String Gas Cosmology (Severin Lüst) ①

Reviews: • hep-th/0808.0746
• hep-th/0510022

I Introduction
II Cosmology with String Gases
III Classical dynamics of string gases
IV Generation of cosmological fluctuations

I Introduction

- Usually string cosmology concentrates on the effective actions obtained by compactifying all extra-dimensions
- String gas cosmology uses intrinsically stringy effects as winding modes and T-duality to describe the cosmology of the early universe before the effective field theory approach becomes valid.
- It gives an dynamical explanation for the dimensionality of space-time.

II Cosmology with String Gases

- Setup: Gas of Strings (at finite temperature) in a classical background
- Assumptions:
  - × Homogeneous fields (only time dependence)
  - × Adiabatic expansion (fields are evolving slow $\rightsquigarrow$ no α'-corrections)
  - × Weak coupling ($g_s \ll 1$)
  - × Toroidal spatial dimensions

Energy and Pressure of a String gas

- Consider strings in a background of the form

$$S_0 = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-G}\, e^{-2\phi}\left(R + c + 4(\nabla\phi)^2 - \frac{1}{12}H^2\right)$$

$$ds^2 = -dt^2 + \sum_{i=1}^{d} a_i^2(t)\, dx_i^2 \qquad B_{\mu\nu} = 0$$

(rescaled dilaton

$$a_i = e^{\lambda_i}, \quad \phi(t) \qquad \varphi = 2\phi - \ln V = 2\phi - \sum_{i=1}^{d}\lambda_i\,)$$

inv. under: $a_i \to \frac{1}{a_i}$, $\lambda_i \to -\lambda_i$, $\varphi \to \varphi$

- total action

$$S = S_0 + S_\sigma \quad \text{non-linear sigma-model} \qquad (1)$$

with $$S_\sigma = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(\sqrt{-\gamma}\,\gamma^{ab}G_{\mu\nu}\partial_a X^\mu\partial_b X^\nu + \varepsilon^{ab}B_{\mu\nu}\partial_a X^\mu\partial_b X^\nu\right)$$

- E.o.m: $(\partial_\tau^2 - \partial_\sigma^2)X^\mu(\sigma,\tau) \approx 0$

together with the constraints

$$G_{\mu\nu}\left(\dot X^\mu \dot X^\nu + X'^\mu X'^\nu\right) = G_{\mu\nu}\dot X^\mu X'^\nu = 0$$

- Solution of the e.o.m:

$$X^\mu = X^\mu_L(\tau+\sigma) + X^\mu_R(\tau-\sigma)$$

$$X^\mu_{L/R} = X^\mu_{L/R} + \sqrt{\frac{\alpha'}{2}}\,p^\mu_{L/R}(\tau\pm\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_{n\,L/R}\, e^{-in(\tau\mp\sigma)}$$

~~Level matching: $p_L^2 - p_R^2 = 4(N_R - N_L + a_R - a_L)$~~

②

- Center of mass momenta

$$p^m_{L/R} = \frac{\sqrt{\alpha'}}{R} n^m \pm \frac{R}{\sqrt{\alpha'}} w^m$$

$\rightsquigarrow$ T-Duality: $\frac{R}{\sqrt{\alpha'}} \longleftrightarrow \frac{\sqrt{\alpha'}}{R}$, $n \leftrightarrow w$

- Energy: $E = \sqrt{\vec{p}^{\,2} + G_{mn}\left(n_m + \frac{w_m}{\alpha'}\right)\left(n_n + \frac{w_n}{\alpha'}\right) + \frac{4}{\alpha'}(N_L + a_L)}$

- Energy density of a string gas:

$$\rho = \sum_s \tilde{n}_s E_s$$

$\tilde{n}_s = N_s V^{-1}$

↳ number density of a certain string state labeled by the quantum numbers $s = (n, w, N_L, N_R)$

- Pressure: $p_i = -\frac{1}{V}\frac{\partial(\rho V)}{\partial \lambda_i}$

× for winding modes: $(w_m \neq 0,\ n_m = 0)$ $\quad \rho_w = \sum_{l=1}^{d} \tilde{n}_w^{(l)} e^{\lambda_l(t)}$

