G_4 fluxes in F-theory models with discrete selection rules

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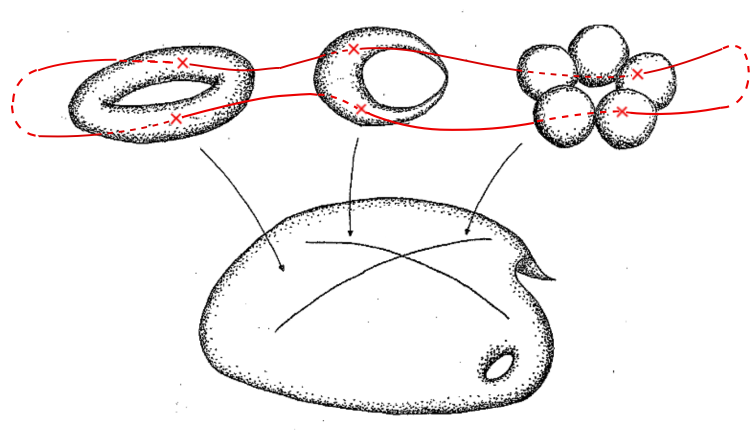
Outline

- Introduction: discrete selection rules in F-theory
- Background: Higgsing as a conifold transition
- G_4 fluxes on both sides of the transition
- Fluxes in models with enhanced gauge symmetry
- Outlook

Discrete selection rules in F-theory

Discrete symmetries are interesting to study both in field theory and geometry

$\mathbb{Z}_k$ symmetries can forbid dangerous couplings e.g proton decay operators in GUT models

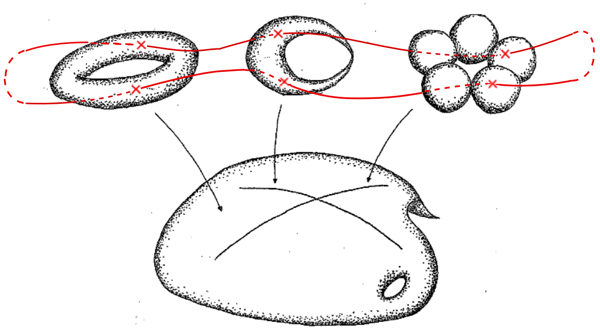


Discrete selection rules in F-theory

Lots of activity in the last year e.g

Klevers/Mayorga Pena/Oehlmann/Piragua/Reuter [1408.4808] *Grimm/Garcia-Etxebarria/Keitel* [1408.6448] *Cvetic/Donagi/Klevers/Piragua/Poretschkin* [1502.06953] *Cvetic/Klevers/Mayorga Pena/Oehlmann/Reuter* [1503.02068] and more.

See also talks by J. Reuter and P. Oehlmann.



Higgsing as conifold transition I

On general grounds, all symmetries are gauged in quantum gravity.

Discrete symmetries as broken gauge symmetries

Higgsing a $U(1)$ with a field of charge n leads to a $\mathbb{Z}_n$ remnant discrete symmetry. [see also talk by C. Mayrhofer]

Higgsing as conifold transition II

Higgsing in F-theory is a complex structure deformation (brane deformation in type IIB theory)

F-theory geometry with an extra $U(1)$: A (resolved) conifold singularity in codimension two.

Conifold transition: Blowing down the exceptional fiber component, and deforming away from the singularity. *Intriligator/Jockers et al* [1203.6662]

Higgsing as conifold transition III, the explicit setup

Elliptic fibration as the smooth hypersurface Y_4 *Morrison/Park* [1208.2695]

$$
\begin{aligned}
sw^2 + & b_0 s u^2 w + b_1 suvw + b_2 v^2 w \\
& + c_0 s^3 u^4 + c_1 s^2 u^3 v + c_2 s u^2 v^2 + c_3 u v^3
\end{aligned}
$$

with the resolved conifold singularity along the curve $C : \{b_2 = c_3 = 0\}$ in the base.

States of $U(1)$ charge 2 is localized along the curve C.

There is also a matter curve hosting states of charge 1

Higgsing as conifold transition III, the explicit setup

By blowing down, $s \to 1$, and deforming to the generic quartic in $\mathbb{P}_{112}$ we get

$$w^2 + b_0 u^2 w + b_1 uvw + b_2 wv^2 \\ + c_0 u^4 + c_1 u^3 v + c_2 u^2 v^2 + c_3 uv^3 + c_4 v^4 \,.$$

Note that this is one of two possible deformations, see talk by C. Mayrhofer. This hypersurface X_4 has no section, but a bi-section, given by the intersection of U : $\{u = 0\}$ with the hypersurface *Braun/Morrison* [1401.7844].

