Exercises to
"Introduction to Effective Field Theories"

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Precision measurements in top-quark and bottom-quark physics
September 24th 2015

RG-improved perturbation theory

Top mass from radiative B decays

Power Counting in NRQCD

RG-improved perturbation theory

aim

demonstrate the difference between n-loop order (fixed-order PT) and N^{n}LO (RG-improved PT) in perturbative calculations

setup

consider a perturbative function $C(m, \mu)$, with a double expansion in $a \equiv \alpha_s(\mu)/(4\pi)$ and $L \equiv \ln(\mu/m)$:

$$C(m, \mu) = \sum_{k=0}^{\infty} a^k \sum_{l=0}^{k} c_{kl} L^l$$

make expansion a bit more transparent:

$\downarrow$ power of a incr.	power of L decr. $\rightarrow$				
					c_{00}
				aLc_{11}	ac_{10}
			$a^2L^2c_{22}$	a^2Lc_{21}	a^2c_{20}
		$a^3L^3c_{33}$	$a^3L^2c_{32}$	a^3Lc_{31}	a^3c_{30}
	$\iddots$		$\dots$		$\vdots$

reminder: explicit calculation of coefficients comes with as many loops as the power of a!

problem: convergence of expansion in aL can be *very slow*

Renormalization Group Equation (RGE) to the rescue!

$$\mu \frac{\mathrm{d}C(m,\mu)}{\mathrm{d}\mu} = \gamma(\mu) C(m,\mu) \qquad \text{(RGE)}$$

where the anomalous dimension only depends on μ via the strong coupling:

$$\gamma(\mu) = \sum_{k=0}^{\infty} a^{k+1}\gamma_k = a\gamma_0 + a^2\gamma_1 + \dots$$

- replace left-hand side with

$$\mu \frac{\mathrm{d}C}{\mathrm{d}\mu} = \mu \frac{\partial C}{\partial \mu} + \mu \frac{\partial a}{\partial \mu}\frac{\partial C}{\partial a} = \frac{\partial C}{\partial L} + \beta \frac{\partial C}{\partial a}$$

- use that β has an expansion in a:

$$\mu \frac{\partial a}{\partial \mu} = \beta(\mu) = \sum_{k=0}^{\infty} a^{k+2}\beta_k = a^2\beta_0 + a^3\beta_1 + \dots$$

- expand both sides in a and L

expansion of left-hand side of (RGE)

$$ac_{11}$$

$$a^2 L(2c_{22} + c_{11}\beta_0) \qquad a^2(c_{21} + c_{10}\beta_0)$$

$$a^3 L^2(3c_{33} + 2c_{22}\beta_0)) \qquad a^3 L(2c_{32} + 2c_{21}\beta_0 + c_{11}\beta_1) \qquad a^3(c_{31} + 2c_{20}\beta_0 + c_{10}\beta_1)$$

expansion of right-hand side of (RGE)

$$ac_{00}\gamma_0$$

$$a^2 Lc_{11}\gamma_0 \qquad a^2(c_{10}\gamma_0 + c_{00}\gamma_1)$$

$$a^3 L^2 c_{22}\gamma_0 \qquad a^3 L(c_{21}\gamma_0 + c_{11}\gamma_1) \qquad a^3(c_{20}\gamma_0 + c_{10}\gamma_1 + c_{00}\gamma_2)$$

solve order by order

coefficients c_{kk} of the leading-logarithm = LO order can be read off:

expansion of left-hand side of (RGE)

		ac_{11}
	$a^2 L(2c_{22} + c_{11}\beta_0)$	$a^2(c_{21} + c_{10}\beta_0)$
$a^3 L^2(3c_{33} + 2c_{22}\beta_0))$	$a^3 L(2c_{32} + 2c_{21}\beta_0 + c_{11}\beta_1)$	$a^3(c_{31} + 2c_{20}\beta_0 + c_{10}\beta_1)$

expansion of right-hand side of (RGE)

		$ac_{00}\gamma_0$
	$a^2 L c_{11}\gamma_0$	$a^2(c_{10}\gamma_0 + c_{00}\gamma_1)$
$a^3 L^2 c_{22}\gamma_0$	$a^3 L(c_{21}\gamma_0 + c_{11}\gamma_1)$	$a^3(c_{20}\gamma_0 + c_{10}\gamma_1 + c_{00}\gamma_2)$

