Anomaly Cancelation from Geometric Symmetries in F-theory

Andreas Kapfer

arXiv:1502.05398 (T. Grimm, AK)

Max-Planck-Institut für Physik

Workshop Beyond the Standard Model

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

1. Abelian anomaly cancelation from invariance under zero-section choices in F-theory
2. Non-Abelian anomaly cancelation from invariance under "zero-node" choices in F-theory

F-theory effective action via M-theory

M-theory

M-theory on (resolved) Calabi-Yau fourfold

$\Longleftrightarrow$

F-theory

4D, $\mathcal{N} = 1$ supergravity on S^1, on the Coulomb branch

F-theory effective action via M-theory

M-theory

M-theory on (resolved) Calabi-Yau fourfold

$\Longleftrightarrow$

F-theory

4D, $\mathcal{N}=1$ supergravity on S^1, on the Coulomb branch

- divisor basis: D_0, D_m, D_I, D_α

$$C_3 = A^0[D_0] + A^m[D_m] + A^I[D_I] + A^\alpha[D_\alpha]$$

A^0 KK-vector
A^m 4D $U(1)$ gauge fields
A^I 4D Cartan gauge fields
A^α dualized axions

F-theory effective action via M-theory

M-theory

M-theory on (resolved) Calabi-Yau fourfold

$\Longleftrightarrow$

F-theory

4D, $\mathcal{N}=1$ supergravity on S^1, on the Coulomb branch

- divisor basis: D_0, D_m, D_I, D_α

$$C_3 = A^0[D_0] + A^m[D_m] + A^I[D_I] + A^\alpha[D_\alpha]$$

A^0 KK-vector
A^m 4D $U(1)$ gauge fields
A^I 4D Cartan gauge fields
A^α dualized axions

- matching of Chern-Simons terms in the 3D theories:
  [Grimm, Hayashi; Bonetti, Grimm; Cvetic, Grimm, Klevers; Grimm, AK, Keitel; Cvetic, Grassi, Klevers, Piragua; Anderson, Garcia-Extebarria, Grimm, Keitel]

$$S_{\mathrm{CS}} = \int \Theta_{\Lambda\Sigma} A^\Lambda \wedge F^\Sigma$$

$$-\frac{1}{4} D_\Lambda \cdot D_\Sigma \cdot [G_4] = \Theta^{\mathrm{M}}_{\Lambda\Sigma} \overset{!}{=} \Theta^{\mathrm{F}}_{\Lambda\Sigma} \begin{cases} \text{classical: 4D axion gauging} \\ \text{quantum: massive modes} \end{cases}$$

Abelian anomalies from zero-section changes in F-theory

F-theory on an elliptically fibered Calabi-Yau fourfold with:

- zero-section σ_0 (corresponds to KK-vector)
- additional sections σ_m (correspond to $U(1)$ vectors)
- no non-Abelian gauge symmetries

Abelian anomalies from zero-section changes in F-theory

F-theory on an elliptically fibered Calabi-Yau fourfold with:

- zero-section σ_0 (corresponds to KK-vector)
- additional sections σ_m (correspond to $U(1)$ vectors)
- no non-Abelian gauge symmetries

Basis of divisors in the dual M-theory geometric setting: D_0, D_m, D_α
[Grimm, Savelli; Park; Cvetic, Grimm, Klevers; Grimm, AK, Keitel; Shioda]

Base divisor shift → KK-vector

$$D_0 = \sigma_0 - \tfrac{1}{2}(\sigma_0 \cdot \sigma_0 \cdot \mathcal{C}^\alpha)\eta^{-1}{}_\alpha{}^\beta D_\beta$$

Shioda map → $U(1)$ vectors

$$D_m = \sigma_m - \sigma_0 - \left[(\sigma_m - \sigma_0)\cdot\sigma_0\cdot\mathcal{C}^\alpha\right]\eta^{-1}{}_\alpha{}^\beta D_\beta$$

Abelian anomalies from zero-section changes in F-theory

F-theory on an elliptically fibered Calabi-Yau fourfold with:

- zero-section σ_0 (corresponds to KK-vector)
- additional sections σ_m (correspond to $U(1)$ vectors)
- no non-Abelian gauge symmetries

