Heterotic MSSM orbifold string models

Fabian Ruehle

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Contents

1 Introduction

We present the construction of MSSM like models in heterotic string theory. Due to time limitations, we only discuss a small subset of the research done in this area, namely model building on abelian heterotic orbifolds [1,2] (for a good review see e.g. [3,4]). For other model building approaches on smooth Calabi–Yau spaces (CYs) which are not resolutions of orbifolds see e.g. [5–8]. There are further model building approaches in other fields like free fermionic models [9], which are dual to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold discussed here at a specific point in moduli space.

2 MSSM model building

Heterotic string theory has been very popular for particle physics beyond the standard model for decades. One of the main reasons is probably that it has by construction a gauge group ($E_8 \times E_8$ or SO(32)) which contains popular GUT groups like SU(5) or SO(10). This is in contrast to the (perturbative) type II string theories. There is no GUT group to begin with and GUT model building is more challenging. In the end, this is related to the fact that it is impossible to build exceptional gauge groups in perturbative type II theories. These are important since one naturally gets the spinor representation **16** of SO(10) (which, as Clemens told us, contains all the SM particles including the right-handed neutrino and whose Yukawa couplings give rise to the MSSM terms) out of the fundamental or adjoint representation of E_6. While it is possible to build SU(5) $\times$ U(1) or SO(10) models in type II theories, they are not so well-suited as GUT groups since on the level of SU(5) the U(1) charge forbids necessary Yukawa couplings and on the level of SO(10) one cannot obtain spinor representations. However, type II theories allow for more different kinds of fluxes (fluxes are vevs of field strengths), which makes them better suited for discussions of cosmological properties and moduli stabilization (moduli are scalar fields without a potential in 4D; they correspond to fields that parameterize the shape of the compactification manifold or the gauge degrees of freedom).

From now on we will focus on heterotic $E_8 \times E_8$ string theory. First we would like to mention that there is no model which has all the properties of the MSSM in all fine prints. So let us first list the properties which we want to impose for MSSM model building:

1. The theory should have $\mathcal{N} = 1$ SUSY in 4D
2. The theory should be free of anomalies
3. The theory should have the MSSM gauge group $SU(3) \times SU(2) \times U(1)$, plus possibly an additional hidden sector gauge group which can be used for SUSY breaking
4. The theory should have the MSSM particle content, i.e. three families of quarks and leptons with one Higgs pair.
5. The theory should give rise to realistic Yukawa textures (mass hierarchies between the families) and solve the μ-problem

We will explain in the following how these points are realized in heterotic model building on orbifolds, using the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold [10] as our working example throughout.

3 Orbifold model building

3.1 $\mathcal{N} = 1$ SUSY in 4D

Toroidal abelian orbifolds are constructed by starting with a six-torus[1] T^6 and modding out a discrete Abelian symmetry group $\mathbb{Z}_N$ or $\mathbb{Z}_N \times \mathbb{Z}_M$. In order to be able to do so, we have to ensure that the torus lattice is compatible with this action. This rules out some discrete symmetries from the very start (e.g. $\mathbb{Z}_5$) and fixes or restricts the complex structure of the tori for other discrete symmetries. It becomes an exercise in 6D lattice crystallography to classify the various possibilities. For the abelian case one finds that $N \in \{2, 3, 4, 6, 8, 12\}$ in the case[2] of $\mathbb{Z}_N$ and that $M, N \in \{2, 3, 4, 6\}$ in the case of $\mathbb{Z}_N \times \mathbb{Z}_M$.

