Multiple Interactions and Parton Rescattering with Pythia

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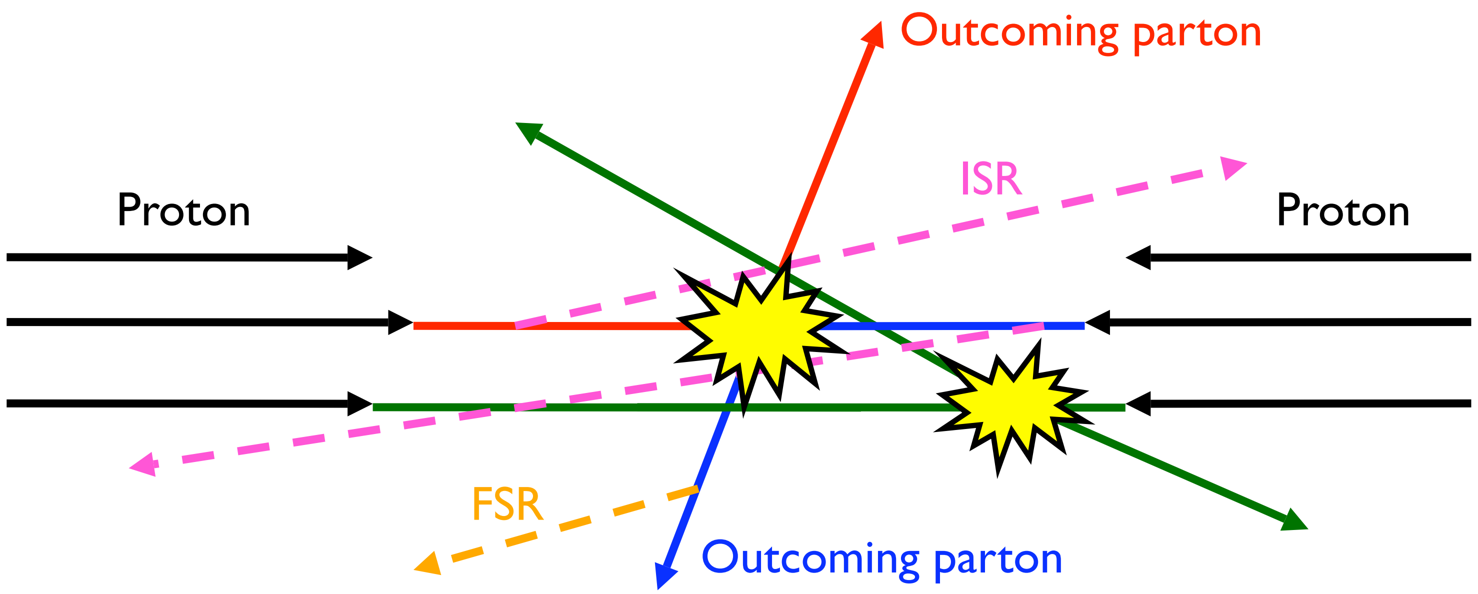
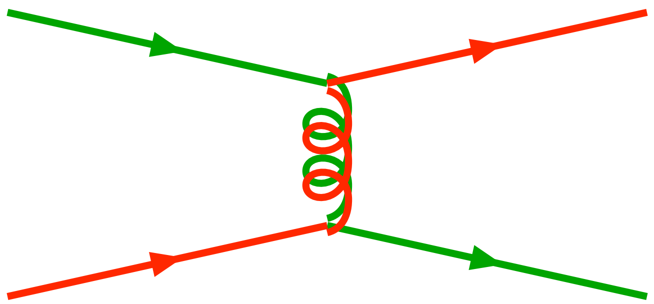
CMS Hamburg Meeting

September 10th 2008



What are MI?

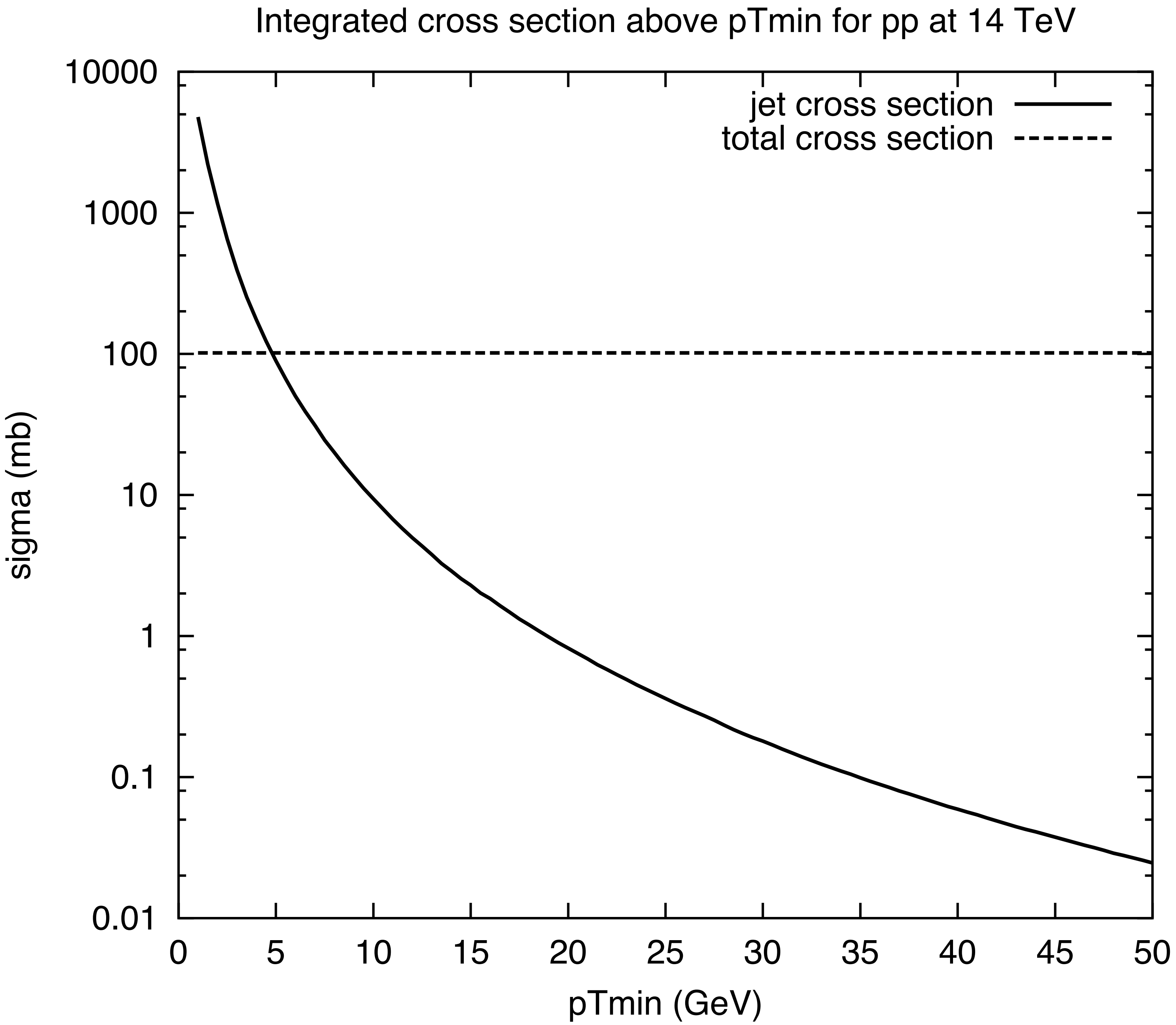
Cross section for $2 \to 2$ interactions is dominated by t-channel gluon exchange, so diverges like $\mathrm{d}\hat{\sigma}/\mathrm{d}p_{\perp}^2 \approx 1/p_{\perp}^4$ for $p_{\perp} \to 0$.



integrate QCD $2 \to 2$

$\mathrm{qq'} \to \mathrm{qq'}$
$\mathrm{q\overline{q}} \to \mathrm{q'\overline{q}'}$
$\mathrm{q\overline{q}} \to \mathrm{gg}$
$\mathrm{qg} \to \mathrm{qg}$
$\mathrm{gg} \to \mathrm{gg}$
$\mathrm{gg} \to \mathrm{q\overline{q}}$

with CTEQ 5L PDF's



Solution I

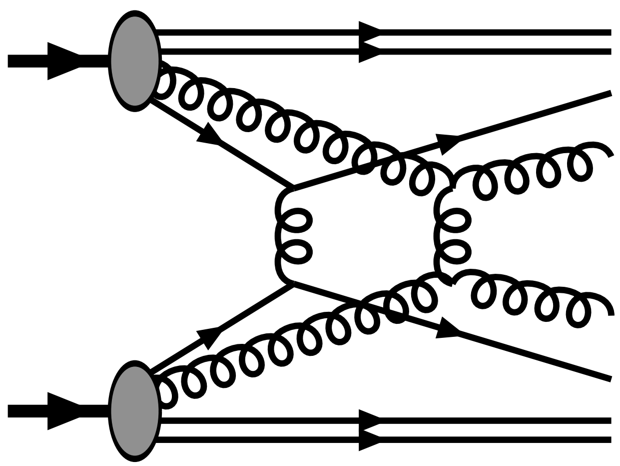
$$\sigma_{\text{int}}(p_{\perp\text{min}}) = \iiint_{p_{\perp\text{min}}} \mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}p_\perp^2\, f_1(x_1,p_\perp^2)\, f_2(x_2,p_\perp^2)\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2}$$

