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Bouncing Cosmologies ①

① Why? (My opinion)

a. Because it's there/why not

b. Inflation cannot be the whole story. It's past incomplete.

c. Theory of initial conditions is lacking. Also low initial entropy of the initial state has to be assumed ⟺ Hot big bang (Liouville measure is conserved)

d. Eternal inflation ⟷ "Measure problem", loss of predictability

Bouncing Cosmologies attempt to continue the evolution back in time before the Big Bang. Many times and also in this lecture, we will be interested in their role as alternative to inflation.

Outline:
- General Characteristics
- Singularity theorems and energy conditions in GR
- Toy model with ~~SEC~~
- ~~NEC~~, some implications
- A concrete example - a ghost condensate with ~~NEC~~
- Some comments about perturbations
- Summary of "generic" predictions

References: 1406.2790 - highly critical and confusing, use mainly as a reference reservoir.

Pre-history: Pre Big Bang scenario, considered to be ruled out, mostly developed by Veneziano & Gasperini Phys. Rept. 373(1)(2003)

② General characteristics

* Singular Bounce: The scale factor passes through zero, $a=0$. Requires (some) understanding of QG regime. Certainly goes beyond GR.

Example: Antigravity By Bars, Steinhardt & Turok, Loop Quantum cosmology

* Non Singular Bounce: a is always finite.
$\dot{a}$ or H pass through zero, $H=0$.
$\Rightarrow$ ~~We don't reach~~ The universe never reaches a classical singularity. ~~[illegible]~~

Example: Cyclic scenario.

* Generation of perturbations:

All bounces except string gas cosmology/S-brane bounce, rely on quantum fluctuations as seeds for structure formation in the Universe.

In that sense these models are not so different ~~th~~ from inflation. The underlying mechanism is the same.

[For the string gas cosmology model - S. Lüst talk]

* Null energy condition violation (at least somewhere in space time)

~~[In 1104.0288, "A simple harmonic Universe", the NEC is not violated but one only gets a classically stable "small universe" and they need a fluid with EOS $-1 < w < -\frac{1}{3}$; inflation like]~~

~~II. Eternal inflation → "Measure problem", loss of predictability~~

② Singularity theorems:

In GR, we can consider arbitrary matter content due to the form of the Einstein's equations: $G_{\mu\nu} = T_{\mu\nu}$. Simply have a metric $g_{\mu\nu}$ and postulate the $T_{\mu\nu}$.

Therefore, one adds specifications of what "reasonable" energy-momentum content is.
For examples, the avoidance of ghosts, superluminality, etc.

Giving some energy conditions on the matter content, we can derive some singularity theorems by proving that they are "geodesically incomplete".

geodesically incomplete means that there is at least one geodesic which is inextendible, but has finite affine length/proper time/proper length.

$\Rightarrow (M, g_{\mu\nu})$ contains at least one incomplete timelike or null geodesic

Theorem: Suppose a spacetime $(M, g_{\mu\nu})$ satisfies:

① $R_{\mu\nu} V^\mu V^\nu \geq 0$ for all timelike and null V^μ, as will be the case if EFE is satisfied with strong energy condition for matter

② timelike and null generic conditions are satisfied, (Every geodesic has at least one point with $R_{\mu\nu\rho\sigma} U^\mu U^\sigma \neq 0$ [timelike] or $R_{\mu\nu} k^\mu k^\nu \neq 0$ [null]

③ No closed timelike curves exists.

④ At least one of the following a. "Closed Universe"
b. A trapped surface
c. $\exists\, p \in M$ such that the expansion of future directed (or past) null geodesics emanating from p becomes negative along each geodesic in this congru

Energy Conditions in GR (4)

Energy momentum tensor of a perfect fluid

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + g_{\mu\nu} p$$

$p \equiv$ pressure

$\rho \equiv$ energy density

$u_\mu \equiv$ four velocity, timelike $u_\mu u^\mu = -1$

(1) Null energy condition (NEC):

$\forall$ future pointing null-vector k^μ; $k^2 = 0$

$0 \leq \rho + p \Leftarrow T_{\mu\nu} k^\mu k^\nu \geq 0$; For $p = w\rho \Rightarrow \begin{cases} w \geq -1 \\ \rho > 0 \end{cases}$ or $\begin{matrix} \rho < 0 \\ w < -1 \end{matrix}$

(2) Weak energy condition (WEC):

$\forall$ time-like vector y^μ; $T_{\mu\nu} y^\mu y^\nu \geq 0$

$y_\mu y^\mu \leq -1$

$\Downarrow$

$w \geq -1 \Leftarrow \rho \geq 0 ; \rho + p \geq 0$

(3) Strong energy condition (SEC):

$\forall$ time-like vector field y^μ; $(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) y^\mu y^\nu \geq 0$

$\Rightarrow \rho + p \geq 0$; $\rho + 3p \geq 0$

(4) Dominant energy condition (DEC):

WEC + $\forall$ future pointing time-like or null vector y^μ

$-T^\mu_\nu y^\nu$ is future pointing. i.e, mass energy cannot be observed flowing faster than light.

