The Wrong-Higgs Couplings of the MSSM

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This talk is based on work that appears in:

H.E. Haber and J.D. Mason, "Hard supersymmetry-breaking 'wrong-Higgs' couplings of the MSSM," *Phys. Rev.* **D77**, 115011 (2008) [arXiv:0711.2890 [hep-ph]].

The wrong-Higgs Yukawa couplings are reviewed in:

M. Carena and H.E. Haber, "Higgs Boson Theory and Phenomenology," Prog. Part. Nucl. Phys. **50** (2003) 63–152.

The decoupling properties of the one-loop corrected Higgs couplings to $b\bar{b}$ in the MSSM are examined:

at $\mathcal{O}(\alpha_s)$ in:

H.E. Haber, M.J. Herrero, H.E. Logan, S. Peñaranda, S. Rigonlin and D. Temes, *Phys. Rev.* **D63**, 055004 (2001) [arXiv: hep-ph/0007006];

at $\mathcal{O}(h_t^2)$ in:

H.E. Haber, H.E. Logan, S. Peñaranda and D. Temes, SCIPP-08/01 (2008), to appear shortly on the arXiv.

Note: Recent work by I. Antoniadis, E. Dudas, D. Ghilencea and P. Tziveloglou (unpublished) obtains wrong-Higgs Yukawa interactions from a general analysis of higher dimensional operators in the MSSM.

Outline

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Introduction

Realistic supersymmetric models of nature require that supersymmetry is broken. The most common framework for SUSY breaking consists of three sectors:

- Visible sector: the fields of the (extended) Standard Model (SM) and their superpartners
- Hidden sector: SU(3)$\times$SU(2)$\times$U(1) neutral fields, where fundamental SUSY-breaking resides
- Messenger sector: messenger fields that communicate the SUSY-breaking of the hidden sector to the visible sector

Let X be a hidden sector superfield. Integrating out the messenger fields (at the mass scale $M \gg m_Z$) results in operators that couple X to SM superfields. The most important operators are:

$$-\mathcal{L} \ni \left(\frac{1}{M}\left[XW^aW^a\right]_F + \frac{1}{M}\left[X\Phi^3\right]_F + \frac{\mu}{M}\left[X\Phi^2\right]_F + \text{h.c}\right) + \frac{1}{M^2}\left[X^\dagger X\Phi^\dagger\Phi\right]_D\,,$$

where Φ is some SM chiral superfield, and W_α is the gauge spinor superfield. Parameterizing the SUSY-breaking by taking $\langle X\rangle = \theta\theta F$, then yields:

$$-\mathcal{L} \ni \left(\frac{F}{M}\lambda^a\lambda^a + \frac{F}{M}\phi^3 + \frac{\mu F}{M}\phi^2 + \text{h.c.}\right) + \frac{|F|^2}{M^2}\phi^\dagger\phi\,,$$

where ϕ is the lowest scalar component of Φ. This corresponds to the standard list of soft-SUSY-breaking operators. If we choose:

$$\frac{F}{M} \equiv M_{\text{SUSY}} \sim 100\text{ GeV—1 TeV}\,,$$

and $\mu \sim M_{\text{SUSY}}$, then the MSSM sparticle masses are of the expected size.

Hard SUSY-breaking operators

Many more effective operators that couple X to SM superfields can be written that exhibit additional M^{-1} suppressions. These have been classified by S.P. Martin [PRD61 (2000) 035004]. Two of interest for this work are:

$$
\begin{aligned}
-\mathcal{L} &\ni \frac{h_{ijk}}{M^4}\left[XX^\dagger\Phi_i^\dagger D^\alpha\Phi_j D_\alpha\Phi_k\right]_D + \frac{g_{ij}^a}{M^4}\left[XX^\dagger\Phi_i D^\alpha\Phi_j W_\alpha^a\right]_D + \text{h.c.} \\
&\ni \frac{|F|^2}{M^4}\left[h_{ijk}\phi_i^\dagger\psi_j\psi_k + g_{ij}^a\phi_i\psi_j\lambda^a + \text{h.c.}\right] ,
\end{aligned}
$$

after putting $\langle X\rangle = \theta\theta F$. Thus, we expect the coefficients of these SUSY-breaking operators to be suppressed by $M_{\rm SUSY}^2/M^2 \ll 1$. If one of the Φ's above is the Higgs superfields, then we call the corresponding SUSY-breaking terms above the "wrong-Higgs" couplings to contrast with the supersymmetric Higgs interactions derived from:

$$
\mathcal{L}_{\rm SUSY} = -\frac{d^2W}{d\phi_i d\phi_j}\psi_i\psi_j + i\sqrt{2}g_a\phi_i^* T_{ij}^a\psi_j\lambda^a + \text{h.c.}
$$

A case study: the wrong-Higgs Yukawa couplings

In the MSSM, the tree-level Higgs–quark Yukawa Lagrangian is supersymmetry-conserving and is given by:

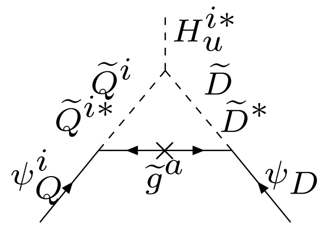
$$\mathcal{L}_{\rm yuk}^{\rm tree} = -\epsilon_{ij} h_b H_d^i \psi_Q^j \psi_D + \epsilon_{ij} h_t H_u^i \psi_Q^j \psi_U + {\rm h.c.}$$

Two other possible dimension-four gauge-invariant non-holomorphic Higgs-quark interactions terms, the so-called wrong-Higgs interactions,

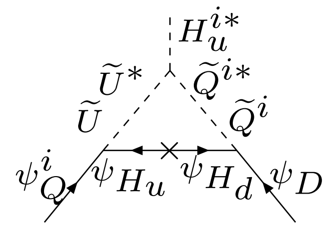
$$H_u^{k*} \psi_d \psi_Q^k \qquad {\rm and} \qquad H_d^{k*} \psi_u \psi_Q^k \,,$$

are not supersymmetric (since the dimension-four supersymmetric Yukawa interactions must be holomorphic), and hence are absent from the tree-level Yukawa Lagrangian.

