“Precision measurements in top-quark and bottom-quark physics”, September 25, 2015

Interpretation of measurements

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Outline

1. Introduction
2. Interpreting BR($B_s \to \mu^+\mu^-$)
   - Standard Model prediction
   - Beyond the Standard Model
3. Towards a global analysis of $b \to s$ transitions
4. Probing top couplings in bottom decays

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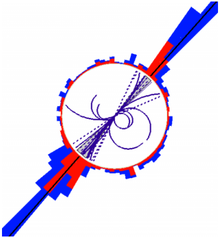
Ideally . . .



$$\Rightarrow \mathcal{L}$$

Realistically . . .

Challenges for theory

1. Observables in top & bottom physics involve *strong interaction*
   $\Rightarrow$ uncertainties from non-perturbative effects
2. Beyond the SM, typically *more free parameters* in $\mathcal{L}$ than observations.
3. . . .

Realistically ...

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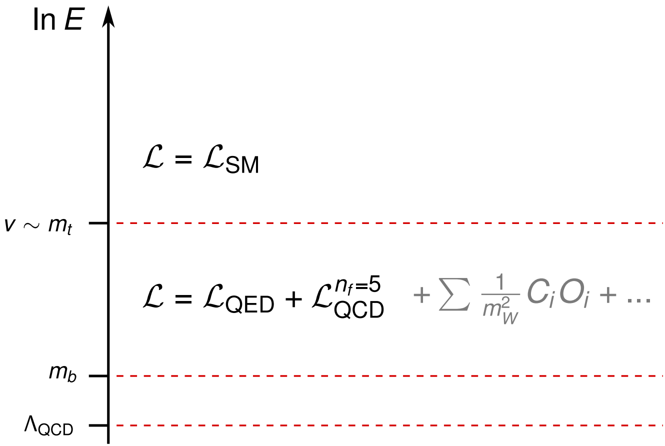
3. ...

How effective field theories can help

1. Allow to *separate* long-distance (QCD) and short-distance (EW, BSM) physics
2. Allow to *parametrize* the ignorance about short-distance physics exploiting the known *symmetries*

Hierarchy of effective theories



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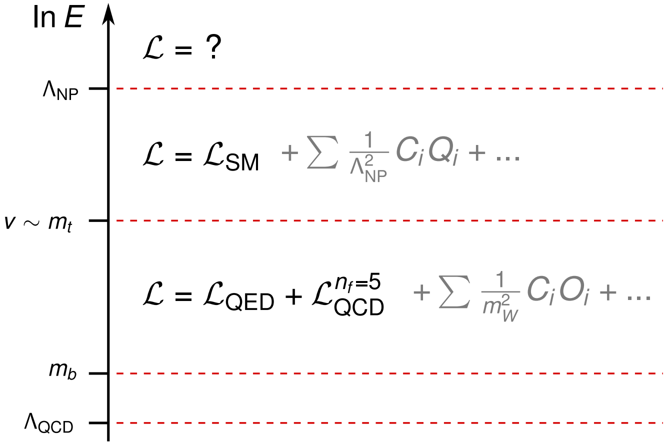

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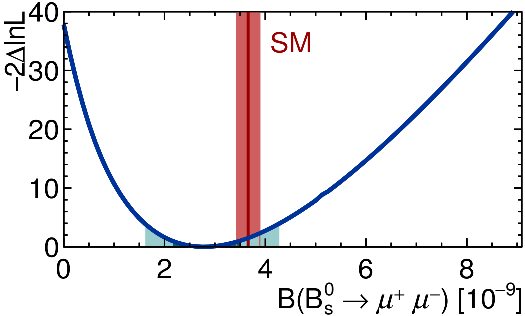
Interpreting a measurement



LHCb & CMS:

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$$

$B_s \to \mu^+\mu^-$ branching ratio in the SM

$$\begin{aligned}\text{BR}(B_s \to \mu^+\mu^-) &= \Gamma(B_s \to \mu^+\mu^-)/\Gamma(B_s \to \text{anything}) \\ &= \tau_{B_s}\,\Gamma(B_s \to \mu^+\mu^-) \\ &= \tau_{B_s}\,\Phi(m_{B_s}, m_\mu)\,|\langle\mu\mu|A|B_s\rangle|^2\end{aligned}$$

- $\tau_{B_s} = 1/\Gamma_s$ – lifetime
- Φ – phase space
- A – amplitude

$$\Phi(m_{B_s}, m_\mu) = \frac{1}{16\pi}\frac{1}{m_{B_s}}\sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}$$

$B_s \to \mu^+\mu^-$ amplitude in the EFT

$$\langle \mu\mu | A | B_s \rangle = i \langle \mu\mu | C_{10} O_{10} | B_s \rangle + O(m_b^2/m_W^2)$$

- $O_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{\mu}\gamma_\mu\gamma_5\mu)$ – semi-leptonic axial vector operator
- C_{10} – Wilson coefficient

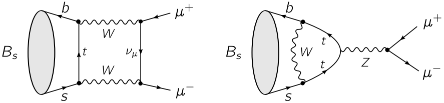
$$\langle \mu\mu | C_{10} O_{10} | B_s \rangle = C_{10} \langle 0 | \bar{s}_L \gamma^\mu b_L | B_s \rangle (\bar{\mu}\gamma_\mu\gamma_5\mu)$$

- $\langle 0 | \bar{s}_L \gamma^\mu b_L | B_s \rangle$ – hadronic matrix element

$$\langle 0 | \bar{s}_L \gamma^\mu b_L | B_s \rangle = \frac{1}{2} \langle 0 | \bar{s} \gamma^\mu b | B_s \rangle - \frac{1}{2} \langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s \rangle = 0 - \frac{1}{2} i f_{B_s} p^\mu$$

