Observables and Parameters

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Observables and Parameters

Theoretical framework

e.g. quantum mechanics, special relativity

Observables

“Experiment”

“Theory”

Model with
certain parameters,
for example the
Standard Model

Observables and parameters

Observables:

- Do not rely on any theory
- Can/should be measured independent from theoretical model

Parameters:

- Only defined within a particular model
- Numerical value depend on precise definition ("renormalization scheme", obvious for coupling constants, holds also true for masses!)

Why do we care about parameters ?

→Need to know them as precise as possible to test theoretical model through comparison:

experimental results ←→ theoretical predictions

How do we extract model parameters ?

❶ Determine theoretical predictions for some set of observables within your favored model

❷ Compare with measured results and extract/fit the model parameters

Need to understand the precise definition of parameters within a model and the role of quantum corrections

→ Renormalization

Plan

1. The physical picture of renormalization
2. Regularization and renormalization
3. The running of the QCD coupling
4. Interlude: The running of the SM couplings and vacuum stability
5. Quark masses: definitions and measurements
6. Interlude: The hierarchy problem
7. Summary

(The technical details are skipped if interested in please ask)

Why do we need renormalization ?

Consider simple example:

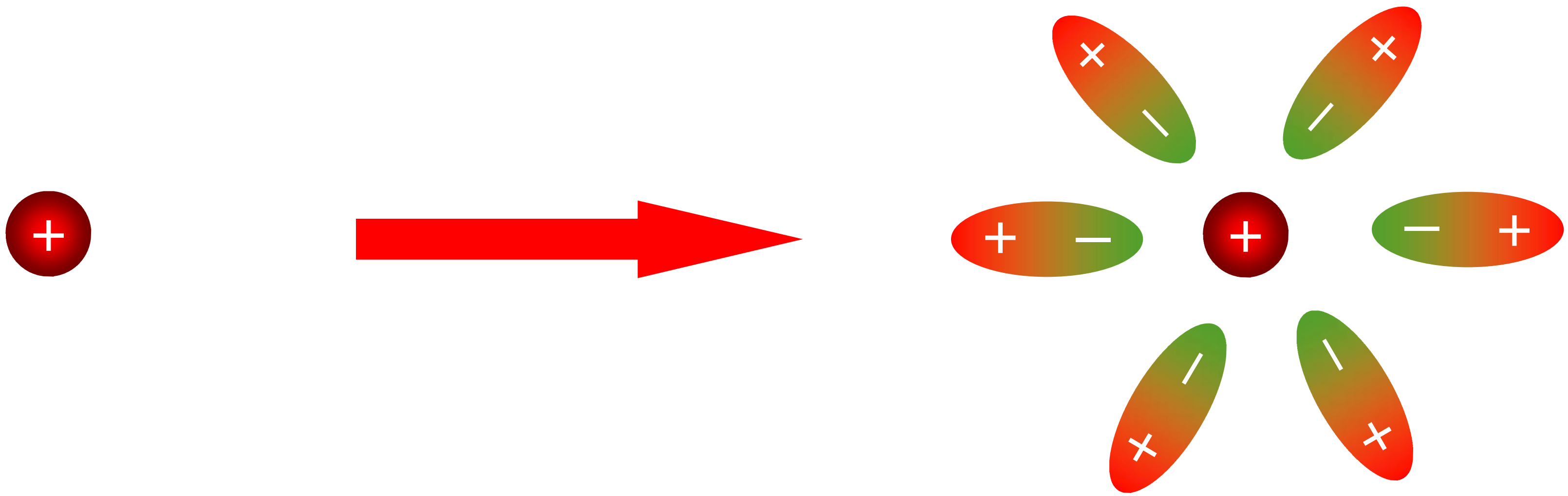
measurement of the electric charge

In classical electrodynamics this is done by the use of a test charge (which should be small…):

we may use for example a positive test charge and measure how the charge is scattered

What happens if we consider quantum field theory ?

the vacuum develops a complicated structure, i.e. we have vacuum polarization



Why do we need renormalization ?

What do we measure when we repeat the experiment with the test charge ?

Clear enough the test charge will see the bare charge surrounded/screened by the vacuum polarization

What we actually measure is thus the bare charge ***together*** with the vacuum polarization

Why do we need renormalization ?

What about the bare charge?

Is it possible to get ride of the vacuum polarization ?

Yes, if we switch off the electric charge

However, switching off the charge, there is now interaction between the test charge and the charge we want to measure

It is impossible to measure the bare charge !

The bare charge is thus meaningless in an interacting QFT. !

Why do we need renormalization ?

Just a problem of the electric charge ?

No, this is a general property of QFT’s

Another example:

self energy corrections to the mass

If we define the mass to be the location of the pole of the propagator (so-called pole-mass) we see that higher order quantum corrections move the pole away from the bare mass!

→ more details later…

Why do we need renormalization ?

The bare parameters are unphysical, they are not experimentally accessible since we cannot switch off the interaction

Renormalization is the step to relate the parameters of our theory to (experimentally) measurable quantities ("observables")

Renormalization has nothing to do with infinities, it is a well defined procedure to relate the parameters to a prescription how to measure them,

we do not put anything under the carpet !

Strictly speaking renormalization is also needed in finite theories !

Parameters versus Observables

Important consequence:

The parameters of the theory (quark masses, strong coupling,…) are in general not observables!

They depend on the scheme to define them

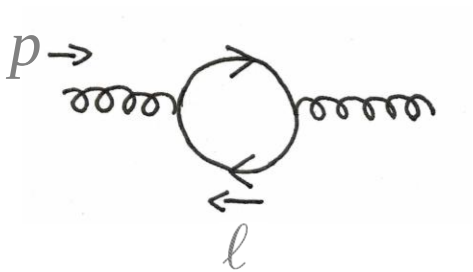
They are related to measurable observables, and are determined through a fit to a specific theory

Sometimes they are called pseudo-observables

(Note that parameters are only defined in the context, of a specific theory!)

Using the (unphysical) bare coupling we may obtain infinite results when calculating loop corrections.

Example:



$$\leftrightarrow \quad -g^2 \int d^4\ell \frac{1}{(\ell^2 - m^2)((\ell+p)^2 - m^2)}$$

Naïve power counting:

$$\int^{\infty} |\ell|^3 d|\ell| \frac{1}{|\ell|^4} = \ln(|\ell|)\Big|_0^{\infty} \quad \text{or} \quad \int^{\Lambda} d|\ell| \frac{|\ell|^3}{|\ell|^4} =\sim \ln(\Lambda)$$

(The infinity at zero only occurs since we ignored the masses…)

Infinities in QFT's

Infinities may be related to the use of the unphysical bare couplings (could also be divergent!)