Nevertheless, the wrong-Higgs interactions can be generated in the effective low-energy theory below the scale of SUSY-breaking. In particular, one-loop radiative corrections, in which supersymmetric particles (squarks, higgsinos and gauginos) propagate inside the loop can generate the wrong-Higgs interactions.



(a)



(b)

One-loop diagrams contributing to the wrong-Higgs Yukawa effective operators. In (a), the cross ($\times$) corresponds to a factor of the gluino mass M_3. In (b), the cross corresponds to a factor of the higgsino Majorana mass parameter μ. Field labels correspond to annihilation of the corresponding particle at each vertex of the triangle.

If the superpartners are heavy, then one can derive an effective field theory description of the Higgs-quark Yukawa couplings below the scale of SUSY-breaking ($M_{\rm SUSY}$), where one has integrated out the heavy SUSY particles propagating in the loops.

The resulting effective Lagrangian is:

$$\mathcal{L}_{\rm yuk}^{\rm eff} = -\epsilon_{ij}(h_b + \delta h_b)\psi_b H_d^i \psi_Q^j + \Delta h_b \psi_b H_u^{k*} \psi_Q^k$$
$$+\epsilon_{ij}(h_t + \delta h_t)\psi_t H_u^i \psi_Q^j + \Delta h_t \psi_t H_d^{k*} \psi_Q^k + \text{h.c.}$$

In the limit of $M_{\rm SUSY} \gg m_Z$,

$$\Delta h_b = h_b \left[\frac{2\alpha_s}{3\pi} \mu M_3 \mathcal{I}(M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, M_g^2) + \frac{h_t^2}{16\pi^2} \mu A_t \mathcal{I}(M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, \mu^2) \right] ,$$

where, M_3 is the Majorana gluino mass, μ is the supersymmetric Higgs-mass parameter, and $\widetilde{b}_{1,2}$ and $\widetilde{t}_{1,2}$ are the mass-eigenstate bottom squarks and top squarks, respectively. The loop integral $\mathcal{I}(a,b,c) \sim 1/\max(a^2, b^2, c^2)$ in the limit where at least one of the arguments of $\mathcal{I}(a,b,c)$ is large.*

Thus, in the limit where $M_3 \sim \mu \sim A_t \sim M_{\tilde{b}} \sim M_{\tilde{t}} \sim M_{\rm SUSY} \gg m_Z$, the one-loop contributions to Δh_b do *not* decouple.

*$\mathcal{I}(a,b,c) = \left[a^2 b^2 \ln(a^2/b^2) + b^2 c^2 \ln(b^2/c^2) + c^2 a^2 \ln(c^2/a^2) \right] / [(a^2 - b^2)(b^2 - c^2)(a^2 - c^2)]$.

Phenomenological consequences of the wrong-Higgs Yukawas

The effect of the wrong-Higgs couplings is a $\tan\beta$-enhanced modification of a physical observable. To see this, rewrite the Higgs fields in terms of the physical mass-eigenstates (and the Goldstone bosons):

$$H_d^1 = \frac{1}{\sqrt{2}}(v\cos\beta + H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta - iG^0\cos\beta)\,,$$

$$H_u^2 = \frac{1}{\sqrt{2}}(v\sin\beta + H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta + iG^0\sin\beta)\,,$$

$$H_d^2 = H^-\sin\beta - G^-\cos\beta\,,$$

$$H_u^1 = H^+\cos\beta + G^+\sin\beta\,,$$

with $v^2 \equiv v_u^2 + v_d^2 = (246~\mathrm{GeV})^2$ and $\tan\beta \equiv v_u/v_d$. The b-quark mass is:

$$m_b = \frac{h_b v}{\sqrt{2}}\cos\beta\left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b)\,,$$

which defines the quantity Δ_b.

In the limit of large $\tan\beta$ the term proportional to δh_b can be neglected, in which case,

$$\Delta_b \simeq (\Delta h_b/h_b)\tan\beta\,.$$

Thus, Δ_b is $\tan\beta$–enhanced if $\tan\beta \gg 1$. As previously noted, Δ_b survives in the limit of large $M_{\rm SUSY}$; this effect does not decouple. It can generate measurable shifts in the decay rate for $h^0 \to b\bar{b}$:

$$g_{h^\circ b\bar{b}} = -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1 + \frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)(1+\cot\alpha\cot\beta)\right]\,.$$

At large $\tan\beta \sim 20$—50, Δ_b can be as large as 0.5 in magnitude and of either sign, leading to a significant enhancement or suppression of the Higgs decay rate to $b\bar{b}$.

Thus, the effect of the wrong-Higgs SUSY-breaking operators can be non-negligible. In the low-energy effective theory, we essentially have $M = M_{\rm SUSY}$, and there is no suppression.

Non-decoupling effects in $h^0 \to b\bar{b}$: a closer look

Working consistently to one-loop order, under the assumption that μ and all SUSY masses are significantly larger than m_Z, a complete diagrammatic computation yields:

$$g_{h^\circ b\bar{b}} = [g_{h^\circ b\bar{b}}]_{\rm tree} \left\{ 1 - \frac{\Delta h_b}{h_b}(\tan\beta + \cot\alpha) \right\} + \mathcal{O}\left(\frac{m_Z^2}{M_{\rm SUSY}^2}\right) ,$$

where

$$[g_{h^\circ b\bar{b}}]_{\rm tree} = [g_{h^\circ b\bar{b}}]_{\rm SM} \left\{ \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha) \right\} .$$