- f_{B_s} decay constant

$B_s \to \mu^+\mu^-$ Wilson coefficient in the SM

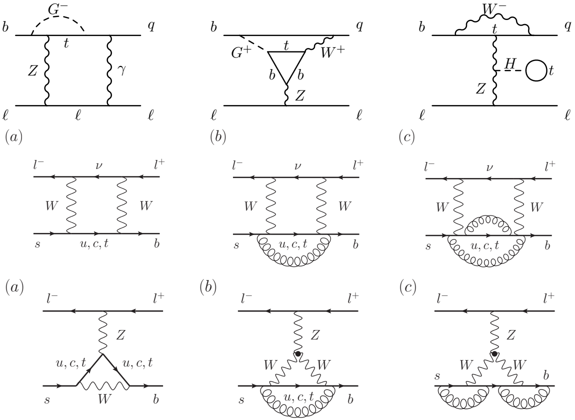


$$C_{10} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \frac{1}{s_w^2} V_{tb} V_{ts}^* \, Y(x_t)$$

- G_F – Fermi constant
- V_{tq} – CKM elements
- $x_t = m_t^2 / m_W^2$
- Y – Inami-Lim function

$$Y(x_t) = Y_0(x_t) \left[1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_{em}) + \ldots\right]$$

Some higher order diagrams



Bobeth, Gorbahn, and Stamou 1311.1348, Hermann et al. 1311.1347

Recipe: how to predict BR($B_s \to \mu^+\mu^-$)$_{\text{SM}}$

$$\text{BR}(B_s \to \mu^+\mu^-)_{\text{SM}} = \tau_{B_s} \frac{G_F^2}{\pi} \left(\frac{\alpha}{4\pi s_w^2} \right)^2 m_\mu^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}\, m_{B_s} f_{B_s}^2 \, |V_{tb} V_{ts}^*|^2 \, Y(x_t)^2$$

- Liftetime τ_{B_s}: take from experiment

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- G_F, α, s_w, $m_{B_s}^2$, m_μ: take from PDG

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  - caveat: which definition to take for α, s_w?

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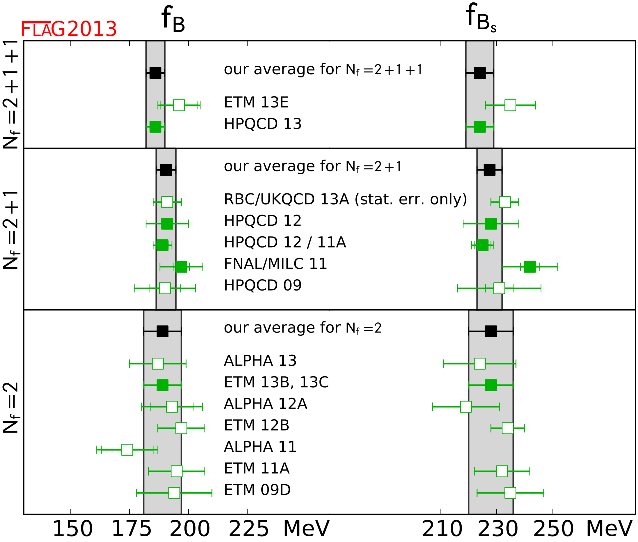
- $Y(x_t)$: include NNLO QCD and NLO EW corrections and RG evolution
- $f_{B_s}^2$: from lattice QCD
- $|V_{tb}V_{ts}^*|^2$: from experiment

Lattice determinations of f_B

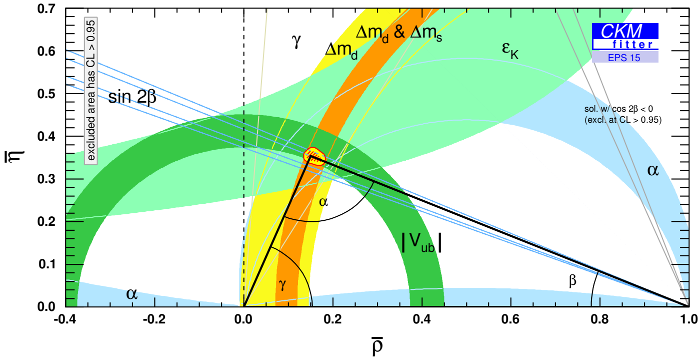


Determining $|V_{tb}V_{ts}^*|$

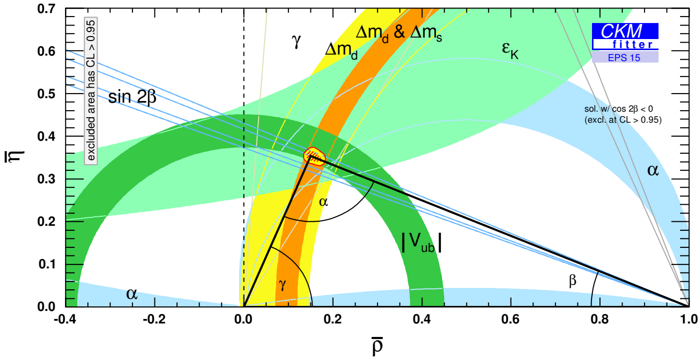
- There is no direct measurement of V_{ts}
- But CKM elements can be extracted from a global fit of the CKM matrix

$$|V_{tb}V_{ts}^*| = A\lambda^2 \left[1 + \lambda^2 \left(\bar{\rho} - \frac{1}{2}\right)\right] + O(\lambda^6)$$

Global CKM fits



Global CKM fits



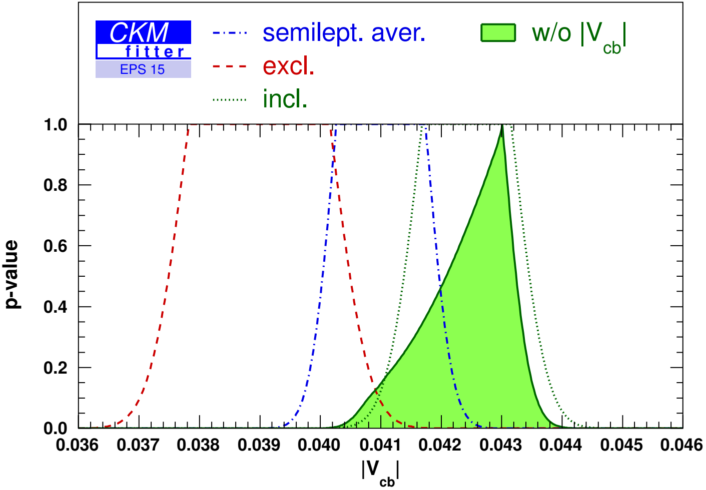
Using the global fit result assumes that neutral meson mixing is free from physics BSM

