Yukawa couplings at the point of E8 in F-theory

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Based on

[1503.02683] with F. Marchesano and G. Zoccarato

also

[1307.8089] with A. Font, F. Marchesano and G. Zoccarato

[1211.6529] with L. Ibáñez, A. Font and F. Marchesano

Motivation

It is difficult to build natural GUTs within perturbative Type II that are phenomenologically viable.

- E_6 and $SO(10)$ cannot be realised.
- $SU(5)$ GUTs forbid a perturbative top Yukawa. Perturbative $U(1)$

F-theory provides a non-perturbative description of Type IIB and it should be possible to build a $SU(5)$ GUT with naturally large top Yukawa.

[Beasley, Heckman, Vafa '08]
[Donagi Wijnholt '08]

Aim of the talk: Discuss the flavour structure in the vicinity of E_8

[Palti '12]

SU(5) GUTs in F-theory

Gauge group: $SU(5)$

Breaking to SM:
$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

Matter content:
$\mathbf{10_M},\, \mathbf{\bar{5}_M},\, \mathbf{5_U},\, \mathbf{\bar{5}_D} \rightarrow Q,\, U,\, E,\, D,\, L,\, H_U,\, H_D$

} Very robust (holomorphic)

[Talks on monday: Mayrhofer, Kapfer, Reuter, Oehlmann, Till]

Yukawa couplings:

$Y_U : \mathbf{10_M} \times \mathbf{10_M} \times \mathbf{5_U} \rightarrow Q \times U \times H_U$

$$Y_{D/L} : \mathbf{10_M} \times \mathbf{\bar{5}_M} \times \mathbf{\bar{5}_D} \rightarrow \begin{cases} Y_D : Q \times D \times H_D \\ Y_L : E \times L \times H_D \end{cases}$$

Non-holomorphic Localisation

Many other issues: moduli stabilisation, SUSY breaking, proton decay, etc.

7-brane gauge theory

[Beasley, Heckman, Vafa '08]
[Donagi Wijnholt '08]

Essentially the same as the gauge theory for a stack of D7-branes

(exceptional gauge groups!)

Field content: Gauge field A + Adjoint complex scalar Φ

$$\left.\begin{aligned} F^{0,2} &= 0 \\ \bar{\partial}_A \Phi &= 0 \end{aligned}\right\} \text{F-terms}$$

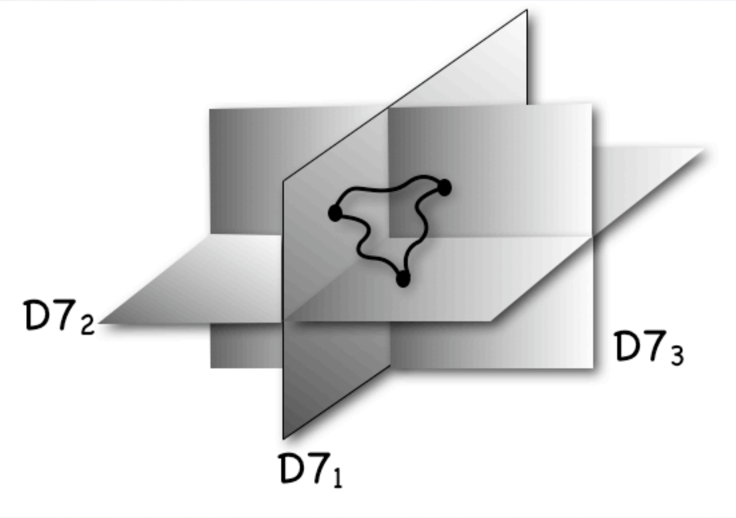
(holomorphic)

$$\omega \wedge F + \frac{1}{2}[\Phi, \Phi^\dagger] = 0 \quad \text{D-term}$$

(physical)