Furthermore, since we want to end up with $\mathcal{N} = 1$ SUSY in 4D, this restricts how the symmetries can act on T^6. Their action has to be such that the resulting orbifold allows for one parallelizable spinor to survive the compactification from 10D $\mathcal{N} = 1$ to 4D and generate the $\mathcal{N} = 1$ SUSY there. Spaces with this property are called CY spaces. There are several equivalent characterizations of CYs as complex Kähler manifolds with at most SU(3) holonomy, with vanishing first Chern class (aka Ricci-flat), or with a nowhere vanishing homomorphic $(3, 0)$ form (aka trivial anti-canonical bundle). The conditions we have to impose on the orbifold action can be seen in the easiest way from the last property. Say we start with three two-tori which are parameterized by the three complex coordinates z_1, z_2, z_3. The $\mathbb{Z}_N$ orbifold action θ will now act on these coordinates by a discrete rotation:

$$\theta : \quad (z_1, z_2, z_3) \mapsto (e^{2\pi i v_1} z_1,\ e^{2\pi i v_2} z_2,\ e^{2\pi i v_3} z_3) \tag{1}$$

[1]For simplicity we discuss here factorizable torus lattices, i.e. $T^6 = T^2 \times T^2 \times T^2$

[2]The case N=2 does not give rise to $\mathcal{N} = 1$ in 4D.

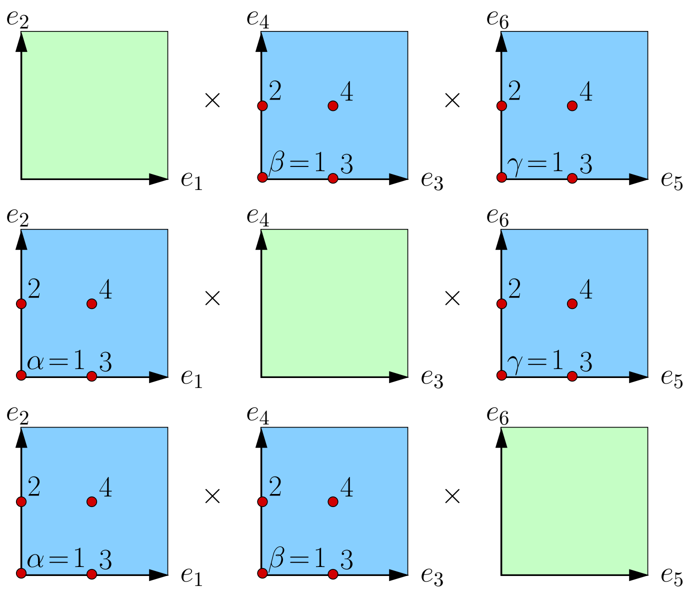

Figure 1: The twisted sectors of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

where the twist vector $v = (v_1, v_2, v_3)$ is quantized as $1/N$. Out of these z_i one can build a holomorphic (3,0)-form by simply wedging their exterior products, $\Omega = \mathrm{d}z_1 \wedge \mathrm{d}z_2 \wedge \mathrm{d}z_3$. Under the orbifold action, this transforms as

$$\theta : \quad \Omega \;\mapsto\; e^{2\pi \mathrm{i}(v_1+v_2+v_3)}\,\Omega\,. \tag{2}$$

This tells us that the resulting space is CY if $v_1+v_2+v_3 \in \mathbb{Z}$. For the case of $\mathbb{Z}_2 \times \mathbb{Z}_2$, we have two generators θ_1 and θ_2, and two associated twist vectors, both of which have to fulfill the aforementioned condition. Thus we are now in a position to write down the orbifold actions for the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold:

$$\begin{gathered} \theta_1 :\ (z_1, z_2, z_3) \mapsto (z_1,\ e^{2\pi \mathrm{i}/2} z_2,\ e^{-2\pi \mathrm{i}/2} z_3)\,, \quad \theta_2 :\ (z_1, z_2, z_3) \mapsto (e^{-2\pi \mathrm{i}/2} z_1,\ z_2,\ e^{2\pi \mathrm{i}/2} z_3)\,, \\ \theta_3 := \theta_1\theta_2 :\ (z_1, z_2, z_3) \mapsto (e^{-2\pi \mathrm{i}/2} z_1,\ e^{2\pi \mathrm{i}/2} z_2,\ z_3)\,. \end{gathered} \tag{3}$$

As Julian explained last week, such a $\mathbb{Z}_2$ action has 4 fixed points on a T^2. Thus we get $4 \cdot 4 = 16$ fixed points from each θ_i for a total of 48 fixed points (actually θ_i leaves the i^{th} torus invariant, so these are fixed tori rather than fixed points). A pictorial presentation can be found in figure 1. Note that for this orbifold the complex structures of the tori are not fixed. We have chosen $\tau_i = \mathrm{i}$ for ease of presentation here.

