Inflation with extra dimensions I (1404.2601, 1104.2659)

Short intro (for completeness): We observe the universe to be

flat, isotropic (i.e. had lots of conformal time), and with LSS

(Flatness Problem) (Horizon problem) (Primordial fluctuations)

$\boxed{\text{early de Sitter phase = inflation}}$

How to get de Sitter? Recall $\mathcal{L} \supset \frac{R}{2} - \Lambda$ (cosmological const.)

This has $\boxed{P_\Lambda = -\rho_\Lambda}$. Postulate scalar field ϕ, for which

$$\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi), \quad P_\phi = \frac{1}{2}\dot\phi^2 - V(\phi) \Longrightarrow \text{Seek } \boxed{\dot\phi \ll V(\phi)}$$

To cut long story short...: Friedmann equation gives

$\left(\frac{\dot a}{a}\right)^2 = H^2 = \frac{V}{3} \sim$ const. One obtains slow-roll parameters

$$\boxed{\varepsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = \frac{V''}{V} < 1} \quad \text{and} \quad \boxed{a(t) \propto e^{Ht}}$$

(to have $P = -\rho$) (for ε_V to be small long enough) (de Sitter)

To summarize:

$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \text{slow roll}$$

$$\longrightarrow ds^2 = dt^2 - \underbrace{a^2(t)}_{\propto e^{Ht}} \bar g_{ij}\, dx^i dx^j$$

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{4D picture}}$

What about UV-Sensitivity? Consider e.g. 4D $N=1$ Sugra set-up

$$V_F = e^{\frac{K}{M_p^2}} \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - \frac{3|W|^2}{M_p^2} \right).$$ Take $K = \phi^2$

and expand $V \to \left(1 + \frac{\phi^2}{M_p^2} + \frac{\phi^4}{2M_p^4}\right) V_0(\phi)$.

Define $x = \phi/M_p$ and calculate η for

$\underline{\text{small-field}}$, i.e. $\phi < 1$: $\eta \simeq \eta_0 + 4x\sqrt{2\epsilon_0} + 2 \simeq \underline{\mathcal{O}(1)}$? (fine tune η_0 (?))

$\underline{\text{large field}}$: $\eta \simeq \eta_0 + \frac{8}{x}\sqrt{2\epsilon_0} + \frac{12}{x^2} << 1$, but EFT invalid !

From Strings to Inflation: $\boxed{S_{10}[C] \xrightarrow{} S_4[\Phi(t)]}$ (compactification)

- C means 10D geometry / fluxes / quantum effects
- $\Phi(t)$ 4D effective scalar fields

} 4D inflaton has geometrical meaning in 10D

Which energy scales are involved? CMB probes $\boxed{H_{inf}}$

For $\boxed{M_s = (\alpha')^{-\frac{1}{2}} > H}$, only massless states excited $\Rightarrow$ 10D sugra.

Drawback: "Stringy" effects out of experimental reach (perhaps axion oscillations?) Compactification introduces: $\boxed{M_{KK}}$

Thus have 4D effective theory for $\boxed{E < M_{KK} < M_s}$

Therefore, usually consider $\boxed{\underbrace{M_{susy} < H}_{\text{(protect against rad. corrections)}} < M_{KK} < M_s < M_{pl}}$

$$\mathcal{V} = \int_{X_6} d^6x \sqrt{g}$$
$$= \frac{1}{6} c_{ijk} t^i t^j t^k$$
$$\tau_i = \frac{\partial \mathcal{V}}{\partial t^i}$$

(For different approach, see David's talk next week) [redacted] Inflaton is extra-dim modulus

Begin with LVS: Essence: Kähler moduli $T_i = \underset{\text{4-cycle volume}}{\tau_i} + i \underset{\text{axionic component}}{b_i}$ stabilized by balancing leading α'-correction to Kähler potential K against non-perturbative superpotential W. (Inflaton will be $\mathrm{Re}\{T_i\}$)
Will choose combination of $\mathrm{Re}\{T_i\}$ orthogonal to volume mode $\mathcal{V}$.

Tree-level K reads: (with leading order α'-corrections)

$$K = \underset{\text{no-scale}}{K_0} + \underset{\text{breaks no-scale}}{\delta K_{(\alpha')}} = -2\ln\left(\mathcal{V} + \frac{\xi}{2 g_s^{3/2}}\right) \simeq -2\ln\mathcal{V} - \frac{\overset{\to\, \mathcal{O}(1)}{\xi}}{g_s^{3/2}\,\mathcal{V}}$$

No-scale K_0: $\boxed{K_0^{i\bar{j}} K_{0,i} K_{0,\bar{j}} = 3} \Longrightarrow$ tree-level τ-directions are flat

↳ τ_i inflaton candidates!