Using tree-level CKM determinations

- $|V_{cb}|$ from inclusive & exclusive $b \to c\ell\nu$
- $|V_{ub}|$ from inclusive & exclusive $b \to u\ell\nu$
- $|V_{us}|$ from $K \to \pi\ell\nu$
- γ from $B \to DK$

$$|V_{tb}V_{ts}^*| = |V_{cb}| \left(1 - \frac{|V_{us}|^2}{2} + \frac{|V_{ub}|}{|V_{cb}|}|V_{us}|\cos\gamma\right) \approx |V_{cb}|(1 - 0.025 + 0.007)$$

Status of V_{cb} measurements



Two subtleties when relating experiment and SM

1. What about the process $B_s \to \mu^+\mu^-\gamma$ with a soft γ escaping detection?
2. What about B_s vs. $\bar{B}_s$ decay? Their lifetimes differ by 12%!

$B_s \to \mu^+\mu^-\gamma$

Two sources of photons

1. *Direct emission* – can be suppressed below the % level by a tight *cut* on $q^2 = m_{B_s}^2$
2. *Bremsstrahlung* – the number we calculated corresponds to the BR *fully inclusive* of bremsstrahlung. This can be taken into account e.g. by simulating bremsstrahlung in the experimental analysis or by imposing a photon energy cut and computing the correction factor

cf. Buras et al. 1208.0934

B_s lifetime difference

Due to B_s-$\bar{B}_s$ mixing, there is a sizable lifetime difference between the two B_s mass eigenstates:

$$\tau_{B_s^L} = \Gamma_{B_s^L}^{-1} = 1.42\,\text{ps} \qquad \tau_{B_s^H} = \Gamma_{B_s^H}^{-1} = 1.61\,\text{ps}$$

$$\tau_{B_s} = \Gamma_{B_s}^{-1} = \left[\frac{1}{2}\left(\Gamma_{B_s^L} + \Gamma_{B_s^H}\right)\right]^{-1}$$

Time-dependent untagged decay rate

$$\Gamma(B_s(t) \to \mu^+\mu^-) = R_H\, e^{-t/\tau_{B_s^H}} + R_L\, e^{-t/\tau_{B_s^L}}$$

So far, we have computed

$$\text{BR}(B_s \to \mu^+\mu^-) = \frac{\tau_{B_s}}{2}\, \Gamma(B_s(t=0) \to \mu^+\mu^-)$$

But experiments actually measure

$$\overline{\text{BR}}(B_s \to \mu^+\mu^-) = \frac{1}{2}\int_0^\infty \Gamma(B_s(t) \to \mu^+\mu^-)\, dt$$

It turns out that De Bruyn et al. 1204.1737

$$\frac{\overline{\text{BR}}(B_s \to \mu^+\mu^-)}{\text{BR}(B_s \to \mu^+\mu^-)} = \frac{\tau_{B_s^H}}{\tau_{B_s}}$$

Result: $B_s \to \mu^+\mu^-$ SM vs. experiment

$$\overline{\text{BR}}(B_s \to \mu^+\mu^-)_{\text{exp}} = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$$

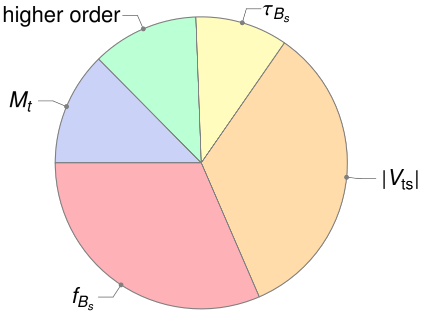
$$\overline{\text{BR}}(B_s \to \mu^+\mu^-)_{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9}$$

$$\Rightarrow R(B_s \to \mu^+\mu^-) = \frac{\overline{\text{BR}}(B_s \to \mu^+\mu^-)}{\overline{\text{BR}}(B_s \to \mu^+\mu^-)_{\text{SM}}} = 0.78 \pm 0.18$$

Summary: SM prediction of BR($B_s \to \mu^+\mu^-$)

- Wilson coefficient
  - Perturbative calculation: a lot of work, but controllable uncertainty
  - CKM elements: *caveat:* ambiguities between full fit, incl. & excl. V_{cb}
- Matrix element
  - Decay constant from lattice: quite precise but error dominated by single computation
- Experiment vs. theory
  - Care has to be taken that what is measured and what is predicted are actually the same thing! (Here e.g.: lifetime effect, soft photons)

BR($B_s \to \mu^+\mu^-$) error budget



Bobeth, Gorbahn, Hermann, et al. 1311.0903

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Physics beyond the SM in $B_s \to \mu^+\mu^-$

Assuming no new particles below 5 GeV, new physics does *not* affect

- Matrix element (f_{B_s})
- CKM extraction based on tree-level decays *
- QCD corrections
- Phase space

All “short-distance” physics enters through modified *Wilson coefficients*

* see however Brod, Lenz, et al. 1412.1446

All possible contributing operators

$$O_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{\mu}\gamma_\mu\gamma_5\mu) \qquad O'_{10} = (\bar{s}_R \gamma^\mu b_R)(\bar{\mu}\gamma_\mu\gamma_5\mu)$$

$$O_S = m_b(\bar{s}_R b_L)(\bar{\mu}\mu) \qquad O'_S = m_b(\bar{s}_L b_R)(\bar{\mu}\mu)$$

$$O_P = m_b(\bar{s}_R b_L)(\bar{\mu}\gamma_5\mu) \qquad O'_P = m_b(\bar{s}_L b_R)(\bar{\mu}\gamma_5\mu)$$