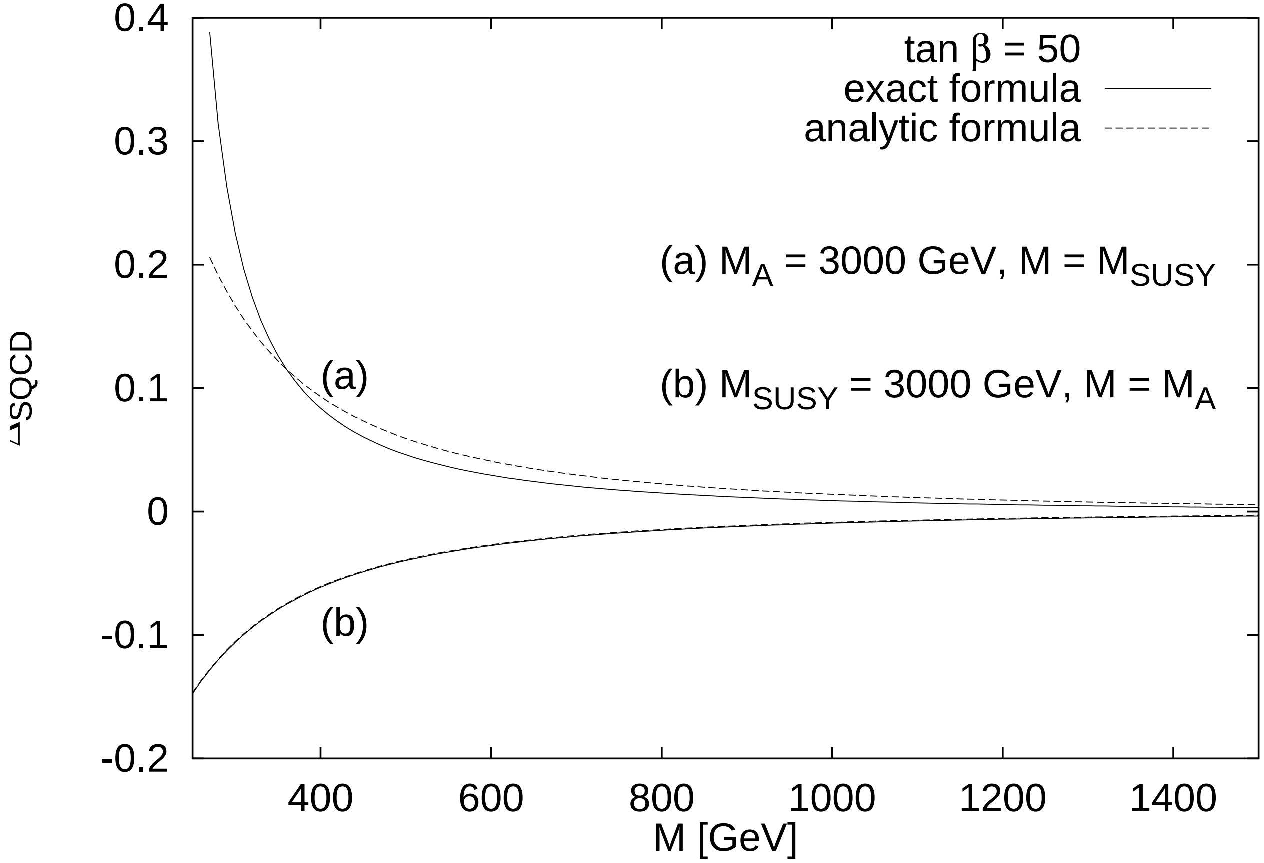
If $m_A \gg m_Z$, then the low-energy theory is an effective one-doublet theory that must coincide with the SM. In this limit,

$$\cos(\beta-\alpha) = \frac{m_Z^2 \sin 4\beta}{2m_A^2} + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right) ,$$

$$\tan\beta + \cot\alpha = -\frac{2m_Z^2}{m_A^2}\tan\beta\cos 2\beta + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right) .$$

In fact, decoupling is delayed—one must have $m_A^2 \gg m_Z^2 \tan\beta$ (even if $M_{\rm SUSY} \to \infty$).

For $m_A \sim \mathcal{O}(m_Z)$, the effective low-energy theory is the most general two-Higgs doublet model (which includes the wrong-Higgs Yukawa couplings). Thus, non-trivial radiative corrections can persist even if $M_{\rm SUSY} \gg m_Z$.



The super-QCD correction, $\Delta_{\rm SQCD}$ [i.e., the $\mathcal{O}(\alpha_s)$ correction to the $h^0 b\bar{b}$ coupling], as a function of particle mass for $\tan\beta = 50$ and $M_{\rm SUSY} = M_{\tilde{g}} = \mu = A_b$. The curves (a) are plotted *vs.* $M_{\rm SUSY}$, with $m_A = 3000$ GeV; whereas the curves (b) are plotted *vs.* m_A, with $M_{SUSY} = 3000$ GeV. Solid lines are based on the exact one-loop formula and dashed lines are based on an analytic approximation given in Haber, Herrero et al.

The decoupling behavior is of the form: $g_{h^\circ b\bar{b}} = [g_{h^\circ b\bar{b}}]_{\rm tree}\,\{1+\Delta_{\rm SM}+\Delta_{\rm SUSY}\}$, where

$$\Delta_{\rm SUSY} \sim C_1 \frac{m_Z^2}{m_A^2}\tan\beta + C_2 \frac{m_Z^2}{M_{\rm SUSY}^2}\,.$$

Integrating out the heavy SUSY spectrum still leaves a contribution proportional to C_1 that is unsuppressed when $m_A \sim m_Z$. At large $\tan\beta$, $C_1\tan\beta \sim \mathcal{O}(1)$ if the heavy SUSY spectrum is roughly degenerate, and corresponds to the effects of the wrong-Higgs Yukawa couplings. If there are non-degeneracies within the heavy SUSY spectrum, then contributions to C_1 may be additionally suppressed by small ratios of SUSY masses.

Example: super-massive squarks

If squarks are significantly heavier than all other SUSY mass parameters, then

$$\Delta_{\rm SUSY} \simeq -(\tan\beta+\cot\alpha)\left\{\frac{2\alpha_s}{3\pi}\frac{\mu M_{\tilde g}}{M_{\tilde b_1}^2-M_{\tilde b_2}^2}\ln\left(\frac{M_{\tilde b_1}^2}{M_{\tilde b_2}^2}\right)+\frac{h_t^2}{16\pi^2}\frac{\mu A_t}{M_{\tilde t_1}^2-M_{\tilde t_2}^2}\ln\left(\frac{M_{\tilde t_1}^2}{M_{\tilde t_2}^2}\right)\right\}$$

$$\simeq -\left(\frac{\tan\beta+\cot\alpha}{M_{\tilde q}^2}\right)\left[\frac{2\alpha_s}{3\pi}\mu M_{\tilde g}+\frac{h_t^2}{16\pi^2}\mu A_t\right]\,,$$

if all squarks are roughly degenerate with mass $M_{\tilde q}$.