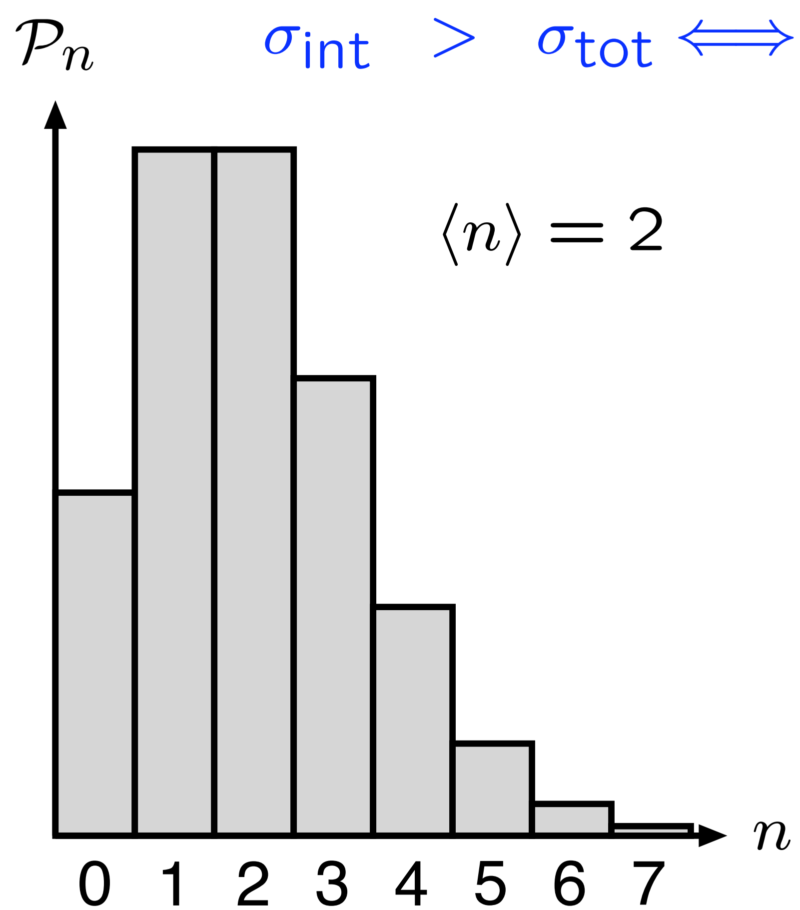
Half a solution to $\sigma_{\text{int}}(p_{\perp\text{min}}) > \sigma_{\text{tot}}$: many interactions per event

$$\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\text{int}} = \sum_{n=0}^{\infty} n\,\sigma_n$$

$$\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1$$





If interactions occur independently
then **Poissonian statistics**

$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!}\, e^{-\langle n \rangle}$$

but energy–momentum conservation
$\Rightarrow$ large n suppressed

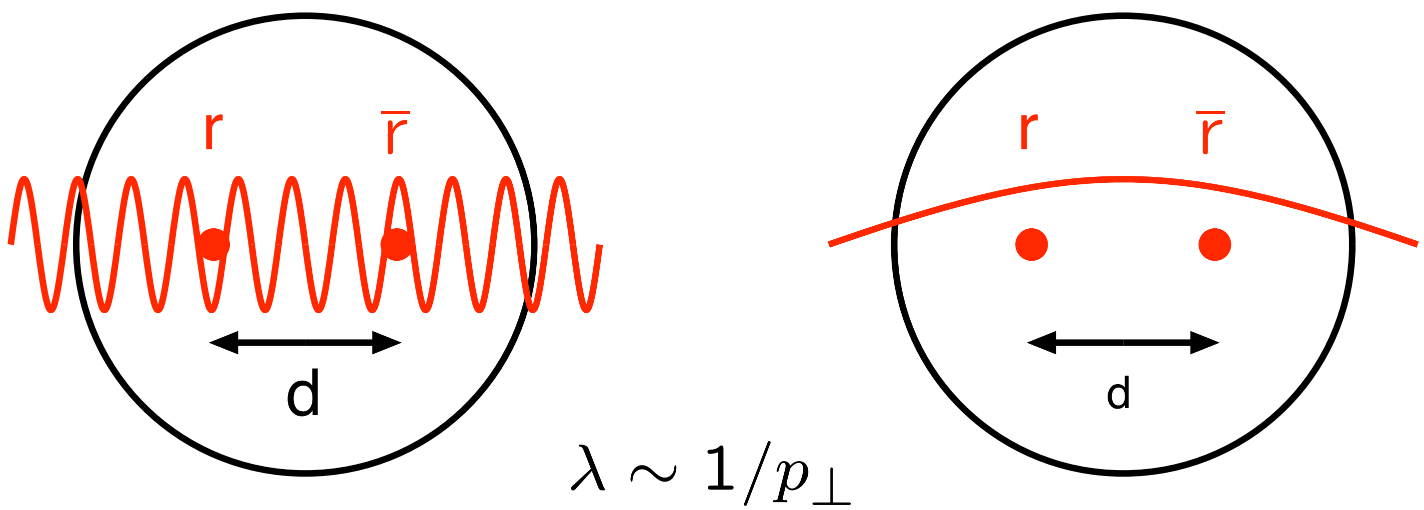
Solution 2

Other half of solution:
perturbative QCD not valid at small $p_\perp$ since q, g not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp\mathrm{min}} \simeq \frac{\hbar}{r_\mathrm{p}} \approx \frac{0.2\ \mathrm{GeV}\cdot\mathrm{fm}}{0.7\ \mathrm{fm}} \approx 0.3\ \mathrm{GeV} \simeq \Lambda_\mathrm{QCD}$$

. . . but better replace r_p by (unknown) colour screening length d in hadron



resolved

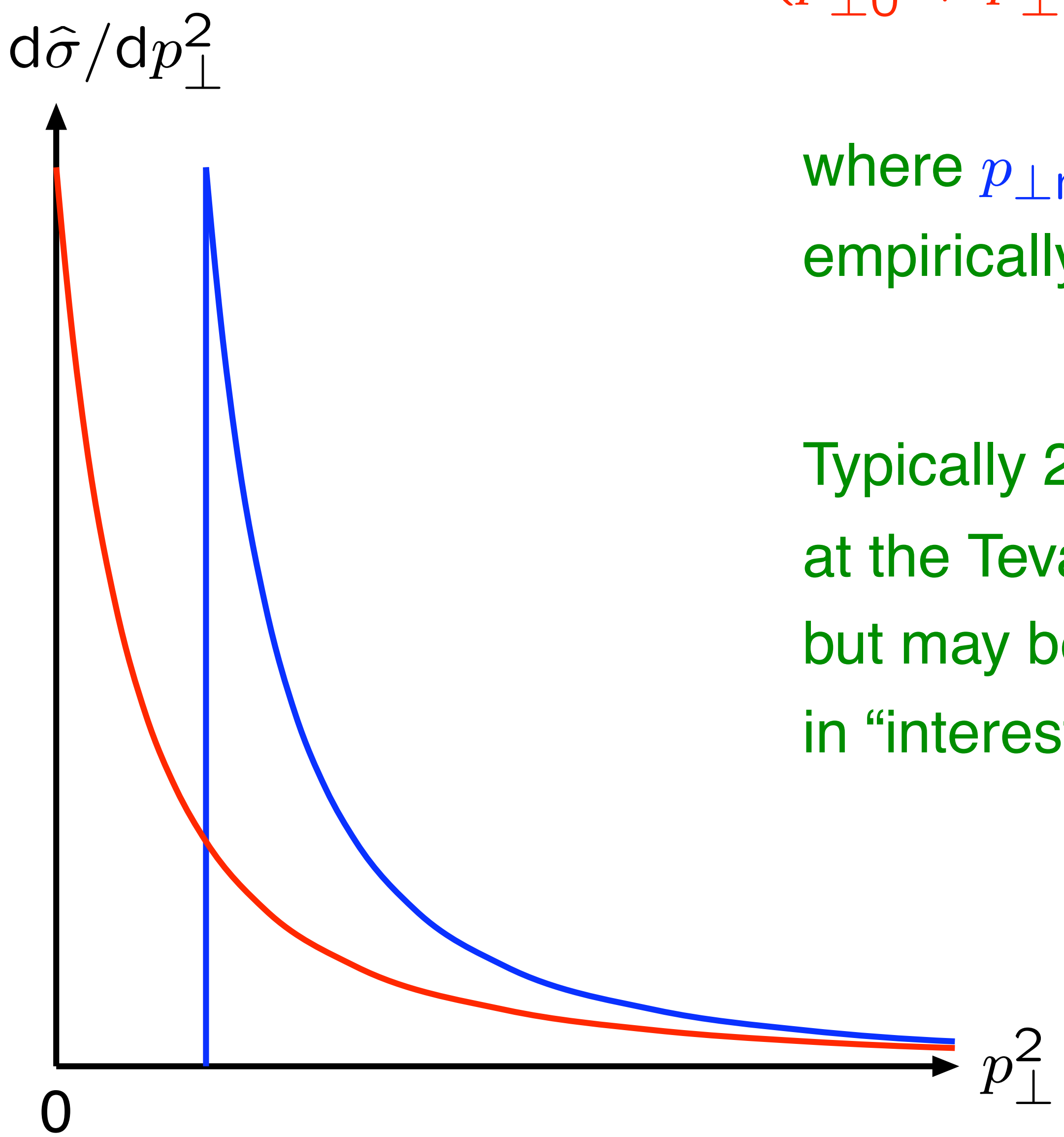
screened

Colour Screening

so modify

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} \propto \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4} \quad \to \quad \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4}\,\theta\left(p_\perp - p_{\perp\mathrm{min}}\right) \qquad \text{(simpler)}$$

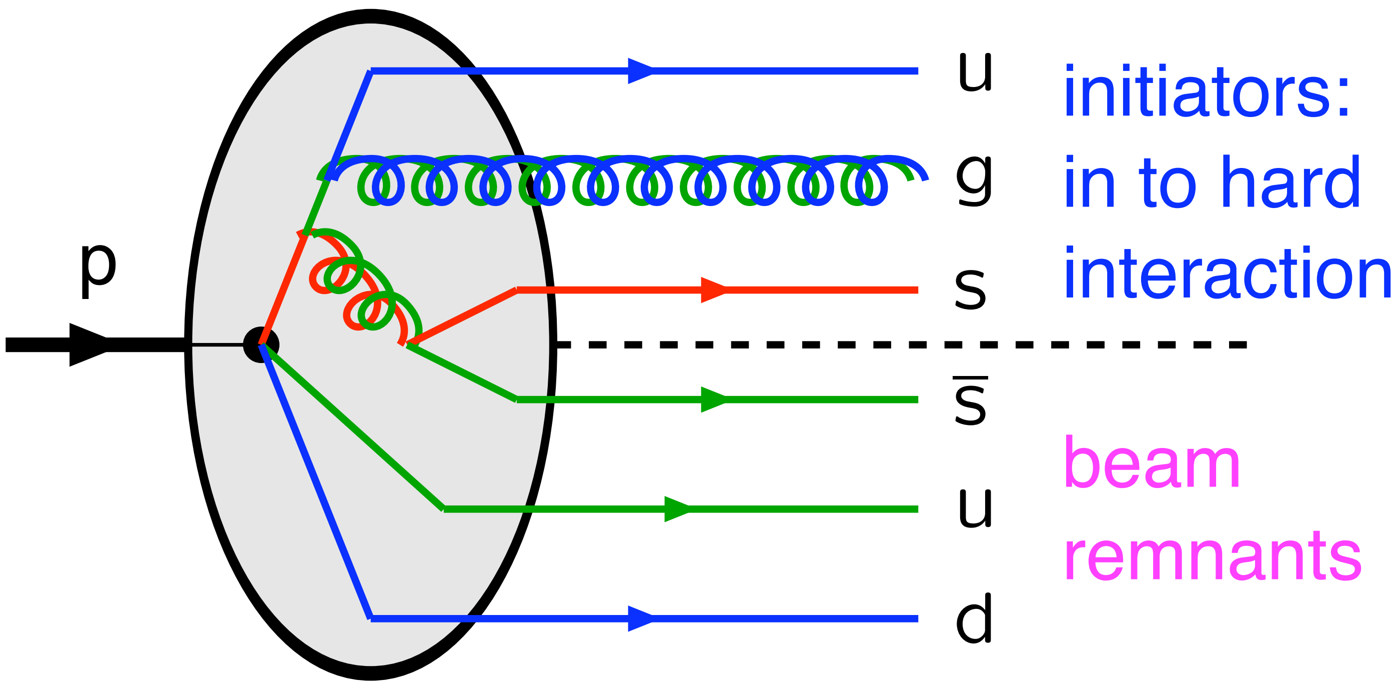
$$\text{or} \quad \to \quad \frac{\alpha_s^2(p_{\perp 0}^2 + p_\perp^2)}{(p_{\perp 0}^2 + p_\perp^2)^2} \qquad \text{(more physical)}$$



where $p_{\perp\mathrm{min}}$ or $p_{\perp 0}$ are free parameters, empirically of order **2 GeV**

Typically 2 – 3 interactions/event at the Tevatron, 4 – 5 at the LHC, but may be more in “interesting” high-$p_\perp$ ones.

