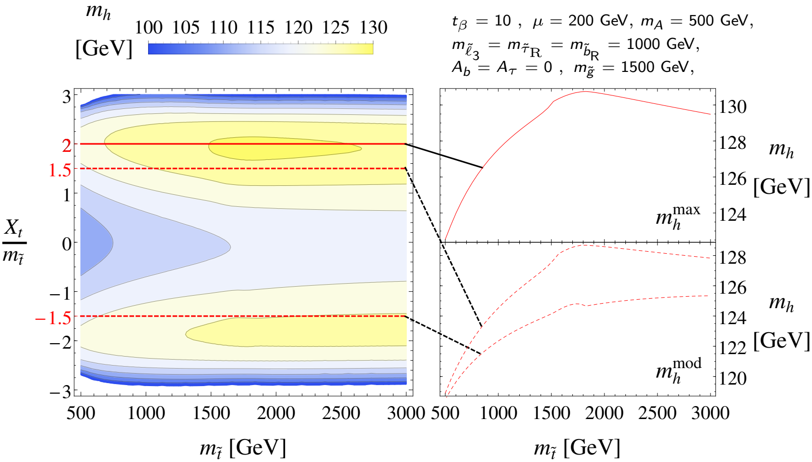
$m_{\tilde{\ell}_3} = m_{\tilde{\tau}_R} = 1000$ GeV

$m_{\tilde{g}} = 1500$ GeV

$A_t = A_b = A_\tau = 1500$ GeV

Maximum of lightest Higgs-boson mass

$t_\beta = 10$, $\mu = 200$ GeV, $m_A = 500$ GeV,
$m_{\tilde{\ell}_3} = m_{\tilde{\tau}_R} = m_{\tilde{b}_R} = 1000$ GeV,
$A_b = A_\tau = 0$, $m_{\tilde{g}} = 1500$ GeV,

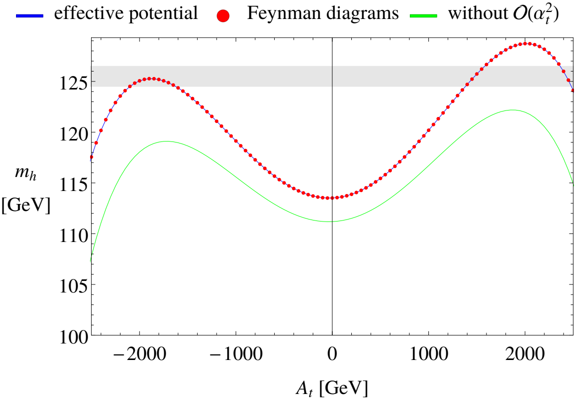


at high $m_{\tilde{t}} \equiv m_{\tilde{q}_3} = m_{\tilde{t}_R}$: missing higher-order corrections,
kinks: thresholds for $\tilde{t}_i \to \tilde{g}\, t$.

Comparison with previous result in the real MSSM

numerical agreement with existing result for real parameters,

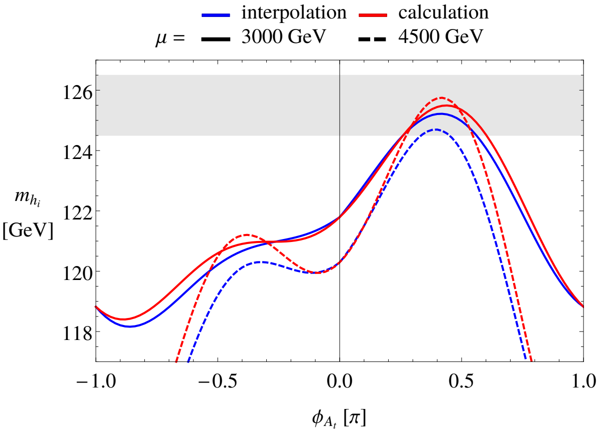
example:



