Workshop seminar   Julian Stöckel   Nov. 4th

Orbifold GUTs

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Sources: - „New ideas in symmetry breaking", Quirós, [0302189]
- Lecture Notes by W. Buchmüller

I Orbifolds

- Orbifold theories include extra dimensions
- Orbifolds are special construction going beyond smooth manifolds that allow us to perform symmetry breaking by compactifying!

To construct an Orbifold we need a discrete group H and a compact space C, on which we have a representation of H that acts non-freely.

Minimal example: $H = \mathbb{Z}_2$

$C = S^1 = (-\pi R, \pi R]$ with $-\pi R \sim \pi R$ coordinate y

$\xi_h(y) = -y$ (non-free: $\exists y \neq 0 : \xi_h(y) = y$)
$\hookrightarrow \xi_h(\pi R) = -\pi R = \pi R$

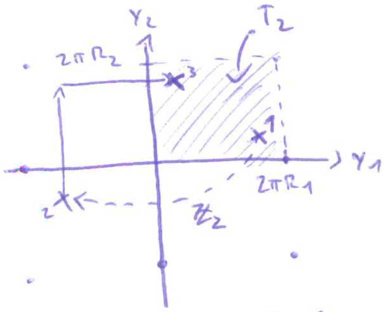
From these ingredients we get an Orbifold C/H by identifying $y \sim \xi_h(y)$.

In our minimal example this means to identify $y \sim -y$

There are two points unaffected by this: $y=0$ and $y=\pi R$ these are called „fixed points" (which I mark with a small circle ∘) in pictures:

S^1 $\mathbb{Z}_2$ action: Result: $S^1/\mathbb{Z}_2$

Another example: $T^2/\mathbb{Z}_2$



$$\mathbb{Z}_2 : \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \to \xi \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -y_1 \\ -y_2 \end{pmatrix}$$

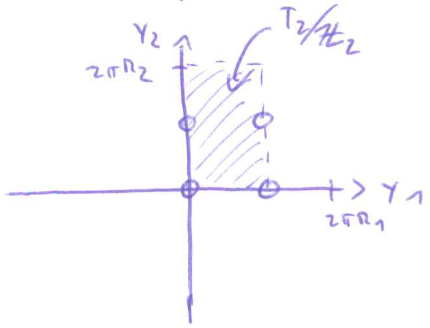
Fixed points: $(0,0)$ $\quad (\pi R_1, 0)$
$(0, \pi R_2)$ $\quad (\pi R_1, \pi R_2)$

Identification: $y \sim -y$

=> take a point (1)

=> mirror it w.r.t. the origin (2)

=> move that back into the fundamental domain.



II Fields on Orbifolds

Our goal is to do field theory on 4D Minkowski space with orbifolded extra dimensions; the spacetime therefore looks like

$$\mathcal{M}_4 \times \mathcal{O} \xrightarrow{\text{minimally}} \mathcal{M}_4 \times S^1/\mathbb{Z}_2 .$$

A scalar field depends generally on all coordinates

$$\phi = \phi(x, y)$$

When implementing the orbifold identification, we can define fields to transform in a representation of H.

$$\phi(x, \xi_n(y)) = P_n \, \phi(x, y)$$

For $\mathbb{Z}_2$ this is especially easy since the only choice is $P_n = \pm 1$!

To be consistent, this needs to be a symmetry of the theory.

On a $\mathbb{Z}_2$ orbifold we call fields even/odd depending on their orbifold parity.

J. Louis showed us the decomposition of a scalar on a compact extra dimension:

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} \phi^{(n)}(x) \, e^{iny \cdot \frac{1}{R}}$$

Let us rearrange the terms:

$$\phi(x,y) = \phi^{(0)}(x) + \sum_{i=1}^{\infty} \Big[\underbrace{\tilde{\phi}_+^{(n)}}_{\propto \phi^{(n)} + \phi^{(-n)}} \big(e^{iny \cdot \frac{1}{R}} + e^{-iny \cdot \frac{1}{R}} \big) + \underbrace{\tilde{\phi}_-^{(n)}}_{\propto \phi^{(n)} - \phi^{(-n)}} \big(e^{iny \frac{1}{R}} - e^{-iny \cdot \frac{1}{R}} \big) \Big]$$

If the field has even parity:

$\phi(x,-y) = +\phi(x,y) \Rightarrow \phi^{(0)}$ and $\tilde{\phi}_+^{(n)}$ as modes with 4d masses

$m_n^2 = \frac{n^2}{R^2} \Rightarrow$ even parity $\leftrightarrow$ massless mode $\phi^{(0)}$

For odd parity we have

$\phi(x,-y) = -\phi(x,y) \Rightarrow$ only $\tilde{\phi}_-^{(n)}$ with a KK mass $m_n^2 = \frac{n^2}{R^2} \geq \frac{1}{R^2}$

> The zero mode of odd fields is „projected out" of the low-energy spectrum!

Vector fields have more components in higher dimensions.

$V_M(x,y)$ with $M = 0,\dots,3,5$

Lorentz transformations in 5D mingle the components: $L_M{}^N \in SO(1,4)$

$$V_M \to L_M{}^N V_N \Rightarrow \Lambda_\mu{}^\nu V_\nu + \boxed{\Lambda_\mu{}^5} V_5$$
$$\boxed{\Lambda_5{}^\mu} V_\mu + \Lambda_5{}^5 V_5$$

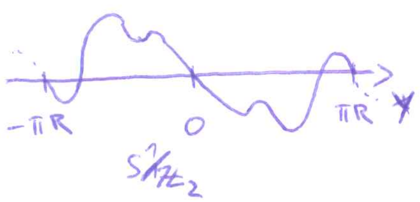
On our product space $M_4 \times S^1/\mathbb{Z}_2$, $\Lambda_\mu{}^5$ and $\Lambda_5{}^\mu$ do not exist!

$\Rightarrow$ In the effective 4D theory, V_5 behaves as a scalar.

- The KK decomposition works just as for scalar fields.

Since for a fixed point $\xi_n(y) = y$ even before identification, odd fields vanish at the fixed points. In addition, in the presence of odd fields there only is an effective 4D limit at the fixed points.

Odd field:
$\phi(-y) = -\phi(y)$



Even field:
$\phi(-y) = \phi(y)$

[Caveat: Gauge fields only need to match the periodicity up to a gauge transformation. $\rightsquigarrow$ Extra dimensions with flux/Wilson lines

$\Rightarrow$ beyond the scope of today's seminar]

III Gauge symmetry breaking

Idea: Use freedom of assigning parities to break a gauge symmetry by making some gauge bosons vanish from the effective action (i.e. heavy).

Gauge group $\mathcal{G}$ with generators $\{T^A\} = g$.

-> $\dim(\mathcal{G})$ gauge bosons $A_M^A(x,y)$

Breaking to a smaller group $\mathcal{H} \subset \mathcal{G}$ with generators $\{T^a\} = h$

-> $\dim(\mathcal{H})$ g.b. with <u>even</u> parity: $A_M^a(x,y) = A_M^a(x,-y)$ if $T^a \in h$

$\dim(\mathcal{G}) - \dim(\mathcal{H})$ g.b with <u>odd</u> parity: $A_M^{\hat{a}}(x,y) = -A_M^{\hat{a}}(x,-y)$ if $T^{\hat{a}} \in g/h$

Limitations: Consistency with the Lie algebra g implies that structure constants can only have an even number of hats:

$\rightarrow f^{abc}, f^{a\hat{b}\hat{c}}$ may exist, but

$f^{ab\hat{c}}$ and $f^{\hat{a}\hat{b}\hat{c}}$ must vanish!

One can always find a matrix P such that

$P T^a P^{-1} = T^a$, but $P T^{\hat{a}} P^{-1} = -T^{\hat{a}}$

With this, one can write the orbifold action on the gauge field concisely:

$$V_M(x,-y) = P V_M(x,y) P^{-1} \quad \Rightarrow \quad \begin{array}{l} + \text{ for } T^a \\ - \text{ for } T^{\hat{a}}. \end{array}$$

If $P \in g$ the rank of the gauge group is preserved in the breaking, else it is reduced.

Example: $SU(5) \rightarrow \mathcal{G}_{SM}$

A_M is a hermitian 5×5 matrix. Breaking to the SM is done via

$P = \mathrm{diag}(-1,-1,-1,1,1) = P^{-1}$.