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coefficients c_{kk} of the leading-logarithm = LO order can be read off:

$$c_{11} = c_{00}\gamma_0$$

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solve order by order

coefficients c_{kk} of the leading-logarithm = LO order can be read off:

$$c_{11} = c_{00}\gamma_0$$

$$c_{22} = \frac{1}{2}c_{11}(\gamma_0 - \beta_0) = \frac{1}{2}c_{00}\gamma_0(\gamma_0 - \beta_0)$$

expansion of left-hand side of (RGE)

$$a c_{11}$$

$$a^2 L(2c_{22} + c_{11}\beta_0) \qquad a^2(c_{21} + c_{10}\beta_0)$$

$$a^3 L^2(3c_{33} + 2c_{22}\beta_0)) \qquad a^3 L(2c_{32} + 2c_{21}\beta_0 + c_{11}\beta_1) \qquad a^3(c_{31} + 2c_{20}\beta_0 + c_{10}\beta_1)$$

expansion of right-hand side of (RGE)

$$a c_{00}\gamma_0$$

$$a^2 L c_{11}\gamma_0 \qquad a^2(c_{10}\gamma_0 + c_{00}\gamma_1)$$

$$a^3 L^2 c_{22}\gamma_0 \qquad a^3 L(c_{21}\gamma_0 + c_{11}\gamma_1) \qquad a^3(c_{20}\gamma_0 + c_{10}\gamma_1 + c_{00}\gamma_2)$$

solve order by order

coefficients c_{kk} of the leading-logarithm = LO order can be read off:

$$c_{11} = c_{00}\gamma_0$$
$$c_{22} = \frac{1}{2}c_{11}(\gamma_0 - \beta_0) = \frac{1}{2}c_{00}\gamma_0(\gamma_0 - \beta_0)$$
$$c_{33} = \frac{1}{3}c_{22}(\gamma_0 - 2\beta_0) = \frac{1}{6}c_{00}\gamma_0(\gamma_0 - \beta_0)(\gamma_0 - 2\beta_0)$$

solve order by order

coefficients c_{kk} of the leading-logarithm = LO order can be read off:

$$\begin{aligned} c_{11} &= c_{00}\gamma_0 \\ c_{22} &= \frac{1}{2}c_{11}(\gamma_0 - \beta_0) = \frac{1}{2}c_{00}\gamma_0(\gamma_0 - \beta_0) \\ c_{33} &= \frac{1}{3}c_{22}(\gamma_0 - 2\beta_0) = \frac{1}{6}c_{00}\gamma_0(\gamma_0 - \beta_0)(\gamma_0 - 2\beta_0) \end{aligned}$$

needs only three ingredients:

- c_{00} from tree-level matching,
- γ_0 from a C-specific one-loop calculation, and
- β_0 from a universal one-loop calculation

expansion of left-hand side of (RGE)

$$ac_{11}$$

$$a^2L(2c_{22} + c_{11}\beta_0) \qquad a^2(c_{21} + c_{10}\beta_0)$$

$$a^3L^2(3c_{33} + 2c_{22}\beta_0)) \qquad a^3L(2c_{32} + 2c_{21}\beta_0 + c_{11}\beta_1) \qquad a^3(c_{31} + 2c_{20}\beta_0 + c_{10}\beta_1)$$

expansion of right-hand side of (RGE)

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solve order by order

coefficients $c_{k(k-1)}$ of the next-to-leading logarithms:

$$c_{21} = c_{10}(\gamma_0 - \beta_0) + c_{00}\gamma_1$$

expansion of left-hand side of (RGE)

$$ac_{11}$$

$$a^2L(2c_{22} + c_{11}\beta_0) \qquad a^2(c_{21} + c_{10}\beta_0)$$

$$a^3L^2(3c_{33} + 2c_{22}\beta_0)) \qquad \boxed{a^3L(2c_{32} + 2c_{21}\beta_0 + c_{11}\beta_1)} \qquad a^3(c_{31} + 2c_{20}\beta_0 + c_{10}\beta_1)$$

expansion of right-hand side of (RGE)