3.2 Anomaly freedom

One big advantage of orbifolds is that they correspond to free superconformal field theories (SCFTs) and thus are (at least in principle) exactly computable. This is not true for general (smooth) CYs, where a lot of properties (like the metric) are unknown and one thus has to rely on topological quantities and work in the limit of heterotic supergravity (which uses only the lowest terms in the α' expansion and truncates the massive string excitations).

Being defined as a SCFT, one can in particular write down the one-loop string partition function. Since a closed string loop has the topology of a torus, the partition function is given in terms of generalized θ- and Dedekind η-functions which depend on the boundary conditions of the worldsheet (WS) fields and the complex structure of the worldsheet torus. The orbifold action fixes the boundary conditions for the fields that parameterize the spacetime. In addition, we can choose the boundary conditions for the gauge degrees of freedom; however, they have to be chosen such that the symmetries of the torus partition function are not spoiled, i.e. such that the torus is modular invariant. For toroidal abelian orbifolds, these boundary conditions can always be chosen such that the orbifold twist θ_i is accompanied by a shift (rather than a twist) in the gauge degrees of freedom which arise from the T^{16} with root lattice $\Lambda_{\mathrm{E}_8\times\mathrm{E}_8}$. These shifts can be parameterized in terms of the shift vectors V_i (each V_i has 16 components specifying the shift in the 16 directions of $\Lambda_{\mathrm{E}_8\times\mathrm{E}_8}$) associated to θ_i. For our orbifold these modular invariance conditions then read:

$$2(V_i^2 - v_i^2) \equiv 0 \bmod 2\,. \tag{4}$$

An obvious solution to this equation is to repeat the entries of the twist vectors v_i in the shift vectors V_i, i.e. in our case

$$\begin{aligned} v_1 &= \frac{1}{2}(0,1,-1)\,, & v_2 &= \frac{1}{2}(-1,0,1) \\ V_1 &= \frac{1}{2}(0,1,-1,0^5)(0^8), & V_2 &= \frac{1}{2}(-1,0,1,0^5)(0^8)\,. \end{aligned}$$

This is the so-called standard embedding (albeit there's nothing standard about it). These modular invariance conditions, which link the gauge dofs to the spacetime dofs, correspond to the Bianchi identities (BIs) for the three-form field-strength H of the two-form Kalb–Ramond field B_2. This field (or rather its behavior under gauge transformations) is vital to the cancellation of anomalies via the so-called Green–Schwarz (GS) mechanism. In other words, if the BIs for H are satisfied, all of the anomalies are cancelled via the GS mechanism. On a generic CY these BIs are very hard to satisfy and for quite some years the only known solution has been this standard embedding, which means identifying the gauge connection with the spin connection. However, this choice does not give rise to the standard model gauge group, which we will turn to in the next section.

For orbifolds, in contrast, finding solutions to (4) is not so hard, and all inequivalent solutions have been classified in [11] for all possible 6D abelian toroidal orbifolds. However, note that in general just choosing V_i is not enough to obtain MSSM-like models, so one has to add in addition so-called Wilson lines, which can be thought of as constant gauge backgrounds around the six fundamental cycles[3] of the underlying T^6. If the underlying orbifold is non-simply connected one can have, in addition to these Wilson lines, a Wilson line associated to the non-trivial first fundamental group. All these Wilson lines correspond again to WS boundary conditions and they have to obey similar modular invariance conditions like the ones in (4).

