Low String Scale and Large Extra Dimensions in String Theory

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Literature

- Witten, 1996 hep-th/9602070
- Lykken, 1996 hep-th/9603133
- ⊙ Antoniadis, 2003 Progress in String, Field and Particle Theory pp 25-74
- Arkani-Hamed, Dimopoulos, Dvali hep-th/9803315 + Antoniadis, 1998 hep-th/9804398

I. Basics

String theory has one free parameter:

$$m_s = \frac{1}{\ell_s} \sim \frac{1}{\sqrt{\alpha'}}$$

But: for realistic 4d models:

→ fix string coupling $\lambda_s = e^{\langle\phi\rangle}$

→ choose internal manifold with volume V

↳ effectively 3 parameters

⇒ should be enough to fit parameters of phenomenological interest, e.g.

- α_{GUT}, M_{pl} (Witten)
- m_{EW}, M_{pl} (Lykken)

But: problems for weakly coupled heterotic string theories possible solutions: • go to strong coupling

⇒ dual theories (M-theory, type I, type II)

⇒ leads to large extra dimensions

⌈ remember: for extra dimensions of size R

- Kaluza-Klein modes $\Delta E^2 \sim \frac{n^2}{R^2}$
- winding modes $\Delta E^2 \sim \frac{n^2 R^2}{\ell_s^4}$, $n \in \mathbb{N}$

(interchanged by T-duality) ⌋

II. Motivation:

Mainly phenomenological interest:
- gauge coupling unification (1)
- SUSY breaking (2)
- hierarchy problem (3)

(1) from models like MSSM:

gauge coupling unification at $m_{GUT} \sim 10^{16}$ GeV

⇒ natural to identify with string scale m_s

⇒ leads to large extra dimensions / strongly coupled heterotic string theory

(2) hard to break SUSY at scales hierarchically lower than m_s (for heterotic string)

⇒ SUSY breaking by compactification (Scherk-Schwarz mechanism)

$$m_{SUSY} \sim \frac{1}{R}$$

⇒ large extra dimensions

(3) as we heard last week: SM-like models achieved by non-supersymmetric configurations ⇝ new mechanism to solve

hierarchy problem

$\Rightarrow$ $m_s \sim$ TeV

$\Rightarrow$ large extra dimensions

III. Problems in (weakly coupled) heterotic string theory

effective 4d action (for gravity / gauge)

$$\boxed{S_{eff}^{het} = \int d^4x \, \frac{V}{\lambda_s^2} \left(\ell_s^{-8} \underbrace{R}_{\text{Ricci-scalar}} + \ell_s^{-6} \, tr F^2 \right)}$$

$\Rightarrow$ matching to 4d Planck mass M_{pe} and gauge coupling g:

$$M_{pe}^2 = \frac{V}{\lambda_s^2} m_s^8 \quad , \quad \frac{1}{g^2} = \frac{V}{\lambda_s^2} m_s^6$$

$$\Rightarrow \quad M_{pe}^2 g^2 = m_s^2 \quad , \quad \lambda_s = g \sqrt{\frac{V}{\ell_s^6}}$$

$\downarrow g \simeq 1$ $\qquad\qquad$ $\downarrow$

$\boxed{M_{pe} \simeq m_s}$ $\qquad$ weakly coupled for $\boxed{V \sim \ell_s^6}$

no large volume possible for weakly coupled heterotic strings

IV. Dual models

use strong/weak coupling dualities for large extra dimensions

M-theory

heterotic $E_8 \times E_8$ on CY_3

dual to

M-theory on $M_4 \times CY_3 \times S^1/\mathbb{Z}_2$

$\uparrow$ $[0, \pi g]$

- gravity propagates in bulk
- gauge theory on fixpoints

$\Rightarrow$ at weak coupling described by 11d SUGRA effective 4d action:

$$\boxed{S_{eff}^{M} = \int d^4x\, V\left(\frac{\rho}{\ell_M^3} R + \frac{1}{\ell_M^6} \operatorname{tr} F^2 \right)}$$