- In the SM, $C'_{10} = C_S = C'_S = C_P = C'_P = 0$
- f_{B_s} remains the only required matrix element because

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}_s\rangle = ip^\mu f_{B_s}, \qquad \langle 0|\bar{s}\gamma_5 b|\bar{B}_s\rangle = -\frac{if_{B_s} m_{B_s}^2}{m_b + m_s},$$

- Other operators (tensor, dipole) have vanishing matrix elements

Branching ratio beyond the SM

$$\text{BR}(B_s \to \mu^+\mu^-) = \text{BR}(B_s \to \mu^+\mu^-)_{\text{SM}} \left[|A|^2 + |B|^2 \left(1 - \frac{4m_\mu^2}{m_{B_s}^2} \right) \right]$$

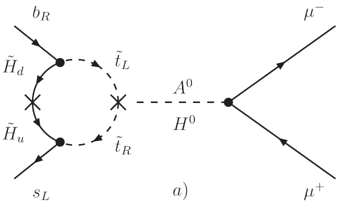
$$A = \frac{1}{C_{10}^{\text{SM}}} \left[\left(C_{10} - C'_{10} \right) + \frac{m_{B_s}^2}{2m_\mu} \left(C_P - C'_P \right) \right]$$

$$B = \frac{1}{C_{10}^{\text{SM}}} \left[\frac{m_{B_s}^2}{2m_\mu} \left(C_S - C'_S \right) \right]$$

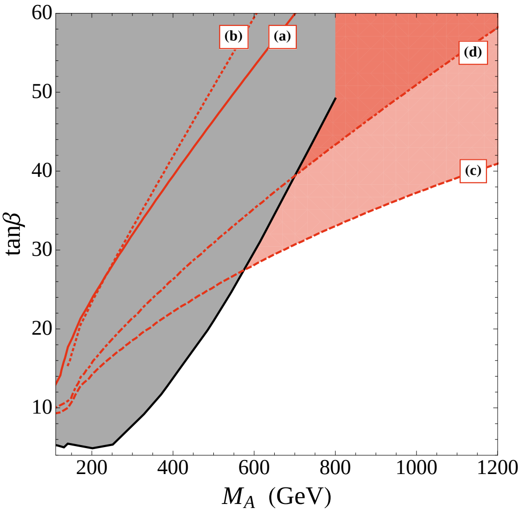
Example new physics models: MSSM

Even for a degenerate spectrum: Higgsino contribution

$$C_S \approx -C_P \approx \frac{G_F^2 m_t^2}{8\pi^2} \frac{m_\mu}{m_A^2} \frac{A_t \mu \tan^3 \beta}{m_{\tilde{t}}^2} f\left(\frac{\mu^2}{m_{\tilde{t}}^2}\right)$$



Complementarity with Higgs searches

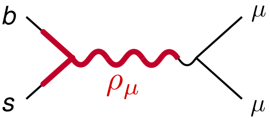


Gray: bound from search for $A^0 \to \tau^+\tau^-$ Altmannshofer, Carena, et al. 1211.1976

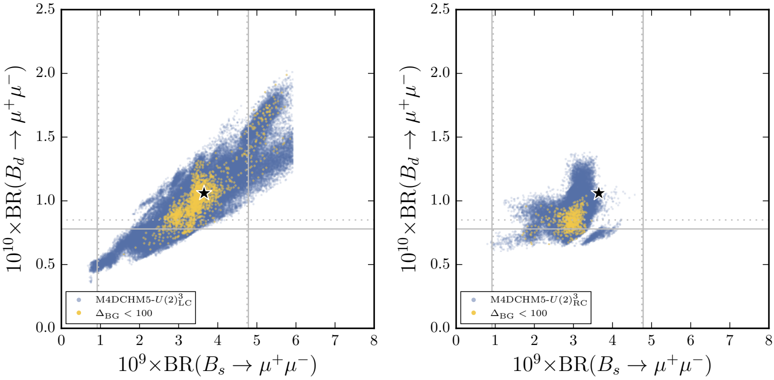
Example 2: Composite Higgs models

$$C_{10}^{(\prime)} \sim \sin\theta^b_{L,R} \sin\theta^s_{L,R}\, g_\rho \frac{1}{m_\rho^2} \frac{g^2}{g_\rho}$$

- Tree-level exchange of heavy vector resonance and modification of Z coupling
- no (pseudo)scalar operators generated in minimal models



$B_q \to \mu^+\mu^-$ in composite Higgs models



two different scenarios for the flavour structure Niehoff et al. 1508.00569

Fitting the Wilson coefficients

- We can obtain model-independent constraints on new physics by considering the χ^2 function

$$\chi^2(C_i) = \frac{(x(C_i) - x_{\text{exp}})^2}{\sigma_{exp}^2 + \sigma_{\text{th}}^2}$$

where $x = \overline{\text{BR}}(B_s \to \mu^+\mu^-)$ and C_i are the Wilson coefficients.

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- For a single real coefficient, the value allowed at 1σ (2σ) is determined by

$$\chi^2(C) - \chi^2(C^*) < 1 \ (< 4)$$

where C^* is the value that minimizes χ^2.

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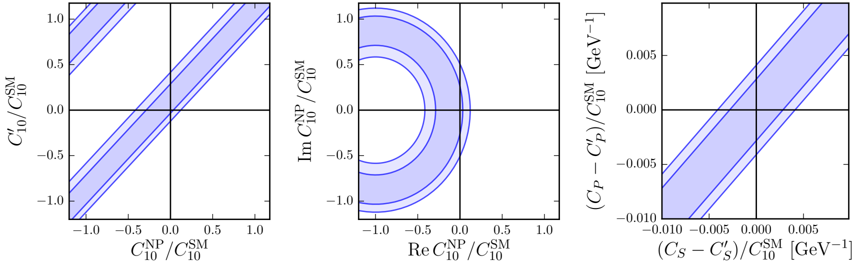
where C^* is the value that minimizes χ^2.