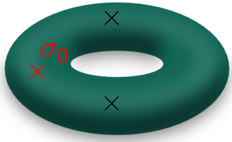
Basis of divisors in the dual M-theory geometric setting: D_0, D_m, D_α
[Grimm, Savelli; Park; Cvetic, Grimm, Klevers; Grimm, AK, Keitel; Shioda]

Base divisor shift → KK-vector

$$D_0 = \sigma_0 - \tfrac{1}{2}(\sigma_0 \cdot \sigma_0 \cdot \mathcal{C}^\alpha)\eta^{-1}{}_\alpha{}^\beta D_\beta$$

Shioda map → $U(1)$ vectors

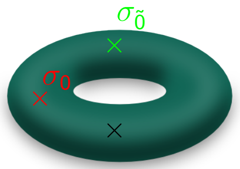
$$D_m = \sigma_m - \sigma_0 - \left[(\sigma_m - \sigma_0) \cdot \sigma_0 \cdot \mathcal{C}^\alpha\right]\eta^{-1}{}_\alpha{}^\beta D_\beta$$



Map under a zero-section change

$$\begin{pmatrix} \tilde{A}^0 \\ \tilde{A}^{\tilde{0}} \\ \tilde{A}^{\tilde{m}} \\ \tilde{A}^{\alpha} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \delta^{\tilde{m}}_{\tilde{n}} & 0 \\ \frac{1}{2} b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{n}} & \delta^{\alpha}_{\beta} \end{pmatrix} \cdot \begin{pmatrix} A^0 \\ A^{\tilde{0}} \\ A^{\tilde{n}} \\ A^{\beta} \end{pmatrix}$$

σ_0: zero-section

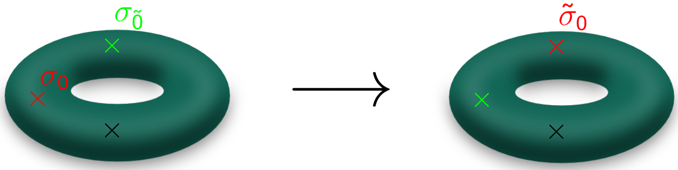


Map under a zero-section change

$$\begin{pmatrix} \tilde{A}^0 \\ \tilde{A}^{\tilde{0}} \\ \tilde{A}^{\tilde{m}} \\ \tilde{A}^{\alpha} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \delta^{\tilde{m}}_{\tilde{n}} & 0 \\ \frac{1}{2} b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{n}} & \delta^{\alpha}_{\beta} \end{pmatrix} \cdot \begin{pmatrix} A^0 \\ A^{\tilde{0}} \\ A^{\tilde{n}} \\ A^{\beta} \end{pmatrix}$$

σ_0: zero-section

$\sigma_{\tilde{0}}$: exchanged $U(1)$-section

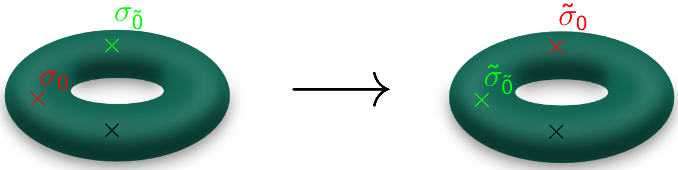


Map under a zero-section change

$$\begin{pmatrix} \tilde{A}^0 \\ \tilde{A}^{\tilde{0}} \\ \tilde{A}^{\tilde{m}} \\ \tilde{A}^{\alpha} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \delta^{\tilde{m}}_{\tilde{n}} & 0 \\ \frac{1}{2} b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}0} & b^{\alpha}_{\tilde{0}\tilde{n}} & \delta^{\alpha}_{\beta} \end{pmatrix} \cdot \begin{pmatrix} A^0 \\ A^{\tilde{0}} \\ A^{\tilde{n}} \\ A^{\beta} \end{pmatrix}$$

σ_0: zero-section

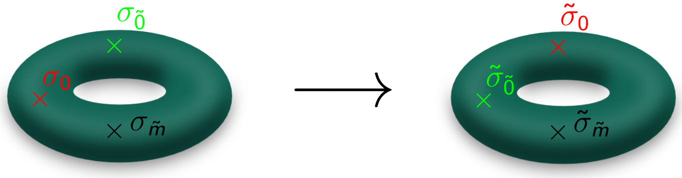
$\sigma_{\tilde{0}}$: exchanged $U(1)$-section