Initiators and Remnants



Need to assign:

- correlated flavours
- correlated $x_i = p_{zi}/p_{z\text{tot}}$
- correlated primordial $k_{\perp i}$
- correlated colours
- correlated showers

- **PDF after preceding MI/ISR activity:**

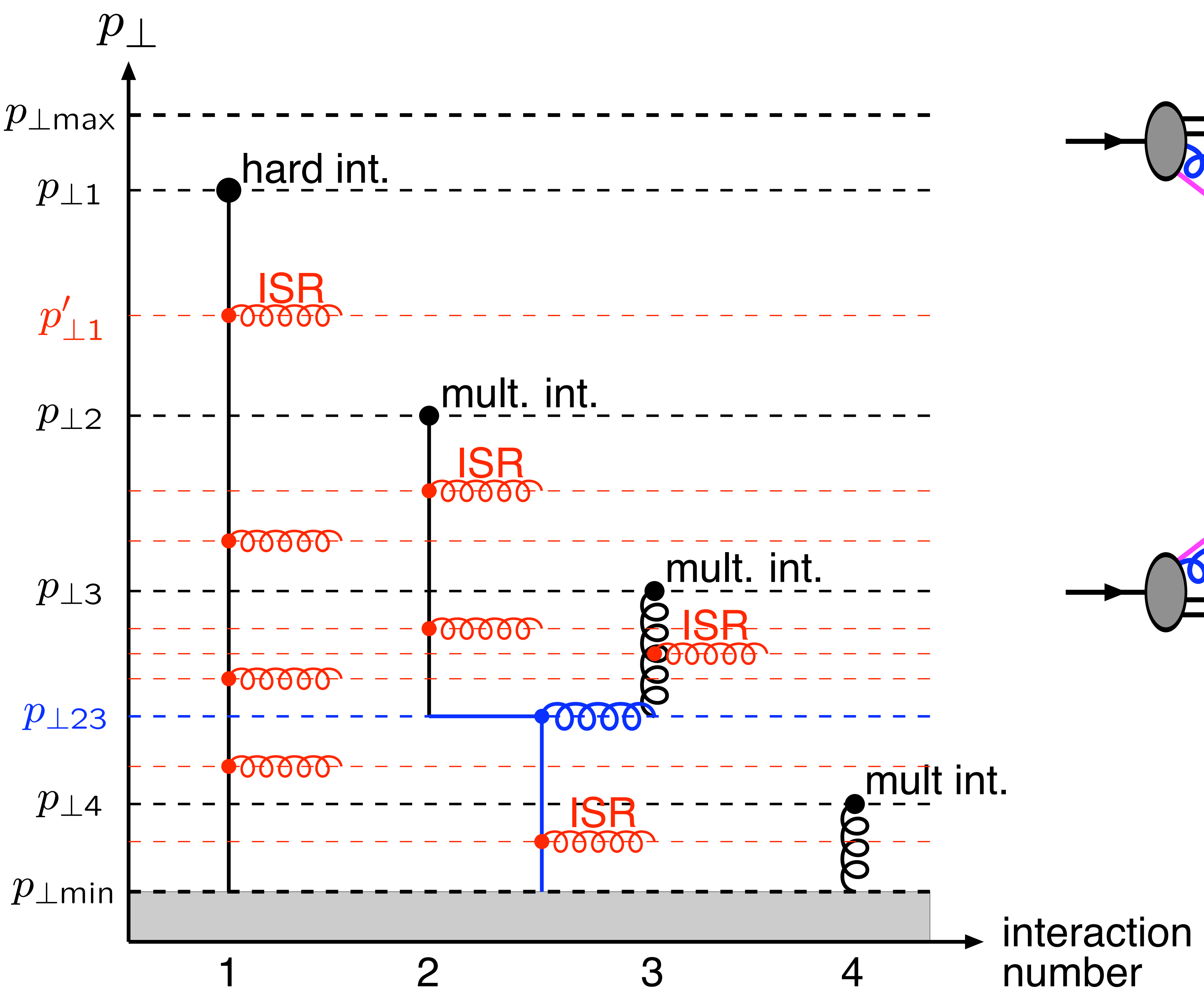
0) Squeeze range $0 < x < 1$ into $0 < x < 1 - \sum x_i$ (ISR: $i \neq i_{\text{current}}$)

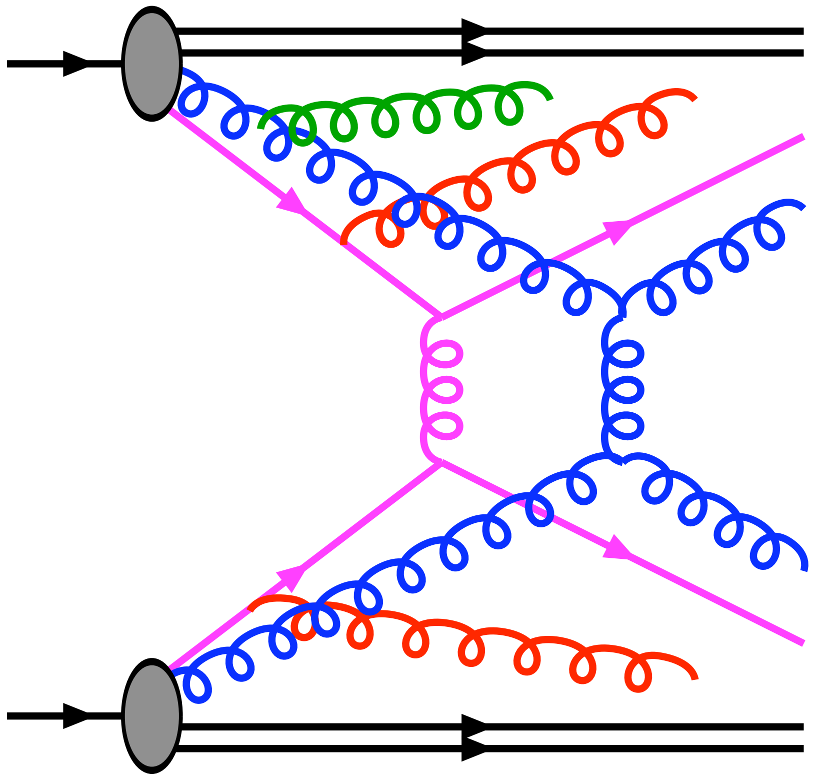
1) Valence quarks: scale down by number already kicked out

2) Introduce companion quark $\mathrm{q}/\bar{\mathrm{q}}$ to each kicked-out sea quark $\bar{\mathrm{q}}/\mathrm{q}$, with x based on assumed $\mathrm{g} \to \mathrm{q}\bar{\mathrm{q}}$ splitting

3) Gluon and other sea: rescale for total momentum conservation

Interleaved MI





MI: New Evolution Equation

	time	evolution	probability
FSR	forwards	$p_\perp \searrow 0$	normal & local
ISR	backwards	$p_\perp \searrow 0$	conditional
MI	simultaneous	$p_\perp \searrow 0$	conditional

ISR + MI: PDF competition $\Rightarrow$ interleaving (PYTHIA 6.3)

FSR: previously at end, now also interleaved (PYTHIA 8.1):

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}p_\perp} = \left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{MI}}}{\mathrm{d}p_\perp} + \sum \frac{\mathrm{d}\mathcal{P}_{\mathrm{ISR}}}{\mathrm{d}p_\perp} + \sum \frac{\mathrm{d}\mathcal{P}_{\mathrm{FSR}}}{\mathrm{d}p_\perp}\right)$$
$$\times \quad \exp\left(-\int_{p_\perp}^{p_{\perp i-1}} \left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{MI}}}{\mathrm{d}p'_\perp} + \sum \frac{\mathrm{d}\mathcal{P}_{\mathrm{ISR}}}{\mathrm{d}p'_\perp} + \sum \frac{\mathrm{d}\mathcal{P}_{\mathrm{FSR}}}{\mathrm{d}p'_\perp}\right) \mathrm{d}p'_\perp\right)$$

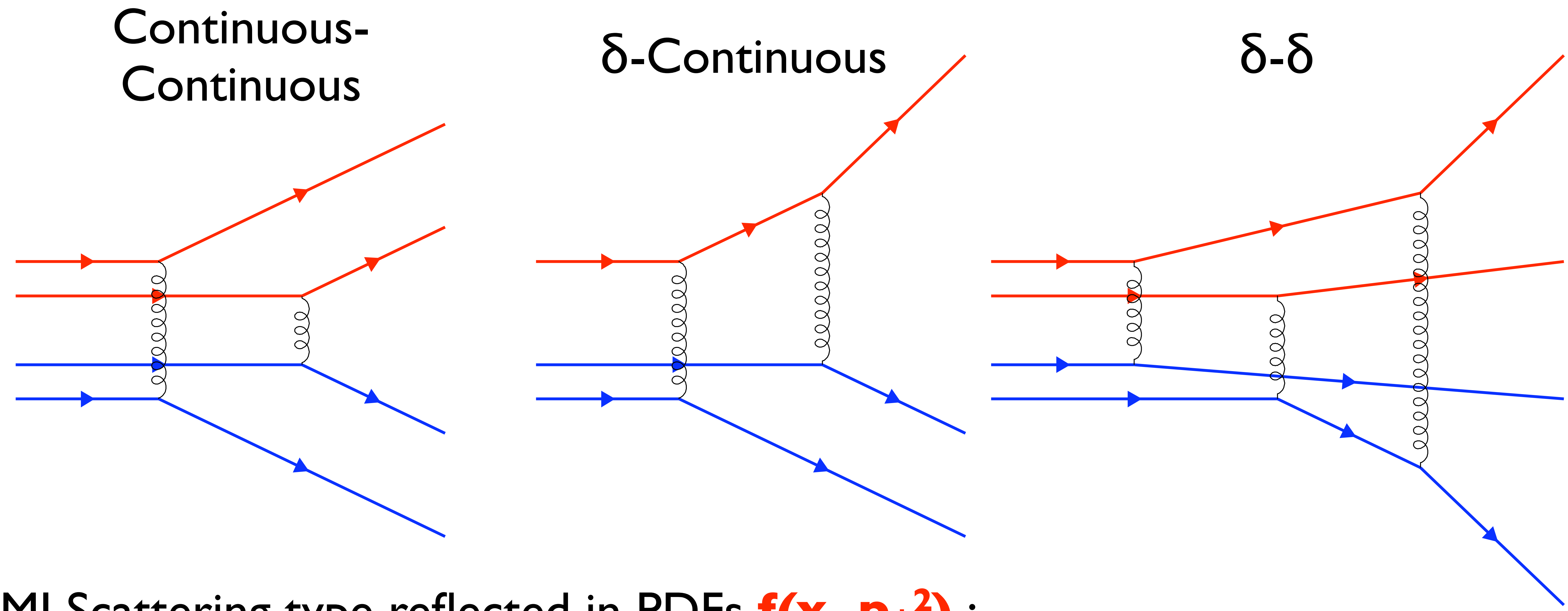
"resolution evolution"