$\langle \Phi \rangle \neq 0$

$\langle A \rangle \neq 0$



Gauge group: singularity at Yukawa point

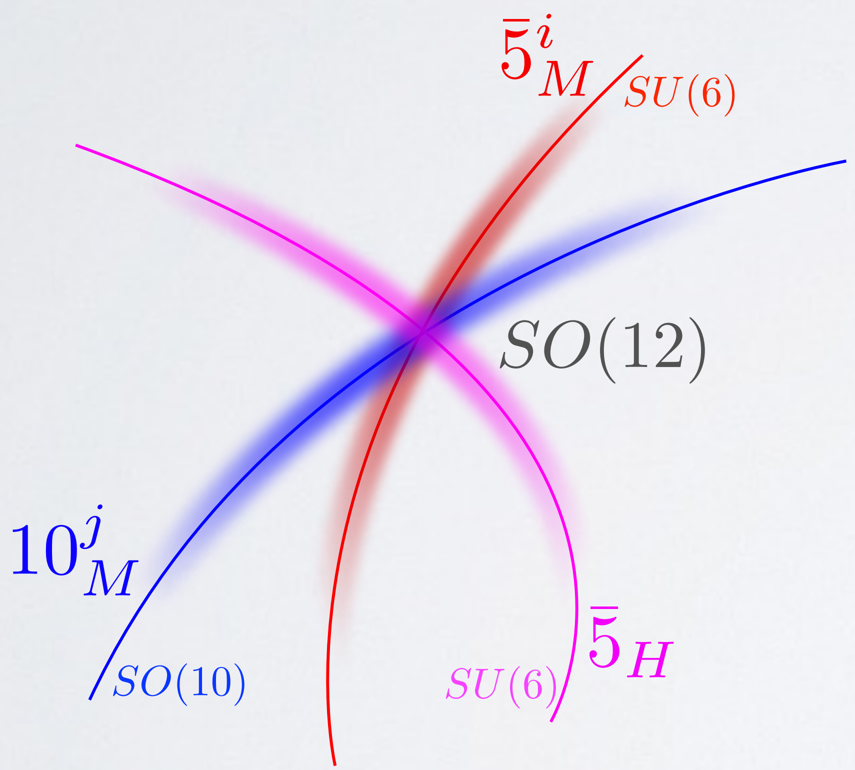
Fluctuations around background. Wavefunctions for MSSM fields, $\vec{\Psi}$

Yukawa couplings

[Heckman, Vafa '08]
[Font, Ibañez '09]
[Cecotti, Cheng, Heckman, Vafa '09]
[Conlon, Palti '09]

$$W = m_* \int_S \mathrm{Tr}(F \wedge \Phi) \supset -i m_* \int_S \mathrm{Tr}(A \wedge A \wedge \Phi)$$

$$Y_{ij} = m_* \int_S \det(\vec{\Psi}_H, \vec{\Psi}^i_M, \vec{\Psi}^j_M)$$



Holomorphic Yukawas are independent of the metric and fluxes. Rank-one problem.

Physical Yukawas when canonical kinetic terms. Depend on the metric and fluxes (hypercharge).

$$K^{ij}_\rho = \int_S \mathrm{Tr}\,(\vec{\Psi}^{\,i\dagger}_\rho \cdot \vec{\Psi}^{\,j}_\rho)\,\mathrm{dvol}_S$$

Beyond rank one

[Cecotti et al '09]
[Marchesano, Martucci '09]
[Aparicio et al '11]
[Font et al '12]

Additional effects are needed to solve the rank-one problem

E3-branes generate a np superpotential for D3-branes and for D7s with induced D3 charge (i.e. $\frac{1}{8\pi^2}\int_S \mathrm{Tr} F\wedge F$)

$$W = m_*^4 \left[\int_S \mathrm{Tr}\,(\Phi \wedge F) + \frac{\epsilon}{2}\int_S \theta_0 \,\mathrm{Tr}\,(F\wedge F) + \text{higher } \theta_n' s \right]$$