⇒ again matching to 4d parameters:

$$\ell_M = (g^2 V)^{1/6} \quad , \quad \rho = g^2 \ell_M^3 M_{pl}^2$$

compare to heterotic: $\ell_M = \ell_s \lambda_s^{1/3} \quad , \quad \rho = \lambda_s \ell_s$

⇒ SUGRA limit $\rho > \ell_M$, $V > \ell_M^6$ corresponds to strong coupling in heterotic

<u>extrem</u>: $\ell_M^{-1} \sim 1\,\mathrm{TeV}$

corresponds to $\rho \sim 10^8\,\mathrm{km}$

→ excluded

(for $\rho \sim 1\mathrm{mm}$ → $\ell_M^{-1} \sim 10^7\,\mathrm{GeV}$)

in fact <u>further restriction</u>: $CY_3 \times S^1/\mathbb{Z}_2$ no SUGRA solution at higher order

(due to 4-form flux)

⇒ V varies linearly in x_{11}-direction

⇒ gauge coupling of E_8-factors should remain small

⇒ $V(0)$, $V(\pi\rho)$ sufficiently large

⇒ $\ell_M^{-1} \gtrsim g^{5/3} M_{pl}$

<u>type I / I'</u>

heterotic $SO(32)$

dual to

type I $\xrightarrow[\text{T-duality}]{}$ type I' (unoriented string theory: closed and open sector)

- closed strings: gravity
- open strings on D-branes: gauge theory

$\Rightarrow$ split internal dimensions in: (our world on Dp-brane)

- longitudinal $(p-3)$: $V_{\parallel}$
- transverse $n = (9-p)$: $V_{\perp}$

effective action in 4d:

$$\boxed{S^{I}_{eff} = \int d^4x \left(\frac{V_{\perp} V_{\parallel}}{\lambda_I^2 \, l_I^8} \mathcal{R} + \frac{V_{\parallel}}{\lambda_I \, l_I^{p-3}} \, tr F^2 \right)}$$

(remember: tension of Dp-brane $\sim \frac{1}{\lambda_I}$)

$$\Rightarrow \quad \lambda_I = g^2 \frac{V_{\parallel}}{l_I^{p-3}} \qquad \text{small if} \quad V_{\parallel} \sim l_I^{p-3}$$

$$M_{pl}^2 = \frac{V_{\perp}}{V_{\parallel} g^4} \, l_I^{4-2n} \sim \frac{1}{g^4} \, l_I^{-2-n} \, \underset{\substack{\uparrow \\ \text{free parameter}}}{V_{\perp}}$$

$\Rightarrow$ $\boxed{l_I \gg l_{pl}}$ for $V_{\perp}$ large

(large extra dimension only seen by gravity $\nearrow$ ADD)

$\Rightarrow$ gravity becomes strong at distances $< l_I$

<u>extrem case:</u> $l_I^{-1} \sim 1\,\text{TeV}$

$\rightarrow$ $\rho_{\perp} \sim 10^8\,\text{km}$ (1 large dimension)

$\rho_{\perp} \sim 0.1\,\text{mm}$ (2 large dimensions)

$\rho_{\perp} \sim 0.1\,\text{fm}$ (6 large dimensions)

<u>Relation to heterotic string theory</u> (from spacefilling D9-branes)

by S-duality: $\lambda_I = \frac{1}{\lambda_S}$, $l_I = \sqrt{\lambda_S}\, l_S$

for k large dimensions R and $6-k$ dimensions $\sim l_s$

we futher find: $\lambda_S = g R^{k/2} l_s^{-k/2} \sim \left(\frac{R}{l_s}\right)^{k/2} \Rightarrow \lambda_I \sim \left(\frac{R}{l_s}\right)^{-k/2}$

$$l_I \sim \left(\frac{R}{l_s}\right)^{k/4} l_s$$

$\Rightarrow$ $\boxed{l_I \gg l_s}$

specifically: for $k<4$: $R > l_I > l_s$

for $k=4$: $R \sim l_I$

for $k>4$: $l_I > R$

$\Rightarrow$ for $l_I > R$ problem to describe geometry

$\Rightarrow$ perform T-duality $\Rightarrow$ string coupling changes $\lambda \to \frac{l}{R}\lambda$