Monte Carlo: winner takes all

+ many other assumptions/models

MI: Rescattering

Distinguish 3 types of multiple interactions:



MI Scattering type reflected in PDFs $\mathbf{f(x, p_\perp^2)}$:

$$\sigma_{\mathrm{int}}(p_{\perp\mathrm{min}}) = \iiint\limits_{p_{\perp\mathrm{min}}} \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}p_\perp^2 \, \mathbf{f_1(x_1, p_\perp^2) f_2(x_2, p_\perp^2)} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2}$$

Reminder: Veto-Algorithm (I)

"Radioactive decay" problem:

Looking for probability that *something* (nucleus decay, parton branch, multiple interaction) happens at time t provided that it did not happen at earlier times t'

$$\mathcal{P}(t) = f(t) \underbrace{\exp\left\{-\int_0^t f(t')\mathrm{d}t'\right\}}_{\to \text{ Sudakov}}$$

Use-case: Interleaved Evolution (Pythia)

- For each step: Ask for p_T of next ISR, FSR, MI
- Choose interaction with largest p_T

Reminder: Veto-Algorithm (2)

Case 1: f(t) has primitive function F(t) with known inverse

$$\int_0^t \mathcal{P}(t')\mathrm{d}t' = 1 - \exp\left\{-\int_0^t f(t')\mathrm{d}t'\right\} = 1 - \mathcal{R} \Leftrightarrow t = F^{-1}(F(0) - \ln \mathcal{R})$$

Case 2: f(t) ≤ g(t) with primitive function G(t) with known inverse

→ use **"Veto-Algorithm"**

1. start with $i = 0$ and $t_0 = 0$;
2. add 1 to i and select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$, i.e. according to $g(t)$, but with the constraint that $t_i > t_{i-1}$,
3. compare a (new) R with the ratio $f(t_i)/g(t_i)$; if $f(t_i)/g(t_i) \leq R$, then return to point 2 for a new try;
4. otherwise t_i is retained as final answer.

δ-δ-Type Rescattering

$$\sigma_{\mathrm{int}}(p_{\perp\min}) = \iiint_{p_{\perp\min}} \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}p_\perp^2 \, \delta(\mathbf{x_1} - \mathbf{x_1^0}, \mathbf{p_\perp^2}) \delta(\mathbf{x_2} - \mathbf{x_2^0}, \mathbf{p_\perp^2}) \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2}$$

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \left| \frac{\mathrm{d}\hat{t}}{\mathrm{d}p_\perp^2} \right| = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \frac{1}{\sqrt{1 - \frac{4p_\perp^2}{\hat{s}}}} \propto \frac{1 - \frac{x}{2}}{x^2 \sqrt{1-x}}$$

$$\uparrow$$

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \propto \frac{N}{2} \left(\frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2} \right) = 2N \left(\frac{2-x}{4p_\perp^4} \right) \quad \text{where } x \equiv \frac{4p_\perp^2}{\hat{s}}$$

Veto Algorithm: $\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} \propto \frac{1 - \frac{x}{2}}{x^2 \sqrt{1-x}} \leq \frac{1}{x^2} + \frac{1}{2\sqrt{1-x}}$

$$\text{pick } x_1 \text{ according to } \frac{1}{x^2} \exp\left(- \int_x^{x_{\max}} \frac{1}{x'^2} \mathrm{d}x' \right)$$

$$\text{pick } x_2 \text{ according to } \frac{1}{2\sqrt{1-x}} \exp\left(- \int_x^{x_{\max}} \frac{1}{2\sqrt{1-x'}} \mathrm{d}x' \right)$$

Winner-takes-all: $x = \max\{x_1, x_2\}$

δ-Cont.-Type Rescattering

$$\sigma_{\text{int}}(p_{\perp\min}) = \iiint\limits_{p_{\perp\min}} \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}p_\perp^2 \, \delta(\mathbf{x_1} - \mathbf{x_1^0}, \mathbf{p_\perp^2}) f_2(x_2, p_\perp^2) \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2}$$

Variable substitution with y_3, y_4 (rapidities of the two produced partons):

$$\int \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} (\mathrm{d}\hat{t} + \mathrm{d}\hat{u}) = \int \mathrm{d}y_3 \mathrm{d}y_4 \mathrm{d}p_\perp^2$$

Result: $$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} = 2 \int \mathrm{d}\Delta y \; x_2 f_2(x_2) \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \text{ with } \Delta y = \frac{1}{2}(y_3 - y_4)$$

Study limits and pick Δy flat in allowed interval:

$$\Delta y_{\max} = \frac{1}{2} \ln \frac{E + p_z}{p_\perp} \leq \ln \frac{\sqrt{\hat{s}_{\max}}}{p_\perp}$$

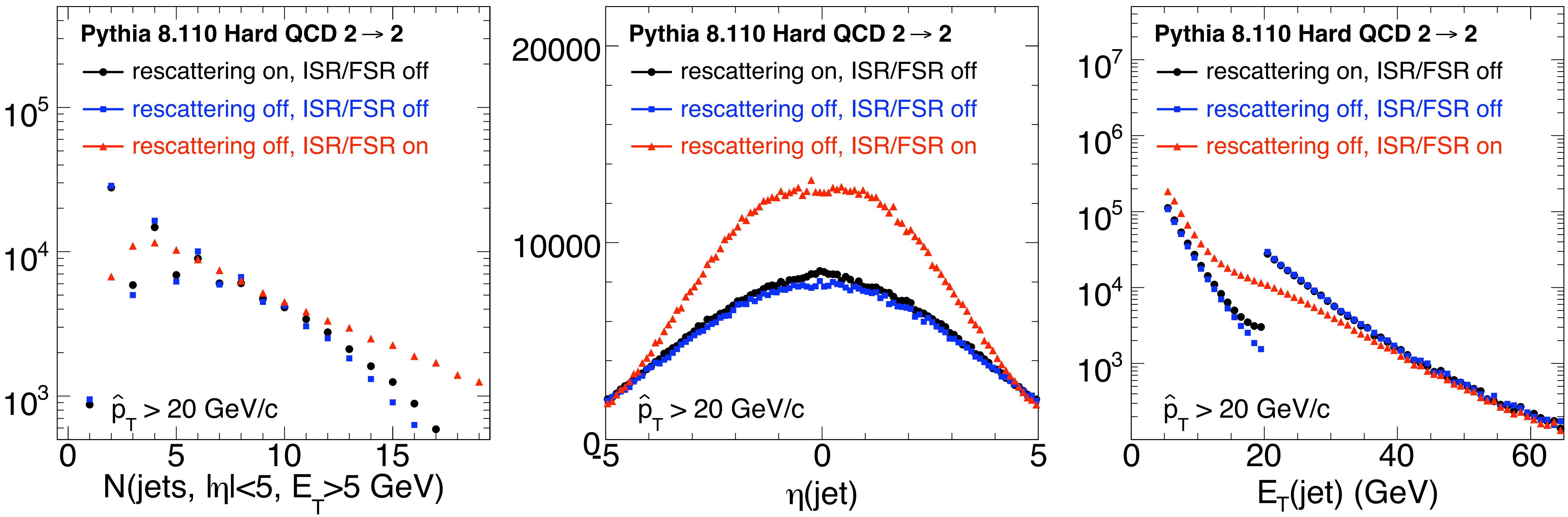
Solve for x_2: $$x_2 = \frac{4 p_\perp^2 \cosh^2 \Delta y}{s x_1^0}$$

Results: Jet-Kinematics

NB: Studies done before Rescattering + PS implemented

Compare three settings:

Pythia with rescattering
without rescattering
Influence of parton shower



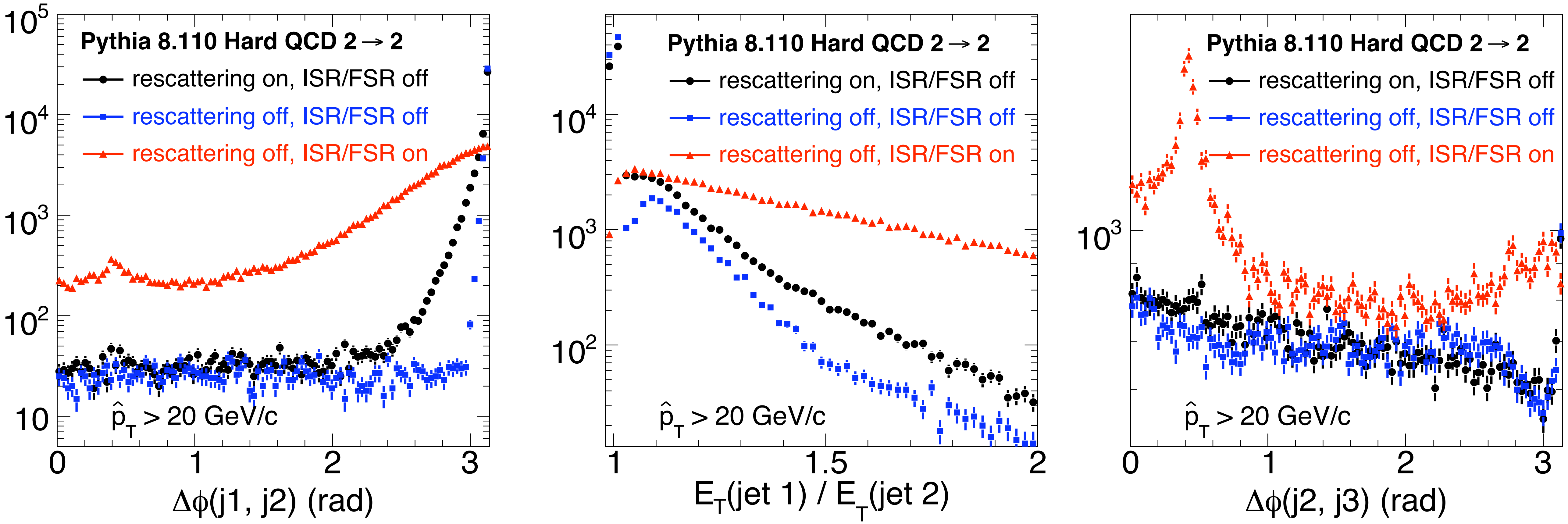
Parton level rescattering effects ~ few percent contribution

Results: Jet-Correlations

NB: Studies done before Rescattering + PS implemented

Compare three settings:

- Pythia with rescattering
- without rescattering
- Influence of parton shower



Parton level rescattering effects ~ few percent contribution

Summary

- **Pythia models multiple interactions in great detail**
- **Rescattering option available in Pythia 8**
- **Validation ongoing**
- **Percent level effects, may be ~10-20% in some distributions**

BACKUP

Modelling MI (I)

T. Sjöstrand, M. van Zijl, PRD36 (1987) 2019: first models for event properties based on perturbative multiple interactions

(1) Simple scenario:
no longer used (no impact-parameter dependence)

(2) More sophisticated scenario:
still in frequent use (Tune A, Tune DWT, ATLAS tune, . . .)