$t_\beta = 30$
$\mu = 200$ GeV
$m_{\tilde{q}_3} = m_{\tilde{t}_R} = m_{\tilde{b}_R} = 1000$ GeV
$m_{\tilde{\ell}_3} = m_{\tilde{\tau}_R} = 1000$ GeV
$\mathrm{m}_{\tilde{g}} = 1500$ GeV
$m_A = 800$ GeV
$A_t = A_b = A_\tau$

Big improvement in the complex MSSM

lightest Higgs-boson mass,
$m_{H^\pm}$ is input parameter, $H^\pm$-boson renormalized on-shell,



$t_\beta = 5$

$m_{\tilde{q}_3} = m_{\tilde{b}_R} = 1500$ GeV

$m_{\tilde{t}_R} = 1.1\, m_{\tilde{q}_3}$

$m_{\tilde{\ell}_3} = m_{\tilde{\tau}_R} = 1000$ GeV

$m_{\tilde{g}} = 0.9\, m_{\tilde{q}_3}\, e^{i\,\pi/2}$

$m_{H^\pm} = 500$ GeV

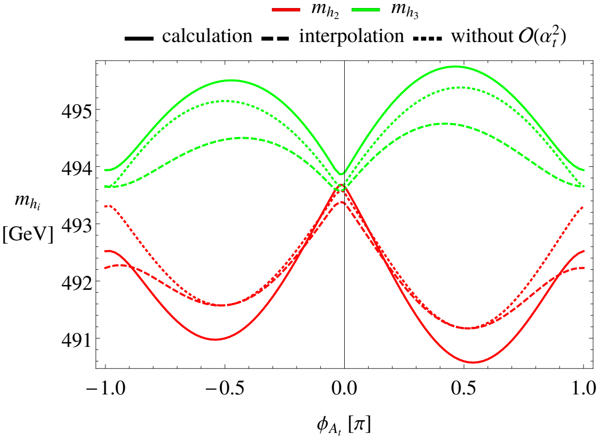
$|A_t| = |A_b| = |A_\tau| = 2\, m_{\tilde{q}_3}$

$\phi_{A_t} = \phi_{A_b} = \phi_{A_\tau}$

Big improvement in the complex MSSM

heavier Higgs-boson masses,

$m_{H^\pm}$ is input parameter, $H^\pm$-boson renormalized on-shell,



$t_\beta = 5$

$\mu = 3000$ GeV

$m_{\tilde{q}_3} = m_{\tilde{b}_R} = 1500$ GeV

$m_{\tilde{t}_R} = 1.1\, m_{\tilde{q}_3}$

$m_{\tilde{\ell}_3} = m_{\tilde{\tau}_R} = 1000$ GeV

$m_{\tilde{g}} = 0.9\, m_{\tilde{q}_3}\, e^{i\,\pi/2}$

$m_{H^\pm} = 500$ GeV

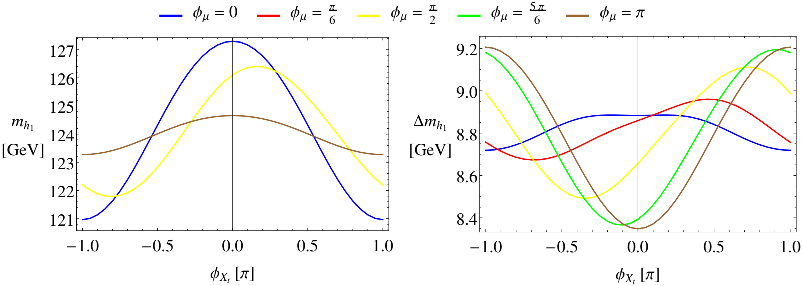
$|A_t| = |A_b| = |A_\tau| = 2\, m_{\tilde{q}_3}$