$\Rightarrow p_w^{(l)} = -\tilde{n}_w e^{\lambda_l(t)}$

× momentum modes: $(w_m = 0,\ n_m \neq 0)$ $\quad \rho_m = \sum_{l=1}^{d} \tilde{n}_m^{(l)} e^{-\lambda_l(t)}$

$p_m = \tilde{n}_m e^{-\lambda_l(t)}$

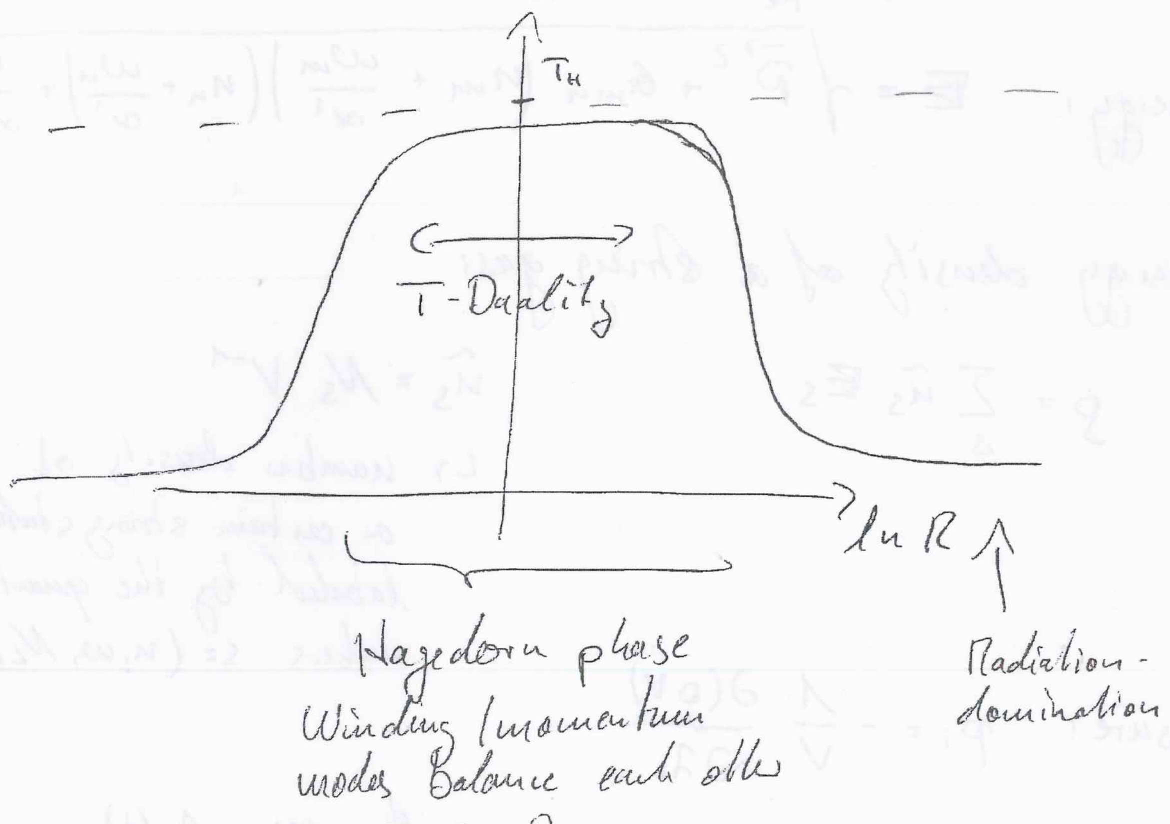
- isotropic distribution:

$p_w = -\frac{1}{d}\rho_w \qquad p_m = \frac{1}{d}\rho_m$

- Three different types of modes
  - × momentum modes $\rightsquigarrow$ positive pressure (radiation)
  - × winding modes $\rightsquigarrow$ negative pressure
  - × oscillatory modes (highly degenerated)

~> Hagedorn Temperature: Limiting temperature for a string gas in thermal equilib.