- Is only a model for nondiffractive events, i.e. for $\sigma_{\mathrm{nd}} \simeq (2/3)\sigma_{\mathrm{tot}}$
- Smooth turn-off at $p_{\perp 0}$ scale
- Require ≥ 1 interaction in an event
- Interactions generated in ordered sequence $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > \ldots$ by "Sudakov" trick (what happens "first"?)

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}p_{\perp i}} = \frac{1}{\sigma_{\mathrm{nd}}} \frac{\mathrm{d}\sigma}{\mathrm{d}p_{\perp}} \exp\left[- \int_{p_{\perp}}^{p_{\perp(i-1)}} \frac{1}{\sigma_{\mathrm{nd}}} \frac{\mathrm{d}\sigma}{\mathrm{d}p'_{\perp}} \mathrm{d}p'_{\perp} \right]$$

- After each interaction rescaled new PDF's for momentum conservation
- Leads to n_{int} narrower than Poissonian, except that . . .

Modelling MI (2)

- Hadrons are extended,
  e.g. double Gaussian ("hot spots"):

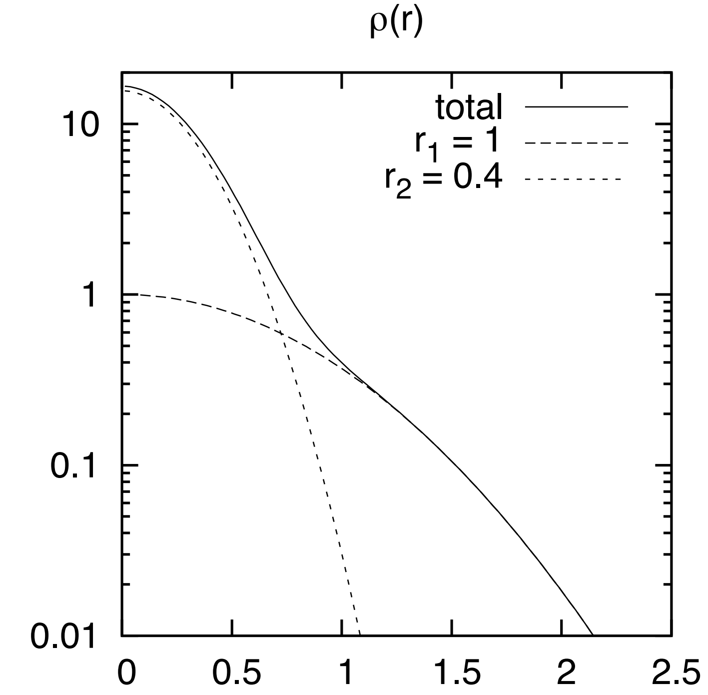
$$\rho_{\text{matter}}(r) = N_1 \exp\left(-\frac{r^2}{r_1^2}\right) + N_2 \exp\left(-\frac{r^2}{r_2^2}\right)$$

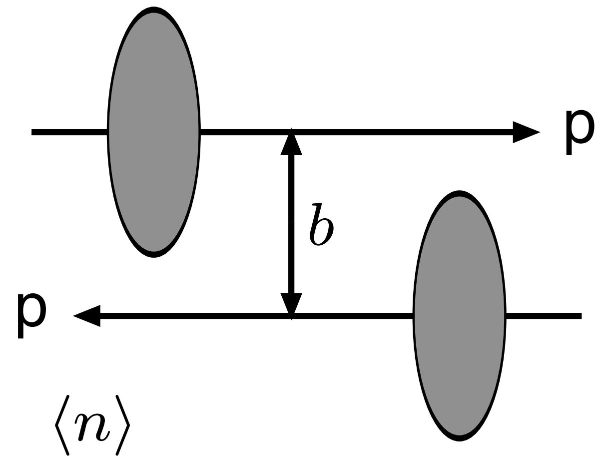
  where $r_2 \neq r_1$ represents "hot spots"
- Events are distributed in impact parameter b
- Overlap of hadrons during collision

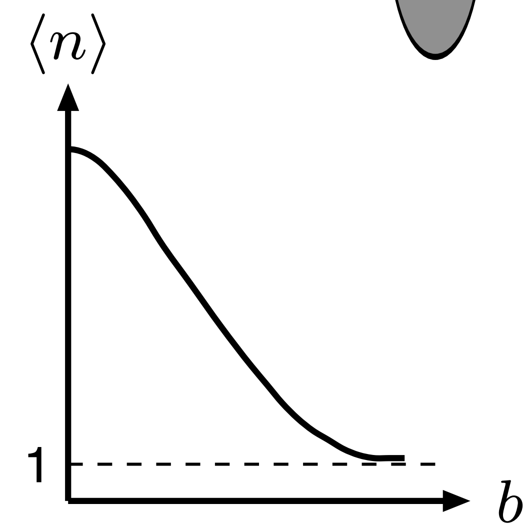
$$\mathcal{O}(b) = \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}t\; \rho_{1,\text{matter}}^{\text{boosted}}(\mathbf{x},t)\rho_{2,\text{matter}}^{\text{boosted}}(\mathbf{x},t)$$

- Average activity at b proportional to $\mathcal{O}(b)$
  $\Rightarrow$ central collisions normally more active
  $\Rightarrow$ $\mathcal{P}_n$ broader than Poissonian

- Time-consuming $(b, p_\perp)$ generation
- Problems if many valence quarks kicked out
  $\Rightarrow$ Simplify after first interaction:
  only gg or $q\bar{q}$ outgoing, no showers, ...







MI: Multiplicity, Correlation (I)