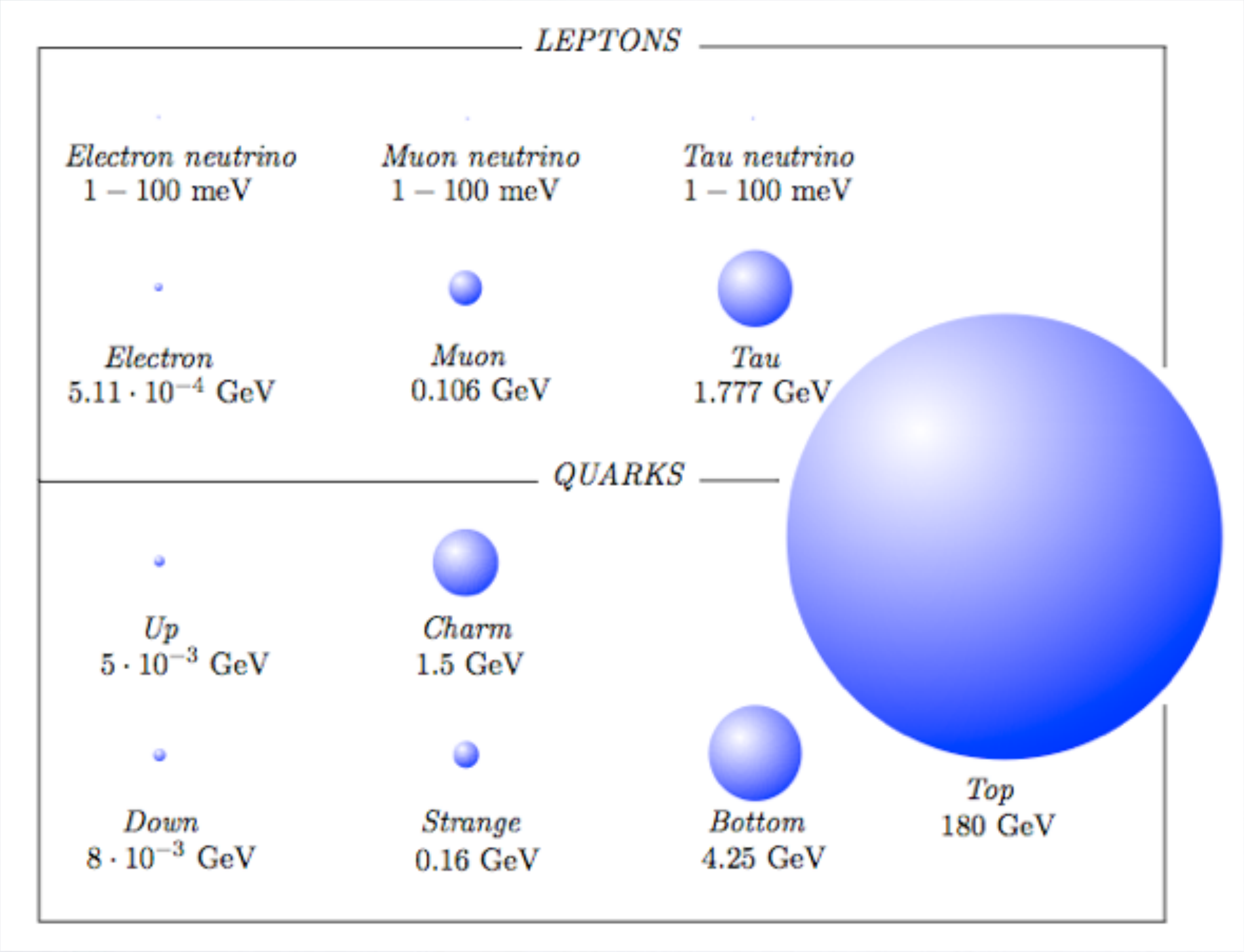
$\epsilon \sim e^{-\mathrm{Vol}_{E3}}$ → Small number (perturbation theory)

$\theta_n \sim \frac{1}{m_*^{2n}} \partial_z^n \log h$ → θ_0 most important

$h = 0$ → Position of E3

Extra contributions to Yukawas
Full rank

What we are looking for



- Hierarchical structure of masses
- Different Yukawas for D-quarks and leptons

$$CKM = \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix}$$

- Almost diagonal CKM

What we are doing

Take E_8 gauge theory on $S_{GUT} = \mathbb{C}^2$ → Ultra-local model for an E_8 stack.

Add deformation to go beyond rank one $W = W_0 + \epsilon W_1$

[Cecotti et al '09]
[Heckman, Tavanfar, Vafa '10]
[Palti '12]

Specify $\langle \Phi \rangle$ in $SU(5)_\perp$, $E_8 \to SU(5)_{GUT} \times SU(5)_\perp$

$$\langle \Phi \rangle \begin{cases} S(U(4) \times U(1))_\perp \\ S(U(3) \times U(2))_\perp \\ S(U(2) \times U(2) \times U(1))_\perp \end{cases}$$

Reconstructible T-branes
Monodromy
$\mathbb{Z}_4,\ \mathbb{Z}_3 \times \mathbb{Z}_2,\ \mathbb{Z}_2 \times \mathbb{Z}_2$

[Cecotti, Cheng, Heckman, Vafa '10]
[Chiou, Faraggi, Tatar, Williams '11]

Focus on the most promising model at the holomorphic level.