$\Rightarrow \rho \geq |p| \Rightarrow |w| \leq 1$

Important: These conditions are valid and well-defined <u>regardless</u> of the perfect fluid description!

~~The precise mathematical formulation can be found in Wald 1984, and other references.~~

(geodesical incompleteness $\xrightarrow[\text{language}]{\text{abuse of}}$ curvature sing.)

For Our purpose: ① Schwarzschild solution – has a curvature singularity if the Null Energy Condition (NEC) holds

② Expanding FLRW – has a curvature singularity if $K=0,-1$ and the NEC holds

③ ~~Expandi~~ FLRW has a curvature singularity if $K=+1$ and the Strong Energy Condition (SEC) holds.

$\Rightarrow$ If we give up SEC, but maintain NEC and $K=+1$, we can avoid the curvature singularity and get a <u>bounce</u>:

1109.0282, 0802.1196, hep-th/0307077

However, in all models one still has an inflation "period" of "fluid" with $-1<w<-\frac{1}{3}$

therefore, we don't consider this as alternative for inflation, but it does partially ~~resolve the initi~~ adress the initial singularity question.

$\Rightarrow$ For a "full" alternative to inflation, we are led to consider a model with $\cancel{NEC}$.

Toy model with ~~SEC~~ ; $k=+1$

$$\begin{cases} \dfrac{\dot{a}^2}{a^2} = \dfrac{\rho}{3} - \dfrac{k}{a^2} \\ \\ \dfrac{\ddot{a}}{a} = -\dfrac{1}{6}(\rho + 3p) \equiv \dot{H} + H^2 \Rightarrow \dot{H} = -\dfrac{1}{2}(\rho + p) + \dfrac{k}{a^2} \end{cases}$$

Using the EOS $p = w\rho$; $-1 < w < -\frac{1}{3}$

and $\rho = \underbrace{\Lambda}_{\text{Negative CC}} + \rho_0 a^{-3(1+w)}$ we can find

oscillatory solutions. Specifically for $w = -\frac{2}{3}$ we

get $a = \dfrac{\rho_0}{2|\Lambda|} + a_0 \cos(\tilde{\omega} t + \varphi)$

↳ arbitrary phase

$\tilde{\omega} = \sqrt{\dfrac{|\Lambda|}{3}}$; $a_0 = \dfrac{1}{2|\Lambda|}\sqrt{12\Lambda + \rho_0^2}$

So $\rho_0^2 > 12|\Lambda|$ for a solution.

$w = -\frac{2}{3}$ is a frustrated domain wall network

For $\frac{a_{max}}{a_{min}} \sim O(1)$ this Universe is stable under linear perturbations.

For large universes $\frac{a_{max}}{a_{min}} \gg 1$ one needs to add another fluid which behaves effectively like curvature, for instance, cosmic strings with $\rho = \frac{c_{str}}{a^2}$; $w = -\frac{1}{3}$ to ensure linear stability $\Rightarrow K_{eff} = k - \frac{c_{str}}{3}$ so the effective curvature is arbitrarily small.

proof of unbounded energy in case of ~~NEC~~ $w<-$

Consider a perfect fluid 1209.2961

$T_{\mu\nu} = (\rho+p)u_\mu u_\nu + p g_{\mu\nu}$ $\Rightarrow$ energy density $\rho = T_{\mu\nu}u^\mu u^\nu$

u_μ - time like, $u_\mu u^\mu = -1$

~~NEC~~ $\Rightarrow \exists$ a null vector $n_\mu n^\mu = 0$ such that

$T_{\mu\nu} n^\mu n^\nu < 0$

Can choose normalization such that $n_\mu u^\mu = -1$

Consider another time-like observer with

$v^\mu = \alpha u^\mu + \beta n^\mu$; $v_\mu v^\mu = -1$ $\Rightarrow$ $\alpha^2 + 2\alpha\beta = 1$

$\Rightarrow \beta = \frac{1-\alpha^2}{2\alpha}$

Consider the energy density $\rho_v \equiv T_{\mu\nu} v^\mu v^\nu$

$\rho_v = \alpha^2 \rho + (1-\alpha^2) T_{\mu\nu} u^\mu n^\nu + \frac{(1-\alpha^2)^2}{4\alpha^2} T_{\mu\nu} n^\mu n^\nu$

To have a minimal energy we need $\frac{\partial \rho_v}{\partial \alpha^2} = 0$

and $\frac{\partial^2 \rho_v}{(\partial \alpha^2)^2} \geq 0$

A direct calculation gives $\frac{\partial^2 \rho_v}{(\partial \alpha^2)^2} = \frac{1}{2\alpha^6} T_{\mu\nu} n^\mu n^\nu < 0$

ρ_v doesn't have a minimum.