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solve order by order

coefficients $c_{k(k-1)}$ of the next-to-leading logarithms:

$$c_{21} = c_{10}(\gamma_0 - \beta_0) + c_{00}\gamma_1$$

$$c_{32} = \frac{1}{2}\left[c_{21}(\gamma_0 - 2\beta_0) + c_{11}(\gamma_1 - \beta_1)\right]$$

$$= \frac{1}{2}\left[c_{00}(\gamma_1(\gamma_0 - 2\beta_0) + \gamma_0(\gamma_1 - \beta_1)) + c_{10}(\gamma_0 - \beta_0)(\gamma_0 - 2\beta_0)\right]$$

solve order by order

coefficients $c_{k(k-1)}$ of the next-to-leading logarithms:

$$
\begin{aligned}
c_{21} &= c_{10}(\gamma_0 - \beta_0) + c_{00}\gamma_1 \\
c_{32} &= \frac{1}{2}\left[c_{21}(\gamma_0 - 2\beta_0) + c_{11}(\gamma_1 - \beta_1)\right] \\
&= \frac{1}{2}\left[c_{00}(\gamma_1(\gamma_0 - 2\beta_0) + \gamma_0(\gamma_1 - \beta_1)) + c_{10}(\gamma_0 - \beta_0)(\gamma_0 - 2\beta_0)\right]
\end{aligned}
$$

needs three additional ingredients:

- c_{10} from one-loop matching,
- γ_1 from a C-specific two-loop calculation, and
- β_1 from a universal two-loop calculation

conclusion

- the RGE implies a very useful pattern in the construction of the N^{m}LO coefficients
  - LO needs only tree-level matching, and one-loop anomalous dimension and β-function
  - NLO needs only one-loop matching, and two-loop anomalous dimension and β-function
  - ...
- this suggests a solution to the RGE that resums all leading logarithms, i.e.: when expanded in a and L, this solution generates the same coefficients c_{kk} as before, but includes all those terms
  - this solution can be constructed from only c_{00}, γ_0 and β_0
  - the game can be continued to next-to-leading logarithms and so on...

Top mass from radiative B decays

aim

assuming the Standard Model (SM) is correct, extract the value of the top-quark mass from radiative B decays (e.g. $\bar{B} \to X_s\gamma$)

setup

for the sake of simplicity, we only consider the short-distance electroweak contribution to this decay, which is encoded in the Wilson coefficient C_7 of the electroweak effective Hamiltonian

$$\mathcal{H}^{\text{eff}}_{b\to s} \supset -\frac{G_F}{\sqrt{2}} V_{tb}V^*_{ts} \frac{e}{8\pi^2} C_7 m_b [\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b] F^{\mu\nu}\,.$$

Consider that model-independently $|C_7|$ has been constrained to $|C_7| = 0.33 \pm 0.05$.

The matching condition for C_7 yields at the electroweak scale (to leading order)

$$C_7(\mu = M_W, x_t) = \frac{3x_t^3 - 2x_t^2}{4(x_t - 1)^4}\ln(x_t) - \frac{8x_t^3 + 5x_t^2 - 7x_t}{24(x_t - 1)^3}$$

where $x_t \equiv m_t/M_W$

We can use this for a fixed-order evaluation, and compare with the experimental constraint.

However, there is also the RG-improved result, which cannot be displayed analytically.

It is obtained from solving the matrix-valued RG equation

$$\mu \frac{\mathrm{d}C_i}{\mathrm{d}\mu} = -\gamma_{ji} C_j$$

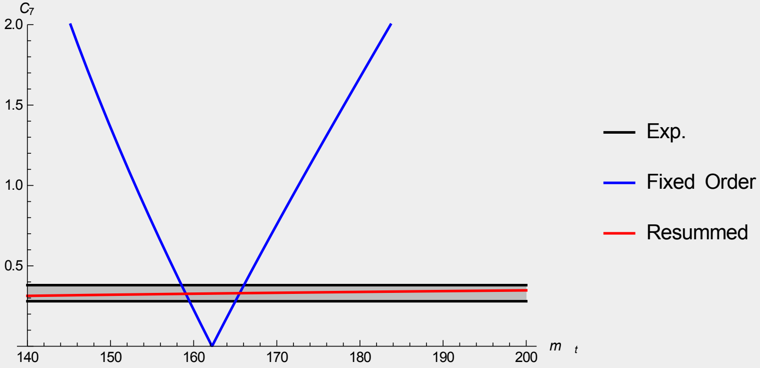
order by order. The anomalous dimension is matrix-valued and non-diagonal! This means that $C_7(\mu = m_b)$ contains contributions from other Wilson coefficients than C_7 at the high scale $\mu \simeq M_W$. The resummed result is known up to N^{2}LO in QCD.