In a renormalizable QFT it is indeed possible to remove all the divergences by using renormalized (measurable) couplings

Two step procedure:

1. Regularize divergences

2. Absorb them through a redefinition of the fields and couplings using renormalized quantities

$$A_0, \Psi_0, g_0, m_0, \ldots \to A_0 = \sqrt{Z_A} A_r, \Psi_0 = \sqrt{Z_\Psi} \Psi_r, g_0 = Z_g g_r, m_0 = Z_m m_r$$

Infinities in QFT's

Note: Renormalizability is a non-trivial property which puts a high constraint on the allowed theories !

→ Only couplings with positive or zero mass dimension are allowed !

Reminder: Mass dimension

$$[S] = [\int d^4x \mathcal{L}] = 1 \to [\mathcal{L}] = 4 (= d)$$

$$[\Psi] = \frac{3}{2} (= \frac{d-1}{2}), \quad [A] = 1 (= \frac{d-2}{2})$$

Example:

$$[m\bar{\Psi}\Psi] = [m] + 2[\Psi] = [m] + 3 = 4 \to [m] = 1$$

$$[g\bar{\Psi}\not{A}\Psi] = [g] + 2[\Psi] + [A] = [g] + 4 = 4 \to [g] = 0$$

In d dimensional space time: $[m] = 1, [g] = \frac{4-d}{2} = \varepsilon, (d = 4 - 2\varepsilon)$

Regularization

Introduce “regulator” such, that integrals are finite,

After the renormalization procedure the regulator is removed

Example: Cut-off regulator

$$\int d^4\ell \to \int^{\Lambda} d^4\ell$$

(after the $\ln(\Lambda)$ are absorbed through renormalization
original theory is restored in the limit $\Lambda \to \infty$

Infinite number of different regularization schemes possible,
Which one should we use ?

Regularization

Note:

Very often regulators break some symmetries of the underlying theory i.e. gauge symmetry, Poincare invariance, SUSY etc

- Calculations become more complicated because simplifying power of symmetries is lost
- Need to make sure that all symmetries are restored when regulator is removed after renormalization

→ A convenient regulator should respect as far as possible the underlying symmetries

Dimensional regularization

Observation: Singularities due to high energy behavior of the measure

$$d^4\ell \sim |\ell|^3 d|\ell|$$

→ Can be improved by lowering the space time dimension

Assume $d < 4$, convention: $d = 4 - 2\varepsilon \; (\varepsilon > 0)$

$$\int d^4\ell \frac{1}{\ell^4} \Rightarrow \int d^d\ell \frac{1}{\ell^4} \sim \int d|\ell| \frac{|\ell|^{d-1}}{|\ell|^4} = \int d|\ell| |\ell|^{d-5} \to \frac{1}{d-4} = -\frac{1}{2\varepsilon}$$

Singularities appear as poles in $\frac{1}{d-4} = -\frac{1}{2\varepsilon}$

The renormalization scale

Remember: $$[g] = \frac{4-d}{2} = \varepsilon, (d = 4 - 2\varepsilon)$$

Introduce arbitrary mass scale μ to keep g dimensionless in d dimensions:

$$g_0 \to \mu^\varepsilon Z_g g_r(\mu)$$

bare coupling, divergent

Renormalized (finite) coupling, depends on unphysical scale μ

Since left hand side is μ independent, μ dependence of $g_r(\mu)$ must cancel the dependence of μ^ε

The running coupling constant

Since Z_g absorbs divergences due to quantum corrections we have:

$$Z_g = 1 + \frac{1}{\varepsilon} g_r^2 \# + O(g_r^4)$$

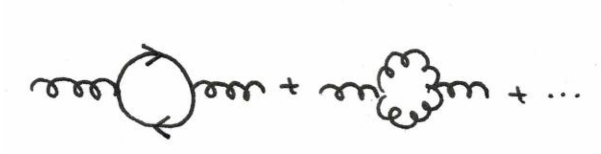
From

$$\mu^{\varepsilon} \frac{1}{\varepsilon} = \frac{1}{\varepsilon} + \ln(\mu) + \ldots$$

we see that g_r depends logarithmically on μ !

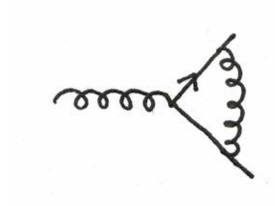
To determine the precise dependence (#) we need to calculate the quantum corrections

In QCD:



$\rightarrow Z_A$

$\rightarrow Z_\Psi$



$\rightarrow Z_g$

Define different couplings

$$Z_g^{\mathrm{MS}} = 1 - \frac{g_s^2}{16\pi^2}\underbrace{\left(\frac{11N}{6} - \frac{1}{3}n_f\right)}_{=2\beta_0}\frac{1}{\varepsilon}$$

N number of colors, n_f number of flavours

$$Z_g^{\overline{\mathrm{MS}}} = 1 - \frac{g_s^2}{16\pi^2}\left(\frac{11N}{6} - \frac{1}{3}n_f\right)\left(\frac{1}{\varepsilon} - \gamma + \ln(4\pi)\right)$$

$$Z_g^{\mathrm{R}} = Z_g^{\mathrm{MS}} + z_R$$

The QCD β-function

Define

$$\alpha_s(\mu) = \frac{g_r(\mu)^2}{4\pi}, \quad a(\mu) = \frac{\alpha_s(\mu)}{\pi}$$

Using Z_g we obtain:

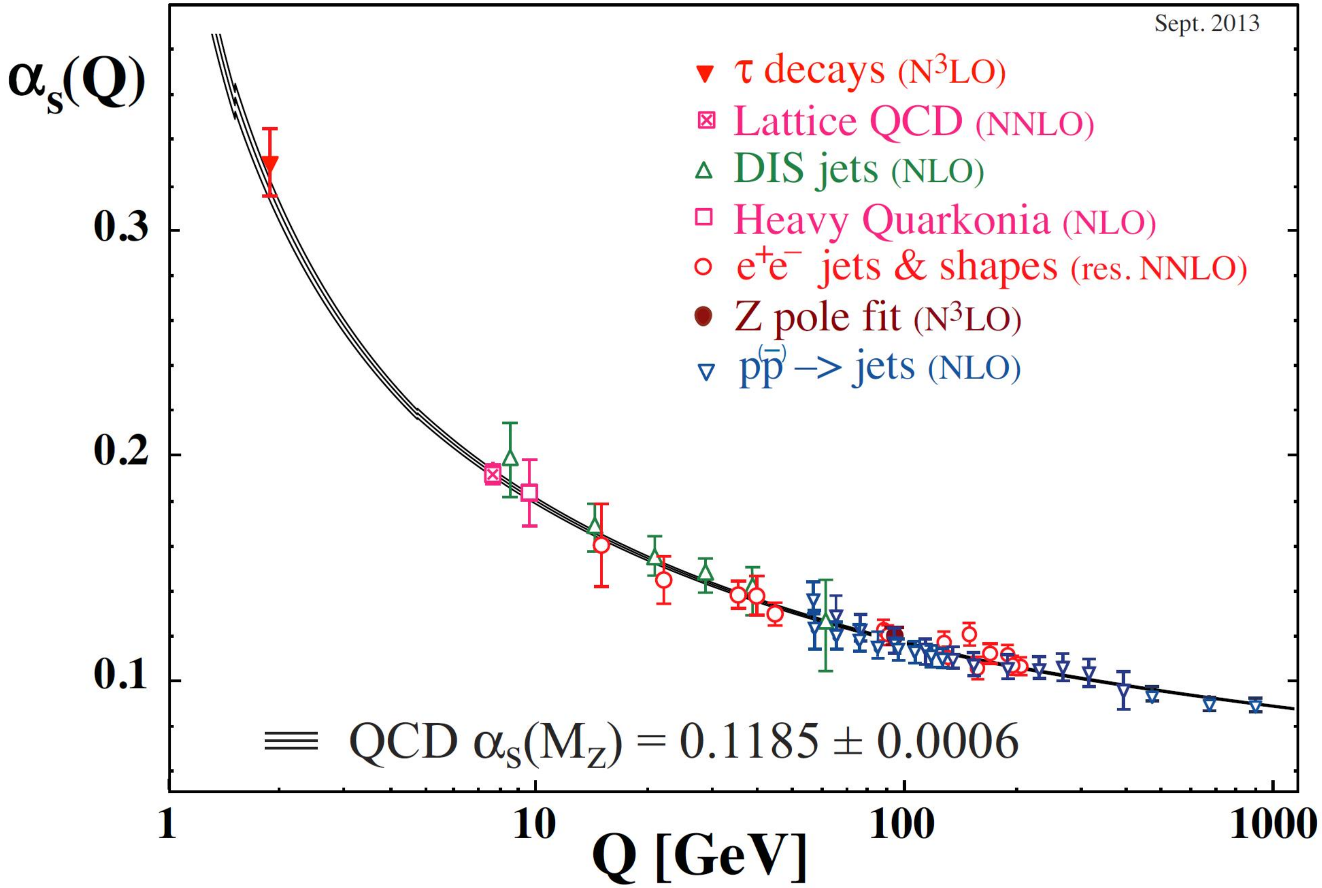
$$\frac{d}{d\ln(\mu^2)} a(\mu) = \mu^2 \frac{d}{d\mu^2} a(\mu) = -\beta_0 a(\mu)^2 + O(a^3)$$

Note:

The value of α_s has to be extracted from experiment. However, knowing the value at one scale, theory tells us how to calculate it at another scale!

→ Just solve the differential equation….

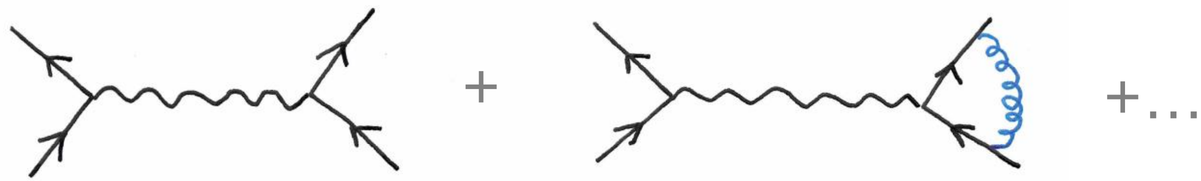
$$a(\mu_1) = \frac{a(\mu_0)}{1 + a(\mu_0)\beta_0 \ln(\mu_1^2/\mu_0^2)}$$



[PDG2014]

The strong coupling constant from e+e- → hadrons*

*) massless



$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2\left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

Possible to extract α_s from measured R value !

However:

At which scale do we measure α_s ??

$$\alpha_s = \alpha_s(\mu)$$

It seems natural to assume $\alpha_s = \alpha_s(\sqrt{s})$, how do we decide?

Strictly speaking not possible since we have only a LO calculation…

What happens if we change the scale ?

$$
\begin{aligned}
R &= 3\sum_q Q_q^2\left(1+\frac{\alpha_s(\sqrt{s})}{\pi}+\dots\right)\\
&= 3\sum_q Q_q^2\left(1+\frac{\frac{\alpha_s(\mu)}{\pi}}{1-\frac{\alpha_s(\mu)}{\pi}\beta_0\ln(\frac{\mu^2}{s})}+\dots\right)\\
&= 3\sum_q Q_q^2\left(1+\frac{\alpha_s(\mu)}{\pi}+\sum_{i=1}^{\infty}\left(\frac{\alpha_s(\mu)}{\pi}\right)^{i+1}\left(\beta_0\ln(\frac{\mu^2}{s})\right)^i+\dots\right)
\end{aligned}
$$

Since s is the only physical scale in the problem we get large higher order corrections if we use μ very different from $\sqrt{s}$!