Particularly, very close to the ~~NEC~~ ray $\alpha \ll 1$

we have $\rho_v \simeq \frac{1}{4\alpha^2} T_{\mu\nu} n^\mu n^\nu < 0$, arbitrarily negative.

It's not clear whether this is a serious physical problem, but it is very uncomfortable and strange.

Additionally, consider the assumptions carefully.

Why do we need ~~NEC~~? 8

For perfect fluid $\dot{H} = -\frac{1}{2}(\rho + p) \Rightarrow$ for $\dot{H} > 0$

we need $\rho + p < 0$ //

Features

* Violation of ~~NEC~~ NEC implies observer sees energies are unbounded from below. (Proof below).

Other (curable) requirements:

1. Stable Poincaré-inv. vacuum
2. The $2 \to 2$ scattering amplitude about the Poincaré vacuum obeys standard analyticity conditions
3. time-dependent homogeneous, isotropic solution that allows a stable ~~NEC~~
4. Subluminal propagation of perturbations around the ~~NEC~~ background
5. Stable against radiative corrections
6. Smooth interpolation between the ~~NEC~~ region and the Poincaré vacuum.

Important note: in 0811.1614, it was shown that certain inflationary models with extra dim. also work only if ~~NEC~~

9) A concrete example of NEC model (Ghost Condensate)

$\cancel{\mathcal{L} = \sqrt{-g}\, M^4 P(X)}$; $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ hep-th/0606090
hep-th/0702165

X - dim-less, $[\phi] = m^{-1}$

$\mathcal{L} = \sqrt{-g}\left[P(X,\phi) - V(\phi)\right]$; $T_{\mu\nu} = [P-V]g_{\mu\nu} + 2P_{,X}\partial_\mu\phi\partial_\nu\phi$

Example: 1) $P(\phi, X) = \frac{1}{8\phi^2}(X-1)^2$

has a minimum at $P=0$; $X=1 \Rightarrow$ $\phi(t) = \dot{\phi}_0 t$; $\phi(x,t) = \dot{\phi}_0 t + \pi(x,t)$

so instead of a shift symmetry in $\phi \to \phi + \text{const}$ we have a symmetry in $\dot{\phi} \equiv \text{const}$.

In the absence of a potential ~~...~~, we start with Minkowski space.

2) To avoid instabilities add $\Delta\mathcal{L} = M_0^2 \frac{\phi_0^2}{\phi^2}(\Box\phi)^2$, negligible at early times.

3) Let's calculate the potential

$$\dot{H} = -\frac{1}{2}\rho + p = -\frac{1}{2}4P_{,XX}\dot{\phi}_0^4\dot{\pi} \equiv -\frac{1}{2}\frac{M_{gc}^2}{t^2}\dot{\pi}$$

assuming $V \ll P(X)$; $\phi \simeq \dot{\phi}_0 t$

Using $a \sim |t|^p$; $H = \frac{p}{t}$ we get, $p \ll 1$

$$\Rightarrow \dot{\pi} = \frac{2p}{M_{gc}^2}.$$ If small we can solve the 1st Friedmann eq.

$$V(t) = 3H^2 + 2\dot{H} = \frac{3p^2 - 2p}{t^2} \Rightarrow V(\phi) \simeq -\frac{(2p - 3p^2)\dot{\phi}_0^2}{\phi^2} + O(\dot{\pi})$$

So this is the potential at the slow-contracting phase.