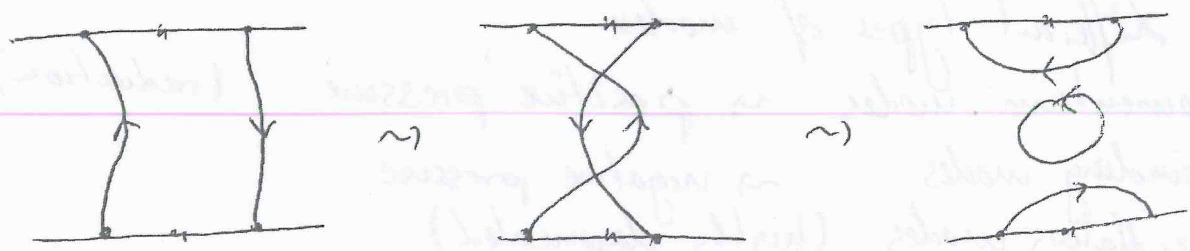
- Temperature of a string gas as a function of R:



The Dimensionality of space-time

(Brandenberger, Vafa: Nucl. Phys B316 (1989) 391-410)

- Initially all (10) space-time dimensions are small
- Winding modes ~> negative pressure
  => Winding modes must decay into radiation to allow for expansion



③

- In more than four space-time dimensions two-dimensional string world-sheets have measure zero intersection probability

  $\rightsquigarrow$ only <u>three</u> space-~~time~~ dimensions are free to expand, the remaining six stay compact

- Dynamical explanation of the dimensionality of space-time (supported by numerically simulations)

III Cosmological Evolution in the Presence of a String gas

~~Moduli Stabilization~~:

- Now: Take the 4+6 split as given
- Background configuration:

$$ds^2 = -dt^2 + \underbrace{a^2(t)\,d\vec{x}^{\,2}}_{\substack{\text{large} \\ \text{(expanding)}}} + \underbrace{b^2(t)\,d\vec{y}^{\,2}}_{\text{compact}}$$

$$a(t) = e^{\lambda(t)}, \quad b(t) = e^{\nu(t)}$$

$$H_3 = h\, dx^1 \wedge dx^2 \wedge dx^3, \quad \phi = \phi(t)$$

- E.o.m from (1):

$$c - 3\dot\lambda^2 - 6\dot\nu^2 + \dot\varphi^2 - \frac{h^2}{2} e^{-6\lambda} = e^{\varphi} E$$

$$\ddot\lambda - \dot\varphi\dot\lambda - \frac{1}{2} h^2 e^{-6\lambda} = \frac{1}{2} e^{\varphi} P_3$$

$$\ddot\nu - \dot\varphi\dot\nu = \frac{1}{2} e^{\varphi} P_6$$

$$\ddot\varphi - 3\dot\lambda^2 - 6\dot\nu^2 = \frac{1}{2} e^{\varphi} E$$

Conservation of sources

$$\dot E + 3\dot\lambda P_3 + 6\dot\nu P_6 = 0$$

• Restricting to the 3+1 dim. case $(\nu = 0)$
for $\phi = \text{const}$ $\left(e^{2\phi} = \frac{16\pi}{M_{pl}^2}\right)$

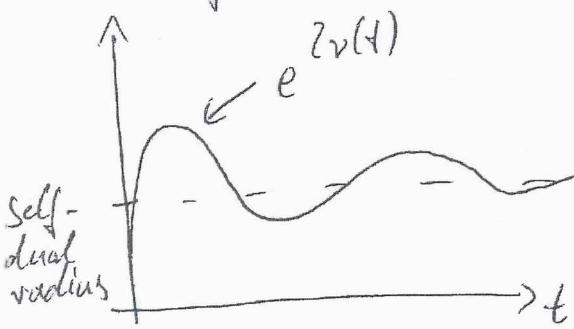
$$\left.\begin{aligned} \dot{\lambda}^2 &= \frac{8\pi}{3M_p^2}\rho \\ \ddot{\lambda} + \dot{\lambda} &= -\frac{4\pi}{3M_p^2}(\rho + 3p) \end{aligned}\right\} \text{Friedmann-Equations } \left(a(t) = e^{\lambda(t)} \Rightarrow H = \frac{\dot{a}}{a} = \dot{\lambda}\right)$$

$$R = \ddot{\lambda} + 2\dot{\lambda}^2 = 0$$

Solution of the e.o.m.   0109165 ~~hep-th/0307044~~

• Evolution of the large dimensions: (hep-th/0109165)
Locking phase phase until the winding modes decayed
$\rightsquigarrow$ afterwards the universe is free to expand