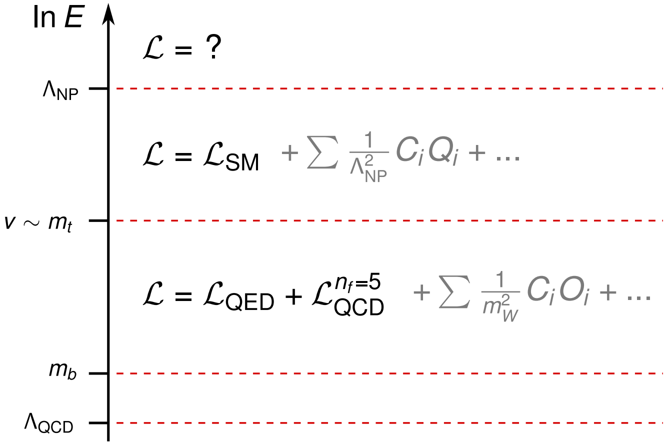
- For two coefficients, the 1σ (2σ) regions are given by

$$\chi^2(\vec{C}) - \chi^2(\vec{C}^*) < 2.3 \; (< 6)$$

Fit results


$C'_{10}/C_{10}^{\rm SM}$
$C_{10}^{\rm NP}/C_{10}^{\rm SM}$
1.0
0.5
0.0
−0.5
−1.0
$\mathrm{Im}\,C_{10}^{\rm NP}/C_{10}^{\rm SM}$
$\mathrm{Re}\,C_{10}^{\rm NP}/C_{10}^{\rm SM}$
$(C_P - C'_P)/C_{10}^{\rm SM}$ [GeV^{-1}]
$(C_S - C'_S)/C_{10}^{\rm SM}$ [GeV^{-1}]
0.005
0.000
−0.005
−0.010

SM effective theory



SM-EFT operators matching onto $O_{10}^{(\prime)}$

$$Q_{Hq}^{(1)} = \left(H^\dagger\, iD_\mu H\right)(\bar{q}_s\gamma^\mu q_b)$$
$$Q_{Hq}^{(3)} = H^\dagger\, iD_\mu^I H(\bar{q}_s\tau^I\gamma^\mu q_b)$$
$$Q_{Hd} = \left(H^\dagger\, iD_\mu H\right)(\bar{s}_R\gamma^\mu b_R)$$

$$Q_{\ell q}^{(1)} = (\bar{\ell}\gamma_\mu\ell)(\bar{q}_s\gamma^\mu q_b), \qquad Q_{\ell q}^{(3)} = (\bar{\ell}\gamma_\mu\tau^I\ell)(\bar{q}_s\gamma^\mu\tau^I q_b),$$
$$Q_{ed} = (\bar{l}_R\gamma_\mu l_R)(\bar{s}\gamma^\mu b_R), \qquad Q_{\ell d} = (\bar{\ell}\gamma_\mu\ell)(\bar{s}\gamma^\mu b_R),$$
$$Q_{qe} = (\bar{q}_s\gamma_\mu q_b)(\bar{l}_R\gamma^\mu l_R)$$

$$C_{10} = C_{qe} - C_{\ell q}^{(1)} - C_{\ell q}^{(3)} + (C_{Hq}^{(1)} + C_{Hq}^{(3)})$$
$$C_{10}' = C_{ed} - C_{\ell d} + C_{Hd}$$

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$$Q_{Hq}^{(3)} = H^\dagger \, iD_\mu^I H (\bar{q}_s \tau^I \gamma^\mu q_b)$$
$$Q_{Hd} = \left(H^\dagger \, iD_\mu H\right) (\bar{s}_R \gamma^\mu b_R)$$

$$Q_{\ell q}^{(1)} = (\bar{\ell}\gamma_\mu \ell)(\bar{q}_s \gamma^\mu q_b), \qquad Q_{\ell q}^{(3)} = (\bar{\ell}\gamma_\mu \tau^I \ell)(\bar{q}_s \gamma^\mu \tau^I q_b),$$
$$Q_{ed} = (\bar{l}_R \gamma_\mu l_R)(\bar{s}\gamma^\mu b_R), \qquad Q_{\ell d} = (\bar{\ell}\gamma_\mu \ell)(\bar{s}\gamma^\mu b_R),$$
$$Q_{qe} = (\bar{q}_s \gamma_\mu q_b)(\bar{l}_R \gamma^\mu l_R)$$

$$C_{10} = C_{qe} - C_{\ell q}^{(1)} - C_{\ell q}^{(3)} + (C_{Hq}^{(1)} + C_{Hq}^{(3)})$$
$$C_{10}' = C_{ed} - C_{\ell d} + C_{Hd}$$

We have not gained anything!

SM-EFT operators matching onto $O_{S,P}^{(\prime)}$

$$Q_{\ell edq} = (\bar{q}_s b_R)(\bar{l}_R \ell) \qquad Q'_{\ell edq} = (\bar{\ell}\, l_R)(\bar{s}_R q_b)$$

$$C_S = -C_P = C_{\ell edq}$$

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$$C_S = -C_P = C_{\ell edq}$$

$$C'_S = C'_P = C'_{\ell edq}$$

- At dimension 6 in the SM-EFT, there are *only 2* independent scalar/pseudoscalar operators (as opposed to 4 in the low-energy EFT).
- The SM gauge symmetries *restrict* the form of scalar NP contributions (valid if $\Lambda_{\text{NP}} \gg v$)

Global analyses of $b \to s$ transitions

- Taking into account also radiative and semi-leptonic decays, more operators become relevant, e.g.

$$O_7^{(\prime)} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_{R(L)}b)F^{\mu\nu} \qquad O_9^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)}b)(\bar{\ell}\gamma^\mu\ell)$$