This means that coefficients other than $C_7(\mu = M_W)$ also contribute to $C_7(\mu = m_b)$!

We can numerically evaluate this using publicly available codes. I will be using EOS:

`http://project.het.physik.tu-dortmund.de/eos`

numerical comparison



conclusion

- fixed-order result has strong dependence on m_t at leading order
- resummed result has only very weak dependence on m_t
- in the end, other quantities than C_7 are better suited to indirectly constrain m_t, e.g. $B - \bar{B}$ mixing observables

Power Counting in NRQCD

aim

Illustrate that potential modes needs to be resummed in top-pair production for $v \ll 1$

setup

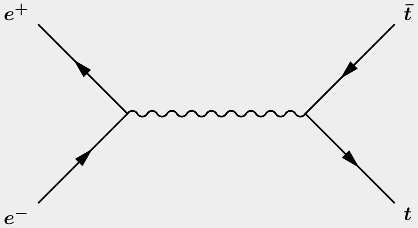
top-pair production at e^+e^- invariant mass $\sqrt{s}$, close to the production threshold:

- velocity v of the top in $t\bar{t}$ CoM frame is small $v \ll 1$
- energy component of the top four momentum in $t\bar{t}$ CoM frame

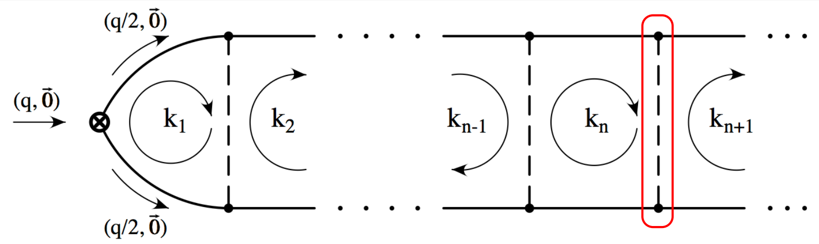
$$p^0 = \frac{\sqrt{s}}{2} = M_t + \frac{M_t v^2}{2} + \mathcal{O}\left(v^4\right)$$

- therefore kinetic energy E of the top pair in the CoM frame is

$$E = 2p^0 - 2M_t = M_t v^2 + \mathcal{O}\left(v^4\right)$$



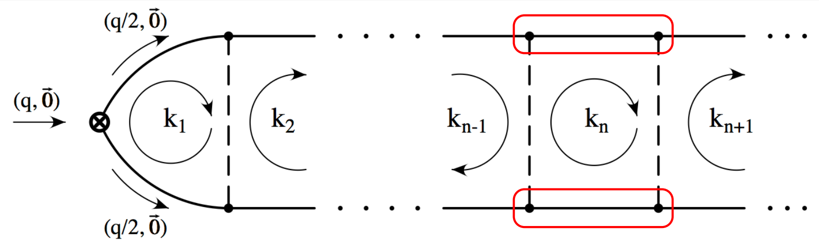
consider now the ladder of Feynman diagrams:



reminder: each new rung with momentum k_n brings along

- one massless propagator: $\frac{1}{k_n^2}$

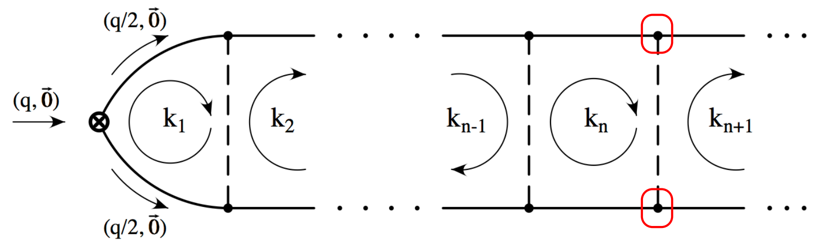
consider now the ladder of Feynman diagrams:



reminder: each new rung with momentum k_n brings along

- one massless propagator: $\frac{1}{k_n^2}$
- two top propagators: $\frac{1}{(p+k_n)^2 - M_t^2}$