We may also say: The proper choice of μ resums a certain class of potentially large logs to all orders

Scale variation as estimate of uncalculated higher orders

Note:

In the order we are calculating theory results must be independent on the renormalization scheme

Changing the scale in the theoretical predictions generates scale dependent terms which are formally of higher order ("residual scale dependence")

→ Higher order corrections must at least cancel this contribution

This is the basis of using scale variation to estimate higher order corrections

Renormalization group equations in the SM

$$\frac{d}{d\ln(\mu^2)}\frac{\alpha_s(\mu)}{\pi} = -\beta_0\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + O(\alpha_s^3)$$

“Renormalization group equation”

In complete SM we have more couplings: $V(\phi) = -\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$

$$\alpha_1, \alpha_2, \alpha_3 = \alpha_s, \lambda, y_t, y_b, y_\tau \ldots$$

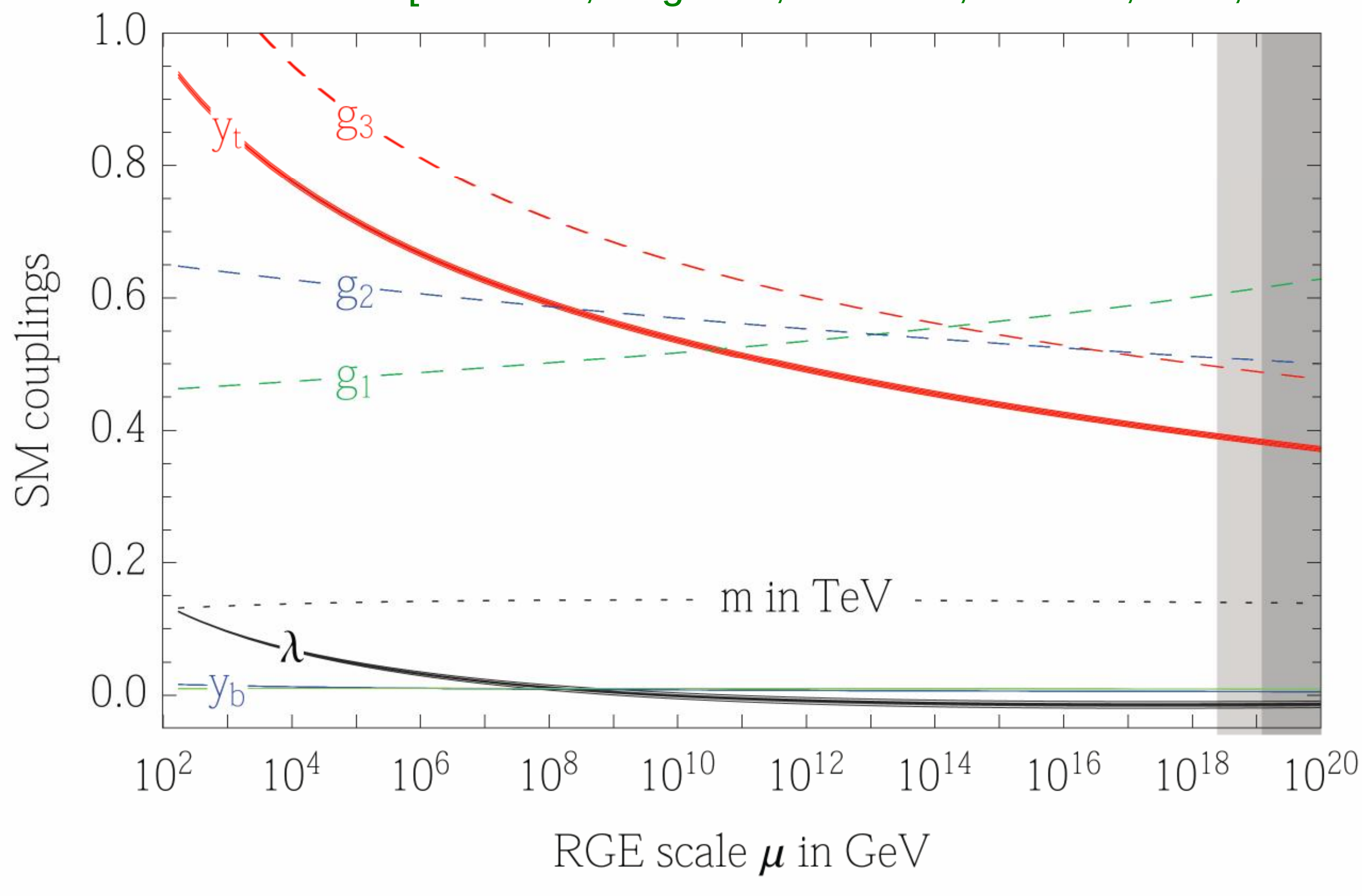
RGE’s are coupled:

$$\frac{d}{d\ln(\mu^2)}\begin{pmatrix} g_1^2 \\ g_2^2 \\ g_3^2 \\ \lambda \\ \vdots \end{pmatrix} = \vec{f}(g_1^2, g_2^2, g_3^2, \lambda, \ldots)$$

can be solved numerically…

Evolution of coupling constants in the SM

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia 14]



Interesting aspects:

- Unification of gauge couplings ?
- Vacuum stability

Vacuum stability

Basic idea:

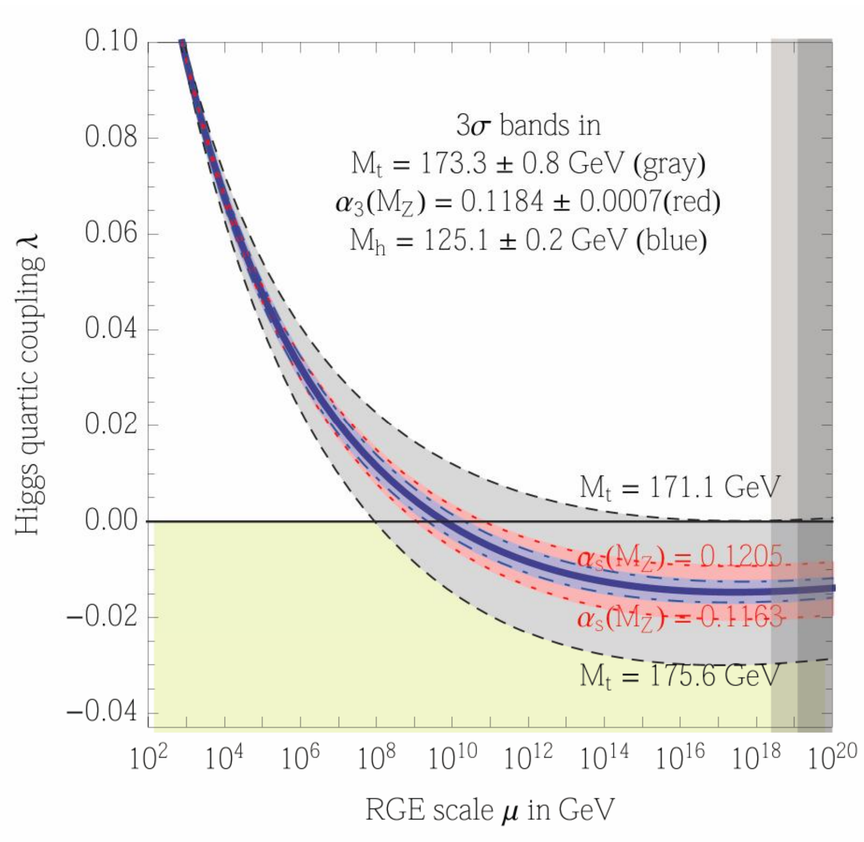
At large field values use $\lambda(\phi)$
instead of $\lambda(\mu)$ at some low scale μ