• Evolution of the 6d-radion: (hep-th/0307044)
Assumption: $n_w^{(6)} = n_m^{(6)}$



$\rightsquigarrow$ damped oscillation of the radion around the self-dual radius

• additional mechanism for dilaton stabilization is needed, e.g. gaugino condensation

• Crucial: running of the dilaton

4d-dynamics and the effective potential

• Transformation to the Einstein frame: $dt_E^2 = e^{\phi/2} dt_s^2$

$$ds_E^2 = -dt_E^2 + e^{\phi/2} a^2(t)\, d\vec{x}^{\,2} + \underbrace{e^{\phi/2} b^2(t)}_{\text{Einstein-frame radion}} d\vec{y}^{\,2}$$

⇝ due to the running of the dilaton the Einstein-frame radion is <u>not</u> stabilized (4)

- 4d-effective ~~potential~~ action:

$$S_4 = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \left[R - \frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi - \frac{1}{2} \nabla_\mu \psi \nabla^\mu \psi \right] - e^{4\phi - \sqrt{\frac{d}{2}}\psi} V_S^{4+d}(1, \varphi, \psi) \right)$$

φ, ψ: 4d-fluctuations of the dilaton / radion

- ~~4D-effective potential:~~
- example: 10d-cosmological constant:

$$V_S^{10} \sim \Lambda \quad \Rightarrow \quad V_E^{(4)} = \frac{e^{4\phi - \sqrt{\frac{d}{2}}\psi}}{(2\pi\sqrt{\alpha'})^4} \Lambda$$

⇝ exp. runaway potential

- wrapped and moving branes in the extra dimensions:

$$V_S^{4+d} = \mu \frac{N \overset{\text{number of strings/branes}}{b^k}}{a^3 b^d} \qquad \mu = (2\pi\sqrt{\alpha'})^{-4}$$

$|k| \leq d$: type of string / brane (e.g. $k = -1$: momentum mode, $k = +1$: winding mode)

$$\Rightarrow V_S^{4+d} = \mu \tilde{n} e^{-3\phi} b^{k + \frac{d}{2}} \qquad (\varphi = 2\phi - d \ln b)$$

$\tilde{n}$: Einstein-frame number density

$$\Rightarrow V_E^{(4)} = \mu \tilde{n} e^{\phi} b^{k - \frac{d}{2}} = \mu \tilde{n} e^{-|\phi|} \exp\left[\left(\frac{2k - d}{2\sqrt{2d}} \right) \psi \right]$$

⇝ confining potential only for $k \geq \frac{d}{2}$,
for winding strings only possible for $d = 1$

• Heterotic string: $(a_L = -1,\ a_R = \frac{1}{2},\ N_R = 0,\ N_R = \frac{1}{2})$

$$\Rightarrow \quad E = \sqrt{G^{mn}\left(n_m + \frac{w_m}{\alpha'}\right)\left(n_n + \frac{w_n}{\alpha'}\right) - \frac{4}{\alpha'}}$$

for $b = \sqrt{\alpha'}$ (self-dual radius):
additional massless states

$$V_S^{4+d} = \mu \tilde{n}\, e^{-3\phi}\, b^{\frac{d}{2}}\, \tilde{E} \leftarrow \text{rescaled Energy to put } \alpha'\text{-dep. into } \mu$$