Map under a zero-section change

$$\begin{pmatrix} \tilde{A}^0 \\ \tilde{A}^{\tilde{0}} \\ \tilde{A}^{\tilde{m}} \\ \tilde{A}^{\alpha} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \delta^{\tilde{m}}_{\tilde{n}} & 0 \\ \frac{1}{2}\, b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{n}} & \delta^{\alpha}_{\beta} \end{pmatrix} \cdot \begin{pmatrix} A^0 \\ A^{\tilde{0}} \\ A^{\tilde{n}} \\ A^{\beta} \end{pmatrix}$$

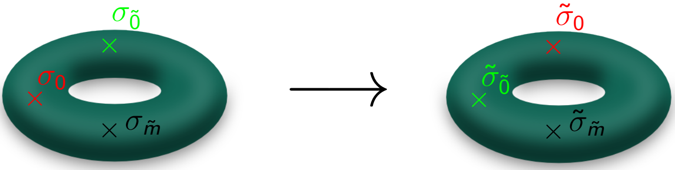
σ_0: zero-section
$\sigma_{\tilde{0}}$: exchanged $U(1)$-section



Map under a zero-section change

$$\begin{pmatrix} \tilde{A}^0 \\ \tilde{A}^{\tilde{0}} \\ \tilde{A}^{\tilde{m}} \\ \tilde{A}^{\alpha} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \delta^{\tilde{m}}_{\tilde{n}} & 0 \\ \frac{1}{2} b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{n}} & \delta^{\alpha}_{\beta} \end{pmatrix} \cdot \begin{pmatrix} A^0 \\ A^{\tilde{0}} \\ A^{\tilde{n}} \\ A^{\beta} \end{pmatrix}$$

σ_0: zero-section
$\sigma_{\tilde{0}}$: exchanged $U(1)$-section
$\sigma_{\tilde{m}}$: "spectator" $U(1)$-sections



Map under a zero-section change

$$\begin{pmatrix} \tilde{A}^0 \\ \tilde{A}^{\tilde{0}} \\ \tilde{A}^{\tilde{m}} \\ \tilde{A}^{\alpha} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \delta^{\tilde{m}}_{\tilde{n}} & 0 \\ \frac{1}{2}\, b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{0}} & b^{\alpha}_{\tilde{0}\tilde{n}} & \delta^{\alpha}_{\beta} \end{pmatrix} \cdot \begin{pmatrix} A^0 \\ A^{\tilde{0}} \\ A^{\tilde{n}} \\ A^{\beta} \end{pmatrix}$$

σ_0: zero-section
$\sigma_{\tilde{0}}$: exchanged $U(1)$-section
$\sigma_{\tilde{m}}$: “spectator” $U(1)$-sections

- appearance of Green-Schwarz couplings b^{α}_{mn}
- mixing of Kaluza-Klein vector with $U(1)$: $\tilde{A}^{\tilde{0}} = A^0 + A^{\tilde{0}}$

Observations:

- transformation contains only field-theoretic quantities
  $\Rightarrow$ application to arbitrary 4D theories
- corresponds to large gauge transformation
- classical Chern-Simons terms invariant (up to $U(1)$ basis transformations)
- interesting things happen for one-loop Chern-Simons terms

Observations:

- transformation contains only field-theoretic quantities
  $\Rightarrow$ application to arbitrary 4D theories
- corresponds to large gauge transformation
- classical Chern-Simons terms invariant (up to $U(1)$ basis transformations)
- interesting things happen for one-loop Chern-Simons terms

Loop calculation in the new basis:

[Grimm, AK, Keitel; Cvetic, Grassi, Klevers, Piragua]

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \sum_q C(q)\; q_m q_n\; (\tilde{l}_q + \frac{1}{2})\; \mathrm{sign}(\tilde{m}^q_{\mathrm{CB}})$$

$C(q)$: 4D chiral multiplets with charge q
$\langle\zeta\rangle$: Wilson line background

$m^q_{\mathrm{CB}} = q\cdot\langle\zeta\rangle\,,\;\; m_{\mathrm{KK}} = \frac{1}{r}\,,\;\; l_q = \left\lfloor \left| \frac{m^q_{\mathrm{CB}}}{m_{\mathrm{KK}}} \right| \right\rfloor$

Loop calculation in the new basis:

[Grimm, AK, Keitel; Cvetic, Grassi, Klevers, Piragua]