$\phi_{A_t} = \phi_{A_b} = \phi_{A_\tau}$

$$\text{in } \mathcal{O}\left(\alpha_t^2\right) \text{ terms} \left[\begin{array}{rl} \text{interpolation:} & \delta m_{H^\pm}^2 = \delta m_A^2 := \Sigma_A(0) \\ \text{calculation:} & \delta m_A^2 = \delta m_{H^\pm}^2 := \Sigma_{H^\pm}(0) \end{array} \right.$$

Influence of phases

dependence of m_{h_1} on ϕ_{X_t} , ϕ_μ ,
$m_{H^\pm}$ is input parameter, $H^\pm$-boson renormalized on-shell,



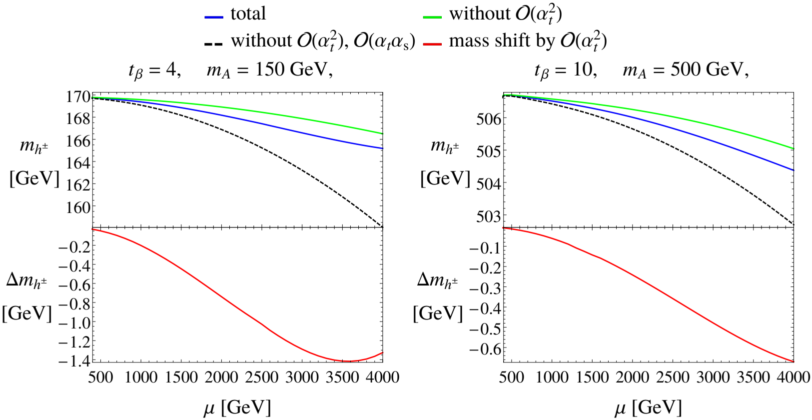
Left: The value of m_{h_1} including all available contributions, with the phase dependence arising from one-loop, $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}\left(\alpha_t^2\right)$ terms.

Right: The contribution Δm_{h_1} to m_{h_1} owing exclusively to the $\mathcal{O}\left(\alpha_t^2\right)$ terms, for different phases.

The input parameters are $m_{H^\pm} = 200$ GeV, $|\mu| = 2500$ GeV, $t_\beta = 10$, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_\mathrm{R}} = 1000$ GeV,
$m_{\tilde{q}_3} = m_{\tilde{t}_\mathrm{R}} = m_{\tilde{b}_\mathrm{R}} = 1500$ GeV, $|X_t| = 2\, m_{\tilde{q}_3}$, $A_b = A_\tau = 0$, $m_{\tilde{g}} = 2000$ GeV.

Charged Higgs-boson mass in the real MSSM

m_A is input parameter, A-boson renormalized on-shell, additional shift to $m_{H^\pm}$ by newly available $\mathcal{O}(\alpha_t^2)$ terms,



The other input parameters are $m_{\tilde{q}_3} = m_{\tilde{t}_R} = m_{\tilde{b}_R} = 1000$ GeV, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_R} = 1000$ GeV, $A_t = A_b = A_\tau = 1.5\, m_{\tilde{q}_3}$, $m_{\tilde{g}} = 1500$ GeV.

Outlook

- implementation into FeynHiggs,
  [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, arXiv:hep-ph/1007.0956, 2010],
- calculation of missing two-loop parts with gauge-couplings.

Input sectors

squarks:

$$\mathbf{M}_{\tilde{q}} = \begin{pmatrix} m_{\tilde{q}_\mathrm{L}}^2 + m_q^2 + M_Z^2 c_{2\beta}(T_q^3 - Q_q s_\mathrm{w}^2) & m_q \left(A_q^* - \mu\kappa_q\right) \\ m_q \left(A_q - \mu^*\kappa_q\right) & m_{\tilde{q}_\mathrm{R}}^2 + m_q^2 + M_Z^2 c_{2\beta} Q_q s_\mathrm{w}^2 \end{pmatrix},$$