- This necessitates a *global* analysis of constraints on Wilson coefficients

Decay	$C_7^{(\prime)}$	$C_9^{(\prime)}$	$C_{10}^{(\prime)}$
$B \to X_s\gamma$	X		
$B \to K^*\gamma$	X		
$B \to X_s\mu^+\mu^-$	X	X	X
$B \to K\mu^+\mu^-$	X	X	X
$B \to K^*\mu^+\mu^-$	X	X	X
$B_s \to \mu^+\mu^-$			X

Interpreting measurements: BR($B \to K\mu^+\mu^-$)

- Similarly to $B_s \to \mu^+\mu^-$,

$$\mathrm{BR}(B \to K\mu^+\mu^-) = \tau_B\,\Phi(m_B, m_K, m_\mu) \sum_i C_i |\langle K\mu\mu|O_i|B\rangle|^2$$

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- *but* there are several additional challenges

1. Wilson coefficients

- There is now more than 1 non-zero Wilson coefficient already in the SM
- These Wilson coefficients are renormalization scale dependent

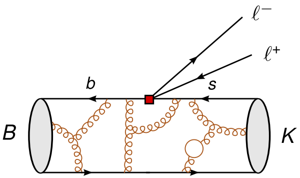
2. Matrix elements

- As in the $B_s \to \mu^+\mu^-$ case, we can factorize the matrix element into a *hadronic* and a *leptonic* part

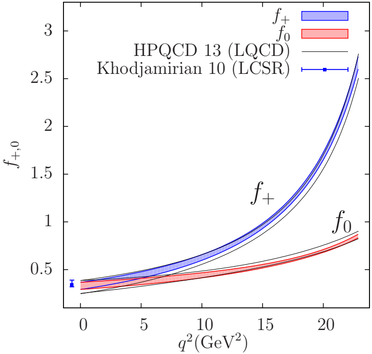
$$\langle K\mu\mu|(\bar{s}\Gamma_i b)(\bar{\mu}\Gamma_j\mu)|B\rangle = \langle K|\bar{s}\Gamma_i b|B\rangle \times \langle\mu\mu|\bar{\mu}\Gamma_j\mu|0\rangle$$

- The hadronic matrix element is a *form factor* depending on the Dirac structure and the momentum transfer

$$\langle K|\bar{s}\Gamma_i b|B\rangle \sim f_i(q^2)$$



Form factors

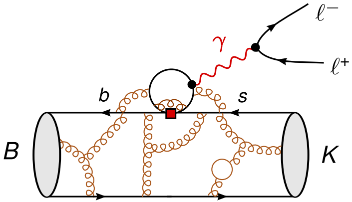


- Lattice QCD: restricted to high q^2
- Light-Cone Sum Rules: restricted to low q^2

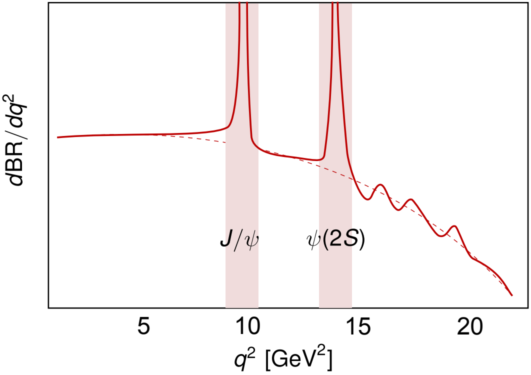
Plot: Bailey et al. 1509.06235

Non-factorizable corrections

- The *naive* factorization is not exact because there are photon-mediated contributions involving *purely hadronic* operators

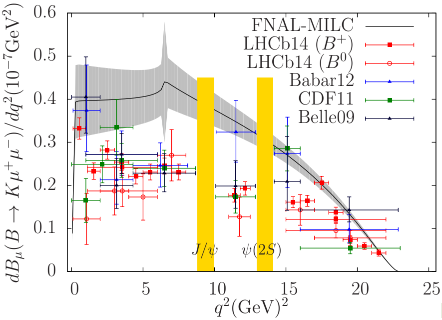


Differential branching ratio (sketch)



- At low q^2: can be computed in the limit $m_b \to \infty$. *Power corrections* $O(\Lambda/m_b)$ notoriously hard to control
- At high q^2: q^2-integrated observables less senitive to duality violation

Predictions vs. data



Bailey et al. 1509.06235

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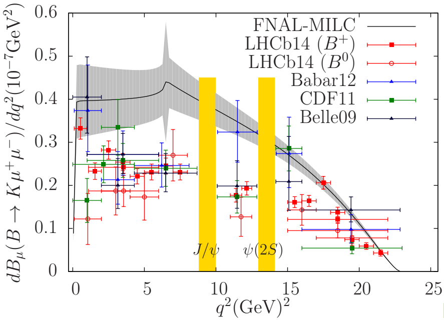


Bailey et al. 1509.06235

- If there is a discrepancy between SM and data, we should keep in mind:
  - Ambiguities in CKM elements (V_{cb})
  - Uncertainties in form factors if they rely on a single method
  - Difficulty to estimate size of non-factorizable (power) corrections

Null tests

- Particularly powerful are measurements of quantities where the SM prediction is basically free from uncertainties. Example:

$$R_K = \frac{\text{BR}(B^+ \to K^+\mu^+\mu^-)}{\text{BR}(B^+ \to K^+e^+e^-)}$$

  - All uncertainties mentioned on previous slide drop out
  - LHCb:

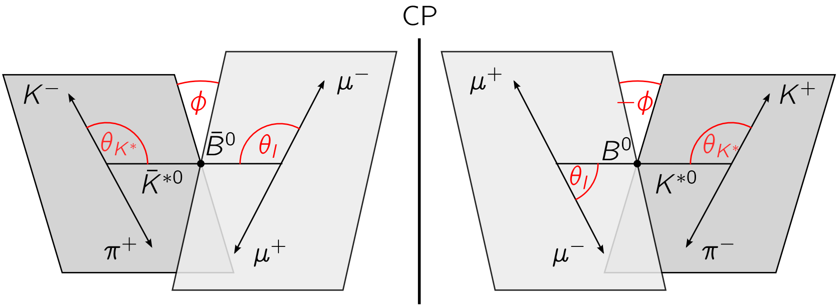
$$R_K \big|_{[1,6]\ \text{GeV}^2} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