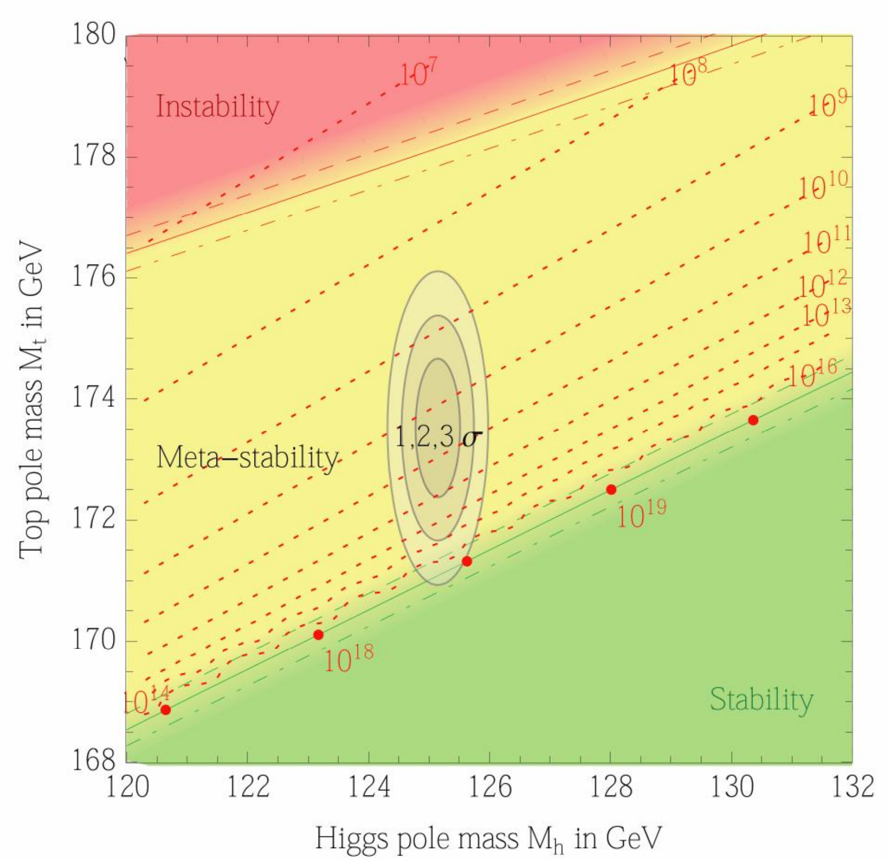
Running of $\lambda(\mu)$ determines potential $V(\phi)$
at large field values

Vacuum becomes instable for $\lambda < 0$

More precise analysis requires to take also higher order corrections to the potential into account

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia 14]





Quark masses in QCD

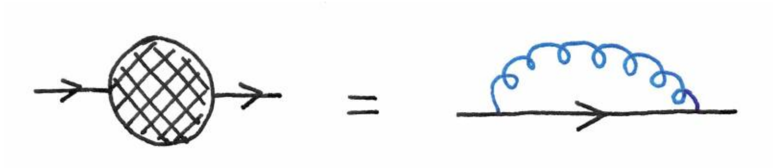
In the SM fermion masses are generated through the Higgs mechanism:

$$m_q = \frac{\lambda_q v}{\sqrt{2}}$$

If we study only QCD we may ignore the Higgs mechanism and use the quark masses as independent parameters

→ Mass renormalization

Self energy corrections and mass renormalization

$-i\Sigma =$  $+ \ldots$

1PI (= one particle irreducible)

From the evaluation of the self energy the field and mass renormalization constant can be determined:

$$\begin{aligned}\Sigma &= \frac{\alpha_s}{4\pi} C_f \not{p} \frac{1}{\varepsilon} + \frac{\alpha_s}{\pi} C_f m_r \frac{1}{\varepsilon} \\ &+ (1 - Z_\Psi^R)\not{p} - (1 - Z_\Psi^R Z_m^R) m_r + \text{finite terms}\end{aligned}$$

Self energy corrections and mass renormalization

Minimal subtraction scheme (MS):

$$Z_\Psi^{\mathrm{MS}} = 1 - \frac{\alpha_s}{4\pi} C_f \frac{1}{\varepsilon}$$

$$Z_m^{\mathrm{MS}} = 1 - \frac{3\alpha_s}{4\pi} C_f \frac{1}{\varepsilon}$$

Modified minimal subtraction scheme $\overline{\mathrm{MS}}$

$$Z_\Psi^{\overline{\mathrm{MS}}} = 1 - \frac{\alpha_s}{4\pi} C_f \left(\frac{1}{\varepsilon} - \gamma + \ln(4\pi) \right)$$

$$Z_m^{\overline{\mathrm{MS}}} = 1 - \frac{3\alpha_s}{4\pi} C_f \left(\frac{1}{\varepsilon} - \gamma + \ln(4\pi) \right)$$

Schemes are technically simple, however, very unintuitive

Self energy corrections and mass renormalization

The renormalized MS mass

$$m_{\overline{\text{MS}}} = \frac{m_0}{Z_m^{\overline{\text{MS}}}}$$

- becomes scale dependent similar to α_s

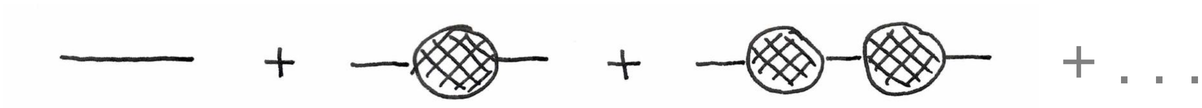
$$\mu^2 \frac{dm_{\overline{\text{MS}}}}{d\mu^2} = -\frac{\alpha_s}{\pi}\frac{3}{4} C_f m_{\overline{\text{MS}}} + O(\alpha_s^2)$$