$$\tilde{E} = \sqrt{\frac{n\cdot n}{b^2} + w\cdot w\, b^2 - 2n\cdot w}$$

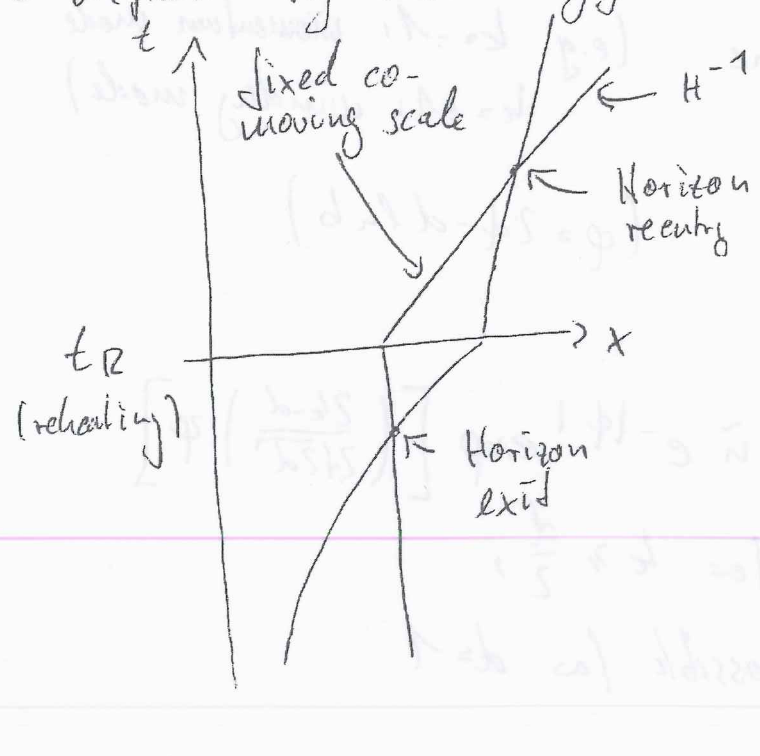
$$= \left|\frac{n}{b} - wb\right| = 2\left|\sinh\left(\frac{\psi}{\sqrt{2d}}\right)\right|$$

$$\Rightarrow \quad V_E^{(4)} = 2\mu\tilde{n}\, e^{\phi - \frac{1}{2}\sqrt{\frac{d}{2}}\psi}\left|\sinh\left(\frac{\psi}{\sqrt{2d}}\right)\right|$$

$\rightsquigarrow$ admits a local minimum
(diluted by running dilaton)

IV Generation of cosmological fluctuations

• Inflationary Cosmology:

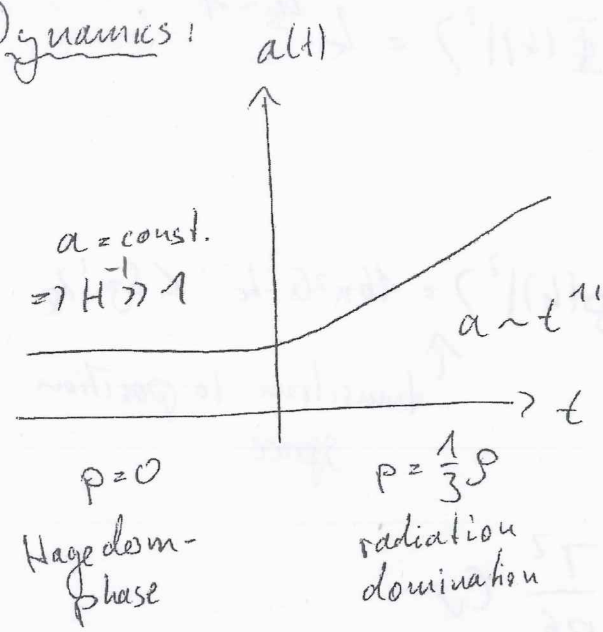


$a(t) \sim t^{1/2}$
(radiation domination)