$B \to K^*\mu^+\mu^-$

New features compared to $B \to K\mu^+\mu^-$

- K^* is a vector meson
  - more amplitudes (depending on K^* polarization)
  - more form factors
- K^* is not stable under strong interactions
  - form factor determinations more difficult (less reliable?)
  - $K^* \to K\pi$ decay gives access to additional decay angle $\Rightarrow$ rich angular distribution

$B \to K^*(\to K\pi)\mu^+\mu^-$ decay distribution



Angular observables

- Huge advantage: considering ratios of angular observables, many uncertainties drop out, BSM sensitivity improves

Angular observables

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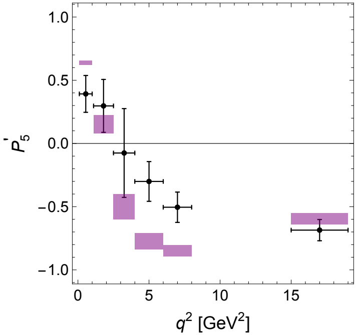
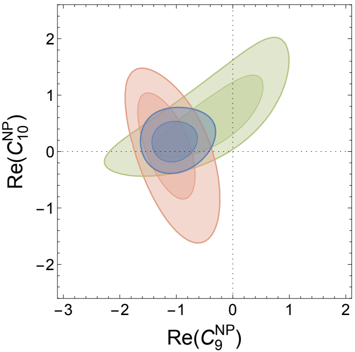
- Some tensions with SM in latest LHCb data, most prominently



Results from a global fit to Wilson coefficients



Altmannshofer and Straub 1411.3161

Some comments on theory uncertainties

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    - statistical and systematic (finite spacing, volume, unphysical quark masses, etc.)
- How to treat them?
  - What is the “likelihood” of a parameter that we only have an order-of-magnitude estimate for?

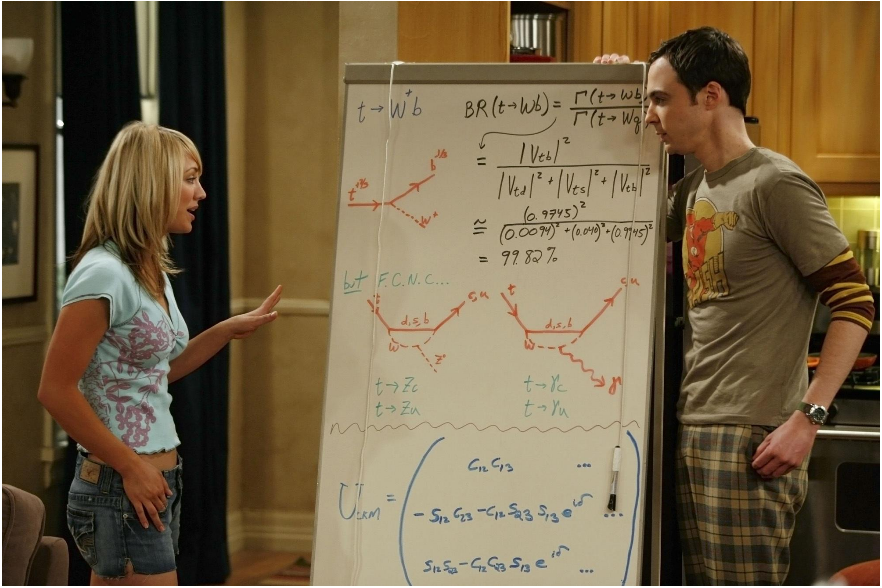
Probing top-Z couplings

- Top couplings to the Z^0 are still poorly known
  - $Z \to t\bar{t}$ not kinematically allowed $\Rightarrow$ only coupling not probed at LEP
  - Even FCNC couplings $Z\bar{t}u$, $Z\bar{t}c$ still allowed to be sizable

$$\text{BR}(t \to qZ) < 5 \times 10^{-4}$$

Chatrchyan et al. 1312.4194

- Many BSM theories predict deviations from the SM in these couplings (e.g.: composite Higgs models)


t → W+ b
BR(t→Wb) =
= |Vtb|² / (|Vtd|² + |Vts|² + |Vtb|²)
≅ (0.9745)² / ((0.0094)² + (0.040)² + (0.9745)²)
= 99.82 %
but F.C.N.C. ...
d,s,b
t → Zc
t → Zu
t → γc
t → γu
U_CKM =

Top vs. bottom couplings

- Let's focus on the couplings of *left-handed* tops:
  - $SU(2)_L$ gauge symmetry relates $t_L \leftrightarrow b_L$, $W^\pm \leftrightarrow Z^0$
  - CKM matrix relates q_L of different generations
- Can use the SM-EFT to find relations between the following couplings

$$\bar{t}_L t_L Z,\ \bar{t}_L c_L Z,\ \bar{b}_L b_L Z,\ \bar{t}_L b_L W^+,\ \bar{c}_L b_L W^+,\ \bar{s} b Z$$

Operators modifying top couplings

- Operators modifying the Z/W couplings of left-handed quarks:

$$\left(Q_{Hq}^{(1)}\right)_{ij} = \left(H^\dagger \, iD_\mu H\right)(\bar{q}_i \gamma^\mu q_j) \qquad \left(Q_{Hq}^{(3)}\right)_{ij} = \left(H^\dagger \, iD_\mu^I H\right)(\bar{q}_i \tau^I \gamma^\mu q_j)$$

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- Work in the basis where the up-type quark mass matrix is diagonal:

$$q_L^i = \begin{pmatrix} u_L^i \\ \sum_j V_{ij}\, d_L^j \end{pmatrix}$$

where u_L, d_L are mass basis fields.