No matter states remain along the curve $b_2 = c_3 = 0$, but the curve of singly charged states in the $U(1)$ model remain, hosting states of odd $\mathbb{Z}_2$ charge.

G_4 flux in F-theory

G_4 is the field strength of the 3-form gauge field in M-theory.

G_4 flux is crucial for a chiral matter spectrum, breaking GUT symmetries and stabilizing moduli.

In an elliptically fibered fourfold M_4 with zero-section Z, the G_4 flux must obey

$$\int_{M_4} G_4 \wedge Z \wedge \pi^* D_a = \int_{M_4} G_4 \wedge \pi^* D_a \wedge \pi^* D_b = 0$$

for all $D_{a,b} \in H^{1,1}(B)$ in order to have a well defined 4d limit.

G_4 flux in the $U(1)$ model

The $U(1)$ gauge field A comes from expanding the M-theory 3-form as $C_3 = A \wedge w_{U(1)} + \ldots$ where $w_{U(1)}$ is a harmonic 2-form, usually called the $U(1)$-generator.

It is given by

$$w_{U(1)} = S - U - \bar{\mathcal{K}} - [b_2]$$

which is the harmonic 2-form dual to the image of the extra section under the Shioda map.

The associated vertical flux is of the form

$$G_4(F) = w_{U(1)} \wedge \pi^* F$$

for $F \in H^{1,1}(B)$ some class in the base.

The D3-tadpole

The Euler characteristics of the two models are related by

$$\chi(Y_4) - 3\chi(C) = \chi(X_4)$$

where C is the curve of conifold singularities. *Gaiotto et al* [0509168] and *Collinucci/Denef/Esole* [0805.1573]. The number of D3-branes

$$n_{D3} = \frac{1}{24}\chi(Y_4) + \frac{1}{2}\int_{Y_4} G_4 \wedge G_4$$

is preserved under the smooth deformation.

If $G_4(F) = 0$ on Y_4, then a flux G_4 on X_4 must appear, such that

$$\frac{1}{2}\int_{X_4} G_4 \wedge G_4 = -\frac{1}{8}\chi(C)$$

The D3 tadpole

By specializing to a locus in complex moduli space where $c_4 = \rho\tau \to$ new 4-cycles, with duals in $H^{2,2}(X_4)$.
Braun/Collinucci/Valandro [1107.5337] *Krause/Mayrhofer/Weigand* [1109.3454]

$$\sigma_0 = \{u = 0\} \cap \{\rho = 0\} \cap \{w = 0\} \subset X_5$$
$$\sigma_1 = \{u = 0\} \cap \{\rho = 0\} \cap \{w + b_2 v^2 = 0\} \subset X_5$$

The flux $G_4(\rho) = [\sigma_1] - \frac{1}{2} U \wedge [\rho]$ satisfies the analogous transversality conditions

$$\int_{X_4} G_4 \wedge U \wedge \pi^* D_a = \int_{X_4} G_4 \wedge \pi^* D_a \wedge \pi^* D_b = 0$$

with the class of the bisection instead of the zero-section.

The D3 tadpole

With no $U(1)$ flux, $G_4(\rho)$ accounts precisely for the tadpole contribution

$$\frac{1}{2}\int_{X_4} G_4(\rho) \wedge G_4(\rho) = -\frac{1}{8}\chi(C)$$

for the choice $[\rho] = [b_2]$.

For a non-zero $U(1)$-flux, this generalizes to

$$\frac{1}{2}\int_{X_4} G_4(\rho) \wedge G_4(\rho) = -\frac{1}{8}\chi(C) + \frac{1}{2}\int_{X_4} G_4(F) \wedge G_4(F)$$

with $[\rho] = 2F + [b_2]$

Since $c_4 = \rho\tau$ the inequality $0 \leq [\rho] \leq [c_4]$ must hold and if violated, no transition is possible since the necessary vector-like pairs of higgs fields fail to exist.

Outlook

Algorithmic and automatized computation of fluxes and indices

Use new technology to do better phenomenology $\rightarrow$ GUT models with R-parity, realistic chiral spectra

Broken non-abelian gauge symmetry $\rightarrow$ Non-abelian discrete symmetries in F-theory?