Specify fluxes to generate chirality and GUT breaking.

Compute physical Yukawa couplings and CKM matrix.

Holomorphic Yukawas

Most promising model:

$$\langle\Phi\rangle = \begin{pmatrix} \lambda & m & 0 & 0 & 0 \\ m^2 x & \lambda & 0 & 0 & 0 \\ 0 & 0 & d(\lambda+\kappa) & \tilde{m} & 0 \\ 0 & 0 & \tilde{m}^2 y & d(\lambda+\kappa) & 0 \\ 0 & 0 & 0 & 0 & -2(1+d)\lambda - 2d\kappa \end{pmatrix}$$

2+2+1 model

$\mathbb{Z}_2 \times \mathbb{Z}_2$

$$E_8 \to SU(5)_{GUT} \times S(U(2)\times U(2) \times U(1))_\perp$$

$$\mathbf{248} \to \underset{\mathbf{10_M}}{(\mathbf{10},\mathbf{2},\mathbf{1})_{3,-2}} \oplus \underset{\mathbf{\bar{5}_M}}{(\mathbf{\bar{5}},\mathbf{2},\mathbf{1})_{1,-4}} \oplus \underset{\mathbf{5_U}}{(\mathbf{5},\mathbf{1},\mathbf{1})_{-6,4}} \oplus \underset{\mathbf{\bar{5}_D}}{(\mathbf{\bar{5}},\mathbf{1},\mathbf{1})_{-4,6}} \oplus \ldots$$

$$Y_U : \mathbf{10_M} \times \mathbf{10_M} \times \mathbf{5_U}$$
$$Y_{D/L} : \mathbf{10_M} \times \mathbf{\bar{5}_M} \times \mathbf{\bar{5}_D}$$

are allowed by gauge invariance (underlying E_8)

[Palti's talk]

This specifies completely the matter curves and holomorphic Yukawas

(Relatively simple algebraic problem)

Holomorphic Yukawas

$$Y_U = \frac{1}{2\rho_m\rho_\mu}\begin{pmatrix} 0 & 0 & \epsilon\frac{1}{2\rho_\mu} \\ 0 & \epsilon\frac{1}{2\rho_\mu} & 0 \\ \epsilon\frac{1}{2\rho_\mu} & 0 & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

$$Y_{D/L} = \frac{1}{2d\,\rho_m\rho_\mu}\begin{pmatrix} -\epsilon\kappa^2\frac{1}{2d\rho_\mu^3} & \epsilon\kappa\frac{1}{d\rho_\mu^2} & \left(\frac{2\kappa^2}{\rho_\mu} - \frac{\epsilon}{d}\right)\frac{1}{2\rho_\mu} \\ \epsilon\kappa\frac{1}{2d\rho_\mu^2} & -\epsilon\frac{1}{2d\rho_\mu} & -\kappa\frac{1}{\rho_\mu} \\ -\epsilon\frac{1}{2d\rho_\mu} & 0 & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

$$Y_t = \frac{1}{2\rho_m\rho_\mu}, \qquad Y_c = \epsilon\,\frac{1}{4\rho_m\rho_\mu^2}, \qquad Y_u = \mathcal{O}(\epsilon^2)$$
$$Y_b = \frac{1}{2d\rho_m\rho_\mu}, \qquad Y_s = \epsilon\,\frac{1}{4d^2\rho_m\rho_\mu^2}, \qquad Y_d = \mathcal{O}(\epsilon^2)$$
$$Y_\tau = \frac{1}{2d\rho_m\rho_\mu}, \qquad Y_\mu = \epsilon\,\frac{1}{4d^2\rho_m\rho_\mu^2}, \qquad Y_e = \mathcal{O}(\epsilon^2),$$

Hierarchical structure (not true for every model)

Do not depend on fluxes ($Y_D = Y_L$)

κ controls the separation between the two Yukawa points (CKM)

Physical Yukawas

Introduce fluxes to generate chirality and break the GUT group (hypercharge)

- Non-primitive fluxes from T-branes (D-term)
- Most general constant primitive fluxes (M_1, M_2, N_1, N_2)
- Hypercharge flux $(N_Y, \tilde{N}_Y)$: $SU(5)_{GUT} \to SU(3) \times SU(2) \times U(1)$

