From torsion to discrete symmetries
From Pati-Salam to the Standard Model

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Based on

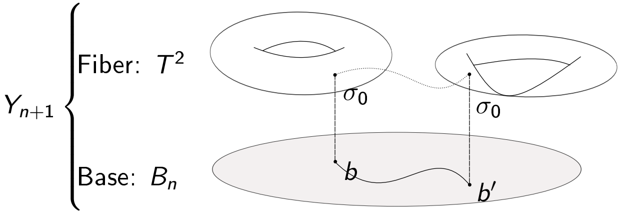
arXiv:1408.4808 with: D. Klevers, D. Mayorga, H. Piragua and J. Reuter

arXiv:1503.02068 with: M. Cvetic, D Klevers, D. Mayorga, J. Reuter

XXVII Workshop Beyond the Standard Model 2015, Bad Honnef

March 16th 2015

Geometric engineering in F-theory

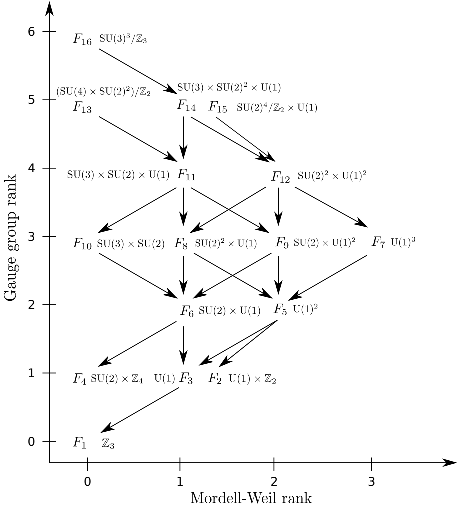


1. **Choose a Fiber:**
   - Codimension 1 singularities: non-Abelian factors
   - Freely acting Mordell-Weil group: Abelian factors

     [Morrison,Park'12;Cvetic,Piragua,Grassi,Klevers,Song;Braun,Grimm,Keitel;'13]
   - Mordell-Weil torsion: Quotient groups [Mayrover,Morrison,Palti,Till,Weigand'14]
   - Discrete $\mathbb{Z}_n$ symmetries: n-sections

     [Braun,Morrison,Mayrhover,Palti,Oskar,Weigand;Anderson,Garcia-Etxebarria,Grimm,Keitel'14]
   - Codimension 2 singularities: Charged matter
   - Coddiemsnion 3 singularities: Yukawa points
2. **Choose a Base space B_n:** Supports divisors for $D7$ branes
3. **Construct the fluxes:** Generate chirality, moduli stabilization...

The Network



Highest Polyhedra:

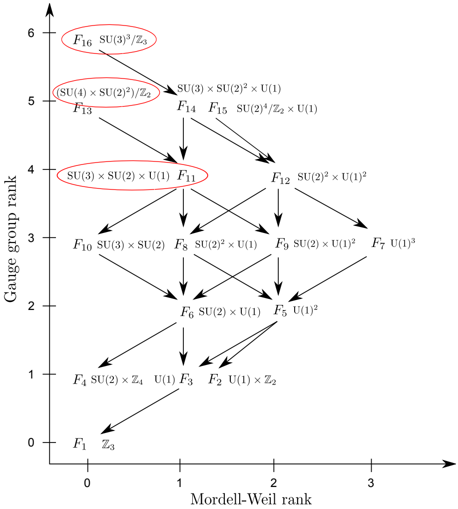
Observe: F_{16}, F_{15}, F_{13}
All feature MW Torsion:
Quotient group factors
Restricted representations

Mirror-Dual? ↕ Higgsing

Lowest Polyhedra

Observe: F_1, F_2, F_4
All are Genus-One fibers
Multi-sections
Discrete group factors

The Network



Standard Model

F_{11}	SM gauge group SM Charge Pattern

↑

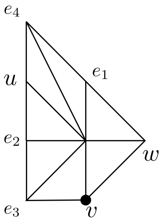
Unified Fibers

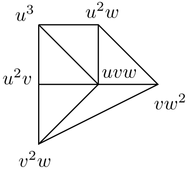
F_{13}	Pati-Salam Group PS Charge Patter
F_{16}	Trinification Group Trinification Charge Patter

The G_4-flux toolbox

1. **Fix fibration geometry** $F_i \to Y_4 \to B_3$
2. **Construct Basis of vertical cohomology** $H_V^{2,2}(Y) \ni D_A \cdot D_B$ With $D_A \in H^{1,1}(Y)$
   Find inequivalent basis: rank(η) $\eta = (D_A \cdot D_B, D_{A'} \cdot D_{B'})$
3. **Expand G_4-flux along a basis (integral?)** [Braun,Watari;Bizet,Klemm,Lopes'14]
   Flux quantization: $G_4 + \frac{c_2(Y)}{2} \in H^4(Y,\mathbb{Z})$
4. Calculate Chiralities of matter curve $\mathcal{C}_{\mathbf{R}}$ with repr. $\mathbf{R}$
   $\chi(\mathbf{R}) = \int_{\mathcal{C}_{\mathbf{R}}} G_4$
5. **Impose M- F-theory Flux match:** $\boxed{\theta_{A,B} = \int G_4 \wedge D_A \wedge D_B \in \mathbb{Z}/2}$
   Impose conditions: No flux along M-theory circle, All 4D gauge groups unbroken...
6. **D3 tadpoles cancelation**: $\frac{\chi(Y)}{24} = n_{D3} + \int G_4 \wedge G_4$
   Only solutions with $n_{D3} \in \mathbb{N}^+$ allowed, guarantee flux quantization
7. **Consistency Checks:** Cancellation of all anomalies