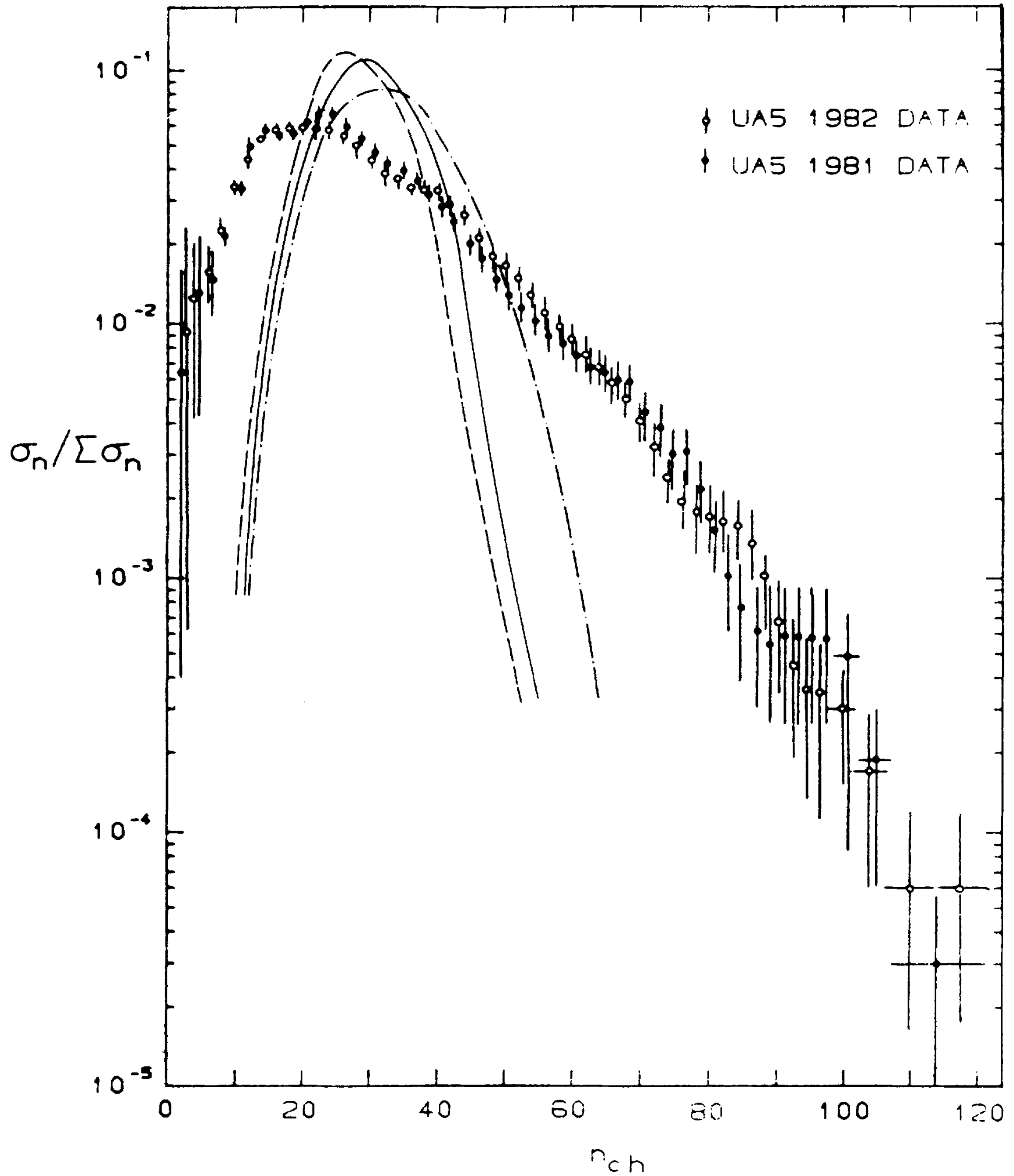


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

without multiple interactions

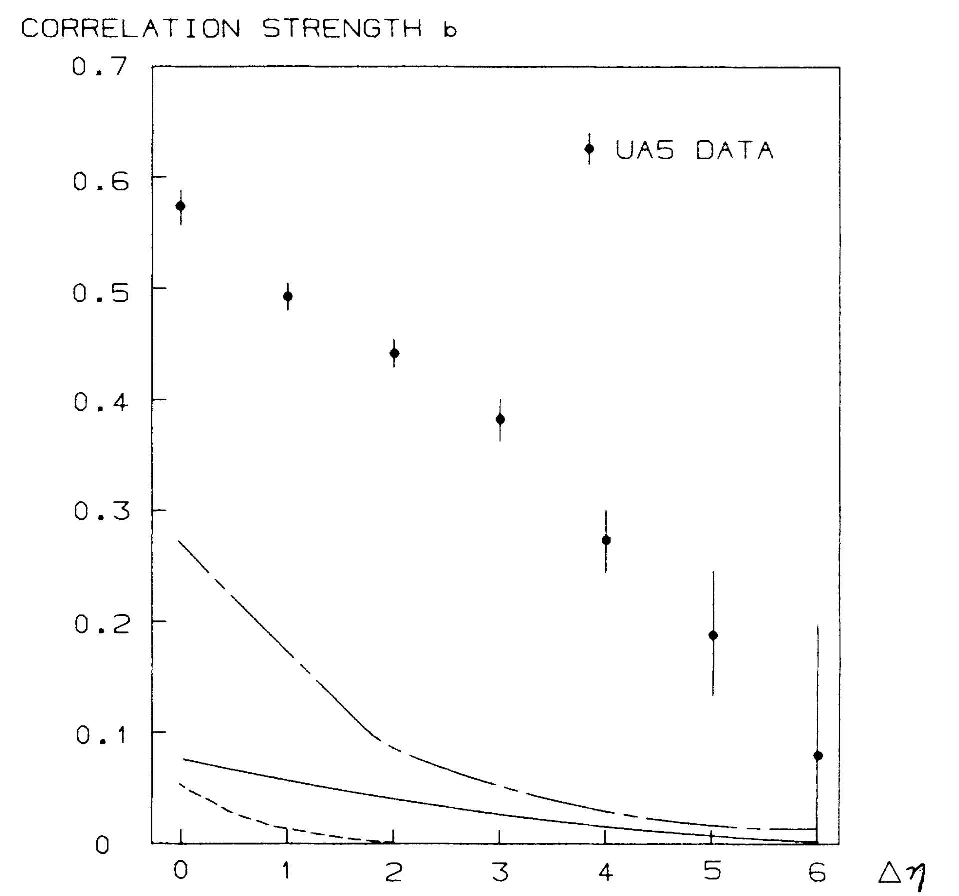


FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.

MI: Multiplicity, Correlation (2)

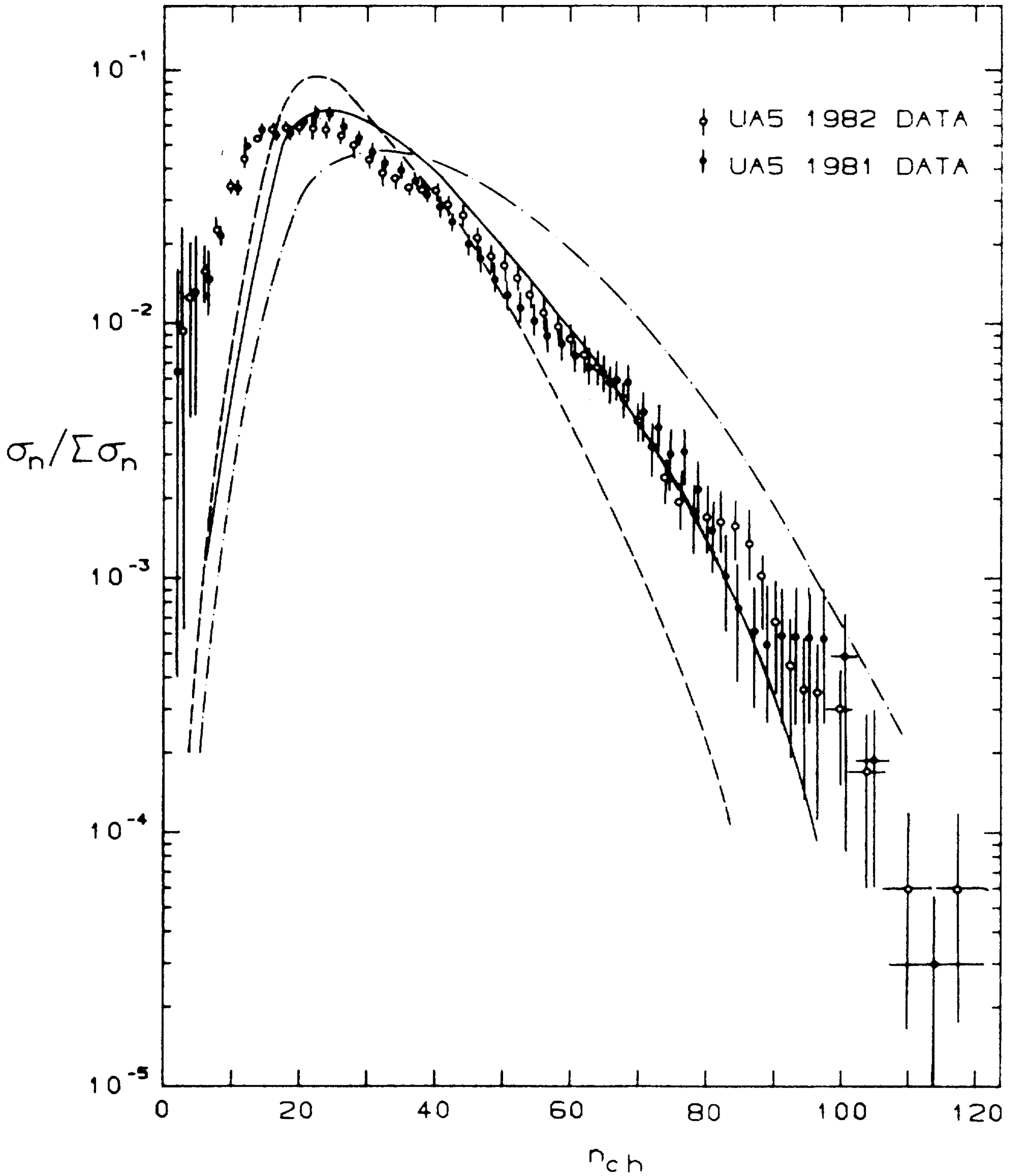


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\min}=2.0$ GeV; solid line, $p_{T\min}=1.6$ GeV; dashed-dotted line, $p_{T\min}=1.2$ GeV.

with multiple interactions

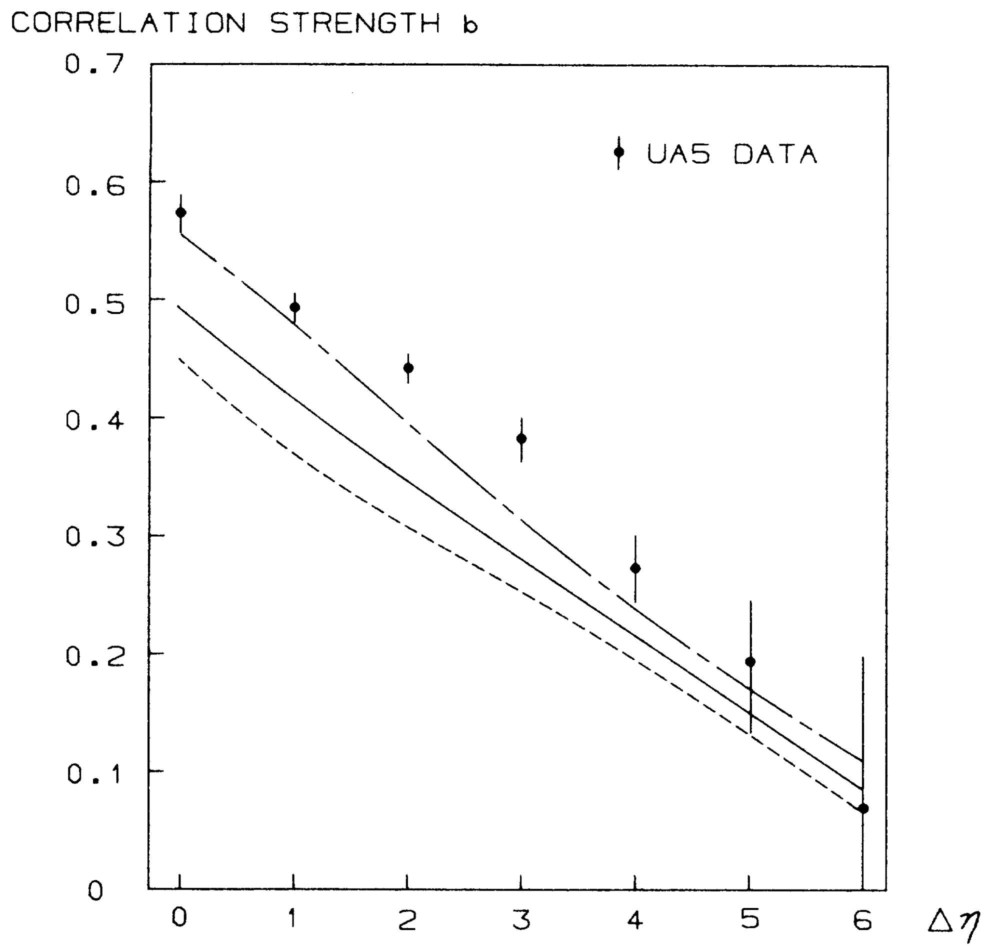


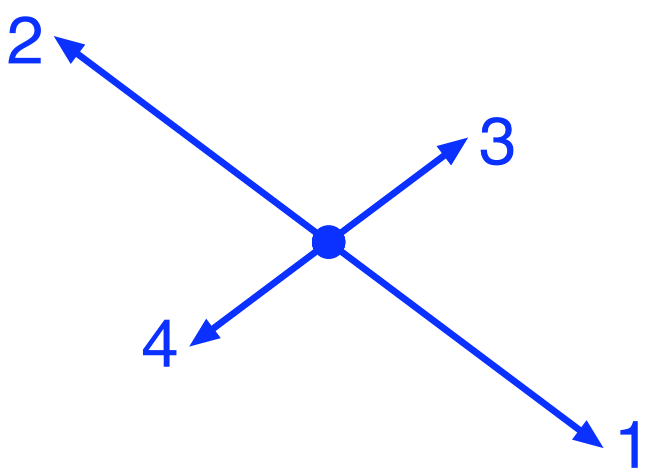
FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

Direct Observation of MI

Four studies: AFS (1987), UA2 (1991), CDF (1993, 1997)