Wrong Higgs gaugino–higgsino operators

In a supersymmetric field theory, the tree-level supersymmetric gaugino–fermion–sfermion interactions originate from the Kähler term:

$$\mathcal{L}_K = \int d^4\theta \, \Phi_i^\dagger (e^{2gV})_{ij} \Phi_j \ni i\sqrt{2} g_a (\phi_i^* T^a_{ij} \psi_j \lambda^a - \bar{\lambda}^a \bar{\psi}_i T^a_{ij} \phi_j) \,.$$

We catalog all possible dimension-four gauge-invariant operators in the gaugino–higgsino–Higgs boson sector that violate supersymmetry. One class of operators includes:

$$\frac{ig_u}{\sqrt{2}} \lambda^a \tau^a_{ij} \psi^j_{H_u} H_u^{*i} + \frac{ig_d}{\sqrt{2}} \lambda^a \tau^a_{ij} \psi^j_{H_d} H_d^{*i} + \frac{ig'_u}{\sqrt{2}} \lambda' \psi^i_{H_u} H_u^{*i} - \frac{ig'_d}{\sqrt{2}} \lambda' \psi^i_{H_d} H_d^{*i} + \mathrm{h.c.} \,,$$

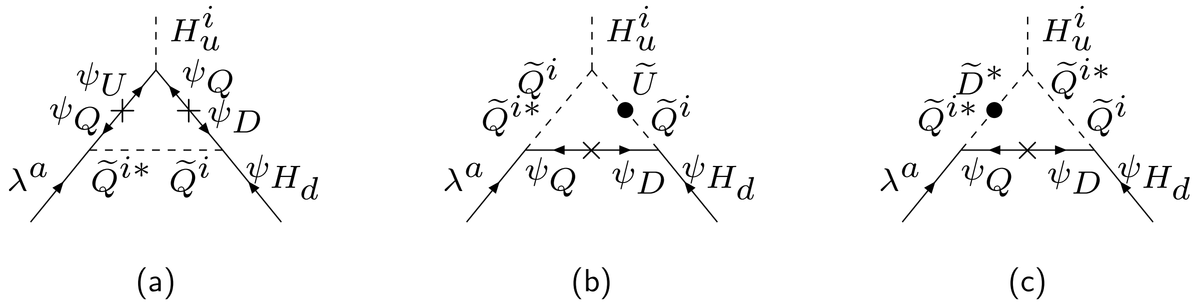
where the coupling g_u, g_d, g'_u and g'_d deviate from their supersymmetric values given by the SU(2) and U$(1)_{\mathrm{Y}}$ gauge couplings, g and g', respectively. Such effects are generated by one-loop corrections and have been studied in detail by Katz et al. and by Kiyoura et al.

Here, we focus on a second class of supersymmetric violating operators:

$$ik_1\lambda^a\tau^a_{ij}\psi^j_{H_u}\epsilon_{ki}H^k_d\,, \qquad ik_2\lambda'\psi^k_{H_u}\epsilon_{ki}H^i_d\,,$$
$$ik_3\lambda^a\tau^a_{ij}\psi^j_{H_d}\epsilon_{ki}H^k_u\,, \qquad ik_4\lambda'\psi^i_{H_d}\epsilon_{ki}H^k_u\,.$$

Integrating out a subset of heavy MSSM fields

Suppose we try to generate these operators in the limit where the squarks are much heavier than the gauginos. Graphs (a), (b) and graph (c) are suppressed by $\mathcal{O}(m_t m_b/M^2_{\rm SUSY})$ and $\mathcal{O}(m_b^2/M^2_{\rm SUSY})$, respectively, and hence decouple when $M_{\rm SUSY} \gg m_Z$.



One-loop diagrams contributing to the wrong-Higgs gaugino operators. The cross ($\times$) indicates the two-component fermion propagator that is proportional to the corresponding Dirac mass. In (b) and (c) the solid dot indicates an insertion of the Higgs vacuum expectation value (vev). Field labels correspond to annihilation at each vertex of the triangle. Replacing the Higgs vev by the appropriate Higgs field, these diagrams actually correspond to dimension-six operators with the expected decoupling behavior.

Gauge-mediated supersymmetry-breaking (GMSB)

Consider a typical GMSB model. SUSY is broken in the hidden sector that is parameterized by the vacuum expectation value of some hidden sector spurion superfield $\widehat{Z}$

$$\widehat{Z} = \langle Z \rangle + \theta\theta F_Z \,.$$

We also introduce messenger superfields $\widehat{\Phi}$ and $\widehat{\overline{\Phi}}$, which transform as complete SU(5) multiplets (e.g. $\mathbf{5} \oplus \mathbf{5^*}$), and embed the SM gauge group inside SU(5) in the usual way. The superpotential connecting the messengers to the hidden sector is:

$$W = \gamma \widehat{Z} \widehat{\Phi}\, \widehat{\overline{\Phi}} \,,$$

where the messenger-Yukawa coupling $\gamma \sim \mathcal{O}(1)$. Inserting the spurion form for $\widehat{Z}$, SUSY-breaking mass-splittings are generated among messenger fermions and their scalar superpartners. Messengers can appear in loop corrections to propagators and vertices involving external MSSM fields. Thus, SUSY-breaking is transmitted to the visible sector.