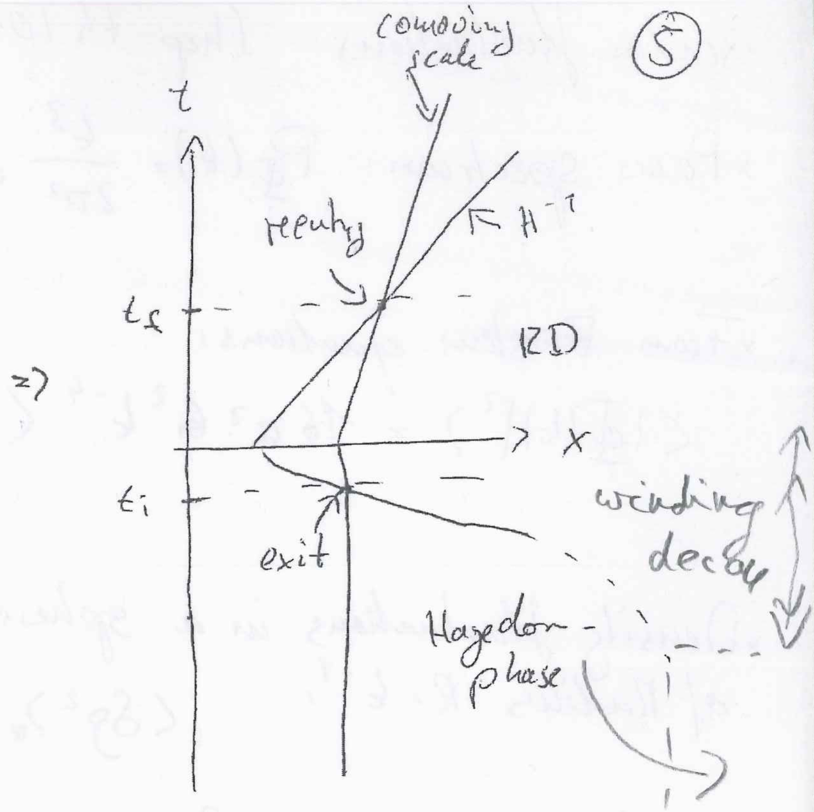
$a(t) \sim e^{Ht}$
$\Rightarrow H = \text{const}$

• String gas cosmology (5)

Dynamics:



$\Rightarrow$



Generation of fluctuations:

• infl. cosmology:
~~fluct~~ cosm. fluctuations arise as quantum fluctuations and are blown up during infl.

• SGC:
cosm. fluctuations come from (classical) thermodyn. fluctuations of the string gas

• spectrum of fluctuations:

4D-metric

$$ds^2 = a^2(\eta)\left[(1+2\Phi)\, d\eta^2 - ((1-2\Phi)\delta_{ij} + h_{ij})\, dx^i dx^j\right]$$

scalar fluct. — tensor fluct.

• Scalar fluctuations (hep-th/0511140)

× Power spectrum: $P_\Phi(k) = \frac{k^3}{2\pi^2}\langle|\Phi(k)|^2\rangle = k^{n_s - 1}$ ← spectral index

× From Einstein equations:

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4}\langle|\delta\rho(k)|^2\rangle = 16\pi^2 G^2 k^{-1}\langle\delta\rho^2\rangle_R$$

↑ transition to position space

× Density fluctuations in a sphere of Radius $R = k^{-1}$:

$$\langle\delta\rho^2\rangle_R = \frac{T^2}{R^6} C_V$$

× Heat capacity of a string gas:

$$C_V \approx 2\frac{R^2/l_s^3}{T(1-\frac{T}{T_H})}$$

$$\Rightarrow P_\Phi(k) = 8G^2\frac{T}{l_s^3}\frac{1}{1-T/T_H} \quad \Rightarrow n_s \approx 1$$

(small ~~blue~~ red-tilt, since T has to be evaluated at $t_i(k)$)

• tensor fluctuations: hep-th/0604126

$$P_h(k) \approx \underbrace{16\pi^2 G^2\frac{T}{l_s^3}}_{\approx\left(\frac{l_{pl}}{l_s}\right)^4 \text{ for } T\approx T_H}\underbrace{(1-T/T_H)}\ln^2\left[\frac{1}{l_s^2 k^2}(1-T/T_H)\right]$$

↳ slight blue tilt

- Cowinks analysis hep-th 10608200 ⑥

× The above analysis was performed in the string frame, not in the Einstein frame

× During the Hagedorn phase the dilaton evolves rapidly $\Rightarrow$ H_E^{-1} and H_S^{-1} do not agree

[notice: H_E^{-1} can only shrink if the null-energy condition is violated]

× Calculation using the Einstein-frame gives

$$n_s = 5 \quad !$$

× Possible solution (?): Stabilization of the dilaton by some mechanism