3.3 MSSM gauge group

In general, in heterotic theories the primordial gauge group $\mathrm{E}_8 \times \mathrm{E}_8$ is broken to the maximal commutant with the gauge bundle. For the standard embedding in the smooth case, the

[3]The Wilson lines have to descend to the orbifold and thus be compatible with the orbifold action, such that there are usually less than six possible Wilson lines; our $\mathbb{Z}_2 \times \mathbb{Z}_2$ example is the only one which has 6 independent Wilson lines.

vector bundle is SU(3), and the maximal subgroup of E_8 that commutes with SU(3) is E_6, so the resulting gauge group would be $E_6 \times E_8$. In the orbifold standard embedding, the holonomy (and thus the gauge shifts) are a discrete subgroup of SU(3), so the commutant is larger than just E_6. In fact, at least U(1) commutes with $\mathbb{Z}_N$, but there could also be larger non-Abelian groups with center $\mathbb{Z}_N$; in any case, the rank of the gauge group is not reduced in this way. In our example, the resulting gauge group of the standard embedding is $E_6 \times U(1)^2 \times E_8$. Thus the name of the game is to find shift vectors and Wilson lines that solve the modular invariance conditions and break $E_8 \times E_8$ to the SM gauge group $SU(3) \times SU(2) \times U(1)$. In addition, since this breaking is rank preserving by construction, we will find other (abelian and non-abelian) gauge groups whose ranks add up to 16. In principle, the set of constraints can be written in terms of diophantic equations and thus be solved in general. In practice, the resulting set of equations is too complicated, so in model scans the shift vectors and Wilson lines are generated randomly and then checked for the correct gauge group and particle content. Furthermore it is convenient (albeit not necessary on orbifolds) to use the shifts and Wilson lines associated with the torus cycles e_i to break to an intermediate SU(5) GUT group (times other stuff), and to break this to the SM gauge group using the Wilson line associated with the non-trivial fundamental group. This ensures that the hypercharge fits into SU(5), allows for doublet-triplet splitting, and sequesters the scales where E_8 is broken to SU(5) from the scale where SU(5) breaks to the SM gauge group, which can be advantageous for GUT threshold corrections. Furthermore, on smooth CYs this mechanism ensures that the hypercharge remains massless and thus a good symmetry at low energies. However, as we have seen before, in order to be able to mod out a symmetry our space has to have the symmetry in the first place. In our case, the freely acting $\mathbb{Z}_2$ symmetry acts as a simultaneous shift along the direction e_2, e_4, e_6. This means that the Wilson lines in these directions have to be equal in such constructions. However, this does not pose a problem, since three torus Wilson lines are already enough to break to the SU(5) GUT group. An example for the choice of shift vectors and Wilson lines that break $E_8 \times E_8$ to $[SU(3) \times SU(2) \times U(1)_Y]_{\text{SM}} \times [SU(3) \times SU(2) \times SU(2)]_{\text{hidden}} \times U(1)^8$ is

$$
\begin{aligned}
V_1 &= \left(\frac{5}{4}, -\frac{3}{4}, -\frac{7}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}, \frac{1}{4}\right) (0,1,1,0,1,0,0,-1)\,, \\
V_2 &= \left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) \left(\frac{1}{2}, \frac{1}{2}, 0,0,0,0,0,4\right)\,, \\
W_1 &= \left(0^8\right)\,\left(0^8\right)\,, \\
W_2 &= \left(\frac{5}{4}, \frac{1}{4}, \frac{3}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}\right) \left(-\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)\,, \\
W_3 &= \left(-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{7}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}\right) \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{3}{4}, \frac{1}{4}, -\frac{3}{4}, \frac{1}{4}\right)\,, \\
W_5 &= \left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \left(\frac{1}{2}, \frac{1}{2}, 0,0,0,0,-\frac{1}{2}, -\frac{1}{2}\right)\,, \\
W_4 &= W_6 = W_2\,, \\
W_{\text{free}} &= \frac{1}{2}(W_2 + W_4 + W_6)\,.
\end{aligned}
\tag{5}
$$

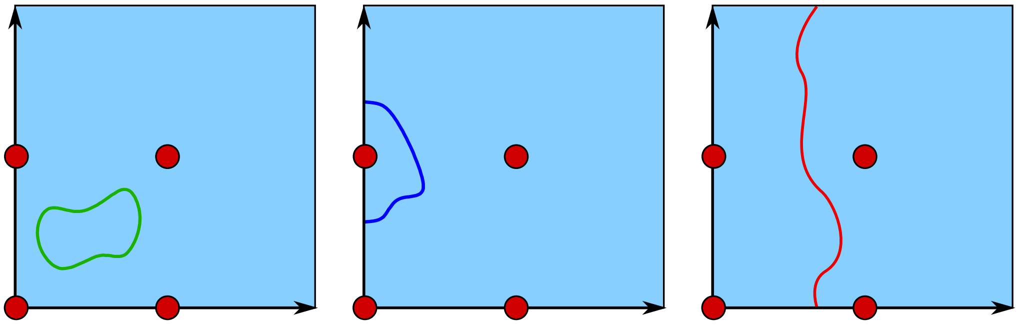

Figure 2: Untwisted, twisted, and massive winding strings on orbifolds.