General expectations for GMSB models

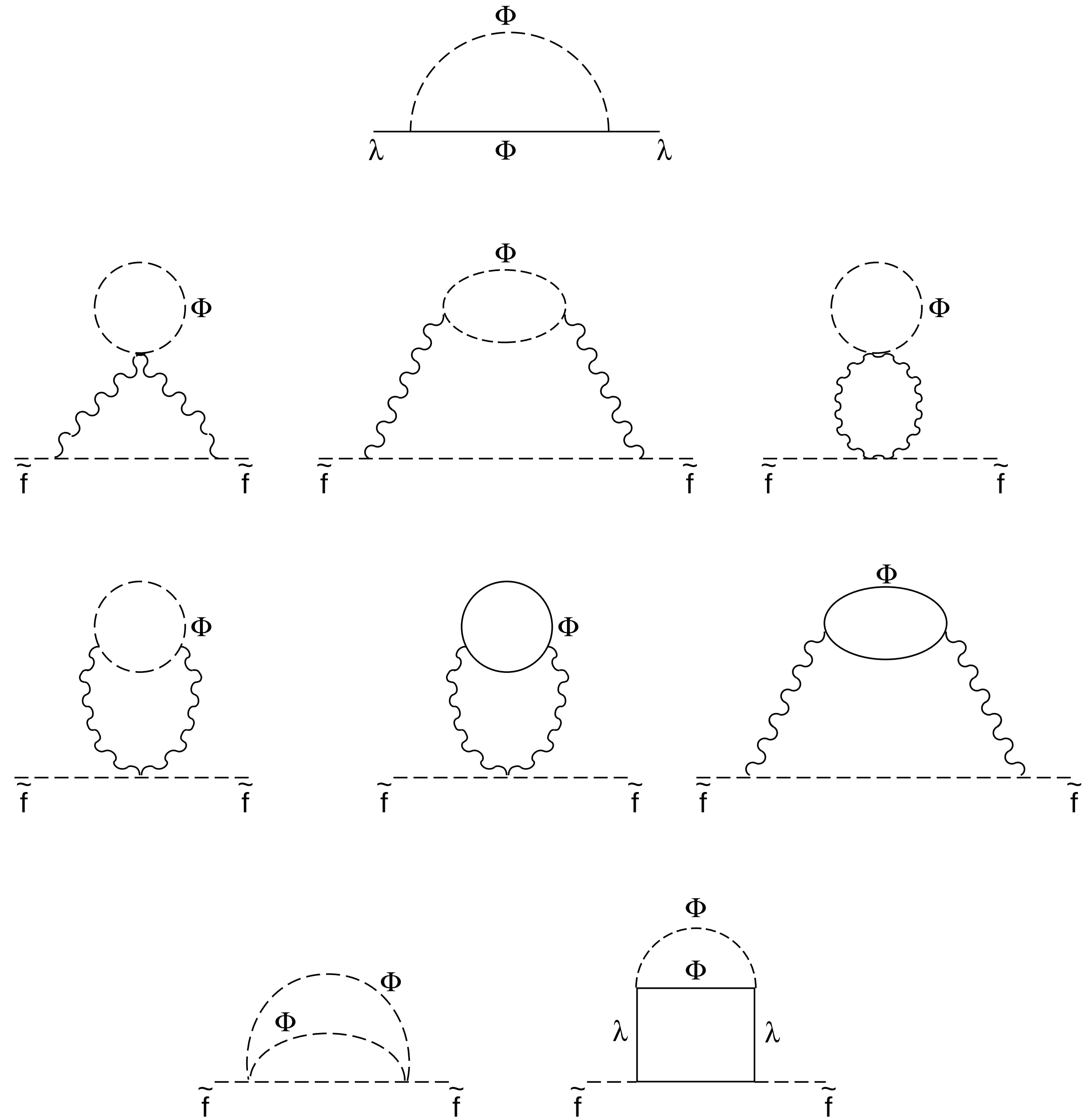
- Gaugino masses are generated at one loop. Squark and slepton (sfermion) masses are generated at two loops:

$$m_\lambda \sim \frac{g^2}{16\pi^2}\frac{F_Z}{\langle Z\rangle}\,, \qquad\qquad m^2_{\tilde f} \sim \left(\frac{g^2}{16\pi^2}\right)^2 \frac{F_Z F_Z^\dagger}{\langle Z\rangle^2}\,.$$

 where g is the appropriate gauge coupling.

- In order that the effective SUSY-breaking scale of the MSSM, $M_{\rm SUSY} \lesssim \mathcal{O}(1~{\rm TeV})$, one usually takes $F_Z/\langle Z\rangle \sim 100$ TeV.

- $\langle Z\rangle$ sets the messenger mass scale. If $F_Z \gtrsim \langle Z\rangle^2$, the squared-mass splitting between scalar messenger pairs is so large that the smallest scalar squared-mass is driven negative. Thus, two limits are of interest:
  - low-scale messengers: $F_Z/\langle Z\rangle^2 \sim 1$,
  - high-scale messengers: $F_Z/\langle Z\rangle^2 \ll 1$.



Feynman diagrams contributing to SUSY-breaking gaugino (λ) and sfermion ($\widetilde{f}$) masses. The scalar and fermionic components of the messenger fields Φ are denoted by dashed and solid lines, respectively; ordinary gauge bosons are denoted by wavy lines. Taken from G.F. Giudice and R. Rattazzi, *Phys. Rept.* **322**, 419 (1999).

Issues for GMSB model building

- The μ and B_μ parameters do not naturally emerge at the correct mass scale. Another mechanism must be invoked.

- It is desirable to have a more fundamental description of SUSY-breaking in the hidden sector. The possible use of metastable vacua [inspired by the paper of Intriligator, Seiberg and Shih (2007)] has spawned a revival of GMSB model-building.

- There is much freedom in the construction of messenger sectors. No single compelling model exists.

- Typically, the messengers only couple to the MSSM via gauge interactions. However, the gauge quantum numbers permit the coupling of (some of the) messengers to the Higgs superfields. Viable models are possible with appropriately chosen messenger and hidden sectors.

Integrating out the messengers in a model of gauge-mediated SUSY-breaking

In models of gauge-mediated SUSY-breaking (GMSB) in which the Higgs fields couple directly to messenger fields, it is possible to generate wrong-Higgs operators by integrating out the messenger fields. Consider the following sector of uncolored messenger fields:

Superfield	SU(3)	SU(2)	$\mathrm{U}(1)_{\mathrm{Y}}$
$\widehat{H}_d$	1	2	-1
$\widehat{H}_u$	1	2	1
$\widehat{M}_1$	1	2	1
$\widehat{\overline{M}}_1$	1	2	-1
$\widehat{M}_2$	1	1	-2
$\widehat{\overline{M}}_2$	1	1	2

Gauge quantum numbers of the Higgs and messenger superfields.

The hidden sector dynamics is governed by a spurion chiral superfield $\widehat{Z} = \langle Z\rangle + \theta\theta F_Z$. In our model, $\widehat{Z}$ couples to the four messenger superfields:

$$W = \gamma_1 \epsilon_{ij} \widehat{Z} \widehat{M}_1^i \widehat{\overline{M}}_1^j + \gamma_2 \widehat{Z} \widehat{M}_2 \widehat{\overline{M}}_2 + \alpha \epsilon_{ij} \widehat{H}_u^i \widehat{M}_1^j \widehat{M}_2 + \beta \epsilon_{ij} \widehat{H}_d^i \widehat{\overline{M}}_1^j \widehat{\overline{M}}_2 \,.$$

To avoid tachyonic messengers, we must take $F_Z \lesssim \langle Z\rangle^2$.