$\delta K_{(\alpha')}$ breaks no-scale, but lifts only volume direction!

Note. $K_0 = -2\ln\mathcal{V} \to$ no inflaton-dependent higher-order terms generated by expansion of e^K

↳ construction evades η-problem!

(S, U fixed supersymmetrically by $D_S W = D_U W = 0$)

So far: flat τ_i-directions. What about $V_{inf}(\phi)$?

There are perturbative corrections: $\boxed{\text{String-loop corrections } \delta K_{(g_s)}}$

$$K = K_0 + \delta K_{(\alpha')} + \delta K_{(g_s)} \leftarrow \text{difficult to compute!}$$

For 1-loop processes of closed strings with KK momentum, conjectured to be

$$\boxed{\delta K_{(g_s)} \sim g_s^2 \sum \frac{t_i}{\mathcal{V}} > \delta K_{(\alpha')}} \quad \overset{\to\, \sim \frac{1}{\mathcal{V}}}{} \quad \text{as } t_i \gg 1$$

$\Longrightarrow$ all τ-directions would be lifted, not just $\mathcal{V}$!

Behold: $\boxed{\delta V_{(g_s)} < \delta V_{(\alpha')}}$ cancellation in scalar potential

Extended no-scale: $W = W_0$, $K = K_0 + \delta K$. If δK is homogeneous function of deg. n in t_i, then: $V \simeq \overset{\nearrow \text{no-scale}}{V_0} + V_1 + V_2 + V_3$

$$V_1 = V_{(g_s)} = -\frac{1}{4}\frac{W_0^2}{\mathcal{V}} n(n+2)\,\delta K \;,\quad V_2 = V_{(g_s^2)} = \left(\frac{W_0}{\mathcal{V}}\right)^2 \sum \frac{\partial^2 K_0}{\partial \tau_i^2}$$

$$V_3 = V_{(\alpha')} = \frac{1}{\mathcal{V}^3} \quad \left\} \begin{array}{l} V_1 \to 0 \text{ as } \mathcal{V} \sim t^3 \\ V_2 \to (\mathcal{V}^3 t)^{-1} << V_3 \to \mathcal{V}^{-3} \end{array} \right.$$

Example: $\mathcal{V} = \tau^{3/2} = t^3$, $t = \sqrt{\tau}$, $W = W_0$

$\hookrightarrow$ $K = -2\ln \mathcal{V} - \frac{\hat{\xi}}{\mathcal{V}} + \frac{\sqrt{\tau}}{\mathcal{V}}$

$\hookrightarrow$ $V = \frac{W_0}{\mathcal{V}^3}\Big(\underset{\text{no-scale}}{0} + \underset{\alpha'}{\hat{\xi}} + \underset{\text{extended no-scale}}{0 \cdot \sqrt{\tau}} + \frac{1}{\sqrt{\tau}} + \frac{1}{\tau^{3/2}}\Big)$

leading / subleading g_s

(Interesting side note: Above may be obtained from CW effective potential $\delta V_{CW} \simeq \Lambda^2 \,\mathrm{STr}(\mathcal{M}^2) + \mathrm{STr}\left[\mathcal{M}^4 \ln\left(\frac{\mathcal{M}^2}{\Lambda^2}\right)\right]$)

So far: can generate potential via string-loops.

Now: Perturbative effects Blow up Inflation (Conlon, Quevedo '05)

Start with $\boxed{\mathcal{V} = \alpha\left(\tau_b^{3/2} - \lambda_1 \tau_\phi^{3/2} - \lambda_s \tau_s^{3/2}\right)}$

(Assume hierarchy $\underset{\text{overall } \mathcal{V}}{\tau_b} \gg \underbrace{\tau_\phi \gg \tau_s}_{\text{blow-ups}}$) and $K = -2\ln\left(\mathcal{V} + \frac{\xi}{g_s^{3/2}\,\mathcal{V}}\right)$

Take $\boxed{W = W_0 + A_\phi e^{-a_\phi T_\phi} + A_s e^{-a_s T_s}}$ $\neq$ const.

(τ_b, τ_s stabilised at $\mathcal{V} \sim \alpha \tau_b^{3/2} \sim e^{a_s \tau_s}$, $\tau_s \gg 1$)

Integrating out τ_b and τ_s yields

$$V = W_0^2 \Big(\overset{\mathcal{O}(1)}{\alpha} \frac{\sqrt{\tau_\phi}\, e^{-2a_\phi \tau_\phi}}{\mathcal{V}} - \beta \frac{\tau_\phi e^{-a_\phi \tau_\phi}}{\mathcal{V}^2} + c \frac{\hat{\xi}}{\mathcal{V}^3} \Big) \qquad (*)$$