Remember from last week (C. Wieck)

$$A_M = \left(\begin{array}{c|c} g_M^a \lambda^a - 2B_M & X, Y \\ \hline X^T, Y^T & W_M^i \tau^i + 3B_M \end{array} \right)$$

Breaking can now be seen „by eye".

The low-energy spectrum then contains:

- Gluons: $G^a_\mu \lambda^a$ and $G^a_5 \lambda^a$
- Ws : $W^i_\mu \tau^i$ and $W^i_5 \tau^i$
- B : B_μ and B_5

$G^a_5 \lambda^a$, $W^i_5 \tau^i$, B_5 → Massless, adjoint scalars without potential due to gauge symmetry in 5D ↯

Giving an opposite parity to A_5 is okay (5D Poincaré is broken anyways by our construction), but leads to massless scalars in the coset $SU(5)/G_{SM}$

$$A_\mu(x,-y) = P A_\mu(x,y) P^{-1}$$
$$A_5(x,-y) = -P A_\mu(x,y) P^{-1}$$

$\Rightarrow$ Gluons, Ws, B $+ X^{\hat a}_5, Y^{\hat a}_5$ ↯

Solution: More orbifolding → use $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$

πR, $\frac{\pi R}{2}$, 0 → $\pi R/2$, 0 Independent Parities!

$\mathbb{Z}_2$: $y \to -y$ $\quad$ $\mathbb{Z}_2'$: $\frac{\pi R}{2} + y \to \frac{\pi R}{2} - y$

$P = \mathbb{1}_5$

$A_\mu(x,-y) = A_\mu(x,y)$

$A_5(x,-y) = -A_5(x,y)$

$P' = \mathrm{diag}(-1,-1,-1,1,1)$

$A_\mu(x, \frac{\pi R}{2}+y) = P' A_\mu(x, \frac{\pi R}{2}-y) P'^{-1}$

$A_5(x, \frac{\pi R}{2}+y) = P' A_5(x, \frac{\pi R}{2}-y) P'^{-1}$

→ Fields with
- $(++)$: Gluons, W, B → massless mode
- $(+-)$: $X^{\hat a}_\mu, Y^{\hat a}_\mu$
- $(-+)$: G^a_5, W^i_5, B_5
- $(--)$: $X^{\hat a}_5, Y^{\hat a}_5$

$(+-)$, $(-+)$, $(--)$: only massive modes ✓

More complicated breakings require more elaborate schemes
cf. Buchmüller, Covi, Asaka 0108021
$SO(10) \to G_{SM} \times U(1)'$ on $T^2/\mathbb{Z}_2^3$

IV Matter and the Higgs

One of the most haunting features of GUTs is proton decay via colored Higgs. Orbifold GUTs solve this in one of two possible ways:

1. Doublet-triplet splitting via parity assignment

→ Higgs field has parity: $H(x,-y) = PH(x,y)$ with $P = \mathrm{diag}(-1,-1,-1,1,1)$

$$\Rightarrow \begin{pmatrix} h_c^1 \\ h_c^2 \\ h_c^3 \\ h^+ \\ h^0 \end{pmatrix} \to \begin{pmatrix} -h_c^1 \\ -h_c^2 \\ -h_c^3 \\ h^+ \\ h^0 \end{pmatrix}$$

- Triplet gets a mass $\mathcal{O}(M_{GUT})$
- Doublet has a massless component.

=> The same parity operator that achieves the correct gauge symmetry breaking also „kills" the triplet.

Alternatively one can have a

2. brane-world scenario

-> say we live on a 3-brane located at an orbifold fixed point

~> Fields that only live on the brane also only have to come in multiplets of the brane gauge symmetry

=> Put the Higgs on the brane -> there is no triplet to start with!

One also has these two options for all matter fields.

-> All fields in the bulk: universal extra dimensions

-> Lots of blocks for happy model building

Summary

- Orbifolds are compact manifolds with fixed points
- The orbifold action (here: parities) can make fields vanish at the fixed points
- Within consistency constraints this can be used for gauge symmetry breaking, which has advantages:
  - -> GUT Higgs potentials are messy! Orbifolds don't need those.
  - -> Doublet-triplet splitting can be solved in an elegant way.
  - (-> The Higgs mechanism only takes place at one scale)
  - -> Susy can be implemented with finite complications