Order 4 jets $\mathbf{p}_{\perp 1} > \mathbf{p}_{\perp 2} > \mathbf{p}_{\perp 3} > \mathbf{p}_{\perp 4}$ and define φ
as angle between $\mathbf{p}_{\perp 1} \mp \mathbf{p}_{\perp 2}$ and $\mathbf{p}_{\perp 3} \mp \mathbf{p}_{\perp 4}$ for AFS/CDF

Double Parton Scattering

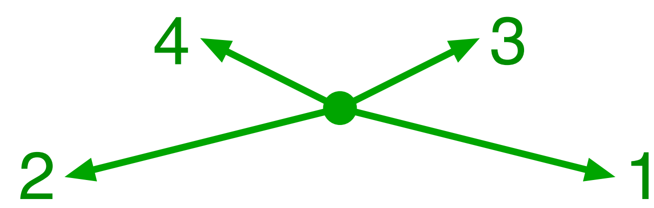


$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \approx 0$

$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \approx 0$

$\mathrm{d}\sigma/\mathrm{d}\varphi$ flat

Double BremsStrahlung



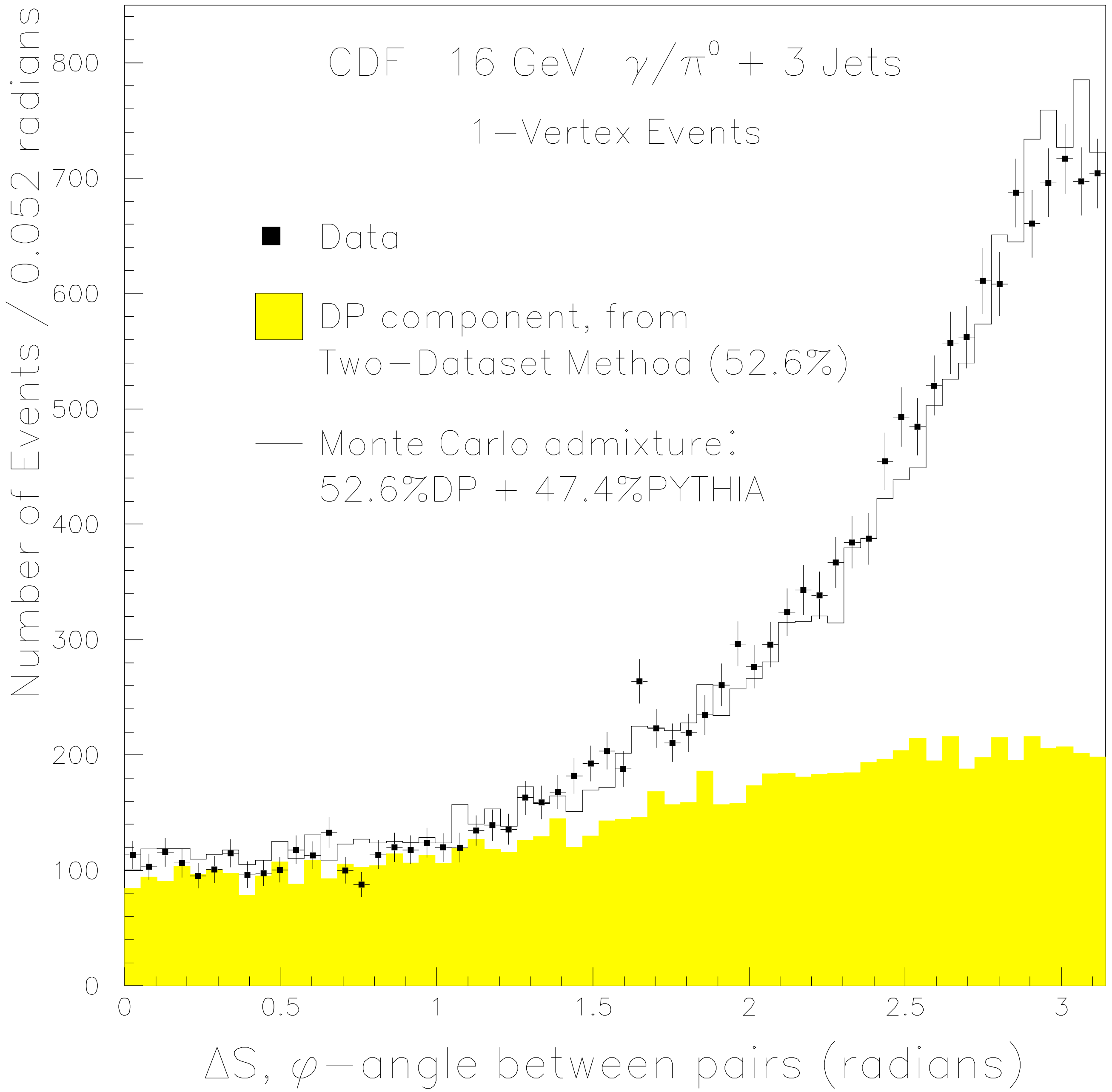
$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \gg 0$

$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \gg 0$

$\mathrm{d}\sigma/\mathrm{d}\varphi$ peaked at $\varphi \approx 0/\pi$ for AFS/CDF

AFS 4-jet analysis (pp at 63 GeV): observe 6 times Poissonian prediction,
with impact parameter expect 3.7 times Poissonian,
but big errors $\Rightarrow$ low acceptance, also UA2

CDF: Photon + 3 Jets



CDF 3-jet + prompt photon analysis

Yellow region = double parton scattering (DPS)

The rest = PYTHIA showers

$$\sigma_{\text{DPS}} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}} \quad \text{for } A \neq B \quad \Longrightarrow \sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}$$

Strong enhancement relative to naive expectations!

Jet Pedestal Effect

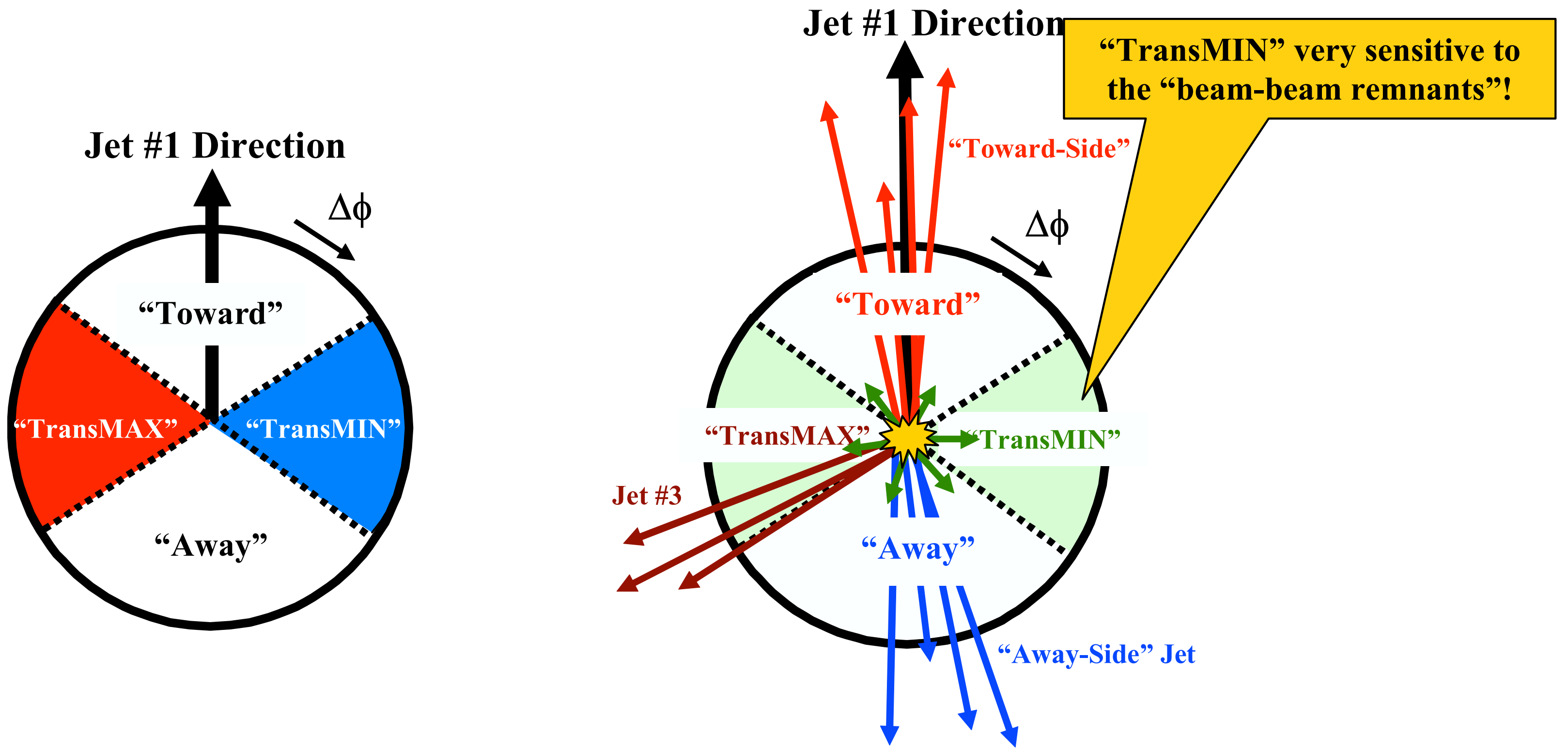
Events with hard scale (jet, W/Z, . . .) have more underlying activity!

Events with n interactions have n chances that one of them is hard,
so “trigger bias”: hard scale $\Rightarrow$ central collision
$\Rightarrow$ more interactions $\Rightarrow$ larger underlying activity.

Centrality effect saturates at $p_{\perp\mathrm{hard}} \sim 10$ GeV.

