# Systematic Uncertainties Exercises

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Discussion on model uncertainties...

## 1 Exercise 1

Which is more accurate a single measurement with an error of 0.2 mm or ten measurements each with an error of 1 mm?

## 2 Exercise 2

How compatible (in terms of a $p$-value or Gaussian $\sigma$) are the following two independent (i.e. uncorrelated) measurements:

$$x = 0.05 \pm 0.20 \pm 0.15$$
$$x = 1.40 \pm 0.20 \pm 0.10$$

What is the average of these two measurements (with its uncertainty of course)? Do you have any comments about the validity of such an average? What about if the two measurements have a statistical correlation of 0.8 and a systematic correlation of 0.6?

## 3 Exercise 3

You do an analysis on a signal of $B^0$ candidates decaying into some final state, XY. You want to publish the branching fraction for $B^0 \to XY$. You find $1265 \pm 23$ signal candidates. The luminosity is 3 fb$^{-1} \pm 3\%$, the cross section for $B^0$ decays in your acceptance is $40 \pm 0.4$ $\mu$b. Your total signal selection efficiency is $40 \pm 2\%$ from the trigger and $89 \pm 3\%$ from your selection cuts (as measured in MC). What result would you quote for the branching fraction of $B^0 \to XY$, give a central value, a statistical and systematic error. Can you see any problems which you think we should address as experimentalists?

## 4 Exercise 4

The following two observables are measured (where the first uncertainty is statistical, the second is systematic):

$$x = 1.12 \pm 0.12 \pm 0.08$$
$$y = 0.03 \pm 0.04 \pm 0.07$$

The correlation between the statistical and systematic uncertainties are $\rho = 0.42$ and $\rho = 0.65$ respectively.
Calculate the full covariance and correlation matrices (stat + syst) for these two measurements (covariance to 4 d.p, correlation to 2 d.p.).

## 5 Exercise 5

Three observables have been measured, $a$, $b$ and $c$. They each have an associated uncertainty (stat + syst combined) of $\sigma_a$, $\sigma_b$ and $\sigma_c$. They are all correlated with each other by $\rho_{ab}$, $\rho_{ac}$, $\rho_{bc}$.

They are related to the very theoretically interesting parameters $x$, $y$ and $z$ by the following relations:

$$x = a(1 + b)$$
$$y = a(1 - b)$$
$$z = \cos(c)$$

Calculate the covaraince matrix, $\mathcal{C}_{xyz}$, in terms of the central values, uncertainties and correlations for $a$, $b$ and $c$. Is this what you expect and why?

# 6 Answers

## 6.1 Exercise 1

One single measurement with an error of 0.2 mm. As error from the others is $\sqrt{10(0.1^2)} = 0.32$ mm.

## 6.2 Exercise 2

The difference of the two should be Gaussian around zero. So take their difference and the propagate the errors. This gives $1.35 \pm 0.336$. This is just over $4\sigma$ ($p = 5.8 \times 10^{-5}$) away from zero. Average is a bit silly if they are this incompatible.

## 6.3 Exercise 3

I get:

$$\mathcal{B} = (2.96 \pm 0.05 \pm 0.20) \times 10^{-8}$$

Surely means we need to work harder as experimentalists to get those systematic errors smaller. Or that's as good as we can do with our given detector. Or thats as good as we can do without generating more MC. Or there is some theory uncertainty in there which we needs theorists to work harder to improve

## 6.4 Exercise 4

Covariance Matrix:

$$\begin{pmatrix} 0.0160 & 0.0059 \\ 0.0059 & 0.0113 \end{pmatrix} \tag{1}$$

Correlation Matrix:

$$\begin{pmatrix} 1.00 & 0.44 \\ 0.44 & 1.00 \end{pmatrix} \tag{2}$$

## 6.5 Exercise 5