Construct the Standard model Fibration





Construct the fibration

Construct Fibration $F_{11} \to Y_4 \to \mathbb{P}^3)$

$K_B^{-1} = -4H_B \quad \mathcal{S}_7 = n_7 H_B$

$\mathcal{S}_9 = n_9 H_B$

Two integer (n_7, n_9) fibration

All base divisors muss be *effective*

$\hookrightarrow$ defines the allowed region of Y_4

Hypersurface and matter curves

Construct the Y_4 as a cubic hypersurface with s_i sections in $\mathcal{S}_7$ and $\mathcal{S}_9$ of the $\mathbb{P}^3$

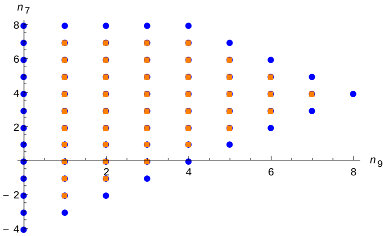
$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 uv^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 uvw + s_9 e_1 vw^2$$

Supports the Standard Model gauge group

$$\boxed{SU(3)_C \times SU(2)_L \times U(1)_Y}$$

and exact MSSM matter charges.

Construct the Standard model Fibration



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3 fluxes

No unbroken gauge group → Family structure with flux solution

$$a^1_{flux} = -\frac{\#_{families}\,(3n_7 + n_9 - 36)}{n_9\,(n_7 - 8)\,(4 + n_7 - n_9)\,(n_7 + n_9 - 12)}$$

$$a^2_{flux} = \frac{6\#_{families}}{(4 + n_7 - n_9)\,n_9}$$

Minimal number of Families where n_{D3} can be canceled?

Construct the Standard model Fibration

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	(27; 16)	–	–			
6	–	(12; 81)	(21; 42)	–	–		
5	–	–	(12; 57)	(30; 8)	–	(3; 46)	
4	(42; 4)	–	(30; 32)	–	–	–	–
3	–	(21; 72)	–	–	–	(15; 30)	
2	(45; 16)	(24; 79)	(21; 66)	(24; 44)	(3; 64)		
1	–	–	–	–			
0	–	–	(12; 112)				
-1	(36; 91)	(33; 74)					
-2	–						

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$\hookrightarrow$ defines the allowed region of Y_4

Three family flux solution

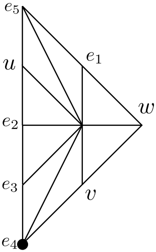
For $(n_7, n_9) = (5, 6)$ there is a three chiral family solution

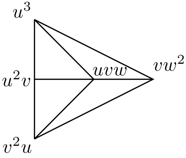
Q_i	$\bar{u}_i$	$\bar{d}_i$	L_i	$\bar{e}_i$	$H_u,\ H_d$
$(\mathbf{3},\mathbf{2})_{1/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	$(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{1},\mathbf{1})_{1}$	$(\mathbf{1},\mathbf{2})_{\pm 1/2}$

With superpotential realized as codim 3 singularities

$$\mathcal{W} \subset Y^u_{i,j} Q_i \overline{u}_j H_u + Y^d_{i,j} Q_i \overline{d}_j H_d + Y^L_{i,j} \overline{e}_i \overline{L}_j H_d$$
$$\lambda^{(0)}_{i,j,k} Q_i \overline{d}_j L_k + \lambda^{(1)}_{i,j,k} \overline{e}_i L_j L_k + \lambda^{(2)}_{i,j,k} \overline{u}_i \overline{d}_j \overline{d}_k + \mu H_u H_d$$

Construct the Pati-Salam Fibration





Construct the fibration

Construct Fibration $F_{13} \to Y_4 \to \mathbb{P}^3$)

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Hypersurface and matter curves

Construct the Y_4 as a cubic hypersurface with s_i sections in $\mathcal{S}_7$ and $\mathcal{S}_9$ of the $\mathbb{P}^3$