Studied in detail by Rick Field, comparing with CDF data:

“MAX/MIN Transverse” Densities



- **Define the MAX and MIN “transverse” regions on an event-by-event basis with MAX (MIN) having the largest (smallest) density.**

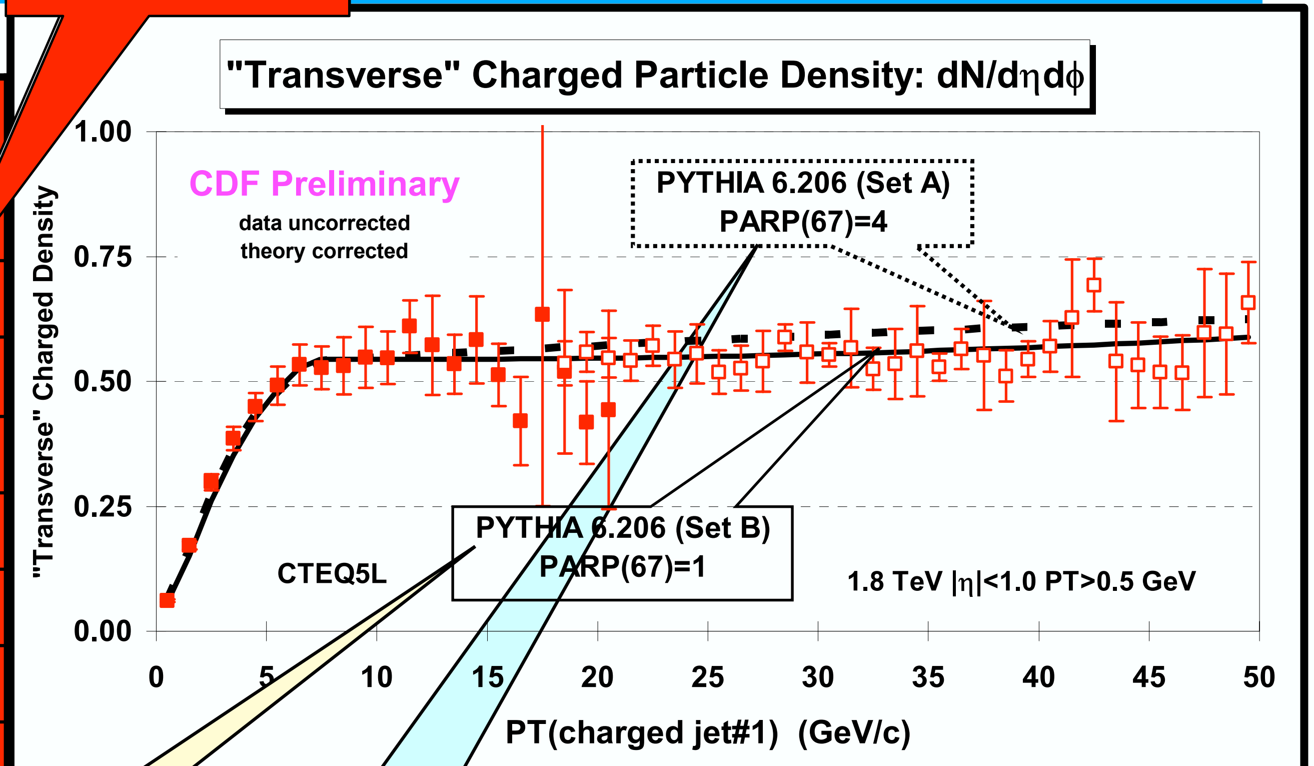
UE Tunes

Tuned PYTHIA 6.206

Tune A CDF
Run 2 Default!

PYTHIA 6.206 CTEQ5L

Parameter	Tune B	Tune A
MSTP(81)	1	1
MSTP(82)	4	4
PARP(82)	1.9 GeV	2.0 GeV
PARP(83)	0.5	0.5
PARP(84)	0.4	0.4
PARP(85)	1.0	0.9
PARP(86)	1.0	0.95
PARP(89)	1.8 TeV	1.8 TeV
PARP(90)	0.25	0.25
PARP(67)	1.0	4.0



Plot shows the "Transverse" charged particle density versus P_T(chgjet#1) compared to the QCD hard scattering predictions of two tuned versions of PYTHIA 6.206 (CTEQ5L, Set B (PARP(67)=1) and Set A (PARP(67)=4)).

New PYTHIA default
(less initial-state radiation)

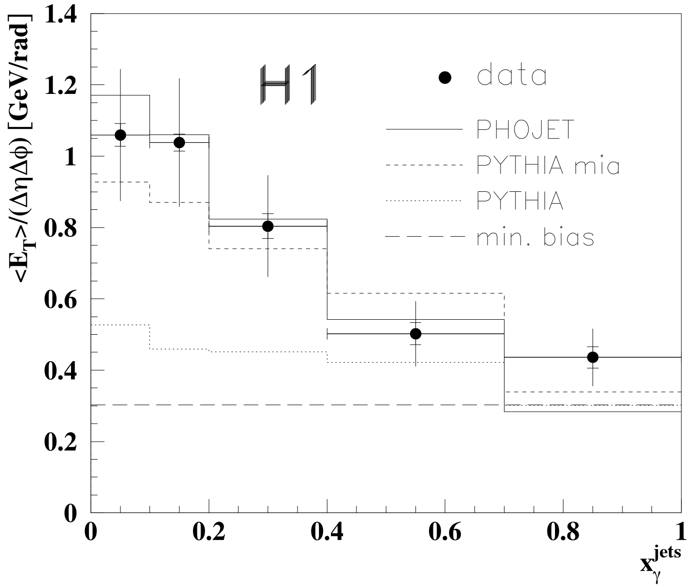
Old PYTHIA default
(more initial-state radiation)

CERN July 31, 2003 *Rick Field - Florida/CDF* *Page 24*

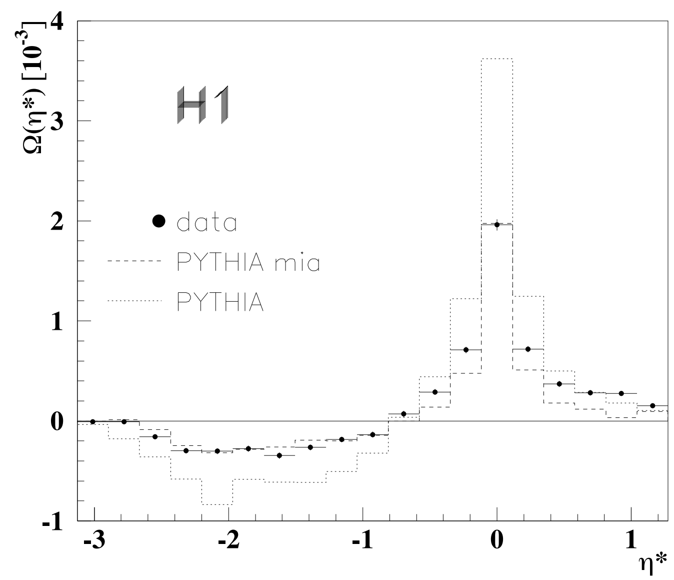
Photoproduction at HERA

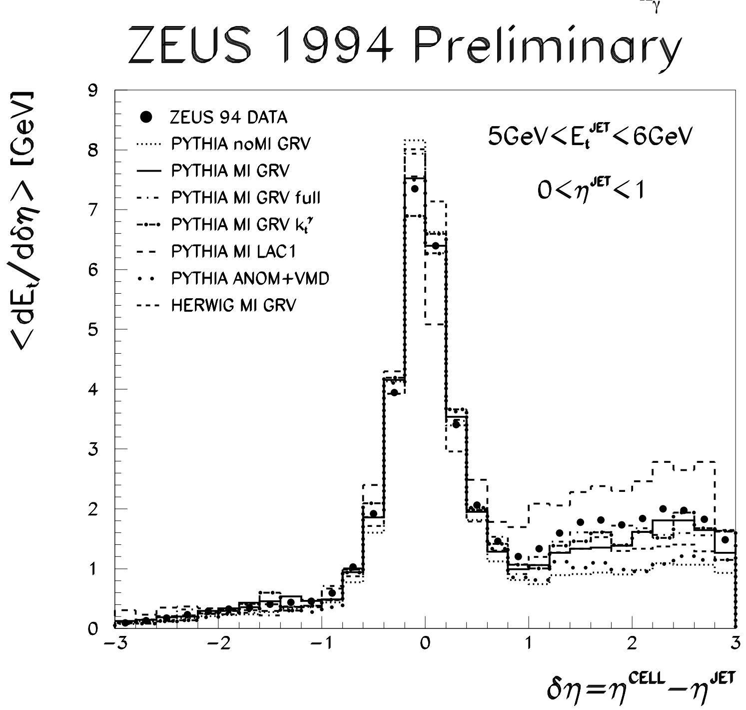
Multiple interactions also preferred by HERA photoproduction data:

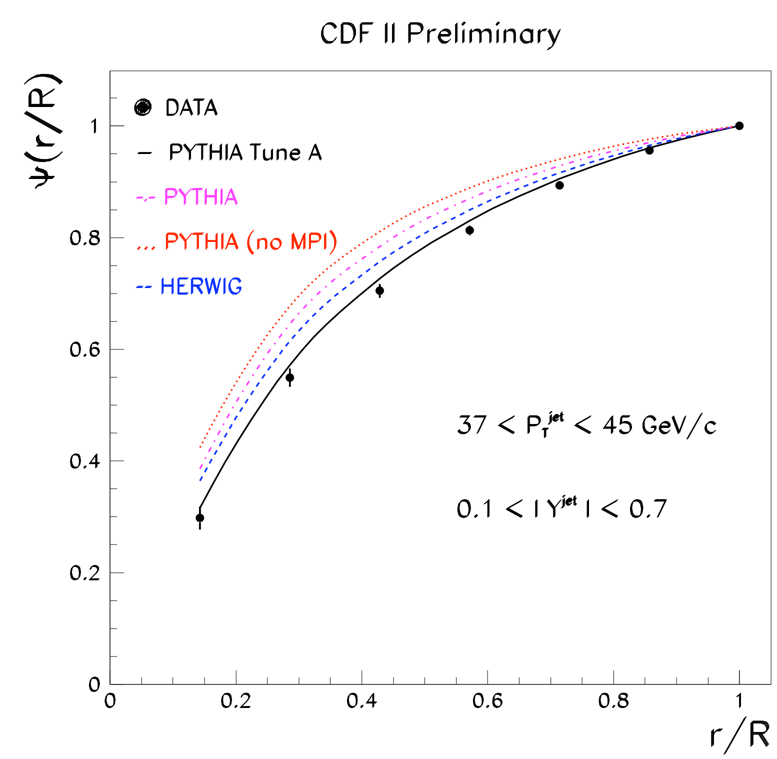
underlying activity in photoproduction vs. DIS



(anti)correlations in energy flow around jet







MI Outlook

Issues requiring further thought and study:

- Multi-parton PDF's $f_{a_1 a_2 a_3 \cdots}(x_1, Q_1^2, x_2, Q_2^2, x_3, Q_3^2, \ldots)$
- Close-packing in initial state, especially small x
- Impact-parameter picture and (x, b) correlations
  e.g. large-x partons more central!, valence quarks more central?
- Details of colour-screening mechanism
- Rescattering: one parton scattering several times
- Intertwining: one parton splits in two that scatter separately
- Colour sharing: two FS–IS dipoles become one FS–FS one
- Colour reconnection: required for $\langle p_\perp \rangle (n_{\text{charged}})$
- Collective effects (e.g. QGP, cf. Hadronization above)
- Relation to diffraction: eikonalization, multi-gap topologies, ...

Action items:

- Vigorous experimental program at LHC
- Study energy dependence: RHIC (pp) $\rightarrow$ Tevatron $\rightarrow$ LHC
- Develop new frameworks and refine existing ones

Much work ahead!