Just to exemplify the calculation of the gauge group branching, we present the calculation of the SU(5) standard model factor, which is then broken to $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ by W_{free}. As explained above, the unbroken gauge group is given in terms of those 248 roots of E_8 which commute with the shifts and Wilson lines. A set of 24 such roots which commute with $V_{1,2}$ and $W_1, \ldots, W_6$ (i.e. which has an integral inner product with the shifts and WLs) is

$$\lambda = (0^3, \underline{1, -1, 0^3})(0^8)\,, \tag{6}$$

where the underline denotes all possible permutations of the underlined entries. These (plus the zero roots corresponding to the Cartan generators) are the 24 roots of the GUT SU(5). If one now takes in addition the freely acting Wilson line W_{free} into account, one sees that (due to the extra factor of 1/2) some of these 24 roots do not yield integral inner products. To be more precise, the 24 roots split into a set of 8 roots and 3 roots which are mutually orthogonal and orthogonal to all shifts and WLs and generate the $\mathrm{SU}(3) \times \mathrm{SU}(2)$ gauge group of the standard model. They are given by

$$\begin{aligned} \lambda_{\mathrm{SU}(3)} &= (0^3, 0, 0, \underline{1, -1, 0})(0^8)\,, \\ \lambda_{\mathrm{SU}(2)} &= (0^3, \underline{1, -1}, 0, 0, 0)(0^8)\,. \end{aligned} \tag{7}$$

The extra U(1) within the SU(5) which is orthogonal to these roots is the usual hypercharge generator

$$T_{\mathrm{U}(1)_Y} = (0^3, \frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})(0^8)\,. \tag{8}$$

3.4 MSSM particle content

Using CFT techniques, one can compute the complete string spectrum from the boundary conditions, i.e. from the orbifold twists, shifts, and Wilson lines. On top of the usual closed strings of heterotic string theory, the orbifold allows for further types of strings: there can be strings that wind around the torus cycles e_i, but these are massive and thus not of our primary interest. The other new type of strings are the so-called twisted strings which appear to be open, but actually close under the orbifold action. Such strings necessarily wind around orbifold fixed points and cannot be moved away from them (since they would not close then), see figure 2 for a cartoon of these types of strings.

In order to analyze the string spectrum it is useful to define the so-called local shifts and twists v_g and V_g, where $g = (\theta_1^k \theta_2^\ell, b_i e_i)$ specifies how the twist in the spacetime dofs and the shift in the gauge dofs acts. In particular we have $g = (\mathbb{1}, 0)$ for untwisted strings, $g = (\theta_1, e_4 + e_5)$ for a twisted string at the fixed point $(\beta = 2, \gamma = 3)$, and so on.

The masses of these strings depend on the momenta P and q of the strings (these are lattice vectors of $\mathrm{E}_8 \times \mathrm{E}_8$ for P and of SO(8) for q, which are in addition shifted by the orbifold shifts, Wilson lines and twists), on internal oscillations $\widetilde{\mathcal{N}}$, and on a zero point energy shift δc. In terms of this data, the massless spectrum reads

$$0 = \frac{M_{\mathrm{L}}^2}{8} = \frac{(P + V_g)^2}{2} + \widetilde{\mathcal{N}} + \delta c - 1\,, \qquad 0 = \frac{M_{\mathrm{R}}^2}{8} = \frac{(q + v_g)^2}{2} + \delta c - \frac{1}{2}\,. \tag{9}$$