In this model, soft Higgs squared-mass terms are generated at one-loop. To avoid significant fine-tuning, we must take $F_Z/\langle Z\rangle \sim 20$ TeV. However, the corresponding two-loop contributions to squark and slepton squared-masses are too small. One must therefore add another source of SUSY-breaking, e.g. additional messenger fields that couple to a different spurion $\widehat{X}$ such that $F_X/\langle X\rangle \sim 100$ TeV, in order to raise SUSY masses above their experimental bounds.

Inserting the form of the spurion field into W yields the the messenger mass-eigenstates and their couplings to the Higgs and higgsinos. The SUSY-couplings of the gauginos to the scalar/fermionic messenger pairs are also easily obtained.

Messenger mass-eigenstates and masses

scalar mass-eigenstates:

$$M^i_{\pm 1R} = \mathrm{Re}(M^i_1 \pm \epsilon_{ij}\overline{M}^j_1)$$

$$M^i_{\pm 1I} = \mathrm{Im}(M^i_1 \pm \epsilon_{ij}\overline{M}^j_1)$$

$$M^i_{\pm 2R} = \mathrm{Re}(M_2 \pm \overline{M}_2)$$

$$M^i_{\pm 2I} = \mathrm{Im}(M_2 \pm \overline{M}_2)$$

scalar squared-masses

$$m^{\pm}_{1R} \equiv \gamma_1^2\langle Z\rangle \pm \gamma_1 F_Z$$

$$m^{\pm}_{1I} \equiv \gamma_1^2\langle Z\rangle \mp \gamma_1 F_Z$$

$$m^{\pm}_{2R} \equiv \gamma_2^2\langle Z\rangle \pm \gamma_2 F_Z$$

$$m^{\pm}_{2I} \equiv \gamma_2^2\langle Z\rangle \mp \gamma_2 F_Z$$

fermion Dirac mass-eigenstates:

$$\Psi_1 = \begin{pmatrix} \psi_{M_1} \\ \overline{\psi}_{\bar{M}_1} \end{pmatrix}$$

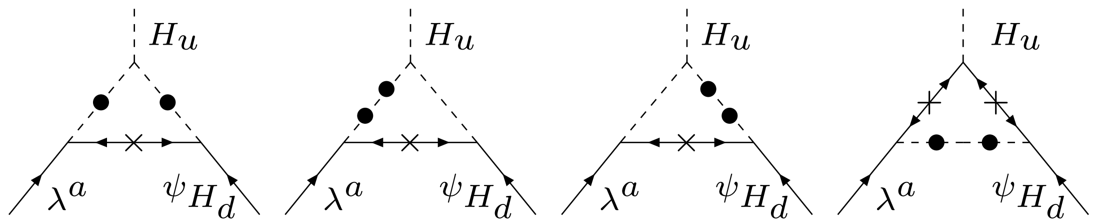
$$\Psi_2 = \begin{pmatrix} \psi_{M_2} \\ \overline{\psi}_{\bar{M}_2} \end{pmatrix}$$

fermion masses

$$m_1 \equiv \gamma_1\langle Z\rangle$$

$$m_2 \equiv \gamma_2\langle Z\rangle$$

If $F_Z \ll \langle Z \rangle^2$, then we can employ the mass insertion approximation to compute the coefficients of the wrong-Higgs gaugino/higgsino operators, k_i.



One-loop diagrams with internal lines consisting of scalar and fermionic messenger fields. The cross ($\times$) indicates the two-component fermion propagator that is proportional to the corresponding fermion mass. The solid dot indicates an F_Z mass-insertion on the scalar messenger line.

For example,

$$k_3 \sim \frac{g}{16\pi^2}\left(\frac{F_Z}{\langle Z \rangle^2}\right)^2 ,$$

whose decoupling properties follow the expected $M^2_{\rm SUSY}/M^2$ behavior identified earlier.

If $F_Z \sim \langle Z \rangle^2$, then messengers are rather “light.” In this case, we evaluate the one-loop diagrams employing the exact scalar messenger mass-eigenstates in the loops.

In the limit where the internal particle masses are much greater than the external momenta,

$$
\begin{aligned}
\frac{k_3}{g} = \frac{\alpha\beta(\gamma_2+\gamma_1)}{128\sqrt{2}\pi^2} m_1 \langle Z\rangle \Big[& \mathcal{I}(m_1, m_{1R}^+, m_{2R}^+) + \mathcal{I}(m_1, m_{1I}^+, m_{2I}^+) + \mathcal{I}(m_1, m_{1R}^+, m_{2I}^-) \\
& + \mathcal{I}(m_1, m_{1I}^-, m_{2R}^+) + \mathcal{I}(m_1, m_{1R}^-, m_{2R}^-) + \mathcal{I}(m_1, m_{1I}^-, m_{2I}^-) \\
& + \mathcal{I}(m_1, m_{1I}^+, m_{2R}^-) + \mathcal{I}(m_1, m_{1R}^-, m_{2I}^+) \Big] \\
+ \frac{\alpha\beta(\gamma_2-\gamma_1)}{128\sqrt{2}\pi^2} m_1 \langle Z\rangle \Big[& \mathcal{I}(m_1, m_{1R}^-, m_{2R}^+) + \mathcal{I}(m_1, m_{1R}^+, m_{2R}^-) + \mathcal{I}(m_1, m_{1I}^+, m_{2R}^+) \\
& + \mathcal{I}(m_1, m_{1I}^-, m_{2R}^-) + \mathcal{I}(m_1, m_{1R}^+, m_{2I}^+) + \mathcal{I}(m_1, m_{1R}^-, m_{2I}^-) \\
& + \mathcal{I}(m_1, m_{1I}^-, m_{2I}^+) + \mathcal{I}(m_1, m_{1I}^+, m_{2I}^-) \Big] \\
- \frac{\alpha\beta m_1 m_2}{32\sqrt{2}\pi^2} \Big[& \mathcal{I}(m_1, m_2, m_{1R}^+) + \mathcal{I}(m_1, m_2, m_{1I}^+) + \mathcal{I}(m_1, m_2, m_{1R}^-) + \mathcal{I}(m_1, m_2, m_{1I}^-) \Big],
\end{aligned}
$$

where the triangle integral $\mathcal{I}$ has been defined previously.