$$\kappa_t = \frac{1}{t_\beta}, \quad \kappa_b = t_\beta,$$

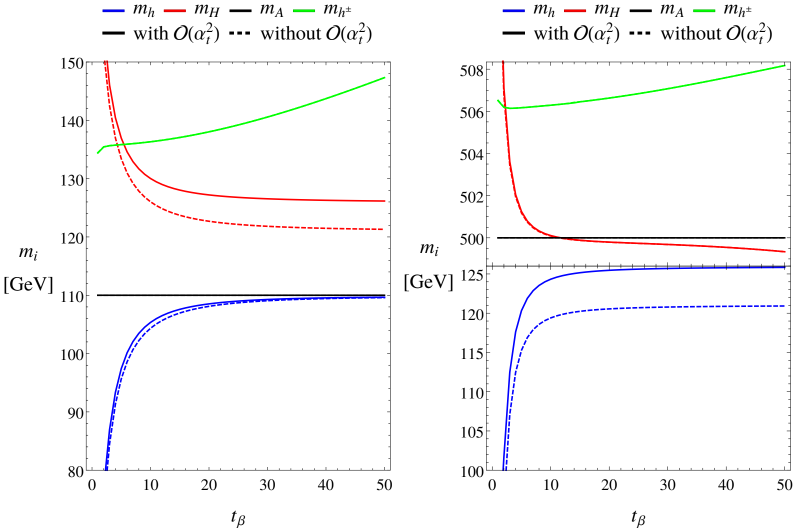
neutralinos:

$$\mathbf{Y} = \begin{pmatrix} M_1 & 0 & -M_Z s_\mathrm{w} c_\beta & M_Z s_\mathrm{w} s_\beta \\ 0 & M_2 & M_Z c_\mathrm{w} c_\beta & M_Z c_\mathrm{w} s_\beta \\ -M_Z s_\mathrm{w} c_\beta & M_Z c_\mathrm{w} c_\beta & 0 & -\mu \\ M_Z s_\mathrm{w} s_\beta & M_Z c_\mathrm{w} s_\beta & -\mu & 0 \end{pmatrix},$$

charginos:

$$\mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}.$$

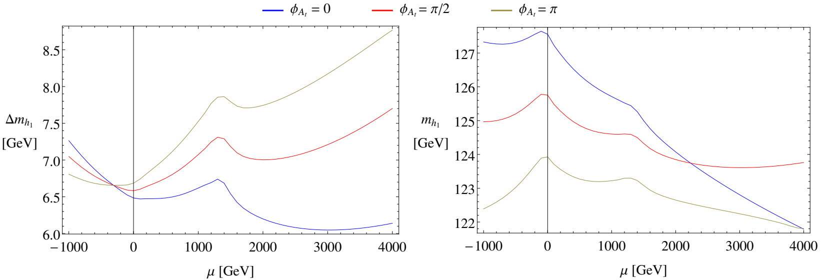
Dependence on tan β in the real MSSM



Left: $m_A = 500$ GeV. Right: $m_A = 110$ GeV. The other parameters are fixed at: $\mu = 200$ GeV, $m_{\tilde{g}} = 1500$ GeV, $m_{\tilde{q}_3} = m_{\tilde{t}_\mathrm{R}} = m_{\tilde{b}_\mathrm{R}} = 1000$ GeV, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_\mathrm{R}} = 1000$ GeV, $A_t = A_b = A_\tau = 1.5\, m_{\tilde{q}_3}$.