$\xrightarrow[\text{large}]{\tau_\phi}$ $V(\phi) \simeq V_0 \big(1 - \mathcal{V}^{\frac{5}{3}} \phi^{\frac{4}{3}} e^{-\mathcal{V}^{\frac{2}{3}} \phi^{\frac{4}{3}}}\big)$

$\xrightarrow{!}$ $\boxed{V_0 (1 - \kappa_1 e^{-\kappa_2 \hat{\phi}})}$ during inflation

($\hat{\phi}$ is expansion around $\langle\phi\rangle$ in minimum of (*))

But: <u>String loops</u> $\delta V_{(g_s)} \sim \frac{1}{\sqrt{\tau_\phi}\, \mathcal{V}^3} \sim \frac{1}{\phi^{\frac{2}{3}} \mathcal{V}^{\frac{10}{3}}}$

$\hookrightarrow$ $\delta\eta \simeq \frac{\mathcal{V}}{\tau_\phi^2} \longrightarrow a_\phi^2 \frac{\mathcal{V}}{\ln(\mathcal{V})^2} \gg 1$

<u>η-problem</u> despite extended no-scale !

Next: <u>Fibre Inflation</u>

choose $\boxed{\mathcal{V} = \alpha(\sqrt{\tau_1}\, \tau_2 - \gamma_3 \tau_3^{\frac{3}{2}})}$

Again, α' corrections $\boxed{\begin{aligned} K &= -2\ln\big(\mathcal{V} + \frac{c}{g_s^{3/2}}\big) \\ W &= W_0 + A_s e^{-a_s \tau_s} \end{aligned}}$ combine with np W:

Obtain V_F which depends on τ_1 only through $\mathcal{V}$

$\hookrightarrow$ flat direction in (τ_1, τ_2)-plane $\Longrightarrow$ Inflation ?

Now, add conjectured string loop corrections:

$$\delta V_{(g_s)} = \left(\frac{W_0}{\mathcal{V}}\right)^2 \left(\alpha \frac{g_s^2}{\tau_1^2} - \beta \frac{1}{\sqrt{\tau_1}\,\mathcal{V}} + c\, \frac{g_s \tau_1}{\mathcal{V}^2} \right)$$

If $\mathcal{V}$ not so large, it is not clear whether scenario entirely robust against higher order corrections

For inflation: τ_1 displaced from minimum, i.e. K3 fiber initially large compared to base and then shrinks!

Upon integrating out τ_1, keeping $\mathcal{V}$ fixed:

$$\boxed{V(\phi) = V_0\left(1 - \frac{4}{3} e^{-\phi/\sqrt{3}} + \frac{1}{3} e^{-\frac{4\phi}{\sqrt{3}}} + \delta \cdot e^{\frac{2\phi}{\sqrt{3}}}\right)}$$

$A_s \sim 10^{-9}$
$\hookrightarrow \mathcal{V} \sim 1700$

Small Latitude: Fibre inflation looks like Starobinsky?

But: During inflation $\Longleftrightarrow$ $f(R) \sim R^2 + R^{2-\frac{1}{\sqrt{2}}}$

Before inflation: $f(R) \sim R^{\sqrt{2}}$

Reason: $\boxed{\kappa_F = \frac{2}{\sqrt{3}} \neq \sqrt{\frac{2}{3}} = \kappa_S}$

Try: $\mathcal{V} \sim \sqrt{\tau_2}\,\tau_1 - \gamma \tau_3^{3/2} \Longrightarrow \kappa = \frac{1}{\sqrt{3}}$ (Inflation - base manifold)

$\mathcal{V} \sim \sqrt{\tau_1 \tau_2 \tau_3} - \gamma \tau_3^{3/2} \Longrightarrow \kappa = 1$ (Torus) arXiv (1411.6010)

Another example Racetrack/Accidental Inflation

Basic idea: $\boxed{K = -3 \ln(T + \bar{T}), \quad W = W_0 + A e^{-aT} + B e^{-bT}}$

Can be shown that for certain parameter values $\mathrm{Im}(T)$ (axion) is stabilised, but inflection point appears in $\mathrm{Re}(T)$ direction. Volume modulus then serves as inflaton.

<u>Brane inflation</u> inflaton is position modulus parametrising <u>separation between</u> two branes.

With $D3/\overline{D3}$ pair: $K = -3 \ln[(T+\bar{T}) - \phi\bar{\phi}]$

$T+\bar{T} \longrightarrow \langle (T+\bar{T}) \rangle$ (KKLT)

$\hookrightarrow \boxed{K = K_0 + 3 \frac{\bar{\phi}\phi}{\langle T+\bar{T} \rangle}}$

$\delta\eta = \mathcal{O}(1)$?

$\hookrightarrow \boxed{V_F = e^{K_0} u(\phi_c)\left(1 + \frac{\phi_c\bar{\phi}_c}{u_1}\right)}$