For example, for $\gamma_1 = \gamma_2 \equiv \gamma$, we find:

$$\frac{k_3}{g} = \frac{\sqrt{2}\alpha\beta}{32\pi^2} f(x)\,, \qquad\qquad x \equiv \frac{F_Z}{\gamma \langle Z \rangle^2}\,,$$

where $f(x) \equiv [(x-2)\ln(1-x) - (x+2)\ln(1+x)]/x^2$.

For small x, $f(x) = \frac{x^2}{3} + \mathcal{O}(x^4)$, and we recover the behavior of k_3 for $x \ll 1$ given previously. Note that $f(x) \to \infty$ as $x \to 1$, which reflects the fact that some of the messenger masses are approaching zero. Thus, we cannot take x as large as 1.

γ_1	γ_2	F_Z	M_-	$16\pi^2 k_3/g$
1	1	(19.8 TeV)^2	2.8 TeV	1.44
0.9	1	(18.8 TeV)^2	2.4 TeV	1.38
1	1	(16.8 TeV)^2	10.9 TeV	0.19
0.75	1	(14 TeV)^2	8.8 TeV	0.15

Sample points in the messenger parameter space. We have fixed $\langle Z \rangle = 20$ TeV and $\alpha = \beta = 1$. The mass of the lightest messenger state is denoted by M_-.

Renormalization group (RG) improvement

We have identified a parameter regime in which

$$k_3 \sim (0.1\text{–}1.4)\,\frac{g}{16\pi^2}\,.$$

This is the value of k_3 at an energy scale just below the scale of fundamental SUSY-breaking, i.e. the threshold scale of the messengers, $\mu_M \sim \langle Z\rangle$. One must then use the RG to run down to the electroweak scale. Keeping only the largest terms in the RG equation for k_3, we find:

$$16\pi^2\,\frac{dk_3}{dt} = 3k_3(h_t^2 + h_b^2)\,.$$

As an example, for $\tan\beta = 50$, we obtain

$$k_3(\mu = 500~\text{GeV}) \simeq 0.86\,k_3(\mu_M = 20~\text{TeV})\,.$$

Typically, we expect the threshold values of the k_i to be reduced by roughly 10% due to RG running.

Implications for chargino observables

After the neutral Higgs bosons acquire their vacuum expectation values, $\langle H_u^0 \rangle = v_u/\sqrt{2}$ and $\langle H_d^0 \rangle = v_d/\sqrt{2}$, the quadratic terms of the effective gaugino Lagrangian are given by:

$$\begin{aligned}\mathcal{L}_{\rm mass} = & \frac{i g_u v_u}{2}\lambda^a \tau^a_{2j}\psi^j_{H_u} + \frac{i g_d v_d}{2}\lambda^a \tau^a_{1j}\psi^j_{H_d} + \frac{i g'_u v_u}{2}\lambda' \psi^2_{H_u} - \frac{i g'_d v_d}{2}\lambda' \psi^1_{H_d} \\ & - M\lambda^a\lambda^a - M'\lambda'\lambda' - \mu\epsilon_{ij}\psi^i_{H_u}\psi^j_{H_d} + \frac{i k_1 v_d}{\sqrt{2}}\lambda^a\tau^a_{2j}\psi^j_{H_u} \\ & - \frac{i k_2 v_d}{\sqrt{2}}\lambda'\psi^2_{H_u} - \frac{i k_3 v_u}{\sqrt{2}}\lambda^a\tau^a_{1j}\psi^j_{H_d} - \frac{i k_4 v_u}{\sqrt{2}}\lambda'\psi^1_{H_d} + \text{h.c.}\end{aligned}$$

The parameters appearing above are effective parameters below the TeV-scale. For example, $g_u = g + \delta g_u$, $g_d = g + \delta g_d$, $g'_u = g' + \delta g'_u$, and $g'_d = g' + \delta g'_d$, where the $\delta g's$ include threshold and renormalization group effects from SUSY breaking below the fundamental SUSY-breaking scale. For M', M, and μ we simply absorb renormalization and threshold corrections into these coefficients.

Isolating the terms that contribute to the chargino matrix, we introduce

$$\psi_i^+ = \begin{pmatrix} -i\lambda^+ \\ \psi_{H_u}^1 \end{pmatrix}, \qquad \psi_i^- = \begin{pmatrix} -i\lambda^- \\ \psi_{H_d}^2 \end{pmatrix},$$

where $\lambda^\pm = \frac{1}{\sqrt{2}}(\lambda^1 \mp i\lambda^2)$. Then, the chargino mass terms are given by:

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}\begin{pmatrix} \psi^+ & \psi^- \end{pmatrix}\begin{pmatrix} 0 & (X^{\rm eff})^T \\ X^{\rm eff} & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + {\rm h.c.}\,,$$

where

$$X^{\rm eff} = \begin{pmatrix} M & (g+\delta g_u)\frac{v_u}{\sqrt{2}}\left(1 - \frac{\sqrt{2}k_1 \cot\beta}{g+\delta g_u}\right) \\ (g+\delta g_d)\frac{v_d}{\sqrt{2}}\left(1 + \frac{\sqrt{2}k_3 \tan\beta}{g+\delta g_d}\right) & \mu \end{pmatrix}$$

with $v_u \equiv v\sin\beta$ and $v_d \equiv v\cos\beta$.

We wish to identify the leading effect at large $\tan\beta$. We can neglect the effects of δg_d as these are one-loop effects with no $\tan\beta$-enhancements.

We shall write:

$$X_{12}^{\text{eff}} = \sqrt{2} m_W \sin\beta \, (1 + \delta_{12}) \,, \qquad\qquad X_{21}^{\text{eff}} = \sqrt{2} m_W \cos\beta \, (1 + \delta_{21}) \,.$$

In the large $\tan\beta$ limit, δ_{21} is $\tan\beta$-enhanced, and provides parametrically the largest of the one-loop corrections to X^{eff}.

$$\delta_{21} \simeq \frac{\sqrt{2} k_3 \tan\beta}{g} \,.$$

The correction to the supersymmetric relation, $X_{21} = gv\cos\beta/\sqrt{2}$ can be as large as 7%—56% for $\tan\beta = 50$ as $\sqrt{F_Z}$ varies between 14—20 TeV.