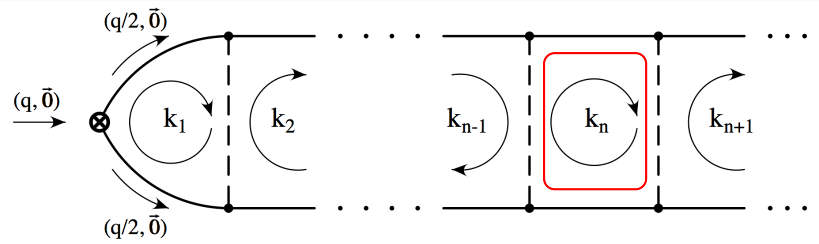
consider now the ladder of Feynman diagrams:



reminder: each new rung with momentum k_n brings along

- one massless propagator: $\dfrac{1}{k_n^2}$
- two top propagators: $\dfrac{1}{(p+k_n)^2 - M_t^2}$
- two powers of the strong coupling: $\alpha_s \sim g_s^2$

consider now the ladder of Feynman diagrams:



reminder: each new rung with momentum k_n brings along

- one massless propagator: $\dfrac{1}{k_n^2}$
- two top propagators: $\dfrac{1}{(p+k_n)^2 - M_t^2}$
- two powers of the strong coupling: $\alpha_s \sim g_s^2$
- the integration measure for the loop momentum k_n: $\mathrm{d}^4 k_n$

hard modes

loop momentum: $k^0 \sim M_t, \quad \vec{k} \sim M_t$

heavy-quark momentum: $(p + k_n)^\mu = (\frac{E}{2} + M_t + k^0, \vec{k})$

massless propagator:

$$\frac{1}{k^2} = \frac{1}{(k^0)^2 - (\vec{k})^2} = \frac{1}{M_t^2} \times \mathcal{O}\left(v^0\right)$$

inverse of the top propagator:

$$\begin{aligned}(p+k)^2 - M_t^2 &= \frac{E^2}{4} + (k^0)^2 + EM_t + Ek^0 + 2M_t k^0 - \vec{k}^2 \\ &= \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^0\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^0\right) + \mathcal{O}\left(v^0\right) \\ &= \mathcal{O}\left(v^0\right)\end{aligned}$$

integration measure:

$$\mathrm{d}^4 k = \mathrm{d}k^0 \times \mathrm{d}^3\vec{k} = \mathcal{O}\left(v^0\right) \times \left[\mathcal{O}\left(v^0\right)\right]^3 = \mathcal{O}\left(v^0\right)$$

total contribution = α_s $\times$ massless prop. $\times$ top prop.^{2} $\times$ int. measure

mode	massless propagator	top propagator	integration measure	total contribution
hard	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$

soft modes

loop momentum: $k^0 \sim M_t v, \quad \vec{k} \sim M_t v$

heavy-quark momentum: $(p+k_n)^\mu = (\frac{E}{2} + M_t + k^0, \vec{k})$

massless propagator:

$$\frac{1}{k^2} = \frac{1}{(k^0)^2 - (\vec{k})^2} = \frac{1}{M_t^2} \times \mathcal{O}\left(v^{-2}\right)$$

inverse of the top propagator:

$$\begin{aligned}(p+k)^2 - M_t^2 &= \frac{E^2}{4} + (k^0)^2 + EM_t + Ek^0 + 2M_t k^0 - \vec{k}^2 \\ &= \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^3\right) + \mathcal{O}\left(v^1\right) + \mathcal{O}\left(v^2\right) \\ &= \mathcal{O}\left(v^1\right)\end{aligned}$$

integration measure:

$$\mathrm{d}^4 k = \mathrm{d}k^0 \times \mathrm{d}^3\vec{k} = \mathcal{O}\left(v^1\right) \times \left[\mathcal{O}\left(v^1\right)\right]^3 = \mathcal{O}\left(v^4\right)$$

total contribution = $\alpha_s \times$ massless prop. $\times$ top prop.^{2} $\times$ int. measure

mode	massless propagator	top propagator	integration measure	total contribution
hard	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$
soft	$\mathcal{O}\left(v^{-2}\right)$	$\mathcal{O}\left(v^{-1}\right)$	$\mathcal{O}\left(v^4\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$

ultrasoft modes

loop momentum: $k^0 \sim M_t v^2, \quad \vec{k} \sim M_t v^2$

heavy-quark momentum: $(p+k_n)^\mu = (\frac{E}{2} + M_t + k^0, \vec{k})$

massless propagator:

$$\frac{1}{k^2} = \frac{1}{(k^0)^2 - (\vec{k})^2} = \frac{1}{M_t^2} \times \mathcal{O}\left(v^{-4}\right)$$