- does not describe the pole of the full propagator

→ Has little to do with our intuitive understanding of a quark mass

The quark propagator and the pole mass scheme

Full propagator



$$= \frac{i}{\not p - m_r} + \frac{i}{\not p - m_r}(-i\Sigma_r)\frac{i}{\not p - m_r} + \frac{i}{\not p - m_r}(-i\Sigma_r)\frac{i}{\not p - m_r}(-i\Sigma_r)\frac{i}{\not p - m_r} + \ldots$$

$$= \frac{i}{\not p - m_r - \Sigma_r(p^2)}$$

In general

$$\Sigma_r(p^2 = m_r^2) \neq 0$$

→ m_r does not describe the location of the pole

Can be enforced using a different renormalization scheme

The quark propagator and the pole mass scheme

In the on-shell / pole mass scheme Z_Ψ and Z_m are chosen such that

a) $\Sigma_r(p^2 = m_r^2) = 0$

b) $\mathrm{Res}_{p^2 \to m_r^2} \frac{1}{\not{p} - m - \Sigma_r} = 1$

Two conditions to fix the finite terms of Z_Ψ and Z_m

Note:

$$Z_m^{\text{on}} = Z_m^{\overline{\text{MS}}} + \text{finite}$$
$$Z_\Psi^{\text{on}} = Z_\Psi^{\overline{\text{MS}}} + \text{finite}$$

can be calculated knowing Σ

Relation between MS and pole mass

$$m_0 = Z_m^{\overline{\mathrm{MS}}} m_{\overline{\mathrm{MS}}} = Z_m^{\mathrm{on}} m_{\mathrm{on}}$$

$$m_{\overline{\mathrm{MS}}} = \frac{Z_m^{\mathrm{on}}}{Z_m^{\overline{\mathrm{MS}}}} m_{\mathrm{on}}$$

$$= m_{\mathrm{on}} \left(1 + \frac{\alpha_s(\mu)}{\pi} C_f \left(1 + \frac{3}{4} \ln\left(\frac{\mu^2}{m^2} \right) \right) + O(\alpha_s^2) \right)$$

In LO:

$$m_{\overline{\mathrm{MS}}} = m_{\mathrm{on}}$$

At least NLO predictions required to distinguish between the two schemes

Relation $m_{\overline{\mathrm{MS}}} \leftrightarrow m_{\mathrm{on}}$ known to 4 loops [Marquard, Smirnov, Smirnov, Steinhauser ’15]

Which mass scheme should we use ?

Naïve answer:

It should not matter

„In theory there is no difference between theory and practice. In practice there is.“ [Yogi Ber(r)a]

In practice:

It does matter !

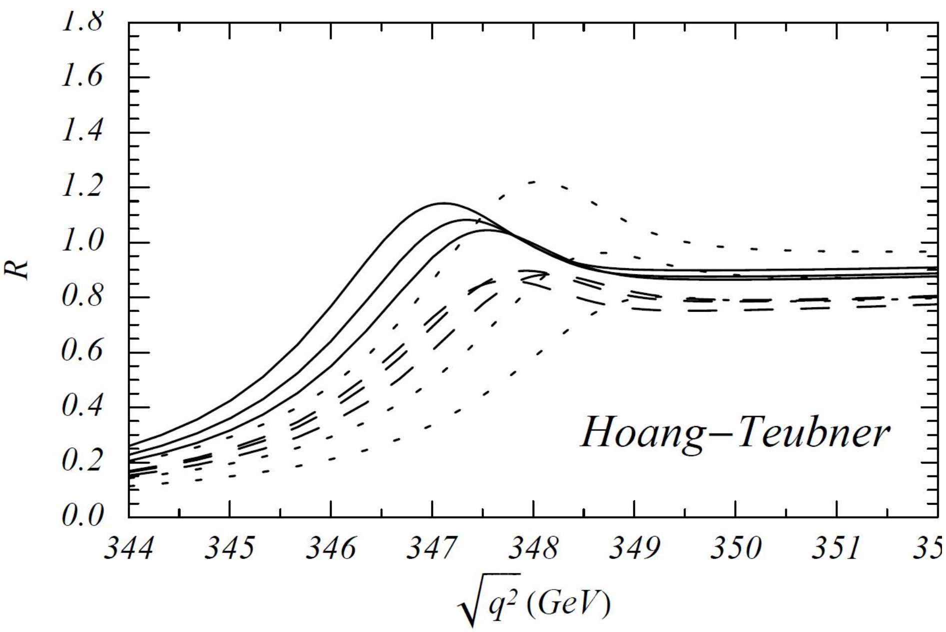
Possible issues:

- Perturbative expansion may converge better
- Conceptual limitation of achievable precision

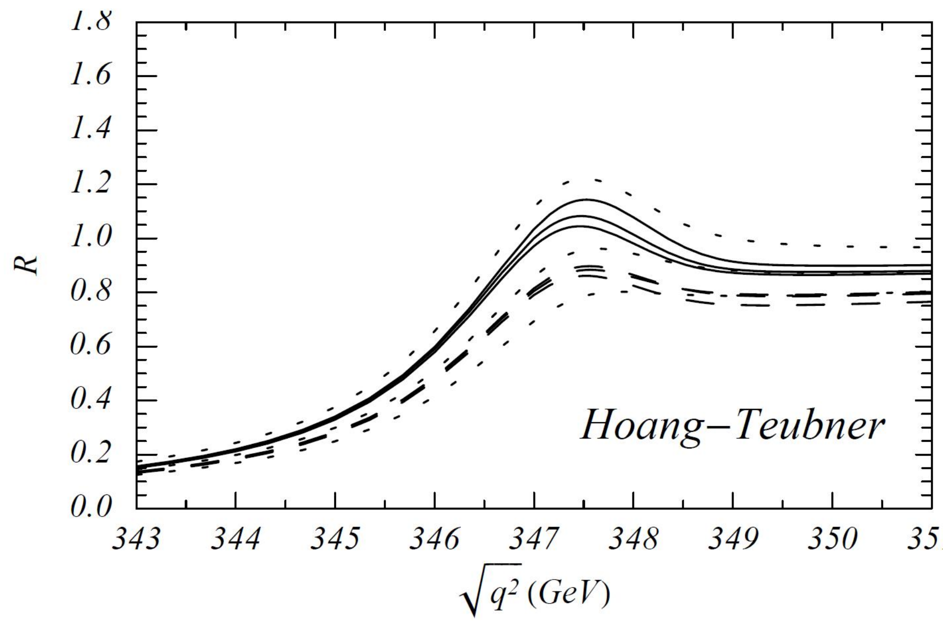
Example: e+e- → tt at threshold

[Hoang et al, 2000]