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- Let's consider a theory with the following non-zero couplings at the electroweak scale:

$$a_{tt} = \left(C_{Hq}^{(1)}\right)_{33} \quad b_{tt} = \left(C_{Hq}^{(3)}\right)_{33} \quad a_{ct} = \left(C_{Hq}^{(1)}\right)_{23} \quad b_{ct} = \left(C_{Hq}^{(3)}\right)_{23}$$

Wilson coefficients vs. couplings

- Setting the Higgs field to its VEV $\langle H \rangle = \frac{1}{\sqrt{2}}(0\ v)^T$ and inserting the explicit form of the covariant derivative, one obtains coupling modifications

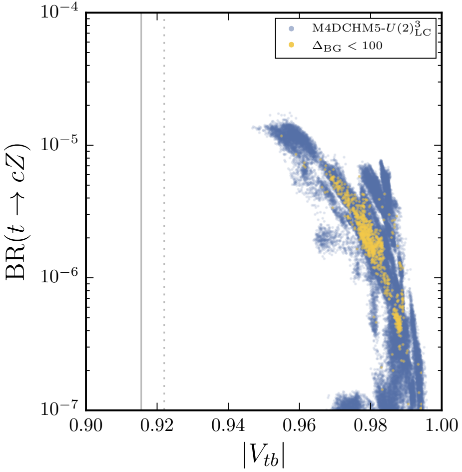
$$\Rightarrow \mathcal{L} \supset \frac{g}{c_w} Z_\mu \delta g^L_{Zqq'} \bar{q}_L q'_L + \left(\frac{g}{\sqrt{2}} W^+_\mu \delta g^L_{Wij} \bar{u}^i_L d^j_L + \text{h.c.} \right)$$

$$\begin{aligned}
\delta g^L_{Ztt} &= b_{tt} - a_{tt} \\
\delta g^L_{Zct} &= b_{ct} - a_{ct} \\
\delta g^L_{Zbb} &= V_{tb}^2 (b_{tt} + a_{tt}) + 2 V_{cb} V_{tb} (b_{ct} + a_{ct}) \\
&\approx (b_{tt} + a_{tt}) + 2 V_{cb} (b_{ct} + a_{ct}) \\
\delta g^L_{Zbs} &= V_{tb} V^*_{cs} (b_{ct} + a_{ct}) + V_{tb} V^*_{ts} (b_{tt} + a_{tt}) + O(\lambda^4) \\
&\approx (b_{ct} + a_{ct}) - V_{cb} (b_{tt} + a_{tt}) \\
\delta g^L_{Wtb} &= V_{tb} b_{tt} + V_{cb} b_{ct}
\end{aligned}$$

Experimental constraints

- If NP only in a_{ij} or b_{ij}, strong constraints from b physics
  - $Z \to b\bar{b}$
  - $B_s \to \mu^+\mu^-$
- Upper bound on $t \to cZ$ – see exercises

Model example: composite Higgs



Niehoff et al. 1508.00569

Custodial protection

- To avoid the strong constraint from $Z \to b\bar{b}$, many BSM models (e.g. warped extra dimensions, composite Higgs) make use of a *custodial protection* that implies $a_{ij} = -b_{ij}$

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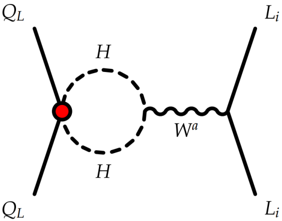
- B physics constraints are still relevant because the protection is spoiled by the *renormalization group running* from the new physics scale Λ_{NP} down to the electroweak scale
- Consider the case where, at Λ_{NP}, only the LH/RH $Z\bar{t}t$ couplings are modified Brod, Greljo, et al. 1408.0792

Operator mixing

Following Brod, Greljo, et al. 1408.0792

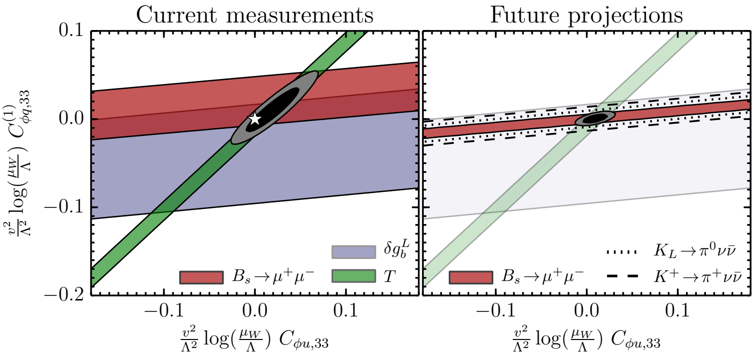
- Under renormalization, the operators $Q_{Hq}^{(3)}$, $Q_{Hq}^{(1)}$, Q_{Hu} mix into each other + the following SM-EFT operators
  - $Q_{\ell q}^{(1)} = (\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma^\mu q)$ ⇒ *rare B and K decays*
  - $Q_{\ell q}^{(3)} = (\bar{\ell}\gamma_\mu \tau^I \ell)(\bar{q}\gamma^\mu \tau^I q)$ ⇒ *rare B and K decays*
  - $Q_{\phi D} = |H^\dagger D^\mu H|^2$ *Electroweak T parameter*



Results

- After running from the new physics scale Λ_{NP} to the electroweak scale, the RG-induced effects are of the form

$$C_i \sim \frac{1}{16\pi^2}\left(g^2 c_{g,ij}^2\, C_j + y_t^2 c_{y,ij}\, C_j\right) \ln \frac{m_W}{\Lambda_{NP}}$$



Conclusions

- Interpreting measurements requires precise SM predictions. Challenges in *B* decays include (among others)
  - Wilson coefficient calculations beyond the leading order
  - Lattice computations of matrix elements (decay constants, form factors, . . .)
  - Calculation or estimation of non-perturbative effects
- EFTs can help to parametrize NP effects model-independently and to correlate different observables
  - NB, in complete generality, this is often not useful – need specific NP model to obtain correlations