To give one example, we look at the fixed point $(\alpha = 1, \beta = 4)$, which corresponds to $g = (\theta_1\theta_2, e_3 + e_4)$. The local twists and shifts are thus

$$\begin{aligned} v_g = &\quad v_1 + v_2 &&= \frac{1}{2}(-1, 1, 0)\,, \\ V_g = &\quad V_1 + V_2 + W_3 + W_4 &&= \left(\frac{5}{4}, -\frac{5}{4}, -\frac{5}{4}, \frac{9}{4}, -\frac{3}{4}, -\frac{3}{4}, -\frac{3}{4}, \frac{1}{4}\right)\left(\frac{1}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{7}{2}\right) \end{aligned}$$

For this sector, the zero point shift is $\delta c = 1/4$, so the masslessness conditions read

$$\frac{3}{2} = (P + V_g) + \widetilde{\mathcal{N}}\,, \qquad \frac{1}{2} = (q + v_g)^2\,. \tag{10}$$

Using the following $\mathrm{E}_8 \times \mathrm{E}_8$ and SO(8) lattice vectors,

$$P = \Big(-1, 1, 1, -3, 0, 1, 1, 0\Big)\Big(-\frac{1}{2}, -\frac{5}{2}, -\frac{5}{2}, -\frac{5}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{7}{2}\Big)\,, \quad q = \Big(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\Big)\,,$$

we readily check that

$$P + V_g = \Big(\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}, -\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\Big)\Big(0, 0, 0, 0, 0, 0, 0, 0\Big)\,, \quad p + v_g = \Big(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\Big)$$

solves (9) with $\widetilde{\mathcal{N}} = 0$. As explained by Clemens, the irreps under which the corresponding strings transform are obtained by calculating the Dynkin labels, which are obtained by taking the inner product with the simple roots of the various non-abelian gauge groups. In particular, we see that the Dynkin labels with respect to SU(5) are $(0, 1, 0, 0)$, which means that the state is the highest weight of the **10**. By using the highest weight procedure and the branching rules, or by calculating the Dynkin labels with respect to $\mathrm{SU}(3) \times \mathrm{SU}(2)$, we see that this state becomes a singlet after breaking the SU(5). The inner product with the hypercharge generator gives 1, which means that the state belongs to the chiral SUSY multiplet of the right-handed electron. As can be seen from the first entry of $+1/2$ in $p + v_g$, this state transforms as a target space spinor, so it is the electron. If we take $q = (0, 0, 0, 0)$, the first entry a 0 and we obtain the selectron. Also note that since $W_1 = 0$, the spectrum at $\alpha = 1$ and $\alpha = 2$ are the same. So we will find the second generation at the other equivalent fixed point $g = (\theta_1\theta_2, e_1 + e_3 + e_4)$. In fact, this degeneracy gives rise to a D_4 flavor group, under which the two generations form a doublet.

Let us remark that there are no known models which give just the particle content of the MSSM and nothing else. Generically there are of the order of 100 extra fields. Some of them are charged under the hidden sector gauge group while others are exotics, i.e. carry standard model charges. This shows one of the main restriction in MSSM model building on orbifolds: from the pure string perspective, the massless spectrum is what it is, and it is not the MSSM spectrum. However, at this point one is in general not at a vacuum of the theory, since this does not include non-perturbative effects, which might generate a potential for some of the fields, which in turn forces them to get a vev in the vacuum. Another condition which imposes that some of the fields get a vev comes from SUSY D-terms. In general, some of the hidden sector gauge fields are extra U(1)s, and one of them has an anomaly which is canceled via the Green–Schwarz mechanism. This is in contrast to smooth CY models where generically all U(1)s are Green–Schwarz anomalous. The reason that on the orbifold only one U(1) can be anomalous is that there is only one axionic zero mode in B_2 (which is the imaginary part of the dilaton). In general, there are $h^{1,1}$ additional axions (which are the imaginary part of the CY Kähler moduli). However, the Kähler moduli are frozen at the orbifold point and hence their axions do not contribute to the anomaly cancellation. Consequently, there is only one axion and thus at most one anomalous U(1). In our example this anomalous U(1) generator reads

$$T_{\mathrm{U}(1)_{\mathrm{anom}}} = (-2,-1,2,1,1,1,1,1)(-\frac{1}{2},-\frac{1}{2},0,0,0,0,-\frac{1}{2},-\frac{1}{2})\,. \tag{11}$$

Most importantly, this U(1) is orthogonal to the hypercharge (8) which means that the latter does not get Green–Schwarz massive.