inverse of the top propagator:

$$\begin{aligned}(p+k)^2 - M_t^2 &= \frac{E^2}{4} + (k^0)^2 + EM_t + Ek^0 + 2M_t k^0 - \vec{k}^2 \\ &= \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^4\right) \\ &= \mathcal{O}\left(v^2\right)\end{aligned}$$

integration measure:

$$\mathrm{d}^4 k = \mathrm{d}k^0 \times \mathrm{d}^3\vec{k} = \mathcal{O}\left(v^2\right) \times \left[\mathcal{O}\left(v^2\right)\right]^3 = \mathcal{O}\left(v^8\right)$$

total contribution = α_s $\times$ massless prop. $\times$ top prop.^{2} $\times$ int. measure

mode	massless propagator	top propagator	integration measure	total contribution
hard	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$
soft	$\mathcal{O}\left(v^{-2}\right)$	$\mathcal{O}\left(v^{-1}\right)$	$\mathcal{O}\left(v^4\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$
ultrasoft	$\mathcal{O}\left(v^{-4}\right)$	$\mathcal{O}\left(v^{-2}\right)$	$\mathcal{O}\left(v^8\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$

potential modes

loop momentum: $k^0 \sim M_t v^2, \quad \vec{k} \sim M_t v$

heavy-quark momentum: $(p + k_n)^\mu = (\frac{E}{2} + M_t + k^0, \vec{k})$

inverse of the massless propagator:

$$k^2 = (k^0)^2 - (\vec{k})^2 = \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^2\right) = \mathcal{O}\left(v^2\right)$$

inverse of the top propagator:

$$\begin{aligned}(p+k)^2 - M_t^2 &= \frac{E^2}{4} + (k^0)^2 + EM_t + Ek^0 + 2M_t k^0 - \vec{k}^2 \\ &= \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^4\right) + \mathcal{O}\left(v^2\right) + \mathcal{O}\left(v^2\right) \\ &= \mathcal{O}\left(v^2\right)\end{aligned}$$

integration measure:

$$\mathrm{d}^4 k = \mathrm{d}k^0 \times \mathrm{d}^3\vec{k} = \mathcal{O}\left(v^2\right) \times \left[\mathcal{O}\left(v^1\right)\right]^3 = \mathcal{O}\left(v^5\right)$$

total contribution = α_s $\times$ massless prop. $\times$ top prop.^{2} $\times$ int. measure

mode	massless propagator	top propagator	integration measure	total contribution
hard	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(v^0\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$
soft	$\mathcal{O}\left(v^{-2}\right)$	$\mathcal{O}\left(v^{-1}\right)$	$\mathcal{O}\left(v^4\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$
ultrasoft	$\mathcal{O}\left(v^{-4}\right)$	$\mathcal{O}\left(v^{-2}\right)$	$\mathcal{O}\left(v^8\right)$	$\mathcal{O}\left(\alpha_s v^0\right)$
potential	$\mathcal{O}\left(v^{-2}\right)$	$\mathcal{O}\left(v^{-2}\right)$	$\mathcal{O}\left(v^5\right)$	$\mathcal{O}\left(\alpha_s v^{-1}\right)$

total contribution = α_s $\times$ massless prop. $\times$ top prop.^{2} $\times$ int. measure

mode	massless propagator	top propagator	integration measure	total contribution
hard	$\mathcal{O}(v^0)$	$\mathcal{O}(v^0)$	$\mathcal{O}(v^0)$	$\mathcal{O}(\alpha_s v^0)$
soft	$\mathcal{O}(v^{-2})$	$\mathcal{O}(v^{-1})$	$\mathcal{O}(v^4)$	$\mathcal{O}(\alpha_s v^0)$
ultrasoft	$\mathcal{O}(v^{-4})$	$\mathcal{O}(v^{-2})$	$\mathcal{O}(v^8)$	$\mathcal{O}(\alpha_s v^0)$
potential	$\mathcal{O}(v^{-2})$	$\mathcal{O}(v^{-2})$	$\mathcal{O}(v^5)$	$\mathcal{O}(\alpha_s v^{-1})$

conclusion

All rungs in the ladder carrying potential modes add a power of $\alpha_s/v \simeq 1$! Integrating out all of these modes yields a Coulomb-like potential, which the top quarks experience. This is why they are called “potential modes”!