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \sum_q C(q)\ q_m q_n\ (\tilde{l}_q + \frac{1}{2})\ \mathrm{sign}(\tilde{m}^q_{\mathrm{CB}})$$

$C(q)$: 4D chiral multiplets with charge q
$\langle\zeta\rangle$: Wilson line background

$m^q_{\mathrm{CB}} = q \cdot \langle\zeta\rangle\,,\quad m_{\mathrm{KK}} = \frac{1}{r}\,,\quad l_q = \left\lfloor \left| \frac{m^q_{\mathrm{CB}}}{m_{\mathrm{KK}}} \right| \right\rfloor$

Loop calculation in the old basis:

axion gauging ↓

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \Theta^{\mathrm{F}}_{mn} + 3\ \Theta^{\mathrm{F}}_{\alpha(\tilde{0}} b^{\alpha}_{mn)} = \sum_q C(q)\ q_m q_n\ (l_q + \frac{1}{2})\ \mathrm{sign}(m^q_{\mathrm{CB}}) + \frac{3}{2}\theta^{\mathrm{ax}}_{\alpha(\tilde{0}}\ b^{\alpha}_{mn)}$$

Loop calculation in the new basis:

[Grimm, AK, Keitel; Cvetic, Grassi, Klevers, Piragua]

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \sum_q C(q)\ q_m q_n\ (\tilde{l}_q + \frac{1}{2})\ \mathrm{sign}(\tilde{m}^q_{\mathrm{CB}})$$

$C(q)$: 4D chiral multiplets with charge q
$\langle\zeta\rangle$: Wilson line background

$$m^q_{\mathrm{CB}} = q\cdot\langle\zeta\rangle\,, \quad m_{\mathrm{KK}} = \frac{1}{r}\,, \quad l_q = \left\lfloor \left| \frac{m^q_{\mathrm{CB}}}{m_{\mathrm{KK}}} \right| \right\rfloor$$

Loop calculation in the old basis:

axion gauging ↓

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \Theta^{\mathrm{F}}_{mn} + 3\ \Theta^{\mathrm{F}}_{\alpha(\tilde{0}} b^{\alpha}_{mn)} = \sum_q C(q)\ q_m q_n\ (l_q + \frac{1}{2})\ \mathrm{sign}(m^q_{\mathrm{CB}}) + \frac{3}{2}\theta^{\mathrm{ax}}_{\alpha(\tilde{0}}\ b^{\alpha}_{mn)}$$

Different results,
depending on the choice
of zero-section?

Loop calculation in the new basis:

[Grimm, AK, Keitel; Cvetic, Grassi, Klevers, Piragua]

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \sum_q C(q)\ q_m q_n\ (\tilde{l}_q + \frac{1}{2})\ \mathrm{sign}(\tilde{m}^q_{\mathrm{CB}})$$

$C(q)$: 4D chiral multiplets with charge q
$\langle\zeta\rangle$: Wilson line background

$$m^q_{\mathrm{CB}} = q\cdot\langle\zeta\rangle\ ,\quad m_{\mathrm{KK}} = \tfrac{1}{r}\ ,\quad l_q = \left\lfloor \left| \tfrac{m^q_{\mathrm{CB}}}{m_{\mathrm{KK}}} \right| \right\rfloor$$

Loop calculation in the old basis:

axion gauging ↓

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \Theta^{\mathrm{F}}_{mn} + 3\ \Theta^{\mathrm{F}}_{\alpha(\tilde{0}} b^{\alpha}_{mn)} = \sum_q C(q)\ q_m q_n\ (l_q + \frac{1}{2})\ \mathrm{sign}(m^q_{\mathrm{CB}}) + \frac{3}{2}\theta^{\mathrm{ax}}_{\alpha(\tilde{0}}\ b^{\alpha}_{mn)}$$

Anomaly cancelation

$$\sum_q C(q) q_m q_n q_{\tilde{0}} = \tfrac{3}{2}\theta^{\mathrm{ax}}_{\alpha(\tilde{0}}\ b^{\alpha}_{mn)}$$

Loop calculation in the new basis:

[Grimm, AK, Keitel; Cvetic, Grassi, Klevers, Piragua]

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \sum_q C(q)\; q_m q_n\; (\tilde{l}_q + \frac{1}{2})\; \mathrm{sign}(\tilde{m}^q_{\mathrm{CB}})$$

$C(q)$: 4D chiral multiplets with charge q
$\langle\zeta\rangle$: Wilson line background