(10) The bounce

Consider a linear H across the bounce

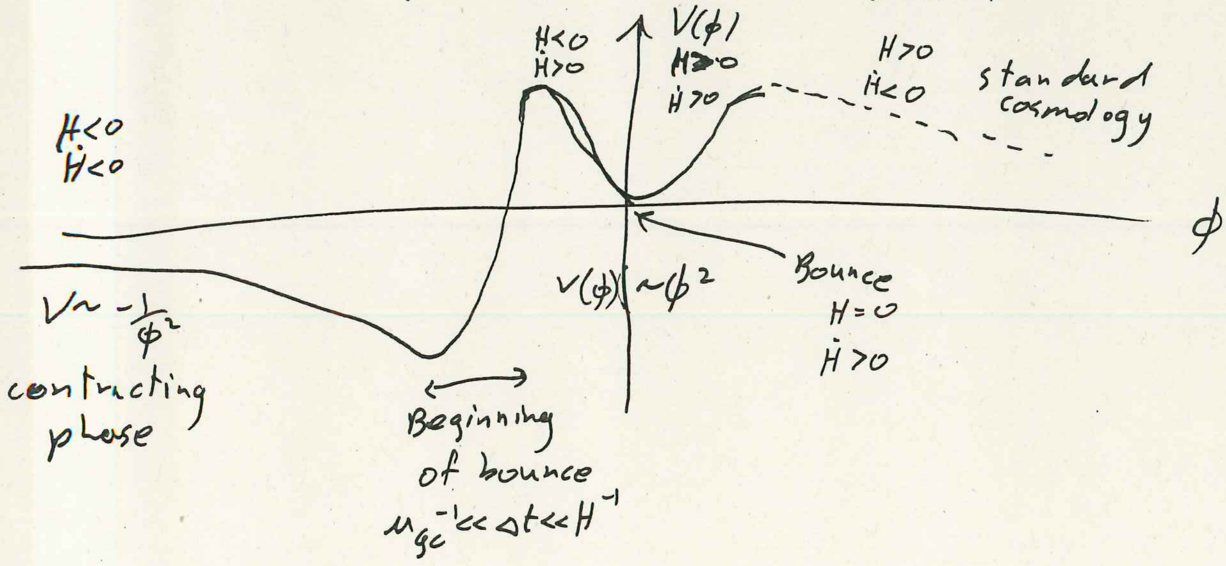
$$H = \frac{t}{T^2} \; ; \; \dot H = \frac{1}{T^2} \Rightarrow \dot\pi \text{ constant, negative}$$

$$\Rightarrow \quad \pi = -\frac{2}{M_{gc}^2 T^2} t \qquad \text{(fixing } t=0 \text{ to be the point where } H=0\text{)}$$

Solving the Friedmann eq. again we have

$$V(\phi) = \frac{2}{T^2} + \frac{3\phi^2}{(M_{gc}T)^4}$$

Schematic picture of the potential



Perturbations:

(1) "matter bounce": pert. are generated during a slow contraction with dust EOS $w \simeq 0$

$$a \simeq (-t)^p \; ; \; u'' + \left(k^2 - \frac{z''}{z}\right)u = 0 \Rightarrow u \sim \sqrt{-k\eta}\, H_\nu(-k\eta)$$

$$\nu = \frac{1-3p}{2(1-p)} \; ; \; n_s - 1 = 3 - 2|\nu| \Rightarrow n_s \simeq 1 \Longleftrightarrow p \simeq \frac{2}{3}$$

One needs to complement this phase with a bouncing part and hope or check that the pert. follow through with marginal modifications

2. Entropic/isocurvature/curvaton pert. that dominate over the adiabatic ones is required in most bouncing models (ekpyrosis $(w \gg 1)$, cyclic, galileons etc.)

Example: $V = -\sum_{I=1}^{2} V_I e^{-c_I \phi_I} \xrightarrow[\text{rotation}]{\text{field}} \underset{\text{adiabatic}}{\phi}, \underset{\text{isocurvature}}{\xi}$

$\Rightarrow n_\phi - 1 \simeq 2 + 4\varepsilon \Rightarrow n_\phi > 3$

$n_\xi - 1 \simeq 4\varepsilon \Rightarrow n_\xi \simeq 1$ and can be made red by further modelling.

Summary of ~~predictions~~ bounces-generic predictions

1. Classical resolution of the Big Bang singularity.
2. No multiverse.
3. Solves flatness and horizon problems.
4. ~~NEC~~ - many instabilities and patholgies to worry about.
5. Adiabatic blue spectrum $n_\phi > 3$, except for a matter bounce with $(w \simeq 0)$, but then more work needed to specify the bounce.
   curvaton spectrum $\Rightarrow$ like inflation, $n_s \lesssim 1$
6. Gravitational waves spectrum. Because $\delta h \sim H$ and $H \ll 1 \Rightarrow r$ is completely negligible on CMB scales. Actually 2nd order GW coming from scalar fluc. $\delta h \sim (\delta\varphi)^2$ expect to dominate giving $r \sim 10^{-6}$.
   Blue tilt $n_T > 0$. contrary to inflation with $n_T < 0$