Pole mass



1S mass

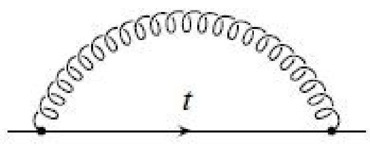


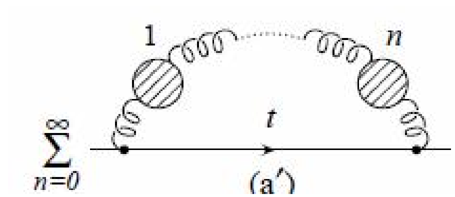
LO dotted, NLO dashed, NNLO solid

Very large corrections using the pole mass scheme

1S mass: Position of would-be 1S boundstate

Renormalon ambiguity in pole mass

$\Sigma^{(1)} =$ 



$$\frac{16 m_R}{3\beta_0}\sum_{n=0}^{\infty} c_n a^{n+1}$$

$$c_n \overset{n\to\infty}{\longrightarrow} e^{-C/2}2^n n!$$

[Bigi, Shifman, Uraltsev, Vainshtein 94 Beneke, Braun,94 Smith, Willenbrock 97]

There is no pole in full QCD !

Pole mass has intrinsic uncertainty of order Λ_{QCD}

→ All e+e- threshold studies use so-called short distance mass free of renormalon ambiguity

Checklist:

- ❑ Observable should show good sensitivity to m

$$\frac{\Delta O}{O} \leftrightarrow \frac{\Delta m_t}{m_t}$$

- ❑ Observable must be theoretically calculable

- ❑ Theory uncertainty must be small

  small non-perturbative corrections

- ❑ Observable must be “experimentally accessible”

- ❑ Well defined mass scheme

Well defined mass scheme using as observable

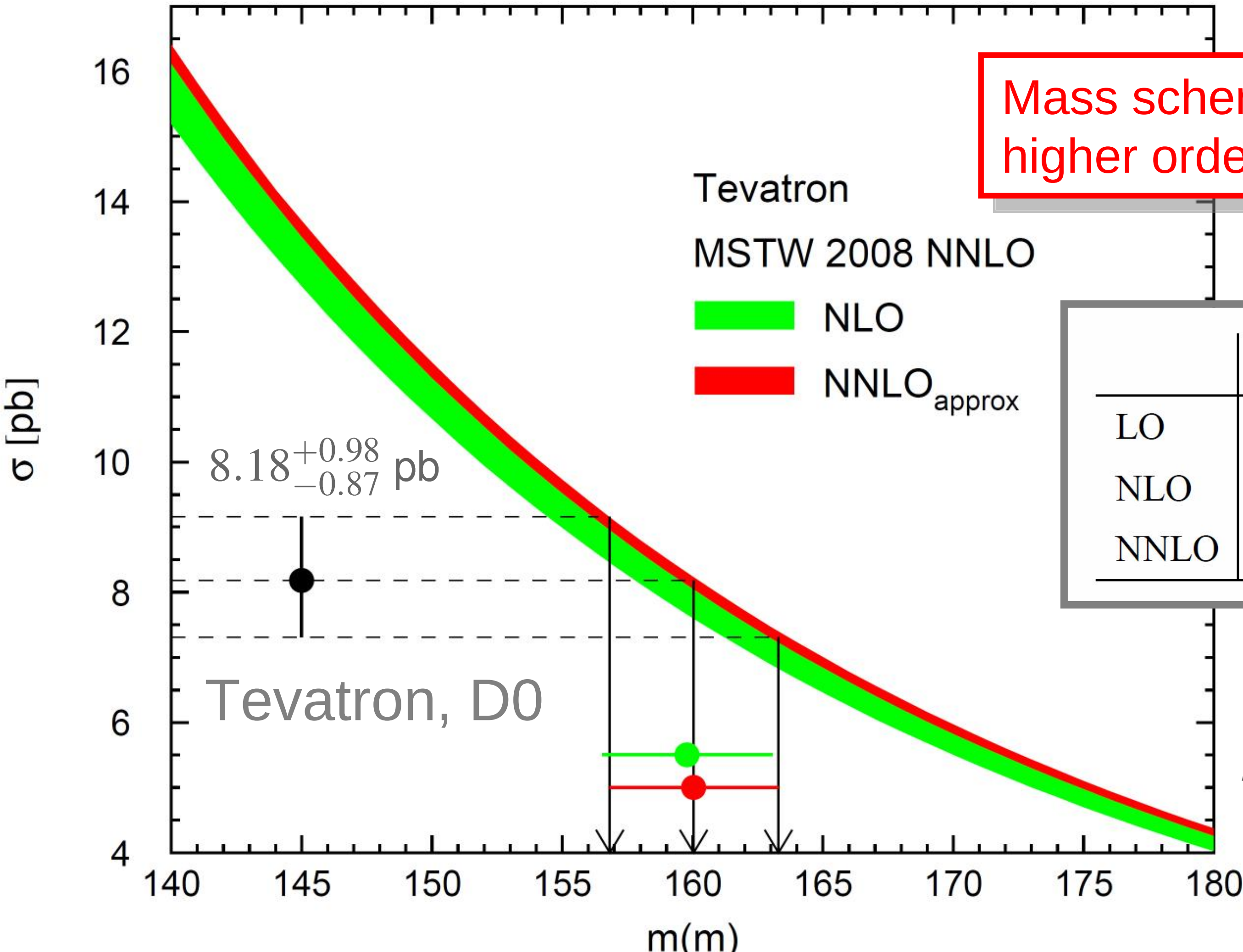
- Inclusive cross section
- m_{lb} distribution
- Top-quark pair production in association with an additional jet (most precise pole mass determination so far)

Mass scheme not well defined in

- Kinematical reconstruction
- Template method

→ The measured mass is the so-called Monte Carlo mass, Not well defined in perturbation theory, however expected to be close to the pole / on-shell mass

$\overline{\text{MS}}$ mass from cross section



Mass scheme well defined, higher orders can be included

	$\overline{m}$ [GeV]	m_t [GeV]
LO	$159.2^{+3.5}_{-3.4}$	$159.2^{+3.5}_{-3.4}$
NLO	$159.8^{+3.3}_{-3.3}$	$165.8^{+3.5}_{-3.5}$
NNLO	$160.0^{+3.3}_{-3.2}$	$168.2^{+3.6}_{-3.5}$