Dependence on μ in the complex MSSM

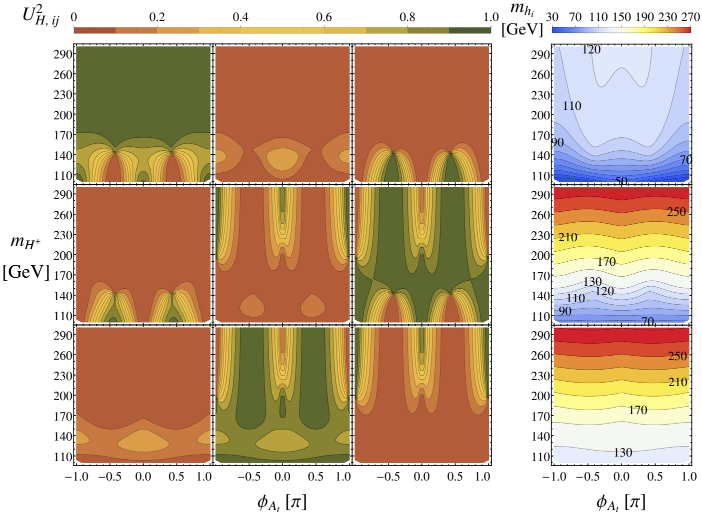
shift of lightest Higgs-boson mass for different ϕ_{A_t}



The parameters are chosen as follows: $t_\beta = 7$, $m_{\tilde{q}_3} = m_{\tilde{t}_\mathrm{R}} = m_{\tilde{b}_\mathrm{R}} = 1500$ GeV, $m_{\tilde{g}} = 1500$ GeV, $m_A = 500$ GeV, $A_t = A_b = A_\tau = 1.6 m_{\tilde{q}_3}$, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_\mathrm{R}} = 1000$ GeV.

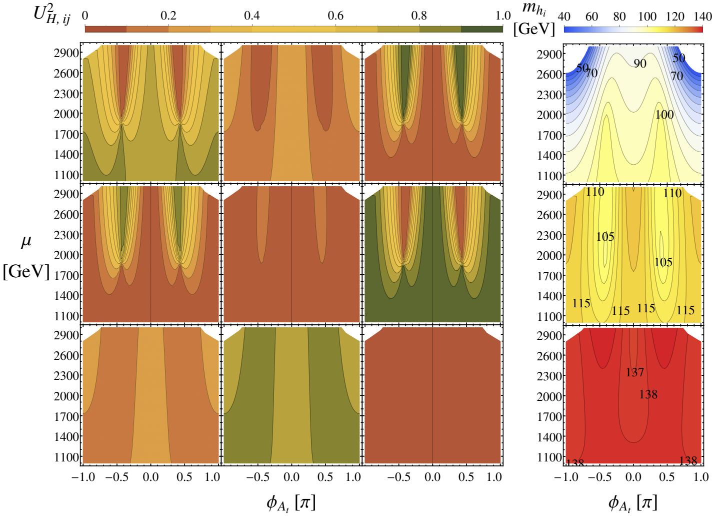
CP mixing in the complex MSSM

dependence on $m_{H^\pm}$



The input parameters are fixed at $\mu = 2000$ GeV, $t_\beta = 5$, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_{\rm R}} = 1000$ GeV,
$m_{\tilde{q}_3} = m_{\tilde{t}_{\rm R}} = m_{\tilde{b}_{\rm R}} = 1000$ GeV, $|A_t| = |A_b| = |A_\tau| = 2\,m_{\tilde{q}_3}$, $m_{\tilde{g}} = 1500$ GeV.

CP mixing in the complex MSSM

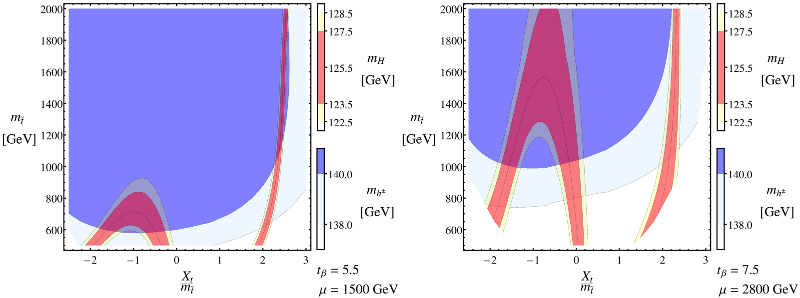


dependence on μ

The input parameters are fixed at $m_{H^\pm} = 140$ GeV, $t_\beta = 5$, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_\mathrm{R}} = 1000$ GeV, $m_{\tilde{q}_3} = m_{\tilde{t}_\mathrm{R}} = m_{\tilde{b}_\mathrm{R}} = 1000$ GeV, $|A_t| = |A_b| = |A_\tau| = 2\, m_{\tilde{q}_3}$, $m_{\tilde{g}} = 1500$ GeV.