Upon canceling the anomaly via the GS mechanism, an FI term ξ is introduced for this anomalous U(1). In order to obtain an overall vanishing D-term,

$$\sum_i q^i_{\mathrm{anom}}|\psi_i|^2 - \xi = 0\,, \tag{12}$$

some fields ψ_i that are charged under the anomalous U(1) have to get a vev as well. Generically, these fields are also charged under other U(1)'s, in which case D-flatness requires other fields to get a vev as well. All in all, this leads to a higgsing of some of the hidden sector gauge symmetries and to mass terms for the particles. However, the vevs of the fields lead to a backreaction on the geometry which resolves the singular orbifold into a smooth CY. But this means that the theory is not a pure orbifold anymore and one should apply the smooth CY analysis and consistency conditions. On the other hand, not all singularities might be resolved and thus the supergravity approximation is not valid everywhere. In any case, all of the following discussion has to be carried out in an effective field theory (which is derived from either the orbifold theory or the heterotic supergravity theory; both can be matched to one another [12, 13]).

Lastly we want to repeat here that orbifolds provide a natural and beautiful solution to the doublet-triplet splitting problem. Since the mechanism was explained by Julian last week, we will be very brief: In our model the shift vectors and torus Wilson lines break the $\mathrm{E}_8 \times \mathrm{E}_8$ among other things to the GUT SU(5). This SU(5) is then broken via the freely acting Wilson line to the SM gauge group and the Higgs triplets are projected out since they are not compatible with the action of the Wilson line.

3.5 Yukawa textures

The string couplings can also be calculated from CFT techniques. While this is in principle doable [14], the calculations become very involved rather quickly. By studying the lowest n-point correlators, rules were derived that have to be fulfilled for such a correlator to be non-vanishing [15]. They mostly correspond to classical conditions in target space like gauge invariance and momentum conservation. However, one of the rules (aka rule 4) seems to not have a corresponding target space symmetry.

Due to the aforementioned fact that some fields have to acquire vevs to cancel the FI term, to decouple the extra states, and to break the extra U(1) symmetries, the discussion is usually carried out in an effective field theory. In this approach, all couplings that comply with the manifest target space symmetries are written down. Due to rule 4, some of these might actually be forbidden; however, since there is no underlying symmetry in the effective field theory that protects these couplings, it is expected that they are generated in any case. So let us now discuss the couplings from the effective field theory point of view.

The mass hierarchies for the Yukawa couplings are generated in a Froggatt-Nielsen type mechanism once some of the extra fields that are charged under the hidden U(1)s get a vev. In addition, some couplings are suppressed due to the localization of the fields: as mentioned before, twisted fields are localized at fixed points. Couplings between fields at different fixed points are generated via non-perturbative instanton effects. These come with an exponential suppression which is proportional to the size of the CY (or rather to the distance between the fixed points) and are consequently small.

Lastly we want to discuss the μ-term. Since the Higgses are vector-like under the standard model, we need a symmetry which forbids the μ-term. If the Higgses carry different charges under the extra hidden U(1) symmetries, these can serve to forbid the μ-term[4]. Another possibility is to use discrete symmetries. In orbifolds they arise from either partially higgsed hidden U(1) symmetries or from discrete remnants of the internal Lorenz transformations (or any combination of the two). Especially the latter can usually be found on orbifolds owing to the high degree of symmetry of the orbifold point. In any case, there are many singlets which might be added to the $H_u H_d$ term to make it gauge invariant even under these additional symmetries. Since the μ-term is extremely small, such singlet insertions have to be sufficiently suppressed. Since this is hard to achieve in general, it is better to identify a symmetry which forbids the μ-term to all orders and to generate it e.g. from the Kähler potential using a Guidice–Masiero-like mechanism.

References

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