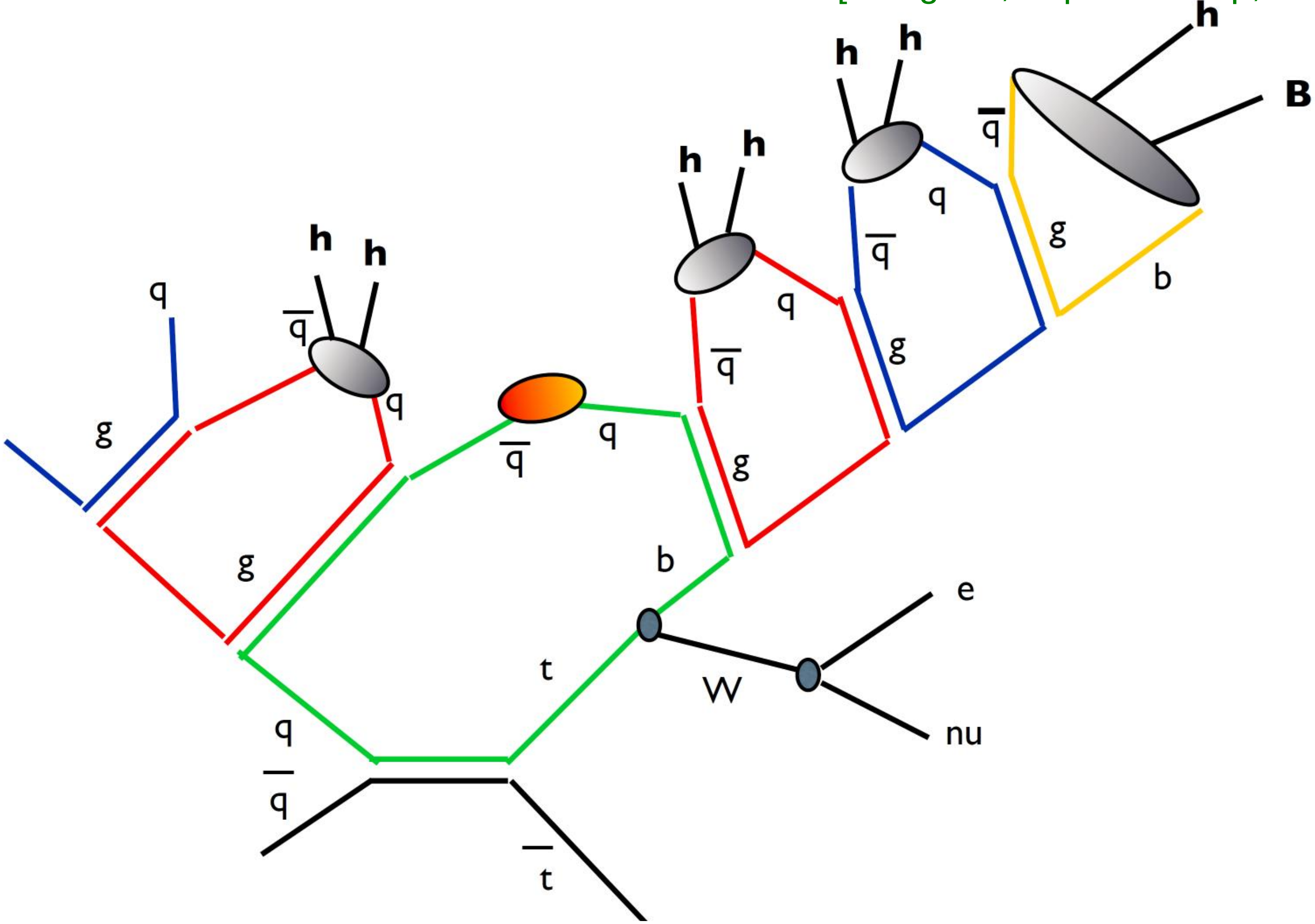
$$m_t = 168.9^{+3.5}_{-3.4}$$

only exp. uncertainties

Drawback: Limited sensitivity to m_t $\quad \frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \approx 5\frac{\Delta m_t}{m_t}$

Top-quark mass from kinematical reconstruction

[Mangano, Top workshop, July 2012, CERN]



$$p_t \neq p_W + \sum_i p^i_{\text{had}}$$

The hierarchy problem

Remember

$$m_r = m_0 + \Sigma_{\text{bare}} + \delta\Sigma_r$$

individually
divergent

→ It doesn't make sense to speculate about the size of Σ_{bare}

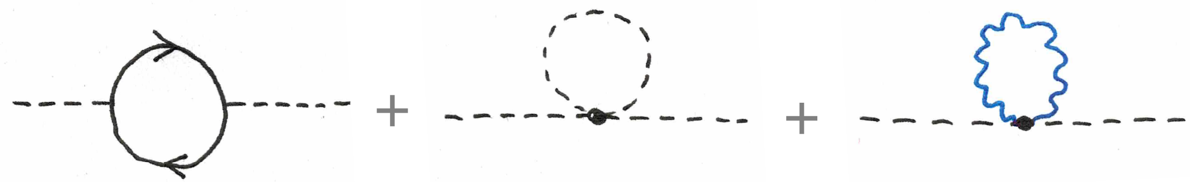
Situation changes if you treat your theory / theoretical model as an effective theory only valid up to some cut-off scale Λ

The hierarchy problem

- → Self energy corrections are finite since divergences are cut-off at scale Λ
- → "no technical need" to introduce renormalization
- → m_0 can be finite and thus physically meaning full

Consider corrections to Higgs mass in theory with cut-off

$\delta m_h =$



$$= -\frac{3y_t^2}{8\pi^2}\Lambda^2 + \frac{3\lambda}{8\pi^2}\Lambda^2 + \frac{9\alpha_w + 3\alpha'}{16\pi}\Lambda^2$$

→ Depending on Λ very large corrections, since Higgs mass is not protected by any symmetry, need fine tuning of m_0 for large Λ

The hierarchy problem

Either we have to fine tune m_0 or we have to assume $\Lambda \approx 1$ TeV to avoid large corrections

If fine tuning is not an option, we should see new physics at 1 TeV

Note:

Argument relies on the assumption of a physical cut-off !

Ignoring problems like neutrino masses, dark matter, gravity, the SM could be valid up to very high scales…

Summary

- The different role of observables and parameters
- The physical picture of renormalization
- Role of quantum corrections and the origin of the scheme dependence
- Examples:
  - the running coupling constant
  - mass renormalization

Detailed understanding of the interplay between measurements and theory is crucial for precision physics and the interpretation of the results !

Thank you for your attention