Inverted Higgs-boson mass hierarchy in real MSSM

interpretation of heavy *CP*-even Higgs boson as measured particle



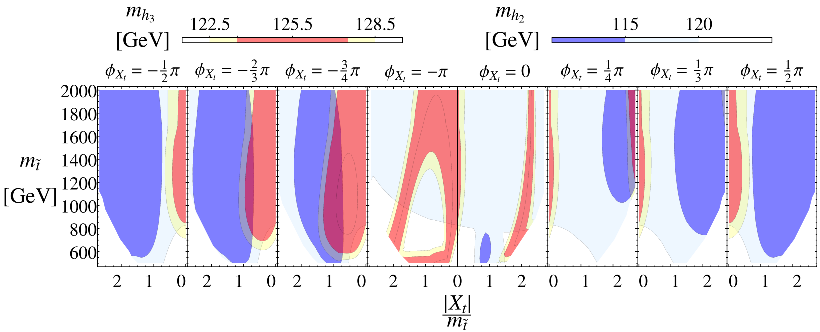
$t_\beta = 5.5$
$\mu = 1500$ GeV

$t_\beta = 7.5$
$\mu = 2800$ GeV

The other parameters are $m_{\tilde{g}} = 1500$ GeV, $A_b = A_\tau = 0$, $m_{\tilde{b}_{\mathrm{R}}} = m_{\tilde{\ell}_3} = m_{\tilde{\tau}_{\mathrm{R}}} = 1000$ GeV.

Inverted Higgs-boson mass hierarchy in complex MSSM

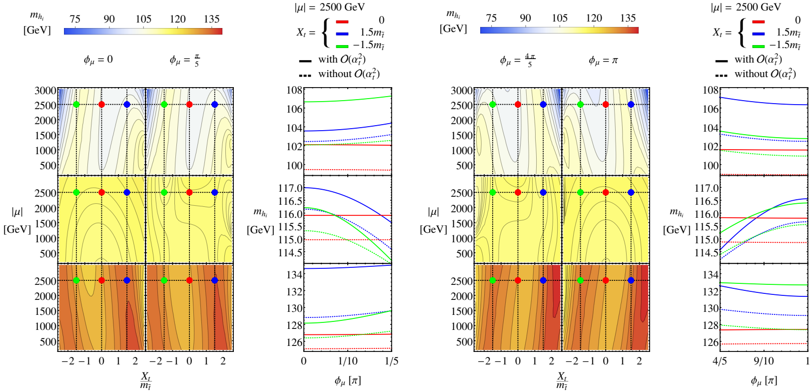
interpretation of heaviest Higgs boson as measured particle



Parameter region at $t_\beta = 7.5$, $\mu = 2800$ GeV.

The other input parameters are $m_{\tilde{b}_\mathrm{R}} = m_{\tilde{\ell}_3} = m_{\tilde{\tau}_\mathrm{R}} = 1000$ GeV, $A_b = A_\tau = 0$, $m_{\tilde{g}} = 1500$ GeV.

Dependence on complex μ



The input parameters are $t_\beta = 7.5$, $m_{H^\pm} = 140$ GeV, $m_{\tilde{t}} \equiv m_{\tilde{q}_3} = m_{\tilde{t}_\mathrm{R}} = m_{\tilde{b}_\mathrm{R}} = 1500$ GeV, $m_{\tilde{\ell}_3} = m_{\tilde{\tau}_\mathrm{R}} = 1500$ GeV, $A_b = A_\tau = 0$, $m_{\tilde{g}} = 1500$ GeV.