Solve for the fluctuations around this background (including the D-term)

- Compute the kinetic terms for the MSSM fields $K_\rho^{ij} = \int_S \mathrm{Tr}\,(\vec{\Psi}_\rho^{\,i\dagger} \cdot \vec{\Psi}_\rho^{\,j})\, \mathrm{dvol}_S$
- Compute the physical Yukawa couplings

(Very complicated system of coupled PDEs)

Physical Yukawas

$$Y_t = \frac{\pi^2\, \gamma_U\, \gamma^Q_{10,3}\gamma^U_{10,3}}{2\rho_m\rho_\mu}, \qquad Y_c = \epsilon\, \frac{\pi^2\, \gamma_U\, \gamma^Q_{10,2}\gamma^U_{10,2}}{4\rho_m\rho_\mu^2}, \qquad Y_u = \mathcal{O}(\epsilon^2)$$
$$Y_b = \frac{\pi^2\, \gamma_D\, \gamma^Q_{10,3}\gamma^D_{5,3}}{2d\rho_m\rho_\mu}, \qquad Y_s = \epsilon\, \frac{\pi^2\, \gamma_D\, \gamma^Q_{10,2}\gamma^D_{5,2}}{4d^2\rho_m\rho_\mu^2}, \qquad Y_d = \mathcal{O}(\epsilon^2)$$
$$Y_\tau = \frac{\pi^2\, \gamma_D\, \gamma^E_{10,3}\gamma^L_{5,3}}{2d\rho_m\rho_\mu}, \qquad Y_\mu = \epsilon\, \frac{\pi^2\, \gamma_D\, \gamma^E_{10,2}\gamma^L_{5,2}}{4d^2\rho_m\rho_\mu^2}, \qquad Y_e = \mathcal{O}(\epsilon^2),$$

The gammas are the normalisation factors. Depend on intersection angles and fluxes (not a simple dependence)

For reasonable values of the parameters, it is possible to (not generic):
- Obtain a large top Yukawa coupling
- Have different Y_D and Y_L at unification scale (hypercharge flux)
- Match measured Yukawas for 2nd and 3rd generations (RGEs)

CKM element:

$$|V_{tb}| \simeq 1 - \frac{|\kappa|^2(\gamma^Q_{10,2})^2}{2|\rho_\mu|^2(\gamma^Q_{10,3})^2} \simeq 0.9991$$

κ controls the distance between Yukawa points

Y_U and $Y_{D/L}$ very close to each other

Conclusions

- We have performed a partial classification of ultra-local models at the point of E_8 in F-theory $SU(5)$ GUTs

- It seems possible (though not generic) to get a satisfactory flavour structure from E_8 in F-theory:
  - Large top Yukawa
  - Hierarchy of masses
  - Different Y_D and Y_L
  - Almost diagonal CKM

Outlook

- Go to next order in the epsilon analysis. Better control over the lightest generation (Cabibbo angle)

- From ultra-local to local. Better control over the normalisation factors. Curvature of S_{GUT}, more constraints, etc.

Thank you!

Yukawas and masses at unification scale

$\tan\beta$	10	38	50
m_u/m_c	$2.7 \pm 0.6 \times 10^{-3}$	$2.7 \pm 0.6 \times 10^{-3}$	$2.7 \pm 0.6 \times 10^{-3}$
m_c/m_t	$2.5 \pm 0.2 \times 10^{-3}$	$2.4 \pm 0.2 \times 10^{-3}$	$2.3 \pm 0.2 \times 10^{-3}$
m_d/m_s	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$
m_s/m_b	$1.9 \pm 0.2 \times 10^{-2}$	$1.7 \pm 0.2 \times 10^{-2}$	$1.6 \pm 0.2 \times 10^{-2}$
m_e/m_μ	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$
m_μ/m_τ	$5.9 \pm 0.2 \times 10^{-2}$	$5.4 \pm 0.2 \times 10^{-2}$	$5.0 \pm 0.2 \times 10^{-2}$
Y_τ	0.070 ± 0.003	0.32 ± 0.02	0.51 ± 0.04
Y_b	0.051 ± 0.002	0.23 ± 0.01	0.37 ± 0.02
Y_t	0.48 ± 0.02	0.49 ± 0.02	0.51 ± 0.04