$m^q_{\mathrm{CB}} = q\cdot\langle\zeta\rangle\,,\quad m_{\mathrm{KK}} = \frac{1}{r}\,,\quad l_q = \left\lfloor \left| \frac{m^q_{\mathrm{CB}}}{m_{\mathrm{KK}}} \right| \right\rfloor$

Loop calculation in the old basis:

axion gauging
↓

$$\tilde{\Theta}^{\mathrm{F}}_{mn} = \Theta^{\mathrm{F}}_{mn} + 3\; \Theta^{\mathrm{F}}_{\alpha(\tilde{0}} b^{\alpha}_{mn)} = \sum_q C(q)\; q_m q_n\; (l_q + \frac{1}{2})\; \mathrm{sign}(m^q_{\mathrm{CB}}) + \frac{3}{2}\theta^{\mathrm{ax}}_{\alpha(\tilde{0}}\; b^{\alpha}_{mn)}$$

Anomaly cancelation

$\sum_q C(q) q_m q_n q_{\tilde{0}} = \frac{3}{2}\theta^{\mathrm{ax}}_{\alpha(\tilde{0}}\; b^{\alpha}_{mn)}$

Identity on the Coulomb branch

$q_{\tilde{0}} = -\left(l_q + \frac{1}{2}\right)\, \mathrm{sign}(m^q_{CB}) + \left(\tilde{l}_q + \frac{1}{2}\right)\, \mathrm{sign}(\tilde{m}^q_{CB})$

difference in the matching: anomaly

difference in the matching: anomaly

$$\Downarrow$$

invariance under zero-section choices for F-theory on Calabi-Yau manifolds
$\Rightarrow$ cancelation of Abelian anomalies in the effective theory

difference in the matching: anomaly

$$\Downarrow$$

invariance under zero-section choices for F-theory on Calabi-Yau manifolds
$\Rightarrow$ cancelation of Abelian anomalies in the effective theory

general 4D circle-reduced theory:
4D anomaly cancelation conditions from 3D one-loop Chern-Simons terms

difference in the matching: anomaly

$\Downarrow$

invariance under zero-section choices for F-theory on Calabi-Yau manifolds
$\Rightarrow$ cancelation of Abelian anomalies in the effective theory

general 4D circle-reduced theory:
4D anomaly cancelation conditions from 3D one-loop Chern-Simons terms

- works similarly in six dimensions (F-theory on Calabi-Yau threefolds)
- in general:

Further geometric symmetries for other anomalies?

Non-Abelian anomalies from “zero-node” changes in F-theory

Issue:

- Abelian theories: zero-section changes $\Rightarrow$ transformation of divisors
- analog geometric symmetry for non-Abelian theories not obvious

Non-Abelian anomalies from “zero-node” changes in F-theory

Issue:

- Abelian theories: zero-section changes $\Rightarrow$ transformation of divisors
- analog geometric symmetry for non-Abelian theories not obvious

Approach:

1. Write down a transformation of divisors for the non-Abelian theory, inspired from the Abelian theory ($\Leftrightarrow$ large gauge transformation)
2. Check if it implies anomaly cancelation upon transforming one-loop Chern-Simons terms
3. Propose a corresponding geometric symmetry of the F-theory compactification

Non-Abelian anomalies from “zero-node” changes in F-theory

Issue:

- Abelian theories: zero-section changes $\Rightarrow$ transformation of divisors
- analog geometric symmetry for non-Abelian theories not obvious

Approach:

1. Write down a transformation of divisors for the non-Abelian theory, inspired from the Abelian theory ($\Leftrightarrow$ large gauge transformation) ✓
2. Check if it implies anomaly cancelation upon transforming one-loop Chern-Simons terms
3. Propose a corresponding geometric symmetry of the F-theory compactification

Non-Abelian anomalies from “zero-node” changes in F-theory

Issue:

- Abelian theories: zero-section changes $\Rightarrow$ transformation of divisors
- analog geometric symmetry for non-Abelian theories not obvious

Approach:

1. Write down a transformation of divisors for the non-Abelian theory, inspired from the Abelian theory ($\Leftrightarrow$ large gauge transformation) ✓
2. Check if it implies anomaly cancelation upon transforming one-loop Chern-Simons terms ✓ $\rightarrow$ very non-trivially
3. Propose a corresponding geometric symmetry of the F-theory compactification