$\Rightarrow$ check that in type I' still weakly coupled:

- for $k<4$: $\lambda_I \to \lambda_{I'} = \lambda_I \left(\frac{l_I}{l_s}\right)^{6-k} \gg 1$ $\to$ string coupling
- for $k \geq 4$: $\lambda_I \to \lambda_{I'} = \lambda_I \left(\frac{l_I}{l_s}\right)^{6-k} \left(\frac{l_I}{R}\right)^{k} \sim 1$

($(6-k)$ transverse dimensions of size $\frac{l_I^2}{l_s}$

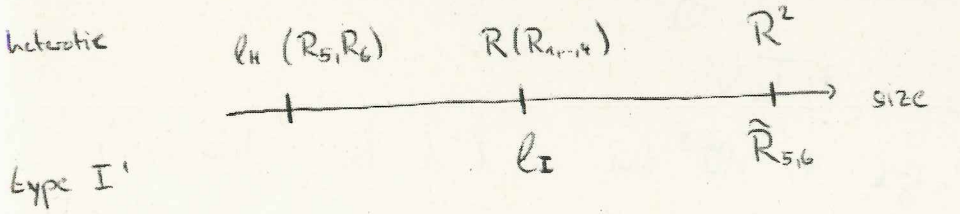
k transverse dimensions of size $\frac{l_I^2}{R}$)

<u>example $k=4$:</u> heterotic string with 4 large dimensions ($\sim$ TeV).

dual to

perturbative type I' with $l_{I'}^{-1} \sim$ TeV and two

extra large dimensions ~ 1mm



<u>type II</u>

more work is required here because:

- type II has twice the number of supercharges

  $\Rightarrow$ compactification on different manifolds

  (e.g. heterotic on T^4 $\leftrightarrow$ II A on K3)

- non-Abelian gauge symmetries arise non-perturbatively

  in type II (wrapping D-branes around ~~van~~ vanishing, non-trivial cycles)

<u>example</u>: IIA on K3 (gauge-sector via wrapping non-trivial vanishing 2-cycles of singular K3)

effective action:

$$S_{IIA} = \int d^{10}x \, \frac{1}{\lambda_{IIA}^2 \, l_{IIA}^8} \, R + \int d^6 x \, \frac{1}{l_{IIA}^2} \, \mathrm{tr} F^2$$

further for T^2- compactifications to 4d:

$$\frac{1}{g^2} = \frac{V_{T^2}}{l_{IIA}^2} \quad , \quad M_{pl}^2 = \frac{V_{T^2} \, V_{K3}}{\lambda_{IIA}^2 \, l_{IIA}^8}$$

$\Rightarrow$ λ_{IIA}, V_{K3} to play with:

$$l_{IIA} = \left(\frac{V_{K3}}{g^2 M_{pl}^2 \lambda_{IIA}^2} \right)^{1/6}$$

V. Theoretical consequences

- Restrictions to extra dimensions due to behavior of moduli fields (e.g. the dilaton could be affected) in background of D-/orientifold planes.
- naively: Unification spoiled for $m_s < m_{GUT}$
  but: possible power law running due to KK-modes
- reformulation of hierarchy problem: why are transverse dimensions so large?
- possibility of non-supersymmetric string theories
  $\rightarrow$ leads to "huge" cosmological constant $\Lambda \sim M_I^2 M_{pl}^2$
- non-supersymmetric strings might even be relevant for electro-weak symmetry breaking if ($m_s \sim$ TeV)

<u>Summary</u>: Interesting <u>but</u> problematic.