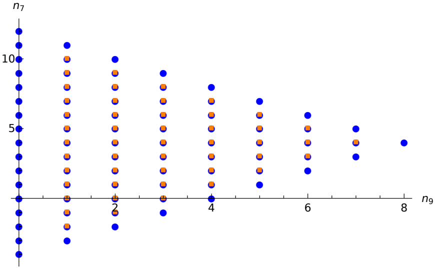
$$\begin{aligned} p_{F_{13}} = {} & s_1 e_1^2 e_2^2 e_3 e_5^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 e_5^2 u^2 v \\ & + s_3 e_2^2 e_3^3 e_4^4 u v^2 + s_6 e_1 e_2 e_3 e_4 e_5 uvw + s_9 e_1 v w^2 \, , \end{aligned}$$

Supports the Pati-Salam unification group

$$\boxed{SU(4)_\times SU(2) \times SU(2)/\mathbb{Z}_2}$$

and their exact matter charges

Construct the Pati-Salam Fibration



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3 Family locations

Generic one parameter flux solution

$$a_{flux} = \frac{4\#_{families}}{n_9 (n_9 - n_7 - 4)(12 - n_7 - n_9)} .$$

Minimal number of Families where n_{D3} can be canceled?

Construct the Pati-Salam Fibration

$n_7 \backslash n_9$	1	2	3	4	5	6	7
10	(13; 204)						
9	–	(11; 140)					
8	(33; 94)	(10; 119)	(9; 90)				
7	–	(9; 100)	(6; 77)	(14; 48)			
6	(15; 108)	(8; 86)	(21; 52)	(12; 46)	(5; 44)		
5	(6; 106)	(35; 44)	–	(30; 16)	–	(3; 44)	
4	(7; 102)	(6; 75)	(15; 50)	(8; 42)	(15; 30)	(6; 41)	(7; 42)
3	(6; 106)	(35; 44)	–	(30; 16)	–	(3; 44)	
2	(15; 108)	(8; 86)	(21; 52)	(12; 46)	(5; 44)		
1	–	(9; 100)	(6; 77)	(14; 48)			
0	(33; 94)	(10; 119)	(9; 90)				
-1	–	(11; 140)					
-2	(13; 204)						

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Three family flux solution

For $(n_7, n_9) = (5, 6)$ there is a three family flux solutions

Name	Representation		SM decomposition
SM Matter	$3 \times (\mathbf{4}, \mathbf{2}, \mathbf{1})_M$	$\to$	$Q_i : (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}, \ L_i : (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$
SM Matter	$3 \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_M$	$\to$	$\overline{d} : (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}, \ \ \overline{u} : (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}, \ \overline{e} : (\mathbf{1}, \mathbf{1})_1, \ \overline{\nu} : (\mathbf{1}, \mathbf{1})_0$

Construct the Pati-Salam Fibration

$n_7 \backslash n_9$	1	2	3	4	5	6	7
10	(13; 204)						
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SM Higgs	$1 \times (\mathbf{1}, \mathbf{2}, \mathbf{2})_H$	$\to$	$H_u : (\mathbf{1}, \mathbf{2})_{1/2}$, $H_d : (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$

Construct the Pati-Salam Fibration

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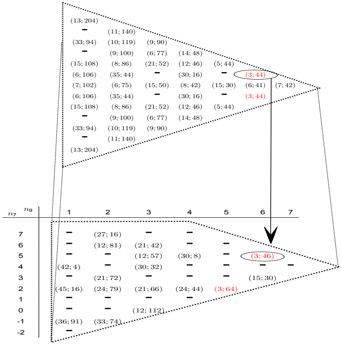
PS Higgs	$1 \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_{H^1}$	$\to$	$\bar{d}_H : (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}, \quad \bar{u}_H : (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}, \ \bar{e}_H : (\mathbf{1}, \mathbf{1})_1, \quad \bar{\nu}_H : (\mathbf{1}, \mathbf{1})_0$
PS Higgs	$1 \times (\mathbf{4}, \mathbf{1}, \mathbf{2})_{H^2}$	$\to$	$d_H : (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}, \ u_H : (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}, \quad e_H : (\mathbf{1}, \mathbf{1})_{-1}, \ \nu_H : (\mathbf{1}, \mathbf{1})_0$



Geometric Restrictions

- Match divisor classes (n_7, n_9): Only horizontal transitions
  $\hookrightarrow$ Geometric transition $\leftrightarrow \langle(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})\rangle = \langle(\mathbf{4}, \mathbf{1}, \mathbf{2})\rangle \neq 0$ **[Klevers,Mayorga,O,Reuter'14]**
- Higgsing does not change chirality: Family number is the same.

A Higgs transition is possible!

Conclusions

What we did:

- We constructed **globally** defined models of particle physics that
  - Have **three** chiral families
  - The exact MSSM, Pati-Salam and Trinification gauge group and matter content
  - All anomalies and n_{D3} tadpoles are canceled
- The models allow for a Higgs transition among them

Open problems

- No distinction between Leptons and H_d $\rightarrow$ proton decay
  $\hookrightarrow$ Additional discrete symmetries [Lin,Weigand'14]
- Vector-like matter sector not visible
  $\hookrightarrow$ Analyze the Chow-Ring [Bies,Mayrhover,Pehle,Weigand'14]

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Thank You!

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