Extracting δ_{21} from precision chargino data

Given the effective chargino matrix $X^{\rm eff}$, the chargino masses and mixing angles are obtained from:

$$U^* X V^{-1} = M_D \equiv {\rm diag}(m_{\chi_1^+}, m_{\chi_2^+})\,,$$

for some suitably chosen unitary matrices U and V, where the elements of the diagonal matrix M_D are real and non-negative.

Let Φ_μ be the relative phase between μ and M (and assume the phases of X_{12} and X_{21} are negligible). Then, the chargino squared-masses and mixing angles θ_L and θ_R are:[†]

$$m^2_{\chi^\pm_{1,2}} = \tfrac{1}{2}\left(M^2 + |\mu|^2 + X_{12}^2 + X_{21}^2 \mp \Delta\right)\,,$$
$$\cos 2\theta_{R,L} = \Delta^{-1}\left[|\mu|^2 - M^2 \pm (X_{12}^2 - X_{21}^2)\right]\,,$$

[†] One can also derive equations for the physical phases that appear in U and V, but these are not needed here.

where the quantity Δ is defined by:

$$\Delta \equiv \left[(M^2 - |\mu|^2 - X_{12}^2 + X_{21}^2)^2 + 4(M^2 X_{12}^2 + |\mu|^2 X_{21}^2 + 2M|\mu| X_{12} X_{21} \cos\Phi_\mu)\right]^{1/2}.$$

Taking δ_{12} and δ_{21} small, and working to first order in these quantities, one obtains two equations for the two unknown δ's. We find:

$$\delta_{21} = \frac{2s_\beta^2 f^{1/2}(\Delta - f^{1/2}) - \frac{1}{2}h\left\{c_{2\beta} + \frac{1}{4m_W^2}\left[(\cos 2\theta_R - \cos 2\theta_L)(m^2_{\chi_2^\pm} - m^2_{\chi_1^\pm})\right]\right\}}{hc_\beta^2 + gs_\beta^2},$$

where f, g and h are complicated (but known) expressions that depend on the two chargino masses, m_W, $\tan\beta$, $\cos 2\theta_{L,R}$, and $\cos\Phi_\mu$. These quantities can in principle be determined at the ILC using precision chargino data [cf. S.Y. Choi *et al.*, EPJC 14 (2000) 535], using measurements of the total production cross-sections for $e^+e^- \to \widetilde{\chi}_i^\pm \widetilde{\chi}_j^\mp$ and asymmetries with polarized beams.

For the morbidly curious

It is convenient to define:

$$C^+_{RL} \equiv -(\cos 2\theta_R + \cos 2\theta_L)\,, \qquad\qquad C^-_{RL} \equiv \cos 2\theta_R - \cos 2\theta_L\,.$$

Then,

$$f = (\tfrac{1}{2}C^+_{RL}\Delta + 2m_W^2 c_{2\beta})^2 + 4m_W^2(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm} - 2m_W^2) - 2m_W^2 C^+_{RL}\Delta c_{2\beta}$$
$$+4m_W^2\Gamma s_{2\beta}\cos\Phi\,,$$

$$g = 2m_W^2 c_\beta^2\Big[4(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm}) + 4m_W^2 c_{2\beta} - 16m_W^2 - C^+_{RL}\Delta + 4\Gamma\tan\beta\cos\Phi$$
$$-\frac{8m_W^2}{\Gamma}(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm} - 2m_W^2)s_{2\beta}\cos\Phi\Big]\,,$$

$$h = 2m_W^2 s_\beta^2\Big[4(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm}) - 4m_W^2 c_{2\beta} - 16m_W^2 + C^+_{RL}\Delta + 4\Gamma\tan\beta\cos\Phi$$
$$-\frac{8m_W^2}{\Gamma}(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm} - 2m_W^2)s_{2\beta}\cos\Phi\Big]\,,$$

where

$$\Gamma \equiv \left[(m^2_{\chi_1^\pm} + m^2_{\chi_2^\pm} - 2m_W^2)^2 - \tfrac{1}{4}(C^+_{RL}\Delta)^2\right]^{1/2}\,.$$

The sub-dominance of non-local effects

Chargino masses and mixing angles are also modified at one-loop due to momentum-dependent radiative corrections in which the MSSM fields propagate in the loop. These "non-local" effects can compete with the local effects of the hard-SUSY-breaking operators in certain regimes of parameter space, and have not been explicitly included in our analysis.

- Virtual squark exchange at one-loop is not competitive in typical GMSB scenarios where $m_{\tilde{e}_R^\pm}/m_{\tilde{q}} \sim g'^2/g_3^2$. The experimental lower bound on $m_{\tilde{e}_R^\pm}$ implies that $m_{\tilde{q}} \gtrsim 800$ GeV, and hence the squark-exchange contributions are sub-dominant.
- Virtual slepton, chargino and neutralino exchange can compete with the wrong-Higgs operators of interest. However, these effects enter with at least two factors of g, g' (for chargino/neutralino exchange) or lepton Yukawa couplings (for slepton exchange). Assuming that the product of messenger-Higgs Yukawa couplings, $\alpha\beta > g^2$, g'^2, the messenger effects will always be parametrically larger than the non-local corrections.

Thus, with the assumptions stated above, a measurement of a significant deviation of δ_{21} from zero means that the measured deviation is coming from effects beyond the MSSM.

Conclusions and future directions

- Hard, dimension-four supersymmetry-breaking wrong-Higgs interactions can modify the expected behavior of certain MSSM masses and couplings.
- At large $\tan\beta$, the presence of wrong-Higgs interactions can be phenomenologically relevant for precision MSSM studies.
- The most dramatic effect of the wrong-Higgs couplings is the shift of the $h^0 b\bar{b}$ coupling at large $\tan\beta$.
- We have identified an analogous $\tan\beta$-enhanced effect that modifies the tree-level chargino mass matrix. A similar effect (not yet computed) also modifies the tree-level neutralino mass matrix.
- Previous studies that derive the MSSM chargino/neutralino parameters from precision MSSM data need to be modified to allow for the possible effects of the wrong-Higgs gaugino/higgsino operators.