Non-Abelian anomalies from "zero-node" changes in F-theory

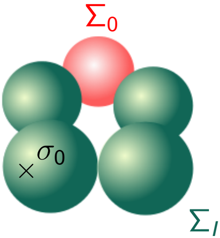
Issue:

- Abelian theories: zero-section changes $\Rightarrow$ transformation of divisors
- analog geometric symmetry for non-Abelian theories not obvious

Approach:

1. Write down a transformation of divisors for the non-Abelian theory, inspired from the Abelian theory ($\Leftrightarrow$ large gauge transformation) ✓
2. Check if it implies anomaly cancelation upon transforming one-loop Chern-Simons terms ✓ $\rightarrow$ very non-trivially
3. Propose a corresponding geometric symmetry of the F-theory compactification ✓ $\rightarrow$ **"zero-node" construction**

Zero-node construction:



- fiber splits into irreducible components
- pick one as the zero-node: Σ_0
- others: Σ_I
- Σ_0 does not have to be the affine node

Generalized base divisor shift → KK-vector

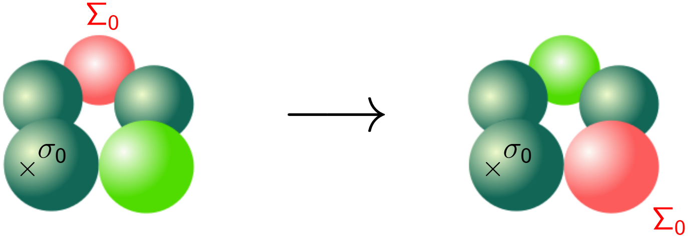
$$D_0 = \sigma_0 - \tfrac{1}{2}(\sigma_0 \cdot \sigma_0 \cdot \mathcal{C}^\alpha)\eta^{-1}{}_\alpha{}^\beta D_\beta + (1 - \sigma_0 \cdot \Sigma_0 \cdot \mathcal{C})\Big[\Sigma_0 - \tfrac{1}{2}(\Sigma_0 \cdot \Sigma_0 \cdot \mathcal{C}^\alpha)\eta^{-1}{}_\alpha{}^\beta D_\beta\Big]$$

"Shioda map" → Cartan vectors

$$\begin{aligned} D_I = \Sigma_I &+ (1 - \sigma_0 \cdot \Sigma_0 \cdot \mathcal{C})\Big[-\Sigma_0 + \Big(\Sigma_0 \cdot (\Sigma_0 - \Sigma_I) \cdot \mathcal{C}^\alpha\Big)\eta^{-1}{}_\alpha{}^\beta D_\beta\Big] \\ &+ (\sigma_0 \cdot \Sigma_I \cdot \mathcal{C})\Big[-\Sigma_I + (\Sigma_0 \cdot \Sigma_I \cdot \mathcal{C}^\alpha)\eta^{-1}{}_\alpha{}^\beta D_\beta\Big] \end{aligned}$$

Properties of this construction:

- If zero-node $\equiv$ affine node: recover the "old" definitions of the base divisor shift and the Cartan divisors
- Classical intersection numbers invariant under the choice of zero-node
- Changing the zero-node induces the desired symmetry transformation, reproducing non-Abelian anomalies from one-loop Chern-Simons terms



Conclusions:

- Invariance of F-theory compactifications on Calabi-Yau manifolds under the choice of the zero-section
  $\Rightarrow$ cancelation of Abelian anomalies in the effective theory
- Introduction of the geometric concept and invariance of "zero-node" choices
  $\Rightarrow$ cancelation of non-Abelian anomalies in the effective theory
- Generalization to field theory: anomaly conditions from applying large gauge transformations to one-loop Chern-Simons terms on the circle

Conclusions:

- Invariance of F-theory compactifications on Calabi-Yau manifolds under the choice of the zero-section
  $\Rightarrow$ cancelation of Abelian anomalies in the effective theory
- Introduction of the geometric concept and invariance of “zero-node” choices
  $\Rightarrow$ cancelation of non-Abelian anomalies in the effective theory
- Generalization to field theory: anomaly conditions from applying large gauge transformations to one-loop Chern-Simons terms on the circle

Future directions:

- Better geometric understanding of the “zero-node” construction
- Identification of further similar geometric symmetries in F-theory compactifications
- Gravitational, conformal anomalies etc.
- Inclusion of multi-sections